Numerical results for multiparticle production in ϕ^4 theory

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1. Overview

Multiparticle production

Multiparticle production — a process which has a few particles with high energy in the initial state and a large $\sim \lambda^{-1}$ number of particles in the final state.

- We consider a weak coupling regime with $\lambda \ll 1$.
- We consider a theory of real scalar field $\lambda\phi^4$.
- We consider the case without spontaneous symmetry breaking.

Main issues

Results for amplitudes $A_{\mathrm{few}\to n}$ are mostly given in following ways:

1. Amplitudes for special choice of kinematics.
2. $|A_{\mathrm{few}\to\mathrm{n}}|^2$ integrated by a phase space of final particles.
3. Estimations and restrictions.

Perturbative results on kinematic threshold and beyond

- Tree-level amplitude behaves $\propto n!\,\lambda^{n/2}$ and given by a solution of spatially-independent field equation with zero energy[1].
- 1-loop correction[2] can be summed[3] into the $A_{1\to n}^{\text{tree}}\exp(B\lambda n^2)$ in the limit $\lambda \to 0, \lambda n = \text{fixed}.$
- Near the threshold amplitude depends on the average kinetic energy in this way[3]: $A_{1\to n}^{\text{tree}}(0)\,\exp\left(-\frac{5}{6}n\varepsilon_{\text{kin}}\right)$.

[1] Brown, 1992

[2] Voloshin, 1993

[3] Libanov, Rubakov, Son, Troitsky, 1994

Motivation

Since tree-level cross-sections grow with growth of n, they do not fit unitarity restrictions for $\lambda n \sim 1$ and larger. They cannot be described using ordinary perturbative approach. There are two main reasons to explore them:

1. Results for multiparticle production probabilities can be applied to processes with Higgs field. There are arguments based on unitarity that the probabilities should be exponentially suppressed[4], however, there are contradicting results for spontaneously broken $\lambda\phi^4$ theory[5] and the topic is under discussion.
2. New non-perturbative approaches will be derived and tested.

[4] Libanov, Rubakov, Troitsky, 1997

[5] Khoze, 2018

Inspiration for semiclassical description

Exponential behavior of amplitudes and the fact that the resulting cross-section in the limit $\lambda \to 0,\ \lambda n, \varepsilon = \text{fixed}$ has the form

$$\sigma_{1\to n} \sim \exp(\lambda^{-1} F[\lambda n, \varepsilon]),\ (1)$$

led to derivation of semiclassical approaches to describe multiparticle production.

We will focus on D.T. Son's method of singular solutions[6].

[6] Son, 1996

2. Semiclassical method of singular solutions

General setup

Our aim is the value :

$$\mathcal{P}_{1\to n}(E) \equiv \sum_f \left|\langle f; E, n|\hat{\mathcal{S}}\,\hat{\phi}(0)|0\rangle\right|^2. \ (2)$$

One is able to compute $\mathcal{P}_{1\to n}(E)$ for $\lambda n \sim 1$ in the limit $\lambda \to 0, \lambda n =$ fixed, $\varepsilon = \frac{E-n}{n} =$ fixed with two assumptions ($m = 1$):

1. Probability is exponentially suppressed $\mathcal{P}_{1\to n}(E) \propto e^{F_{1\to n}/\lambda}$ with $F_{1\to n} < 0$.
2. The answer do not depend on few-particle operator acting on vacuum with exponential accuracy.

Calculation of $\mathcal{P}_{1\to n}(E)$

To calculate the multiparticle production probability one can take the following limit[6] :

$$\mathcal{P}_{1\to n}(E) \approx \lim_{J\to 0} \frac{\lambda^2}{J^2} \sum_f |< f; n, E| \mathrm{e}^{-J\hat{\phi}(0)/\lambda} |0>|^2 = \lim_{J\to 0} \frac{\lambda^2 \mathcal{P}_{\mathcal{J}}(E,n)}{J^2}. \quad (3)$$

Result is independent of particular choice of J term in exponent and in our research we choose it to be

