HEAVY TETRAQUARKS IN THE RELATIVISTIC QUARK MODEL

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Experimental data on hidden-charm exotic mesons

State	J^{PC}	M (MeV)	Γ (MeV)	Observed in	Experiment
$X(3872)$	1^{++}	3871.69 ± 0.17	< 1.2	$B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}J/\psi$	Belle
$Z_c(3900)$	1^{+-}	3887.2 ± 2.3	28.2 ± 2.6	$e^{+}e^{-} \to \pi^{+}\pi^{-}J/\psi$	BESIII
$Z_c(4020)^{\pm}$	1^{+-}	4024.1 ± 1.9	13 ± 5	$e^{+}e^{-} \to \pi^{+}\pi^{-}h_c$	BESIII
$Z_c(4050)^{\pm}$	$?^{?+}$	4051^{+24}_{-40}	82^{+50}_{-28}	$\bar{B}^0 \to K^{-}\pi^{+}h_c$	Belle
$Z_c(4055)^{\pm}$	$?^{?-}$	4054 ± 3.2	45 ± 13	$Y(4360) \to \psi(2S)\pi^{+}\pi^{-}$	Belle
$Z_c(4100)^{\pm}$	$?^{??}$	4096 ± 28	152^{+80}_{-70}	$B^0 \to K^{+}\pi^{-}\eta_c$	LHCb
$X(4140)$	1^{++}	4146.8 ± 2.4	22^{+8}_{-7}	$\gamma\gamma \to \phi J/\psi$	CDF
$Z_c(4200)^{\pm}$	1^{+-}	$4196 \pm 18^{+35}_{-32}$	370^{+100}_{-150}	$\bar{B}^0 \to K^{-}\pi^{+}J/\psi$	Belle
$Y(4230)$	1^{--}	4218^{+5}_{-4}	59^{+12}_{-10}	$e^{+}e^{-} \to \omega\chi_{c0}$	BESIII
$Z_c(4240)^{\pm}$	0^{--}	4239^{+50}_{-21}	220^{+120}_{-90}	$B^0 \to K^{+}\pi^{-}\psi(2S)$	LHCb
$Z_c(4250)^{\pm}$	$?^{?+}$	4248^{+190}_{-50}	177^{+320}_{-70}	$\bar{B}^0 \to K^{-}\pi^{+}\chi_{c1}$	Belle
$Y(4260)$	1^{--}	4230 ± 8	55 ± 19	$e^{+}e^{-} \to \gamma_{\rm ISR}\pi^{+}\pi^{-}J/\psi$	BaBar
$X(4274)$	1^{++}	4274^{+8}_{-6}	49 ± 12	$B^{+} \to J/\psi\phi K^{+}$	CDF, LHCb
$Y(4360)$	1^{--}	4368 ± 13	96 ± 7	$e^{+}e^{-} \to \gamma_{\rm ISR}\pi^{+}\pi^{-}\psi(2S)$	Belle
$Y(4390)$	1^{--}	4392 ± 7	140^{+17}_{-7}	$e^{+}e^{-} \to \pi^{+}\pi^{-}h_c$	BESIII
$Z_c(4430)^{\pm}$	1^{+-}	4478^{+15}_{-18}	181 ± 31	$B \to K\pi^{\pm}\psi(2S)$	Belle
$X(4500)$	0^{++}	4506^{+16}_{-19}	92 ± 29	$B^{+} \to J/\psi\phi K^{+}$	LHCb
$Y(4660)$	1^{--}	4643 ± 9	72 ± 11	$e^{+}e^{-} \to \gamma_{\rm ISR}\pi^{+}\pi^{-}\psi(2S)$	Belle
$X(4700)$	0^{++}	4704^{+17}_{-26}	120 ± 50	$B^{+} \to J/\psi\phi K^{+}$	LHCb
$X(6900)$	$?^{?+}$	$6905 \pm 11 \pm 7$	$80 \pm 19 \pm 33$	$pp \to J/\psi J/\psi X$	LHCb
$X(4740)$	$?^{?+}$	4741 ± 9	53 ± 19	$B_s \to J/\psi\phi\pi^{+}\pi^{-}$	LHCb

Important feature: Most of these states are close to thresholds of open and/or hidden flavor meson production

Main theoretical interpretations

Conventional:

- Heavy quarkonium states influenced by open flavor thresholds

Exotic:

- Molecules (two loosely bound heavy mesons $(Q\bar{q})(\bar{Q}q)$)
- Tetraquarks (tightly bound four-quark states)
- Hybrids ($Q\bar{Q}$-gluon with excited gluonic degrees of freedom)
- Hadro-quarkonium (compact quarkonium states $Q\bar{Q}$ embedded in an excited light-quark matter)
- Kinematic or rescattering effects at corresponding thresholds

TETRAQUARKS in diquark-antidiquark picture

• Main assumption: Tetraquarks — diquark and antidiquark in color $\bar{3}$ and 3 configurations bound by color forces $\Longrightarrow$

⋆ four-body calculation reduces to two-step two-body calculation:

- diquark d (antidiquark $\bar{d}$) as qq' ($\bar{q}\bar{q}'$) bound state (as in baryons) $\longrightarrow$ only color triplet configuration contributes since there is a repulsion between quarks in color sextet
- tetraquark as the $d\bar{d}'$ bound state where constituents interact as a whole (no separate interactions between quarks and antiquarks)

⋆ typical hadronic size

⋆ rich spectroscopy — radial and orbital excitations between diquarks

Diquarks

• Diquark is a composite (qq') system:

– diquark is not point-like object: Its interaction with gluons is smeared by the form factor expressed through the overlap integral of diquark wave functions

• Pauli principle for ground state diquarks:

– (qq') diquark can have $S = 0, 1$ (scalar $[q, q']$, axial vector $\{q, q'\}$)

– (qq) diquarks can have only $S = 1$ (axial vector $\{q, q\}$)

• Both light and heavy quarks and diquarks are considered fully relativistically without nonrelativistic (v/c) expansion

- Heavy tetraquarks $(Qq)(\bar{Q}\bar{q}')$ with hidden charm and bottom

- Neutral X should be split into two states ($[Qu][\bar{Q}\bar{u}]$ and $[Qd][\bar{Q}\bar{d}]$) with ΔM $\sim$few MeV
- existence of charged partners $X^+ = [Qu][\bar{Q}\bar{d}]$, $X^- = [Qd][\bar{Q}\bar{u}]$
- existence of tetraquarks with open $X_{s\bar{q}} = [Qs][\bar{Q}\bar{q}]$ and hidden $X_{s\bar{s}} = [Qs][\bar{Q}\bar{s}]$ strangeness

