ON SCATTERING OF WAVES BY NULL COSMIC STRINGS

D.V. Fursaev, I.G.Pirozhenko

BLTP JINR and Dubna State University

July 18, 2022

QFT, HEP, and Cosmology, Dubna, 18-22 July 2022

Null Strings

A null cosmic string is a one-dimensional object whose points move along trajectories of light rays, orthogonally to the string itself.
A. Schild. Classical Null Strings. Phys. Rev. D, 16:1722, 1977. (Schild equations)

Null strings are characterised by their optical properties. The strings behave as one-dimensional null geodesic congruences characterized by a complex optical scalar which is determined by an analogue of the Sachs' optical equation.
D.V. Fursaev, Phys. Rev. D103 (2021) no.12, 123526

The origin of null strings may be related to physics of fundamental strings at the Planckian energies.
F. Xu, JHEP 10 (2020) 045

- The study is motivated by possible effects of null strings in cosmology. World-sheets of null strings develop caustics which accumulate large amounts of energy.

E.A. Davydov, D.V. Fursaev, V.A.Tainov, Phys. Rev. D105 (2022) no.8, 083510

Null strings (massless, tensionless)

From a massive cosmic string at rest along z-axis

$$ds^2 = -dt^2 + dz^2 + dr^2 + (1-4G\mu)^2 r^2 d\Theta^2, \quad r^2 = x^2 + y^2$$

$\longrightarrow$ Aichelburg-Sexl boost

$$\cosh\chi = (1 - v^2/c^2)^{-1/2} \to \infty, \quad E = mc^2 \cosh\chi \to \mathit{finite}$$

$\longrightarrow$ Kerr-Schield metric

$$ds^2 = -dudv + \omega|y|\delta(u)du^2 + dy^2 + dz^2, \quad \omega \equiv 8\pi G\varepsilon$$

ε - energy per unit length, $u = t - x$, $v = t + x$.

C. Barrabes, P.A. Hogan, W. Israel, Phys.Rev. D66 (2002) 025032.

The problem of particle (wave) in the field of null cosmic strings is related to the movement in the impulsive (shock) gravitational wave background. R. Penrose, Part of General relativity : Papers in honour of J.L. Synge, 101-115 (1972)

The gravitational shock wave of a massless particle attracted a considerable interest in the context of black hole formation in high energy particle collisions.

T. Dray and G. 't Hooft. NPB, 253:173–188, 1985. G. 't Hooft., Phys. Lett. B, 198:61–63, 1987.

- To our knowledge the field effects in the background of a null cosmic string are not yet comprehensively studied.

Classical and quantum FT for scalar fiels only on general shock wave background was given by C. Klimcik [PLB'1988], and in the scatterring matrix context by C.O. Lousto, N. G. Sanchez, Nucl.Phys.B 355 (1991) 231-249.

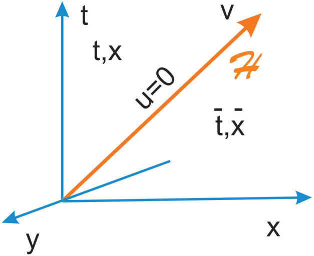
- Our aim is to derive some physical effects using the holonomy property of the null string spacetime.

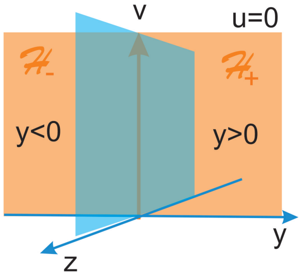
Null string dynamics

We consider a null straight string moving in the Minkowski spacetime in the direction of x axes and parallel to z axes,

$$ds^2 = -dudv + \omega|y|\delta(u)du^2 + dy^2 + dz^2, \quad \omega \equiv 8\pi GE.$$

The delta-function in the metric indicates a singularity of the coordinate chart along the hypersurface $\mathcal{H} : u = 0$, which is the event horizon of the string.





- The string equations of motion are $t - x = 0, \quad y = 0.$
- Null string world surface is $u = y = 0.$

Null Holonomies

- The null string yields the nontrivial null holonomy. The parallel transport of a vector V along a closed contour around the string results in a null rotation,

$$V' = M(\omega)V, \quad \omega \equiv 8\pi GE.$$

Null holonomy belongs to the parabolic subgropup of the Lorentz transformations (null rotations), $x'^{\mu} = M^{\mu}_{\nu}(\omega)x^{\nu}$, and acts on u, v, y, z coordinates in $R^{1,3}$ as follows:

$$u' = u, \quad v' = v + 2\omega y + \omega^2 u, \quad y' = y + \omega u, \quad z' = z.$$

The group parameter of the holonomy, $\omega \equiv 8\pi GE$, is determined by the energy of a string per unit length, E.

These transformations do not move points on the string world surface.

M. van de Meent, Geometry of Massless Cosmic Strings, Phys. Rev. D87 (2013) no.2, 025020, e-Print: arXiv:1211.4365 [gr-qc].