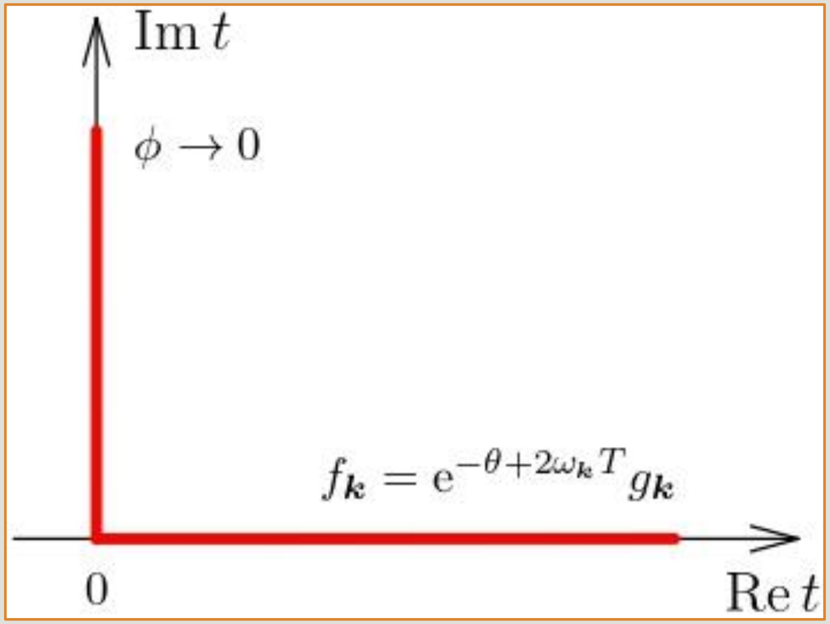
$$\int d^4x\, J(x)\hat{\phi}(x), \qquad J(x) = j\delta(t)\exp\left(-\frac{x^2}{2\sigma^2}\right). \quad (4)$$

Calculation of $\mathcal{P}_{\mathcal{J}}(E,n)$

$\mathcal{P}_{\mathcal{J}}(E,n)$ can be calculated via solving saddle-point equation

$$\partial^2\phi(x) + \phi(x) + \phi^3(x) = iJ(x) \quad (5)$$

on the contour



Why solutions are singular?

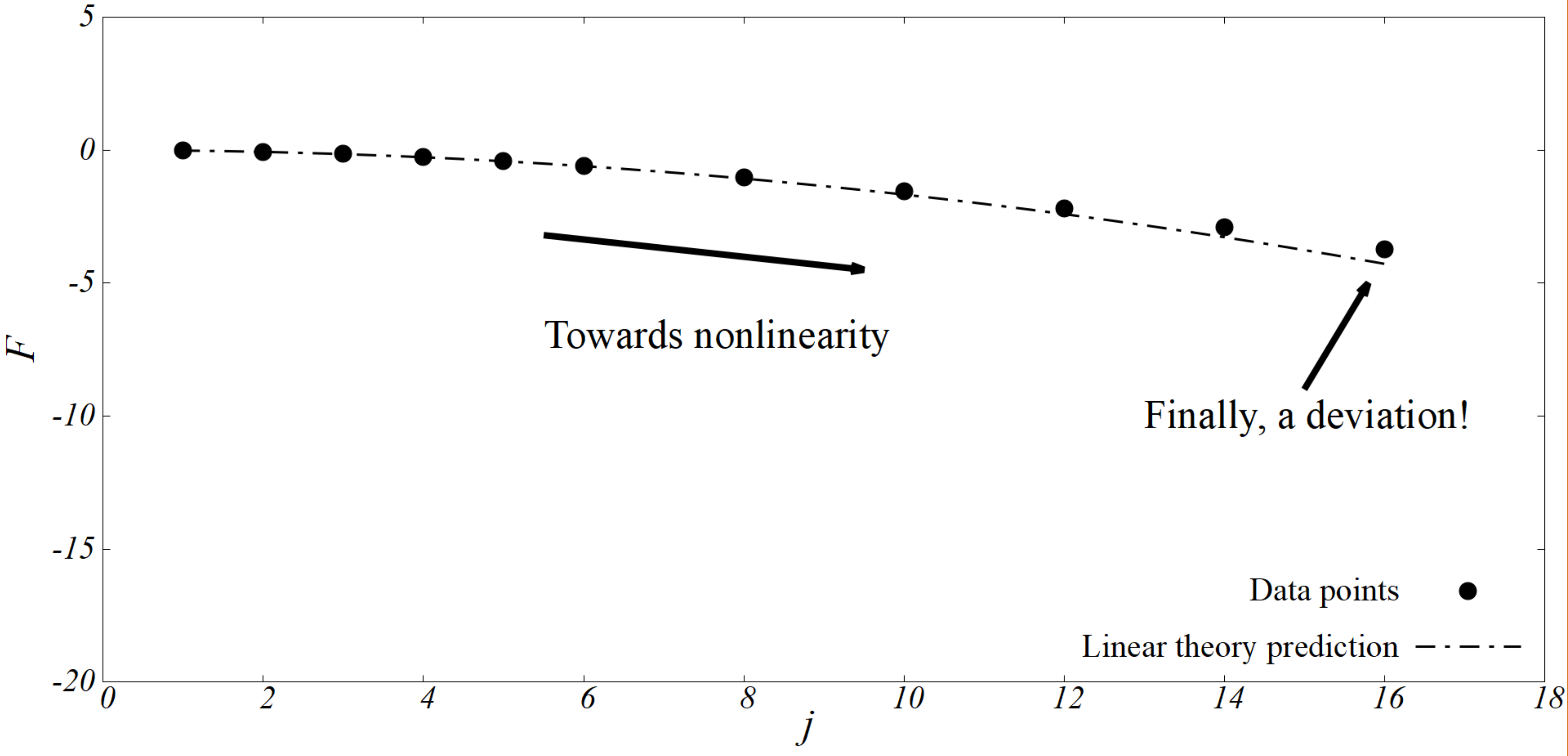
- Due to jump in energy, solutions have jump in time derivative.
- In the limit $j \to 0$ with fixed ε and λn solutions become singular at $t = 0$.
- In the complex time plate solutions have a singularity surface $\tau_0(r)$.