- Doubly heavy tetraquarks $(QQ')(\bar{q}\bar{q}')$ with open charm and bottom

- explicitly exotic states with heavy flavor number equal to 2

$\Longrightarrow$ their observation would be a direct proof of existence of multiquark states

- estimates of the production rates of such tetraquarks indicate that they could be produced and detected at present and future facilities.
- we considered the doubly heavy $(QQ')(\bar{q}\bar{q}')$ tetraquark ($Q = b, c$ and $q = u, d, s$) as the bound system of the heavy diquark (QQ') and light antidiquark $(\bar{q}\bar{q}')$

- Heavy tetraquarks $(cq)(\bar{b}\bar{q}')$ with open charm and bottom

- we considered heavy $(cq)(\bar{b}\bar{q}')$ tetraquark ($q = u, d, s$) as the bound system of the heavy-light diquark (cq) and heavy-light antidiquark $(\bar{b}\bar{q}')$

- $QQ\bar{Q}\bar{Q}$ tetraquarks composed from heavy ($Q = c, b$) quarks only

- new structures in double-J/ψ spectrum observed by LHCb
- absence of narrow structures in the Υ-pair production at CMS and LHCb
- we considered heavy $(QQ')(\bar{Q}\bar{Q}')$ tetraquark as the bound system of the doubly heavy diquark (QQ') and doubly heavy antidiquark $(\bar{Q}\bar{Q}')$

RELATIVISTIC QUARK MODEL

Quasipotential equation of Schrödinger type:

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_M(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p},\mathbf{q};M)\Psi_M(\mathbf{q})$$

$\mathbf{p}$ - relative momentum of quarks
M - bound state mass ($M = E_1 + E_2$)
μ_R - relativistic reduced mass:

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}$$

$b(M)$ - on-mass-shell relative momentum in cms:

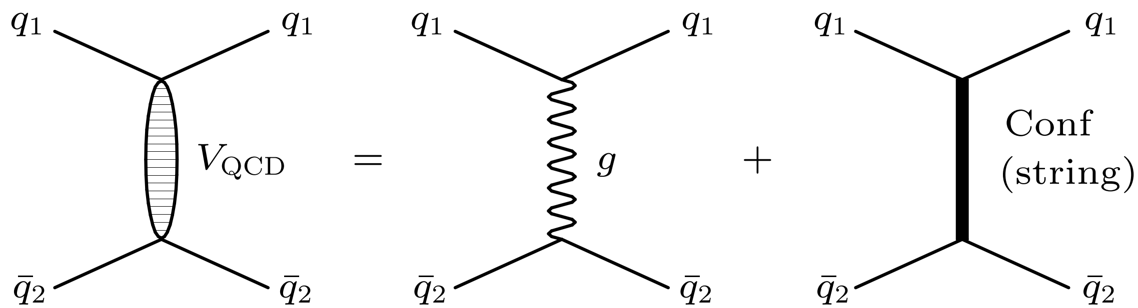
$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}$$

$E_{1,2}$ - center of mass energies:

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}$$

- Parameters of the model fixed from meson sector

- $q\bar{q}$ **quasipotential**



$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\left\{\frac{4}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^\mu\gamma_2^\nu + V^V_{\rm conf}(\mathbf{k})\Gamma_1^\mu\Gamma_{2;\mu} + V^S_{\rm conf}(\mathbf{k})\right\}u_1(q)u_2(-q)$$

$\mathbf{k} = \mathbf{p} - \mathbf{q}$
$D_{\mu\nu}(\mathbf{k})$ - (perturbative) gluon propagator
$\Gamma_\mu(\mathbf{k})$ - effective long-range vertex with Pauli term:

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}k^\nu,$$

κ - anomalous chromomagnetic moment of quark,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}} \begin{pmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\mathbf{p}}{\epsilon(p)+m} \end{pmatrix} \chi^\lambda,$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

- Lorentz structure of $V_{\rm conf} = V^V_{\rm conf} + V^S_{\rm conf}$

In nonrelativistic limit

$$\left.\begin{array}{rcl} V^V_{\rm conf} &=& (1-\varepsilon)(Ar+B) \\ V^S_{\rm conf} &=& \varepsilon(Ar+B) \end{array}\right\} \quad \text{Sum}: \quad (Ar+B)$$

ε - mixing parameter

$$V_{\rm NR}(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar + B$$

$$V_{\rm Coul}(r) = -\frac{4}{3}\frac{\alpha_s}{r}$$

Parameters A, B, κ, ε and quark masses fixed from analysis of meson masses and radiative decays:

$\varepsilon = -1$ from heavy quarkonium radiative decays ($J/\psi \to \eta_c + \gamma$) and HQET

$\kappa = -1$ from fine splitting of heavy quarkonium ^3P_J states and HQET

$(1+\kappa) = 0 \Longrightarrow$ vanishing long-range chromomagnetic interaction !

Freezing of α_s for light quarks

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln \frac{\mu^2 + M_0^2}{\Lambda^2}}, \quad \beta_0 = 11 - \frac{2}{3} n_f, \quad \mu = \frac{2 m_1 m_2}{m_1 + m_2},$$

$$M_0 = 2.24\sqrt{A} = 0.95 \text{ GeV}$$

Quasipotential parameters:

$A = 0.18$ GeV^2, $B = -0.30$ GeV,

$\Lambda = 0.413$ GeV (from M_ρ)

Quark masses:

$m_b = 4.88$ GeV $m_s = 0.50$ GeV

$m_c = 1.55$ GeV $m_{u,d} = 0.33$ GeV

- **Tetraquarks in diquark-antidiquark picture**

(Qq)-interaction: $V_{Qq} = \frac{1}{2} V_{Q\bar{q}}$

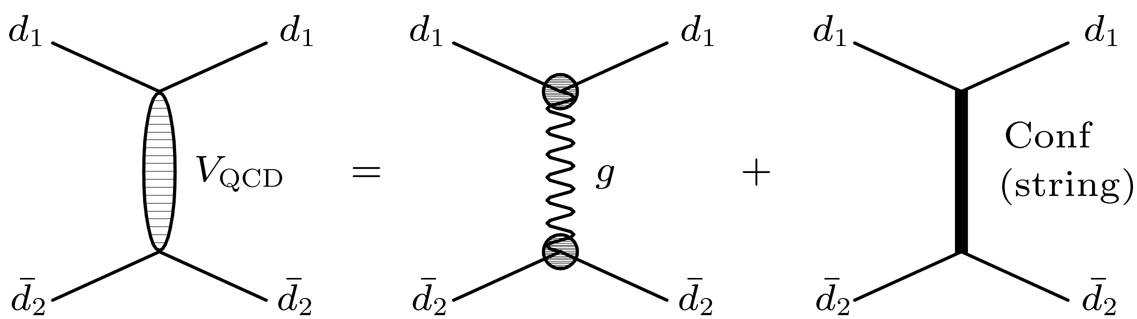
$$V(\mathbf{p},\mathbf{q};M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p},\mathbf{q};M)u_1(q)u_2(-q),$$

where

$$\mathcal{V}(\mathbf{p},\mathbf{q};M) = \frac{2}{3}\alpha_S D_{\mu\nu}(\mathbf{k})\gamma_1^{\mu}\gamma_2^{\nu} + \frac{1}{2}V^V_{\rm conf}(\mathbf{k})\Gamma_1^{\mu}\Gamma_{2;\mu} + \frac{1}{2}V^S_{\rm conf}(\mathbf{k})$$

$(d_1\bar{d}_2)$-interaction: $d = (Qq)$

$$V(\mathbf{p},\mathbf{q};M) = \frac{\langle d_1(P)|J_\mu|d_1(Q)\rangle}{2\sqrt{E_{d_1}E_{d_1}}}\frac{4}{3}\alpha_S D^{\mu\nu}(\mathbf{k})\frac{\langle d_2(P')|J_\nu|d_2(Q')\rangle}{2\sqrt{E_{d_2}E_{d_2}}}$$
$$+\psi^*_{d_1}(P)\psi^*_{d_2}(P')\left[J_{d_1;\mu}J^{\mu}_{d_2}V^V_{\rm conf}(\mathbf{k}) + V^S_{\rm conf}(\mathbf{k})\right]\psi_{d_1}(Q)\psi_{d_2}(Q'),$$



$J_{d,\mu}$ – effective long-range vector vertex of diquark:

$$J_{d;\mu} = \begin{cases} \dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for scalar diquark} \\ -\dfrac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \dfrac{i\mu_d}{2M_d}\Sigma_\mu^\nu k_\nu & \text{for axial vector diquark} \\ & (\mu_d = 0) \end{cases}$$

μ_d - total chromomagnetic moment of axial vector diquark
diquark spin matrix: $(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu)$
$\mathbf{S}_d$ - axial vector diquark spin: $(S_{d;k})_{il} = -i\varepsilon_{kil}$

$\psi_d(P)$ – diquark wave function:

$$\psi_d(p) = \begin{cases} 1 & \text{for scalar diquark} \\ \varepsilon_d(p) & \text{for axial vector diquark} \end{cases}$$

$\varepsilon_d(p)$ – polarization vector of axial vector diquark

$\langle d(P)|J_\mu|d(Q)\rangle$ – vertex of diquark-gluon interaction:

$$\langle d(P)|J_\mu(0)|d(Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}^d_P(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi^d_Q(\mathbf{q}) \Rightarrow F(k^2)$$

Γ_μ – two-particle vertex function of the diquark-gluon interaction:

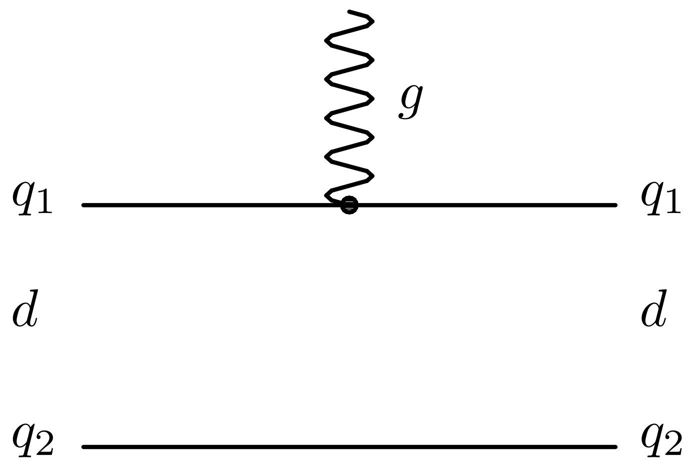


Figure 1: The vertex function Γ of the diquark-gluon interaction in the impulse approximation. The gluon interaction only with one quark is shown.

DIQUARKS

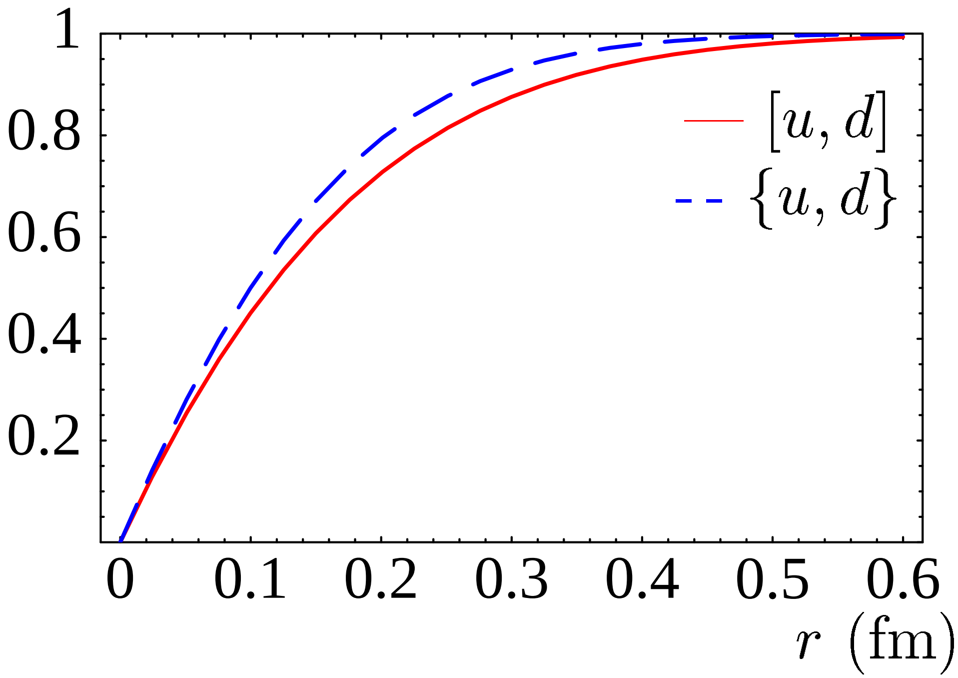
Table 1: Masses of light ground state diquarks (in MeV). S and A denotes scalar and axial vector diquarks antisymmetric $[q, q']$ and symmetric $\{q, q'\}$ in flavor, respectively.