The Holonomy Method

- We decompose the space-time into two parts by the light surface $u = 0$. This surface is the string event horizon $\mathcal{H}$, as the events that occur in the half-space $u > 0$ do not affect the events in the area $u < 0$, and vice versa.
For particles and light rays we define the ingoing trajectories ($u < 0$) and the outgoing trajectories ($u > 0$).
We consider two parts of the event horizon, left and right with respect to the string: $\mathcal{H}_L$ ($y < 0$), and $\mathcal{H}_R$ ($y > 0$).

- To describe the outgoing fields, we introduce a coordinate chart which has a cut on $\mathcal{H}_L$.

- The string horizon is considered as a Cauchy hypersurface where initial data for outgoing trajectories are determined. The 'right' trajectories ($y > 0$) behave smoothly across $\mathcal{H}$ while the 'left' trajectories ($y < 0$) are shifted along the v coordinate and change their direction under the null rotation.

- At $y < 0$, $u = 0$, the coordinate transformations are reduced to

$$v' = v + 2\omega y, \quad y < 0.$$

D. V. Fursaev, Phys. Rev. D, 96(10):104005, 2017.

Extension of the holonomy method to fields

Consider a field φ in $R^{1,3}$ with the transformation law with respect to null rotations

$$\varphi'(x') = S(\omega)\varphi(x)$$

where the element $S(\omega)$ belongs to some representation of the Lorentz group.

Extension of the holonomy method to fields is:

$$\varphi(x) = \bar{\varphi}(x)|_{u=0,y>0}, \quad \varphi(x) = S(\omega)\bar{\varphi}(\bar{x})|_{u=0,y<0} \quad (\star)$$

where $\bar{\varphi}$ is the ingoing field at $u < 0$.

- These transition conditions guarantee that physical quantities measured by the 'left' observers behave smoothly across $\mathcal{H}$.

Scalar plane wave in the null string background

A plane wave, $\varphi_{in}(x) = e^{ik_\mu x^\mu}$, scattered by a null cosmic string remains unchanged for $y > 0$, and for $y < 0$ it undergoes the transformation. Let $\phi_{in}|_{u=0}$ be the value of the field if the horizon is approached from $u < 0$.

Then the initial data for φ_{out} is

$$\begin{aligned}\varphi_{out}(x)|_{u=0,y>0} &= \varphi_{in}(x)|_{u=0,y>0}, \\ \varphi_{out}(x)|_{u=0,y<0} &= M(\omega)\varphi_{in}(x)|_{u=0,y>0}.\end{aligned}$$

The holonomy transforms the field as follows

$$M(\lambda)\varphi_{in}(x)|_{u=0,y>0} \equiv \exp(ik\bar{x})|_{u=0,y>0} = \exp(i\bar{k}x)|_{u=0,y>0}.$$

Here $\bar{k}_\mu$ are momenta transformed by holonomy,

$$\bar{k}_u = k_u + \omega k_y + \omega^2 k_v, \quad \bar{k}_v = k_v, \quad \bar{k}_y = k_y + 2\omega k_v \quad \bar{k}_z = k_z.$$

Solution for a plane wave scattered by a null string

We search for the solution to the wave equation

$$\Box\varphi(x) = 4\left[\partial_u\partial_v - \frac{1}{4}(\partial_y^2 + \partial_z^2)\right]\varphi(x) = 0$$

in the null cosmic string space time.
Then the scattered (outgoing) wave can be written in the form

$$\varphi_{out}(x) = \varphi_+(x) + \varphi_-(x).$$

The boundary condition at the string world surface,

$$\begin{aligned} \varphi_+(0,\mathbf{x}) &\equiv \varphi_{in}(x)|_{u=0,y>0} = \theta(y)\exp\{i\mathbf{k}_+\cdot\mathbf{x}\}, \\ \varphi_-(0,\mathbf{x}) &\equiv \varphi_{in}(\bar{x})|_{u=0,y<0} = \theta(-y)\exp\{i\mathbf{k}_-\cdot\mathbf{x}\}, \end{aligned}$$

were $k_+ = k$, $k_- = M(\omega)k$, and $\mathbf{x} = (v, y, z)$, $\mathbf{k} = (k_v, k_y, k_z)$.

The solution for $k_v > 0$ is

$$\varphi_\pm(u,\mathbf{x}) = \left\{\theta(\pm f_\pm) + \frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dt}{t-(\pm f_\pm + i\epsilon)e^{-i\frac{\pi}{4}}}\, e^{-\frac{t^2}{4k_v u}}\, e^{-\frac{if_\pm^2}{4k_v u}}\right\} e^{ik_\mu^\pm x^\mu}.$$

where $k_+ = k$, $k_- = M(\omega)k$, and

$$f_\pm(x) = uk_y^\pm + 2k_v^\mp.$$

The surfaces $f_\pm(x) = 0$ set the wave front boundaries. The normal vectors $n^\pm$: $df_\pm = n_\mu^\pm dx^\mu$, are orthogonal to the wave vectors $(n^+ \cdot k) = (n^- \cdot \bar{k}) = 0$. The angle between these surfaces is given by

$$\cos\alpha = \frac{(4k_v^2 + k_y(k_y + 2\omega k_v))}{(4k_v^2 + k_y^2)^{1/2}(4k_v^2 + (k_y + 2\omega k_v)^2)^{1/2}}.$$

This is the angle of the wave vector rotation due to holonomy.