Numerical approach

To solve saddle-point equations along with boundary conditions we do the following steps:

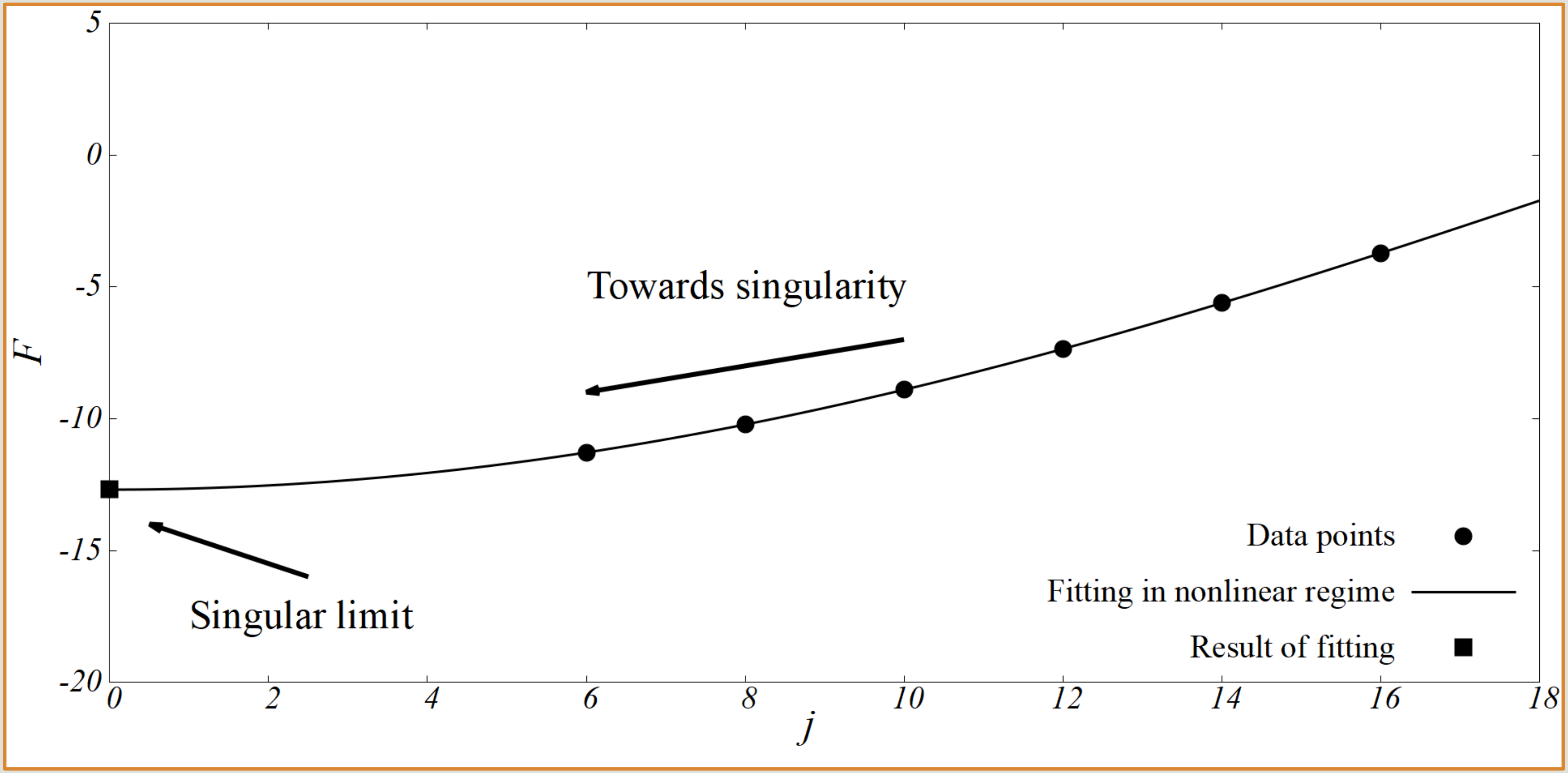
1. Consider only spherically-symmetrical solutions.
2. Define theory on space-time lattice.
3. Start from solutions that can be described with equations without self-interaction that can be solved analytically.
4. Use Newton-Raphson numerical method to converge to strongly interacting solutions with fixed $\lambda n, \varepsilon$.
5. Take limit $j \to 0, \frac{\sigma}{j} = \text{fixed}$.