Quark content	Diquark type	Mass				
		our RQM	Ebert et al. NJL	Burden et al. BSE	Maris BSE	Hess et al. Lattice
$[u, d]$	S	710	705	737	820	694(22)
$\{u, d\}$	A	909	875	949	1020	806(50)
$[u, s]$	S	948	895	882	1100	
$\{u, s\}$	A	1069	1050	1050	1300	
$\{s, s\}$	A	1203	1215	1130	1440	

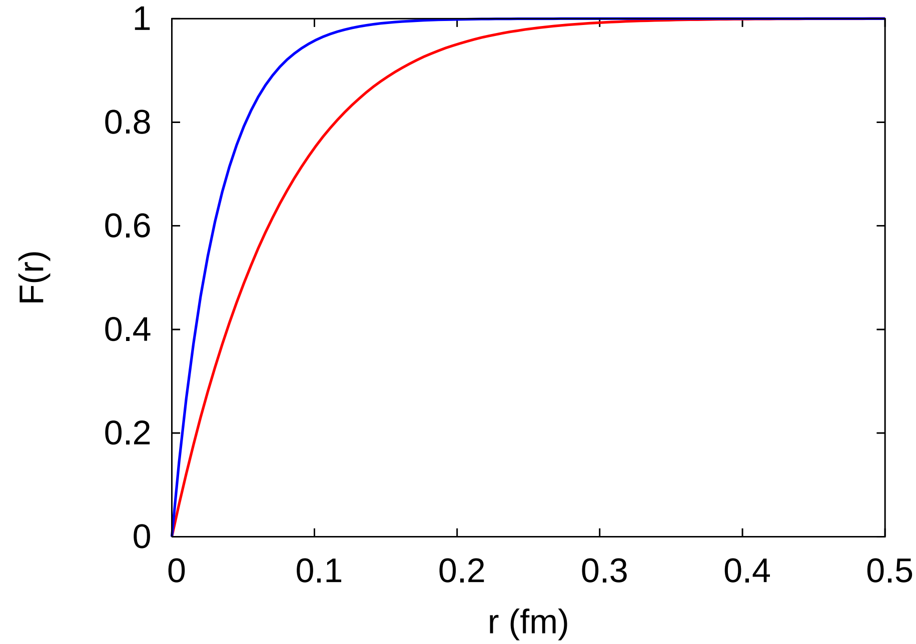
Table 2: Masses of heavy-light and doubly heavy diquarks (MeV).

Quark content	Diquark type	Mass	
		$Q = c$	$Q = b$
$[Q, q]$	S	1973	5359
$\{Q, q\}$	A	2036	5381
$[Q, s]$	S	2091	5462
$\{Q, s\}$	A	2158	5482
$[Q, c]$	S		6519
$\{Q, c\}$	A	3226	6526
$\{Q, b\}$	A	6526	9778

The form factors $F(r)$ for the scalar $[u, d]$ (red solid line) and axial vector $\{u, d\}$ (blue dashed line) diquarks:



The form factors $F(r)$ for $\{c, q\}$ (red line) and $\{b, q\}$ (blue line) axial vector diquarks.



TETRAQUARKS

The potential of the diquark-antidiquark interaction

$$V(r) = V_{\rm Coul}(r) + V_{\rm conf}(r) + \frac{1}{2}\Biggl\{ \left[\frac{1}{E_1(E_1+M_1)} + \frac{1}{E_2(E_2+M_2)}\right] \frac{\hat{V}'_{\rm Coul}(r)}{r} - \left[\frac{1}{M_1(E_1+M_1)}\right.$$

$$\left. + \frac{1}{M_2(E_2+M_2)}\right] \frac{V'_{\rm conf}(r)}{r} + \frac{\mu_d}{2}\left(\frac{1}{M_1^2} + \frac{1}{M_2^2}\right)\frac{V'^{V}_{\rm conf}(r)}{r}\Biggr\} \mathbf{L}\cdot(\mathbf{S}_1+\mathbf{S}_2)$$

$$+ \frac{1}{2}\Biggl\{ \left[\frac{1}{E_1(E_1+M_1)} - \frac{1}{E_2(E_2+M_2)}\right] \frac{\hat{V}'_{\rm Coul}(r)}{r} - \left[\frac{1}{M_1(E_1+M_1)} - \frac{1}{M_2(E_2+M_2)}\right]$$

$$\times \frac{V'_{\rm conf}(r)}{r} + \frac{\mu_d}{2}\left(\frac{1}{M_1^2} - \frac{1}{M_2^2}\right)\frac{V'^{V}_{\rm conf}(r)}{r}\Biggr\} \mathbf{L}\cdot(\mathbf{S}_1-\mathbf{S}_2) + \frac{1}{E_1E_2}\Biggl\{ \mathbf{p}\left[V_{\rm Coul}(r) + V^{V}_{\rm conf}(r)\right]\mathbf{p}$$

$$- \frac{1}{4}\Delta V^{V}_{\rm conf}(r) + V'_{\rm Coul}(r)\frac{\mathbf{L}^2}{2r} + \frac{1}{r}\left[V'_{\rm Coul}(r) + \frac{\mu_d}{4}\left(\frac{E_1}{M_1} + \frac{E_2}{M_2}\right)V'^{V}_{\rm conf}(r)\right]\mathbf{L}(\mathbf{S}_1+\mathbf{S}_2)$$

$$+ \frac{\mu_d}{4}\left(\frac{E_1}{M_1} - \frac{E_2}{M_2}\right)\frac{V'^{V}_{\rm conf}(r)}{r}\mathbf{L}(\mathbf{S}_1-\mathbf{S}_2)$$

$$+ \frac{1}{3}\left[\frac{1}{r}V'_{\rm Coul}(r) - V''_{\rm Coul}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\left(\frac{1}{r}V'^{V}_{\rm conf}(r) - V''^{V}_{\rm conf}(r)\right)\right]\left[\frac{3}{r^2}(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r}) - \mathbf{S}_1\mathbf{S}_2\right]$$

$$+ \frac{2}{3}\left[\Delta V_{\rm Coul}(r) + \frac{\mu_d^2}{4}\frac{E_1E_2}{M_1M_2}\Delta V^{V}_{\rm conf}(r)\right]\mathbf{S}_1\mathbf{S}_2\Biggr\},$$

where $\qquad V_{\rm Coul}(r) = -\frac{4}{3}\alpha_s\frac{F_1(r)F_2(r)}{r}$

Table 3: Masses of the ground state hidden charm tetraquark states (in MeV).

