Black bounces and remnants in dilaton gravity

2202.00023 [gr-qc]

Fitkevich M.D.

Institute for Nuclear Research of RAS
Moscow Institute of Physics and Technology

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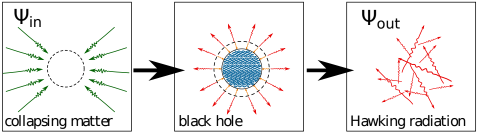
Intro

- Information paradox: apparent violation of unitarity.
  $\hat{\rho}_{in} = |\Psi_{in}\rangle\langle\Psi_{in}| \ \mapsto \ \hat{\rho}_{out} = \mathrm{Tr}_{BH}\left(|\Psi_{ext}\rangle|\Psi_{BH}\rangle\langle\Psi_{BH}|\langle\Psi_{ext}|\right), \ \mathrm{Tr}(\hat{\rho}^2_{out}) < 1$



- Gauge/string duality.
  - AMPS-firewall: unitarity vs equivalence principle.
  - Islands: unitary Page curve.

    1911.12333 [hep-th] Almheiri, 1905.08255 [hep-th] Pennington ...
  - Dynamics: S-matrix.
- Non-singular models for gravity
  - Limiting curvature ${R_{\mu\nu\rho\sigma}}^2 < \Lambda^2$. De Sitter core.

    Markov, 2111.14318 [gr-qc] Frolov ...
  - Other models: Bardeen's black hole, black bounces (timeholes), planck stars...

    1812.07114 [gr-qc] Visser, 1802.04264 [gr-qc] Rovelli...

Models with linear dilaton vacuum $\phi = -\lambda r$

Action

$$S_{\rm LDV} = \int d^2x\sqrt{-g}\left(W(\phi)R + W''(\phi)\left((\nabla\phi)^2 + \lambda^2\right)\right) + S^{\rm m}$$

Field equations

$$W'(\phi)R = 2W''(\phi)\Box\phi + W'''(\phi)\left((\nabla\phi)^2 - \lambda^2\right) \ ,$$
$$g_{\mu\nu}\left(W''(\phi)((\nabla\phi)^2 - \lambda^2) + 2W'(\phi)\Box\phi\right) - 2W'(\phi)\nabla_\mu\nabla_\nu\phi = T^{\rm m}{}_{\mu\nu} \ ,$$

where $T^{\rm m}{}_{\mu\nu} = (-2/\sqrt{-g})\delta S^{\rm m}/\delta g^{\mu\nu}$.

General solution

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} \ , \qquad \phi = -\lambda r \ , \qquad f(r) = 1 + \frac{M}{\lambda W'(\phi)} \ .$$

Sinh-CGHS model

Action

$$S_{\sinh} = -2\int d^2x\,\sqrt{-g}\,\sinh(2\phi)\left(R + 4(\nabla\phi)^2 + 4\lambda^2\right)\ ,$$

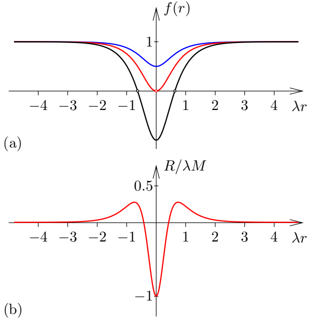
Vacuum solution

$\phi = -\lambda r\ ,\qquad ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2$

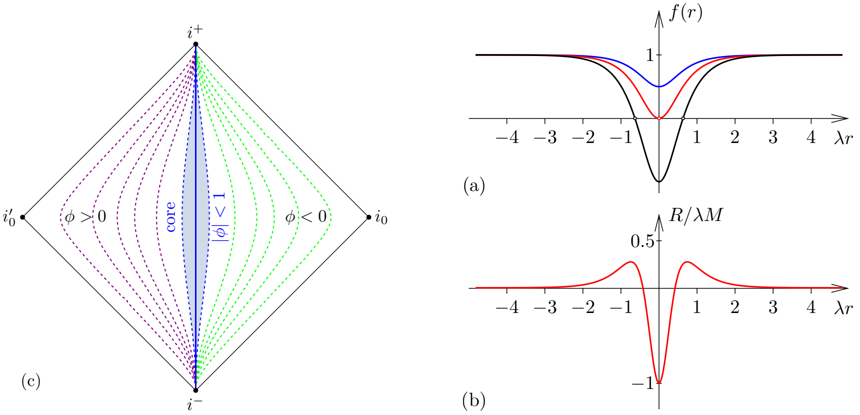
with metric function

$$f(r) = 1 - \frac{M}{4\lambda\cosh(2\lambda r)}$$

Ricci scalar $R = -\partial_r^2 f(r)$ is finite everywhere.