3. Numerical results



Reaching nonlinear regime for ε = 3.0

(here $\lambda n \propto j^2, \sigma = \text{fixed}, \varepsilon = \text{fixed}$)



Taking the singular limit for λn ≈ 2.51, ε = 3.0

(here $\sigma \propto j, \lambda n = \text{fixed}, \varepsilon = \text{fixed}$)

Comparison with tree-level results at $\lambda n \ll 1$

For $\lambda n \ll 1$ suppression exponent has the form[3,6]

$$F(\lambda n, \varepsilon) = \lambda n \ln\left(\frac{\lambda n}{16}\right) - \lambda n + \lambda n f(\varepsilon) + O(\lambda^2 n^2)\,, (23)$$

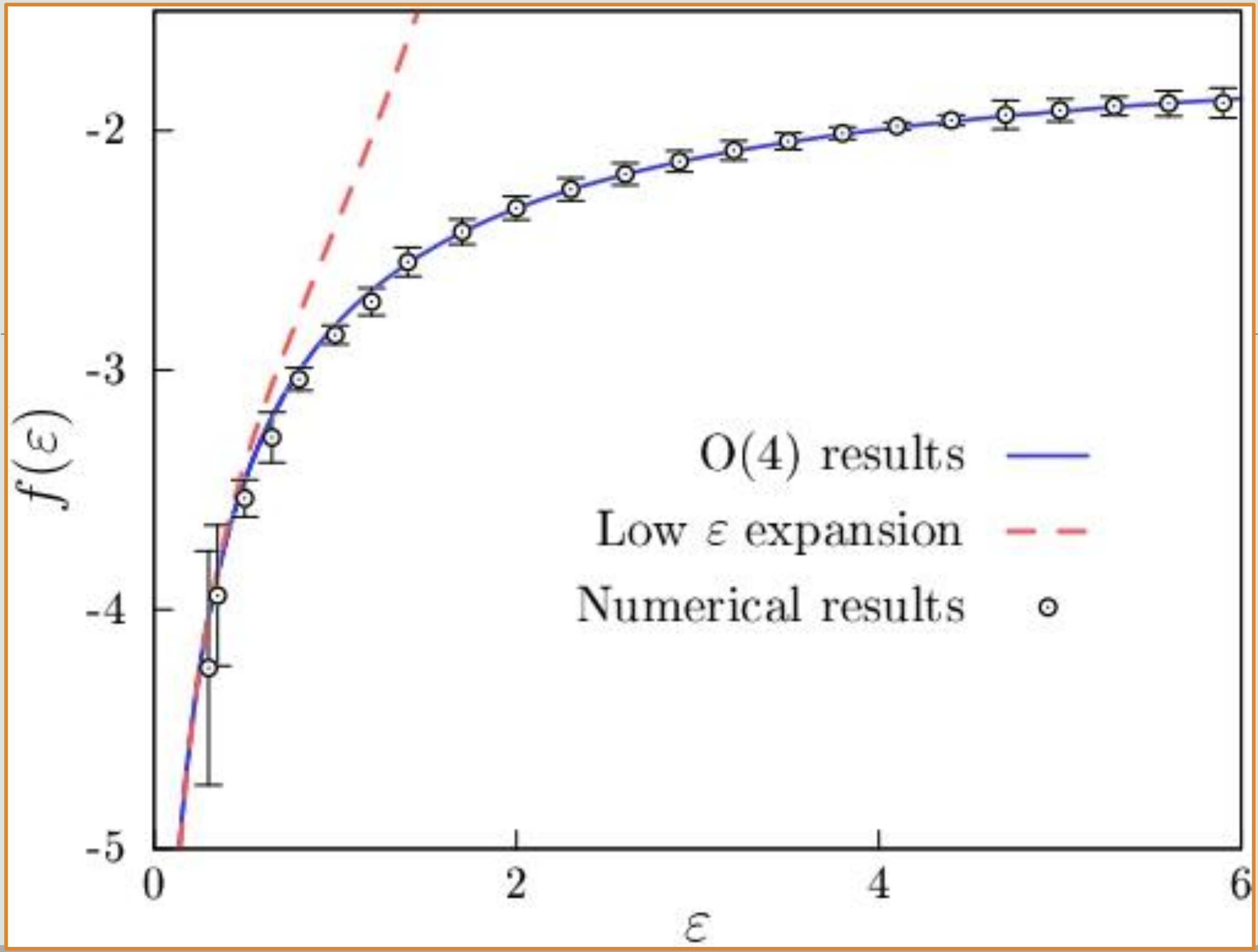
and for $\varepsilon \ll 1$, $f(\varepsilon)$ can be represented[7] as