State J^{PC}	Diquark content	Tetraquark mass $cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cs\bar{c}\bar{q}/cq\bar{c}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080

Table 4: Thresholds for open charm decays and nearby hidden-charm thresholds.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$D^0\bar{D}^0$	3729.4	$D_s^+D_s^-$	3936.2	$D^0D_s^\pm$	3832.9
D^+D^-	3738.8	$\eta' J/\psi$	4054.7	$D^\pm D_s^\mp$	3837.7
$D^0\bar{D}^{*0}$	3871.3	$D_s^\pm D_s^{*\mp}$	4080.0	$D^{*0}D_s^\pm$	3975.0
$\rho J/\psi$	3872.7	$\phi J/\psi$	4116.4	$D^0D_s^{*\pm}$	3976.7
$D^\pm D^{*\mp}$	3879.5	$D_s^{*+}D_s^{*-}$	4223.8	$K^{*\pm}J/\psi$	3988.6
$\omega J/\psi$	3879.6			$K^{*0}J/\psi$	3993.0
$D^{*0}\bar{D}^{*0}$	4013.6			$D^{*0}D_s^{*\pm}$	4118.8

Table 5: Masses of charm diquark-antidiquark excited $1P$, $2S$ states (in MeV). S and A denote scalar and axial vector diquarks; $\mathcal{S}$ is the total spin of the diquark and antidiquark. (C is defined only for $q = q'$).

State J^{PC}	Diquark content	$\mathcal{S}$	Mass $cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{s}$
$1P$					
1^{--}	$S\bar{S}$	0	4244	4466	4350
$0^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4269	4499	4381
$1^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4284	4514	4396
$2^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4315	4543	4426
1^{--}	$A\bar{A}$	0	4350	4582	4461
0^{-+}	$A\bar{A}$	1	4304	4540	4419
1^{-+}	$A\bar{A}$	1	4345	4578	4458
2^{-+}	$A\bar{A}$	1	4367	4598	4478
1^{--}	$A\bar{A}$	2	4277	4515	4393
2^{--}	$A\bar{A}$	2	4379	4610	4490
3^{--}	$A\bar{A}$	2	4381	4612	4492
$2S$					
0^{++}	$S\bar{S}$	0	4375	4604	4481
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4431	4665	4542
0^{++}	$A\bar{A}$	0	4434	4680	4547
1^{+-}	$A\bar{A}$	1	4461	4703	4572
2^{++}	$A\bar{A}$	2	4515	4748	4625

Table 6: Masses of charm diquark-antidiquark excited $1D$, $2P$ states (in MeV).

	State J^{PC}	Diquark content	$\mathcal{S}$	Mass $cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{s}$
$1D$	2^{++}	$S\bar{S}$	0	4506	4728	4611
	$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4553	4779	4663
	$2^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4559	4785	4670
	$3^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4570	4794	4680
	2^{++}	$A\bar{A}$	0	4617	4847	4727
	1^{+-}	$A\bar{A}$	1	4604	4835	4714
	2^{+-}	$A\bar{A}$	1	4616	4846	4726
	3^{+-}	$A\bar{A}$	1	4624	4852	4733
	0^{++}	$A\bar{A}$	2	4582	4814	4692
	1^{++}	$A\bar{A}$	2	4593	4825	4703
	2^{++}	$A\bar{A}$	2	4610	4841	4720
	3^{++}	$A\bar{A}$	2	4627	4855	4736
	4^{++}	$A\bar{A}$	2	4628	4856	4738
$2P$	1^{--}	$S\bar{S}$	0	4666	4884	4767
	$0^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4684	4909	4792
	$1^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4702	4926	4810
	$2^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4738	4960	4845
	1^{--}	$A\bar{A}$	0	4765	4991	4872
	0^{-+}	$A\bar{A}$	1	4715	4946	4826
	1^{-+}	$A\bar{A}$	1	4760	4987	4867
	2^{-+}	$A\bar{A}$	1	4786	5011	4892
	1^{--}	$A\bar{A}$	2	4687	4920	4799
	2^{--}	$A\bar{A}$	2	4797	5022	4903
	3^{--}	$A\bar{A}$	2	4804	5030	4910

Table 7: Masses of hidden bottom tetraquark states (in MeV).

State	Diquark	Tetraquark mass		
J^{PC}	content	$bq\bar{b}\bar{q}$	$bs\bar{b}\bar{s}$	$bs\bar{b}\bar{q}/bq\bar{b}\bar{s}$
$1S$				
0^{++}	$S\bar{S}$	10471	10662	10572
$1^{+\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	10492	10682	10593
0^{++}	$A\bar{A}$	10473	10671	10584
1^{+-}	$A\bar{A}$	10494	10686	10599
2^{++}	$A\bar{A}$	10534	10716	10628
$1P$				
1^{--}	$S\bar{S}$	10807	11002	10907
$0^{-\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	10820	10917	11011
$1^{-\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	10824	10922	11016
$2^{-\pm}$	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	10834	10932	11026
1^{--}	$A\bar{A}$	10850	10947	11039
0^{-+}	$A\bar{A}$	10836	10934	11026
1^{-+}	$A\bar{A}$	10847	10945	11037
2^{-+}	$A\bar{A}$	10854	10952	11044
1^{--}	$A\bar{A}$	10827	10925	11017

Table 8: Thresholds for open bottom decays.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$B\bar{B}$	10558	$B_s^+B_s^-$	10739	BB_s	10649
$B\bar{B}^*$	10604	$B_s^{\pm}B_s^{*\mp}$	10786	B^*B_s	10695
$B^*\bar{B}^*$	10650	$B_s^{*+}B_s^{*-}$	10833	$B^*B_s^*$	10742

Table 9: Masses of charm diquark-antidiquark states (in MeV) and possible experimental candidates.