Solutions: gravitational kink



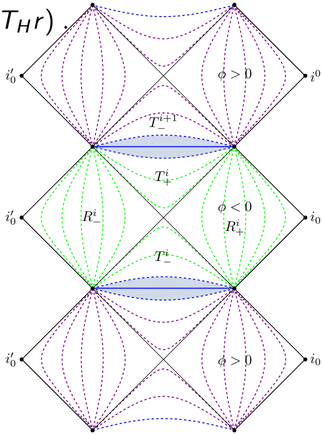
Solution: black bounce* $M > M_{ext}$

Coordinate extension:

$$g(r) = \frac{\left(1 + \frac{M_{\text{ext}}}{M}\right)\tanh(\lambda r) - 2\pi T_H/\lambda}{\left(1 + \frac{M_{\text{ext}}}{M}\right)\tanh(\lambda r) + 2\pi T_H/\lambda}\exp(4\pi T_H r) \ ,$$

$$T = \sqrt{g(r)}\sinh(2\pi T_H t) \ ,$$

$$R = \sqrt{g(r)}\cosh(2\pi T_H t)$$



(a)

Metric takes a form

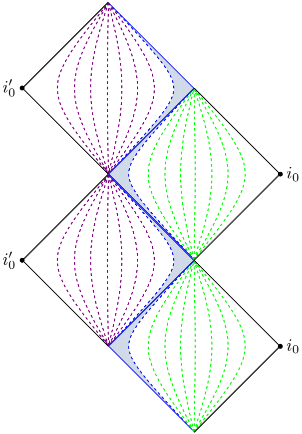
$$\boxed{ds^2 = \frac{f(r)}{4\pi^2 {T_H}^2 g(r)}\left(-dT^2 + dR^2\right)}$$

Maps $(V_i, U_i) = (T_i + R_i, T_i - R_i)$ are identified

$$V_{i+1} = -\kappa/V_i \ , \qquad U_{i+1} = -1/\kappa U_i$$

* *1812.07114 [gr-qc] Simpson, Visser*

Solutions: extremal black bounce $M = M_{ext}$



(b)

$$ds_E{}^2 = f(r)dt_E{}^2 + \frac{dr^2}{f(r)}\,, \qquad 0 \le t_E < \beta_H\,,$$

Euclidean solution has imaginary time period

$$\beta_H = T_H{}^{-1} = 4\pi/f'(r_\mathrm{h})$$

so that no conifold singularity at $r = r_\mathrm{h}$. Derive black hole temperature and entropy

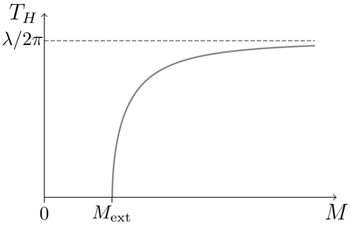
$$\boxed{T_H = \frac{\lambda^2 W''(\phi_\mathrm{h})}{4\pi M}}$$

$$\boxed{S_\mathrm{BH}(M) = 4\pi W(\phi_\mathrm{h}) - 4\pi W(\phi_\mathrm{h,\,ext})}$$

Sinh-CGHS case

$$S_{BH} = \frac{2\pi}{\lambda} M \sqrt{1 - \frac{M^2_\mathrm{ext}}{M^2}}$$

$$T_H = \frac{\lambda}{2\pi}\sqrt{1 - \frac{M^2_\mathrm{ext}}{M^2}}$$



Is core stable?

Example: conformal matter

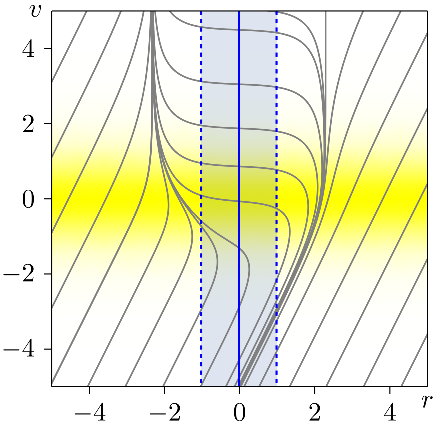
$$T_{\mathrm{m}\,\mu\nu} = \nabla_\mu f \nabla_\nu f - \frac{1}{2} g_{\mu\nu} (\nabla f)^2 \ ,$$

Vaidya ansatz
$ds^2 = -F(v,r)dv^2 + 2dvdr$ with incident wavepacket $f(v)$ has solution

$$F(v,r) = \left(1 - \frac{\mathcal{M}(v)}{4\lambda \cosh(2\lambda r)}\right) \ ,$$

with Bondi mass

$$\mathcal{M}(v) = \int_{-\infty}^{v} dv' (\partial_v f(v'))^2 \ .$$



Vaidya solution with coordinates $(r,\ v)$.