$$f(\varepsilon) = \frac{3}{2}\ln\frac{\varepsilon}{3\pi} + \frac{3}{2} - \frac{17}{12}\,\varepsilon + \frac{1327-96\pi^2}{432}\,\varepsilon^2 + O(\varepsilon^3)\,. (24)$$

We also compared our results with results for solutions with $O(4)$-symmetrical singularity surface[8].

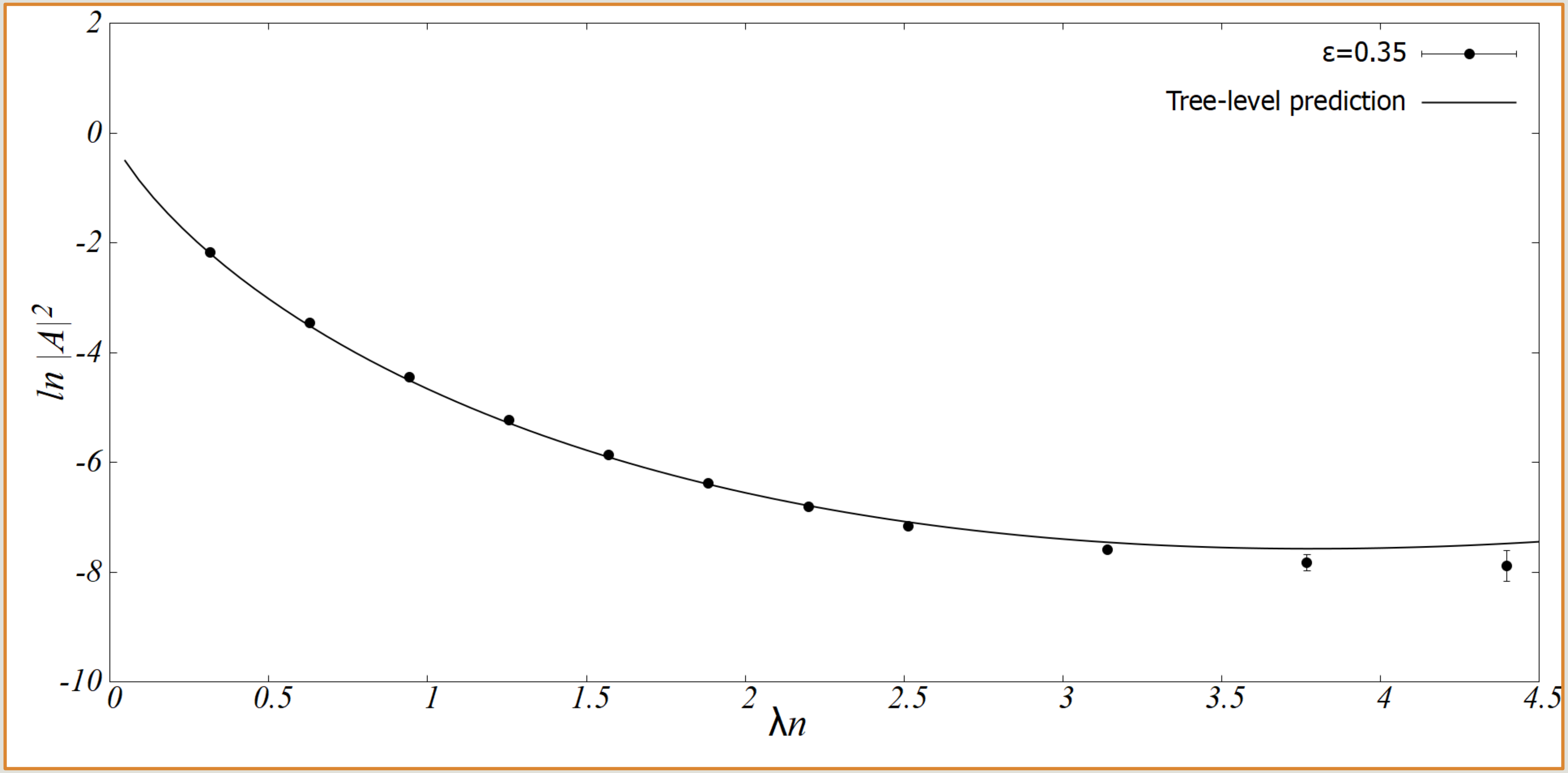
[7] Bezrukov, Libanov, Son, Troitsky, 1995