State J^{PC}	Diquark content	Theory $cq\bar{c}\bar{q}$	Theory $cs\bar{c}\bar{s}$	Experiment state	Experiment mass	Theory $bq\bar{b}\bar{q}$
$1S$						
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	3871		$X(3872)$	3871.69 ± 0.17	10492
1^{+-}	$A\bar{A}$	3890		$Z_c(3900)$	3887.2 ± 2.3	10494
1^{++}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$		4113	$X(4140)$	4146.8 ± 2.4	10682
2^{++}	$A\bar{A}$	3968		$?^{??}$ $X(3940)$	3942 ± 9	10534
$1P$						
1^{--}	$S\bar{S}$	4244		$Y(4230)$	4218^{+5}_{-4}	10807
1^{--}	$A\bar{A}$	4277		$Y(4260)$	4230 ± 8	10827
0^{--}	$(S\bar{A}-\bar{S}A)/\sqrt{2}$	4269		$Z_c(4240)$	4239^{+50}_{-21}	10820
0^{-+}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	4263 }		$?^{?+}$ $Z_c(4250)$	4248^{+190}_{-50}	10820
1^{-+}	$(S\bar{A}+\bar{S}A)/\sqrt{2}$	4284 }				10824
1^{--}	$A\bar{A}$	4350		$Y(4360)$	4368 ± 13	10850
$2S$						
1^{+-}	$(S\bar{A}\pm\bar{S}A)/\sqrt{2}$	4431 }		$Z_c(4430)$	4478^{+15}_{-18}	10939
	$A\bar{A}$	4461 }				10951
0^{++}	$S\bar{S}$		4604	$X(4500)$	4506^{+16}_{-19}	11111
0^{++}	$A\bar{A}$		4680	$X(4700)$	4704^{+17}_{-26}	11133
2^{++}	$A\bar{A}$		4748	$?^{?+}X(4740)$	4741 ± 9	11159
$2P$						
1^{--}	$S\bar{S}$	4666		$Y(4660)$	4643 ± 9	11122

Exotic charmonium-like states

• The mass of $X(3872)$ coincides with the predicted mass of the ground state 1^{++} neutral charm tetraquark state.

• $Z_c(3900)$ can be its 1^{+-} partner state, then $Z_c(4430)$ is its first radial excitation.

• Charged $Z_c(4020)$, $Z_c(4050)$, $Z_c(4055)$, $Z_c(4100)$ and $Z_c(4200)$ have masses inconsistent with our results. They could be hadro-charmonium states.

• Charged $Z(4430)$ can be the 1^+ $2S$-wave tetraquark state and $Z_c(4240)$ – 0^- $1P$-wave tetraquark state.

• Vector $Y(4230)$, $Y(4260)$, $Y(4360)$ and $Y(4660)$ can be the 1^{--} P-wave tetraquark states. We have no tetraquark candidate for $Y(4390)$.

• Axial vector $X(4140)$ can be the $[cs][\bar{c}\bar{s}]$ ground state tetraquark with 1^{++} while scalar $X(4500)$ and $X(4700)$ can correspond to its first radially excited 0^{++} states. If $X(4740)$ very recently observed by LHCb (arXiv:2011.01867) is different from $X(4700)$ it can be $2S$ excitation with 2^{++}. No tetraquark candidate for $X(4274)$.

Exotic botomonium-like states

• We do not have tetraquark candidates for charged $Z_b(10610)$ and $Z_b(10650)$, which are probably molecular states.

• The ground states of tetraquarks with hidden bottom are predicted to have masses below the open bottom threshold and thus should be narrow.

• Predictions for the masses of bottom counterparts to the charm tetraquark candidates are given.

Doubly heavy tetraquarks with open charm and bottom $(QQ')(\bar{q}\bar{q}')$:

Table 10: Masses M of heavy-diquark (QQ')–light-antidiquark $(\bar{q}\bar{q})$ states. T is the lowest threshold for decays into two heavy-light $(Q\bar{q})$ mesons and $\Delta = M - T$. All values are given in MeV.

System	State	$Q = Q' = c$			$Q = Q' = b$			$Q = c$, $Q' = b$		
	$I(J^P)$	M	T	Δ	M	T	Δ	M	T	Δ
$(QQ')(\bar{u}\bar{d})$										
	$0(0^+)$							7239	7144	95
	$0(1^+)$	3935	3871	64	10502	10604	-102	7246	7190	56
	$1(1^+)$							7403	7190	213
	$1(0^+)$	4056	3729	327	10648	10558	90	7383	7144	239
	$1(1^+)$	4079	3871	208	10657	10604	53	7396	7190	206
	$1(2^+)$	4118	4014	104	10673	10650	23	7422	7332	90
$(QQ')(\bar{u}\bar{s})$										
	$\frac{1}{2}(0^+)$							7444	7232	212
	$\frac{1}{2}(1^+)$	4143	3975	168	10706	10693	13	7451	7277	174
	$\frac{1}{2}(1^+)$							7555	7277	278
	$\frac{1}{2}(0^+)$	4221	3833	388	10802	10649	153	7540	7232	308
	$\frac{1}{2}(1^+)$	4239	3975	264	10809	10693	116	7552	7277	275
	$\frac{1}{2}(2^+)$	4271	4119	152	10823	10742	81	7572	7420	152
$(QQ')(\bar{s}\bar{s})$										
	$0(1^+)$							7684	7381	303
	$0(0^+)$	4359	3936	423	10932	10739	193	7673	7336	337
	$0(1^+)$	4375	4080	295	10939	10786	153	7683	7381	302
	$0(2^+)$	4402	4224	178	10950	10833	117	7701	7525	176

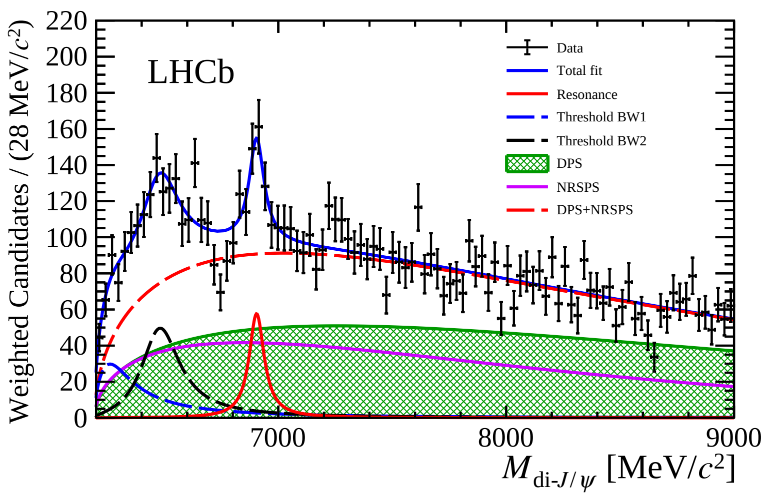
Heavy tetraquarks $(cq)(\bar{b}\bar{q}')$ with open charm and bottom