Is there mass inflation?

CGHS regime $M \gg M_{ext}$,

$$e^{-2\rho} = e^{-2\phi} = -\lambda^2 vu + g(v) + h(u) \ ,$$

$$g(v) = \frac{1}{2}\int_0^v dv' \int_{v'}^{+\infty} dv'' (\partial_v f(v''))^2 \ ,$$

$$h(u) = -\frac{1}{2}\int_{-\infty}^u du' \int_{-\infty}^{u'} du'' (\partial_u f(u''))^2 \ ,$$

For wavepacket tail profile at late times $f(v) \simeq f_0 \cdot (\lambda v)^{-\alpha}$ $v \to +\infty$

$$g(v) \simeq \frac{M}{2\lambda} - \frac{g_\infty}{(\lambda v)^{2\alpha}} \ , \qquad \alpha > 0 \ ,$$

After crossing the core

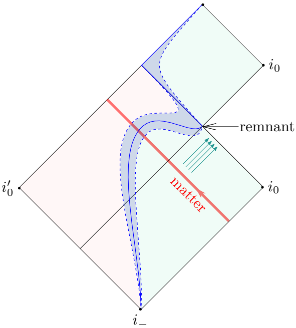
$$f(v) \mapsto f_0 \cdot (-\lambda v)^{\alpha} \ ,$$

Ricci scalar near Cauchy horizon

$$R \simeq 4\lambda^2 e^{2\phi} \left(\frac{M}{2\lambda} + (2\alpha+1) g_\infty (-\lambda v)^{2\alpha} + \frac{\mathcal{E}_{\rm out}(u)}{2\lambda} + \right. \\ \left. + \frac{2\alpha+1}{2\alpha-1} \frac{2\alpha g_\infty}{\lambda} (-\lambda v)^{2\alpha-1} \partial_u h(u) \right)$$

is finite if $\alpha > 1/2$.

Do remnants form?



2D Stefan–Boltzmann (adiabatic approximation):

$$\frac{dM}{dt} = -\frac{\pi}{12} T_H^2(M) \ ,$$

For a mass function one gets

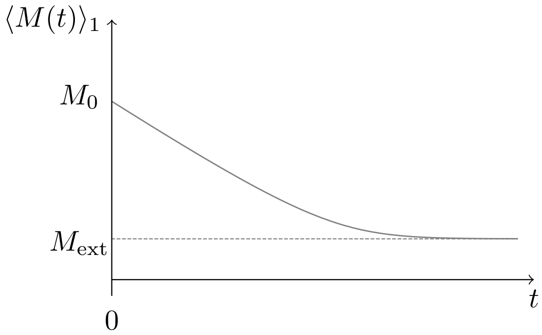
$$M(t) + \frac{M_{\rm ext}}{2} \log\left(\frac{M(t) - M_{\rm ext}}{M(t) + M_{\rm ext}}\right) = M_0 - \frac{\lambda^2 t}{48\pi} \ ,$$

with initial value $M_0 \gg M_{\rm ext}$.
Remnants stage is

$$M \simeq M_{\rm ext}\left(1 + \exp\left(-\frac{\lambda^2 t}{24\pi M_{\rm ext}}\right)\right) \ ,$$

Average black hole mass

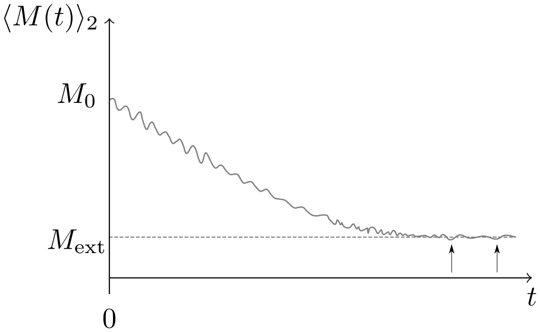
Adiabatic = mean-field approximation with one loop.
Two stages of evaporation: CGHS stage and remnant formation as asymptotics:



Fluctuations of Hawking flux

Effects of thermal/quantum fluctuations.

gr-qc/9905012 Wu, Ford



Are remnants metastable?