[8] Bezrukov, Libanov, Troitsky, 1995



Comparison with tree-level results[9]

[9]Demidov, Farkhtdinov, Levkov, 2021



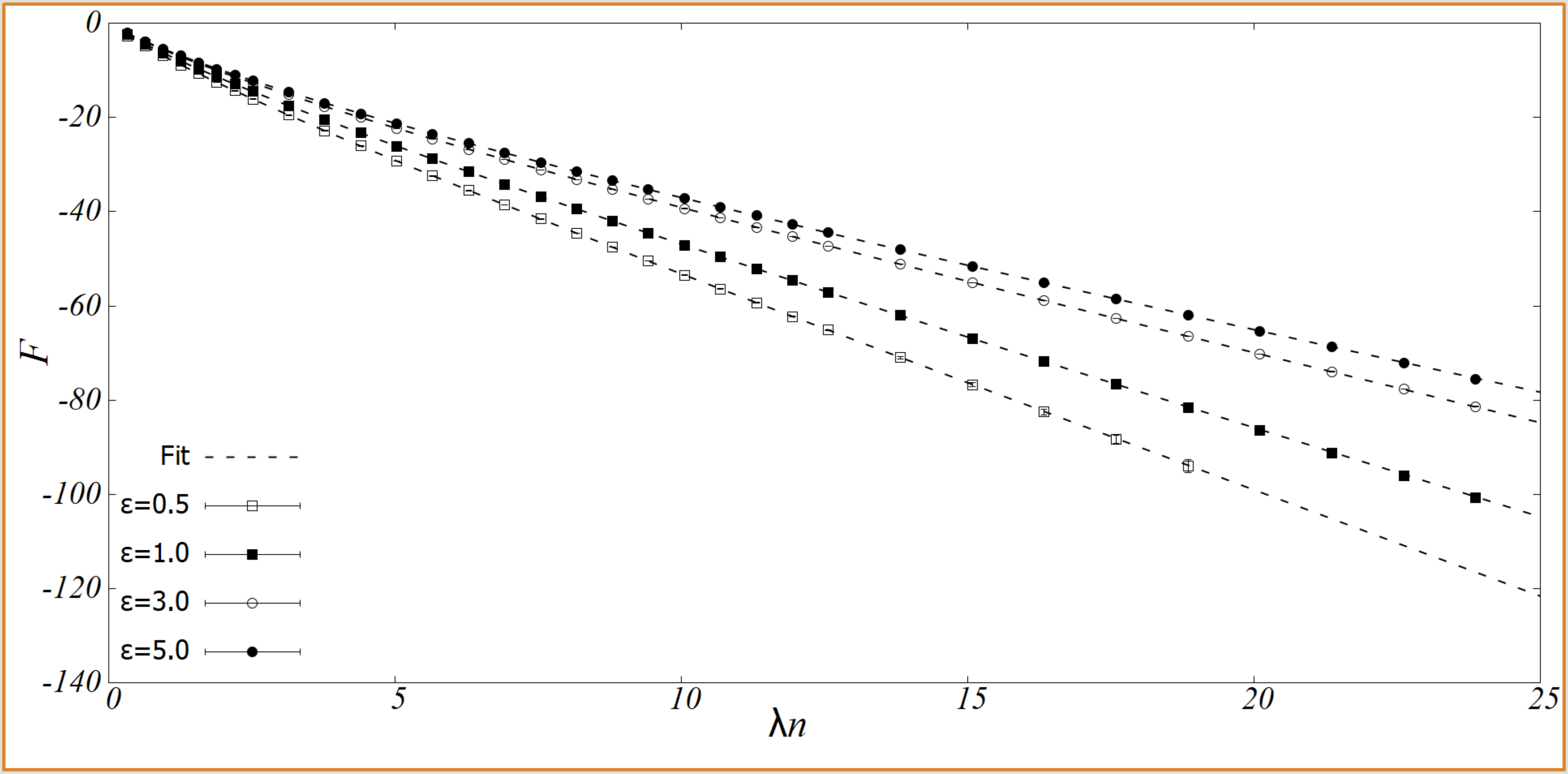
Comparison of $\ln|A_{1\to n}|^2$ with tree-level predictions for low ε

$$\ln\left|A_{1\to n}^{\text{tree}}\right|^2 \to n\left(2\ln\frac{\lambda n}{2\sqrt{2}} - 2 - \frac{5}{3}\varepsilon\right)$$