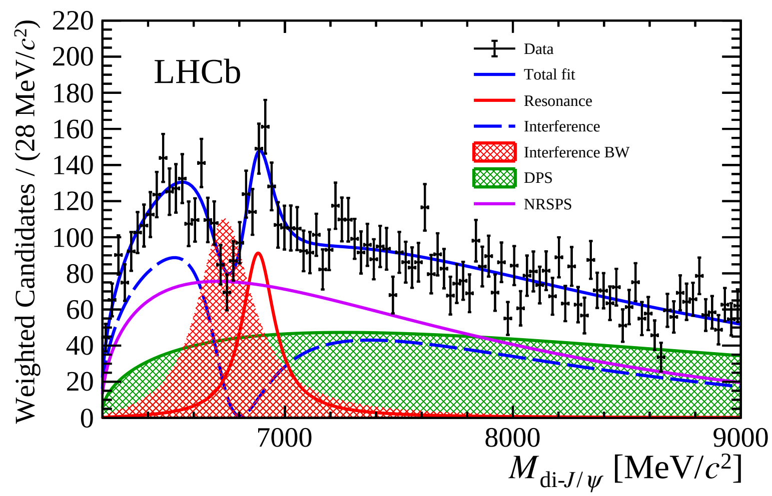
Table 11: Masses M of diquark (cq')–antidiquark $(\bar{b}\bar{q})$ states. T is the lowest threshold for decays into two heavy-light $(Q\bar{q})$ mesons and $\Delta = M - T$; T' is the threshold for decays into the $B_c^{(*)}$ and a light meson $(q'\bar{q})$, and $\Delta' = M - T'$. All values are given in MeV.

System	State J^P	$q'=u$ M	T	Δ	T'	Δ'	$q'=s$ M	T	Δ	T'	Δ'
$(cq')(\bar{b}\bar{q})$											
	0^+	7177	7144	33	6818	359	7294	7232	62	6768	526
	1^+	7198	7190	8	6880	318	7317	7277	40	6820	497
	1^+	7242	7190	52	6880	362	7362	7277	85	6820	542
	0^+	7221	7144	77	6818	403	7343	7232	111	6768	575
	1^+	7242	7190	52	6880	362	7364	7277	87	6820	544
	2^+	7288	7332	-44	7125	163	7406	7420	-14	7228	178
$(cq')(\bar{b}\bar{s})$											
	0^+	7282	7247	35	6768	514	7398	7336	62	6818	580
	1^+	7302	7293	9	6820	482	7418	7381	37	6880	538
	1^+	7346	7293	53	6820	526	7465	7381	84	6880	585
	0^+	7325	7247	78	6768	557	7445	7336	109	6818	627
	1^+	7345	7293	52	6820	525	7465	7381	84	6880	585
	2^+	7389	7437	-48	7228	161	7506	7525	-19	7352	154

Table 12: Masses M of the neutral heavy diquark (QQ)-antidiquark $(\bar{Q}\bar{Q})$ states. T is the threshold for the decays into two heavy-$(Q\bar{Q})$ mesons and $\Delta = M - T$. All values are given in MeV.

Composition	$d\bar{d}$	J^{PC}	M	Threshold	T	Δ
$cc\bar{c}\bar{c}$	$A\bar{A}$	0^{++}	6190	$\eta_c(1S)\eta_c(1S)$	5968	222
				$J/\psi(1S)J/\psi(1S)$	6194	-4
		1^{+-}	6271	$\eta_c(1S)J/\psi(1S)$	6081	190
		2^{++}	6367	$J/\psi(1S)J/\psi(1S)$	6194	173
$bb\bar{b}\bar{b}$	$A\bar{A}$	0^{++}	19314	$\eta_b(1S)\eta_b(1S)$	18797	517
				$\Upsilon(1S)\Upsilon(1S)$	18920	394
		1^{+-}	19320	$\eta_b(1S)\Upsilon(1S)$	18859	461
		2^{++}	19330	$\Upsilon(1S)\Upsilon(1S)$	18920	410





LHCb (2020): Narrow structure around 6.9 GeV matching the lineshape of a resonance $X(6900)$ and a broad structure above twice the J/ψ mass. In the same region more resonances may be present.

Table 13: Masses M of excited $cc\bar{c}\bar{c}$ tetraquarks (in MeV); $\mathcal{S}$ is the total spin of the diquark and antidiquark.

State	J^{PC}	$\mathcal{S}$	M	State	J^{PC}	$\mathcal{S}$	M
1P				1D			
	1^{--}	0	6631		2^{++}	0	6921
	0^{-+}	1	6628		1^{+-}	1	6909
	1^{-+}	1	6634		2^{+-}	1	6920
	2^{-+}	1	6644		3^{+-}	1	6932
	1^{--}	2	6635		0^{++}	2	6899
	2^{--}	2	6648		1^{++}	2	6904
	3^{--}	2	6638		2^{++}	2	6915
2S					3^{++}	2	6929
	0^{++}	0	6782		4^{++}	2	6945
	1^{+-}	1	6816				
	2^{++}	2	6868				

Experiment (LHCb 2020):

$M^{\text{exp}}(X(6900)) = 6905 \pm 11 \pm 7$ (MeV) no interference with NRSPS continuum

$M^{\text{exp}}(X(6900)) = 6886 \pm 11 \pm 11$ (MeV) with interference with NRSPS continuum

Table 14: Masses M of the neutral heavy diquark (cb)-antidiquark $(\bar{c}\bar{b})$ states. T is the threshold for the decays into two heavy-$(Q\bar{Q}')$ mesons and $\Delta = M - T$. All values are given in MeV.