Thermal estamate

$$\langle M\rangle = -\frac{1}{Z}\frac{\partial Z}{\partial\beta}\,, \quad \langle M^2\rangle = \frac{1}{Z}\frac{\partial^2 Z}{\partial\beta^2}\,,$$

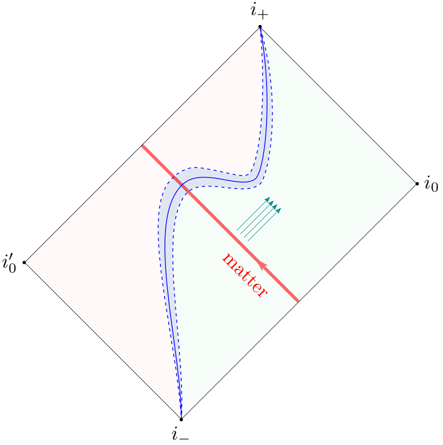
$$\langle(\Delta M)^2\rangle = -\frac{\partial\langle E\rangle}{\partial\beta}$$

for remnants

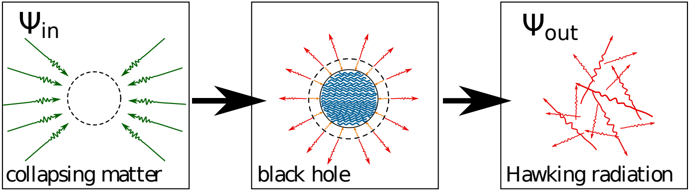
$$M_{\rm dec} - M_{\rm ext} = \sqrt{\langle(\Delta M)^2\rangle} = \frac{\lambda^2}{M_{\rm ext}}O(1)$$

assuming $\delta M \ll M_{\rm ext}$.
Characteristic decay time

$$\boxed{t_{\rm dec} \simeq 48\pi\frac{M_{\rm ext}}{\lambda^2}\log\left(\frac{M_{\rm ext}}{\lambda}\right)}$$



Next step: semiclassical S-matrix



$$\langle \Psi_{out} | \hat{S} | \Psi_{in} \rangle = \int \mathcal{D}\Phi \, \Psi^*_{out} \Psi_{in} \exp\{\frac{i}{\hbar} S[\Phi]\} \,, \quad \Phi = \{g_{\mu\nu}, \phi, f\}$$

- Semiclassics $\Rightarrow \frac{\delta}{\delta\Phi} S = 0 \Rightarrow$ with Φ_s with flat asymptotics. Trivial if $E < E_{thr.}$
- Idea: find saddles at $E > E_{thr.}$ by analytic continuation avoiding singularities.
    - Problem: complexification of spacetime is ambiguous.
- Non-singular model can help!

Semiclassical S-matrix

- Typical semiclassical state Ψ - localized wavepacket into remnant
- For any typical Ψ one can find non-typical Ψ':

$$\langle \Psi' | \hat{T}_{\mu\nu}(x) | \Psi^* \rangle < 0$$

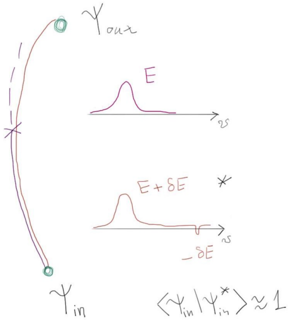
Fulling, Davies (1976)

gr-qc/9711030 Roman, Ford

- Non-typical Ψ' cause remnant decay.

Common QFT counterpart: tunnelling through sphaleron. Neat example:

0903.3916 [quant-ph] Levkov, Panin



Test of unitarity

Consider

$$\langle b_k|\hat{S}|a_k\rangle = \int dc_k^* dc_k \, \langle b_k|\hat{S}_{\rm reg}|c_k\rangle\langle c_k|a_k\rangle \approx e^{iS[c_k]}e^{-\Gamma[c_k]}$$

$\hat{S}_{reg}$ is S-matrix on subspace with topologically trivial spacetimes. Saddle point:

$$i\frac{\delta S}{\delta c_k} = \frac{\delta \Gamma}{\delta c_k}$$

Unitarity check:

$$\int dc_k^* dc_k \, \langle b_k|\hat{S}^\dagger|c_k\rangle\langle c_k|\hat{S}|a_k\rangle = \langle b_k|a_k\rangle \qquad (\hat{S}^\dagger\hat{S} = 1)$$

Conclusion

- 2d dilaton gravity models with regular holes are introduced;
- sinh-CGHS: global geometry, thermodynamical properties;
- backreaction from evaporation - remnants formation;
- conjecture: remannts are metastable;
- horizons are dissolve in unitary evaporation process (similar proposals from 't Hooft, Rovelli etc.)