Limit $\lambda n \to +\infty, \varepsilon = \text{fixed}$

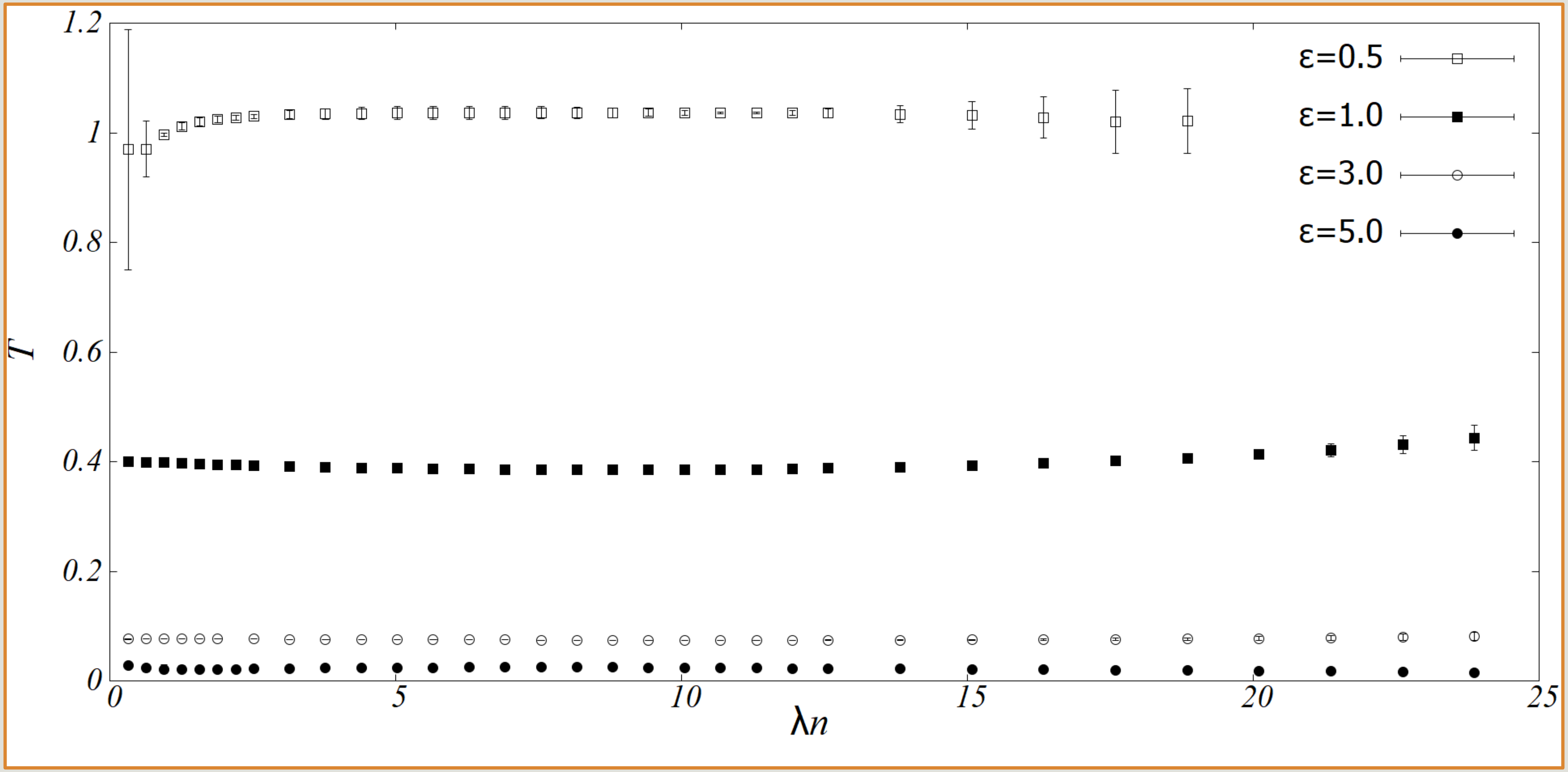
We have obtained limit $\lambda n \to +\infty$ for a set of considered ε in our numerical data. This limit has a set of features:

1. $\lambda n(\boldsymbol{k})$ distribution in the momentum space divided by the value of λn becomes constant.
2. Suppression exponent F starts to behave like a linear function $F = f_\infty(\varepsilon)\lambda n + g_\infty(\varepsilon)$ with negative $f_\infty(\varepsilon)$.
3. T and θ tend to constant.
4. With growth of ε, T tends to zero and f_∞ tends to constant.