Composition	$d\bar{d}$	J^{PC}	M	Threshold	T	Δ
$cb\bar{c}\bar{b}$	$A\bar{A}$	0^{++}	12813	$\eta_c(1S)\eta_b(1S)$	12383	430
				$J/\psi(1S)\Upsilon(1S)$	12557	256
				$B_c^{\pm}B_c^{\mp}$	12550	263
				$B_c^{*\pm}B_c^{*\mp}$	12666	147
		1^{+-}	12826	$\eta_c(1S)\Upsilon(1S)$	12444	382
				$J/\psi(1S)\eta_b(1S)$	12496	330
				$B_c^{\pm}B_c^{*\mp}$	12608	218
				$B_c^{*\pm}B_c^{*\mp}$	12666	160
		2^{++}	12849	$J/\psi(1S)\Upsilon(1S)$	12557	292
				$B_c^{*\pm}B_c^{*\mp}$	12666	183
	$\frac{1}{\sqrt{2}}(A\bar{S} \pm S\bar{A})$	1^{++}	12831	$J/\psi(1S)\Upsilon(1S)$	12557	274
				$B_c^{\pm}B_c^{*\mp}$	12608	223
				$B_c^{*\pm}B_c^{*\mp}$	12666	165
		1^{+-}	12831	$\eta_c(1S)\Upsilon(1S)$	12444	387
				$J/\psi(1S)\eta_b(1S)$	12496	335
				$B_c^{\pm}B_c^{*\mp}$	12608	223
				$B_c^{*\pm}B_c^{*\mp}$	12666	165
	$S\bar{S}$	0^{++}	12824	$\eta_c(1S)\eta_b(1S)$	12383	441
				$J/\psi(1S)\Upsilon(1S)$	12557	267
				$B_c^{\pm}B_c^{\mp}$	12550	274
				$B_c^{*\pm}B_c^{*\mp}$	12666	158

Table 15: Masses M of the charged heavy diquark–antidiquark states. T is the threshold for the decays into two heavy $(Q\bar{Q}')$ mesons and $\Delta = M - T$. All values are given in MeV.

Composition	$d\bar{d}$	J^P	M	Threshold	T	Δ
$cc\bar{c}\bar{b}$, $cb\bar{c}\bar{c}$	$A\bar{A}$	0^+	9572	$\eta_c(1S)B_c^{\pm}$	9259	313
				$J/\psi(1S)B_c^{*\pm}$	9430	142
		1^+	9602	$\eta_c(1S)B_c^{*\pm}$	9317	285
				$J/\psi(1S)B_c^{\pm}$	9372	230
				$J/\psi(1S)B_c^{*\pm}$	9430	172
		2^+	9647	$J/\psi(1S)B_c^{*\pm}$	9430	217
	$A\bar{S}$, $S\bar{A}$	1^+	9619	$\eta_c(1S)B_c^{*\pm}$	9317	302
				$J/\psi(1S)B_c^{\pm}$	9372	247
				$J/\psi(1S)B_c^{*\pm}$	9430	189
$cc\bar{b}\bar{b}$, $bb\bar{c}\bar{c}$	$A\bar{A}$	0^+	12846	$B_c^{\pm}B_c^{\pm}$	12550	296
				$B_c^{*\pm}B_c^{*\pm}$	12666	180
		1^+	12859	$B_c^{\pm}B_c^{*\pm}$	12608	251
				$B_c^{*\pm}B_c^{*\pm}$	12666	193
		2^+	12883	$B_c^{*\pm}B_c^{*\pm}$	12666	217
$cb\bar{b}\bar{b}$, $bb\bar{c}\bar{b}$	$A\bar{A}$	0^+	16109	$B_c^{\pm}\eta_b(1S)$	15674	435
				$B_c^{*\pm}\Upsilon(1S)$	15793	316
		1^+	16117	$B_c^{\pm}\Upsilon(1S)$	15735	382
				$B_c^{*\pm}\eta_b(1S)$	15732	385
				$B_c^{*\pm}\Upsilon(1S)$	15793	324
		2^+	16132	$B_c^{*\pm}\Upsilon(1S)$	15793	339
	$S\bar{A}$, $A\bar{S}$	1^+	16117	$B_c^{\pm}\Upsilon(1S)$	15735	382
				$B_c^{*\pm}\eta_b(1S)$	15732	385
				$B_c^{*\pm}\Upsilon(1S)$	15793	324

SUMMARY

- Masses of tetraquarks with heavy quarks are calculated in the diquark-antidiquark picture.
- Dynamical approach based on the relativistic quark model is used, where both diquark and tetraquark masses are obtained by numerical solution of the quasipotential equation with the corresponding relativistic potentials.
- The diquark size is taken into account with the help of the diquark-gluon form factor in terms of diquark wave functions.
- No free adjustable parameters are introduced.
- It is found that masses of $X(3872)$, $Z_c(3900)$, $X(4140)$, $Z_c(4240)$, $Z_c(4250)$, $Y(4260)$, $Y(4360)$, $Z_c(4430)$, $X(4500)$, $Y(4660)$, $X(4700)$, $X(4740)$ are compatible with the masses of hidden-charm tetraquark states with corresponding quantum numbers. Note that most of these states were observed after our predictions.
- The ground states of tetraquarks with hidden bottom are predicted to have masses below the open bottom threshold and thus should be narrow.
- We do not have tetraquark candidates for charged $Z_b(10610)$ and $Z_b(10650)$, which are probably molecular states.
- Predictions for the masses of bottom counterparts to the charm tetraquark candidates are given
- All the $(cc)(\bar{q}\bar{q}')$ tetraquarks are predicted to be above the decay threshold into the open charm mesons.
- Only the $I(J^P) = 0(1^+)$ state of $(bb)(\bar{u}\bar{d})$ is found to lie below the BB^* threshold.
- All $QQ\bar{Q}\bar{Q}$ ground state tetraquarks have masses above thresholds of decays to two heavy quarkonia. Therefore such tetraquarks can be observed as broad structures decaying dominantly to quarkonia.

- The broad structure recently observed by LHCb in the mass spectrum of J/ψ-pairs can correspond to the 2^{++} state of the $cc\bar{c}\bar{c}$ tetraquark. The narrow state $X(6900)$ could be its excited $2S$ or $1D$ state.
- All ground state $bb\bar{b}\bar{b}$ tetraquarks have masses significantly (400-500 MeV) higher than corresponding thresholds and thus expected to be very broad. This is in accord with the absence of the narrow beautiful tetraquarks in the Υ-pair production reported by CMS and LHCb.

Thank you!