For $k = k_v$, $\cos\alpha = (1+\omega^2)^{-\frac{1}{2}}$

Wave front boundaries

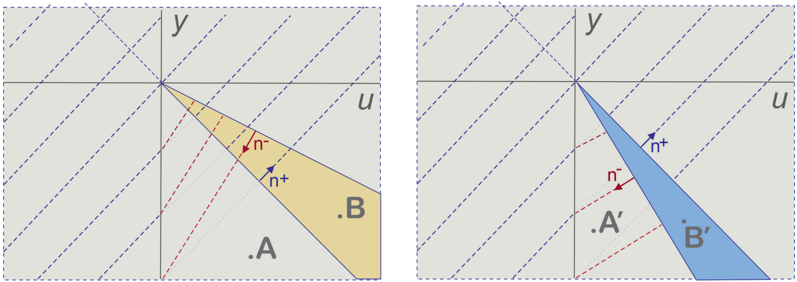


Fig. Wavefronts perturbed by a null string. The world surface of the string is the point $u = 0$, $y = 0$. The picture corresponds to θ-terms of the solution for $k_v < 0$ (left) and $k_v < 0$ (right)

When a null string crosses a wavefront, the left side of the latter does not undergo any changes, while the wave vector of the right side is rotated by the angle α, and the frequency of the wave is shifted as $\delta k_0 = \omega(k_y + \omega k_v)$ (here $\omega = 8\pi GE$)

Observable effects

- The near fieled $(u \to 0)$

The effects are complicated because of the form-factor which appears in the solution due to the string. After scattering on the string a monochromatic wave is not monochromatic anymore.

- The far field $(u \to \infty)$

$$\begin{aligned} A &= A_+ + A_-, \\ A_\pm &= \theta(\pm f_\pm) e^{ik^\pm_\mu x^\mu} + \text{"tail"}, \quad \text{"tail"} \sim \mathcal{O}(u^{-1/2}). \end{aligned}$$

A polarized wave $\hat{e}\, e^{ikx}$ transforms to

$$\theta(\pm f_+)\hat{e}_+\, e^{ik^\mu_+ x_\mu} + \theta(-f_-)\hat{e}_-\, e^{ik^\mu_- x_\mu} + \text{"tail"},$$

where $k_+ = k$, $\hat{e}_+ = \hat{e}$ for the right observer, and

$$k_- = M(\omega)k, \quad \hat{e}_- = S(\omega)\hat{e}.$$

Observable effects in far field region

From an observational point of view, the interaction of a null string with electromagnetic and gravitational waves, is of interest. When they are scattered by a null string the following effects emerge:

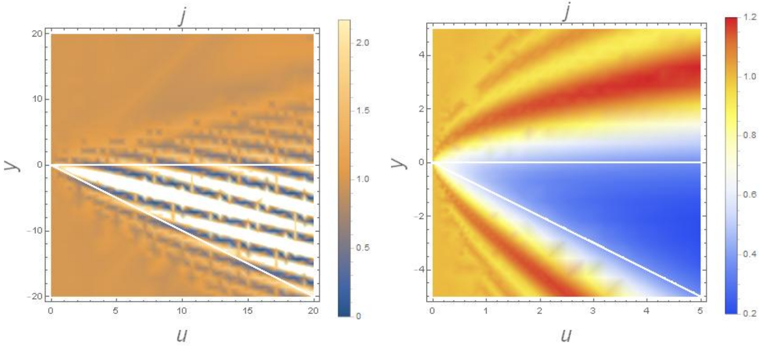
- refraction of the part of the wave that propagates behind the passing string with respect to the observer. All "left" vectors turn in the same way due to holonomy.

- diffraction on the string, when the "right" part of the wave partially overlaps the "left" region. Diffraction is accompanied by interference, and depends on on the sting energy and wavelength (conditons $f_{\pm} = 0$);

- string shadow effect for waves with $k_v < 0$.

The density plots where the value of the current is given by color,

$$j_v = i(\varphi_{in}^* \partial_v \varphi_{in} - \varphi_{in} \partial_v \varphi_{in}^*)$$



The current j_v in the region $u > 0$ is normalized to the current in the absence of a string, and $\lambda = 1$. The left picture corresponds to $k = k_v < 0$ and demonstrates a "string shadow" (dark blue) between wave front boundaries. The right picture manifests the diffraction region, which appears if the energy of the ingoing wave is positive, $k_v > 0$.

Conclusion

- We have developed the holonomy method for fields. It allows to study different fields (fiber bundles) in the null string background.
- We predict physical effects related to wave propagating in the gravitational field of a straight null string in Minkowsky spacetime $R^{1,3}$.
- Electrodynamics in the the null string background is work in progress.