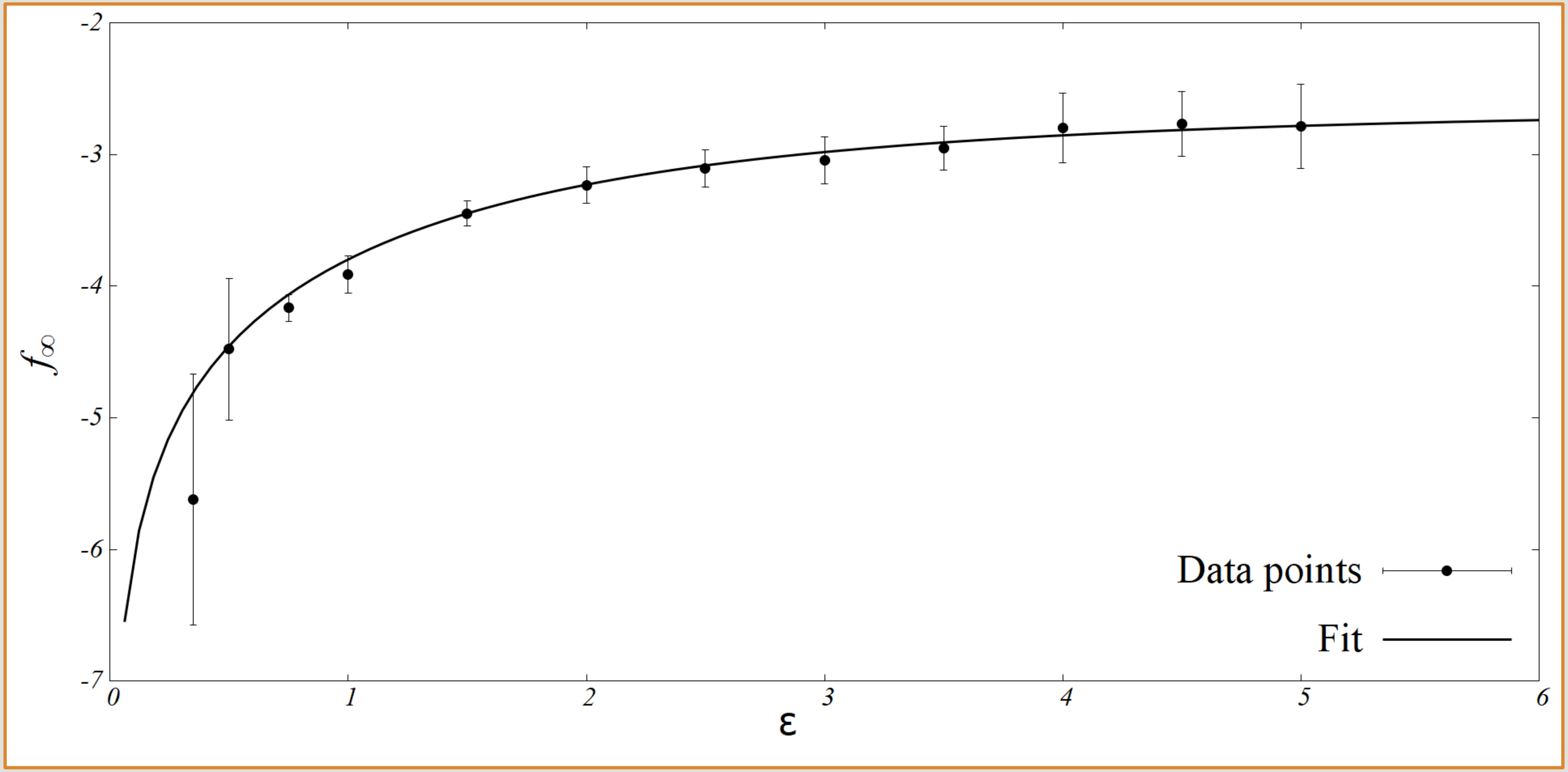


Limit $\lambda n \to +\infty, \varepsilon = \text{fixed}$

(fitting function: $F(\lambda n, \varepsilon) = -\frac{\lambda n}{2} \ln\left[\left(\frac{16}{\lambda n}\right)^2 + \frac{a_\varepsilon}{\lambda n} + b_\varepsilon\right] + (f(\varepsilon) - 1)\lambda n$)



$T(\lambda n)$ for a set of fixed ε



Results for $f_\infty(\varepsilon)$

Fitting function $f_\infty(\varepsilon) = -\frac{1}{2}\ln\left[\left(\frac{u}{\varepsilon}\right)^2 + v\right]$

Summary

1. Numerical implementation of D.T. Son's semiclassical method was constructed and successfully verified for low λn.
2. Multiparticle production probabilities are exponentially suppressed at $\lambda n \to +\infty$ for all considered ε and seem to be suppressed for all ε.
3. There are indications that in the limit $\varepsilon \to +\infty$ one can omit mass term in saddle-point equations.

Thank you for your attention!