Presymplectic structures and Batalin-Vilkovisky formalism beyond jet-bundles

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Introduction

- Theories of fundamental interactions (Gravity, YM, Strings, M-Theory, Higher-spin theories ...) are inevitably gauge theories. We are mostly intrested in Lagrangian gauge theoires! Powerful Batalin-Vilkovisky (BV) formalism is available.

- Mathematical setup to handle local gauhge theories is that of jet-bundles. BV formalism on jet-bundles: *Henneaux, Barnich, Brandt,........* Applies to variational PDEs. Moreover, analyzing local BRST cohomology brings us beyond jet-bundles. E.g. transgression formulas, generalized connections, etc. *Stora, Baulieu, Brandt.....*

- Both from the fundamental perspective and applications in gravity, asymptotic symmetries, holography, higher spin gauge theories, string field theory, etc. it is highly desierable to develop a version of BV comprising on-shell description (BV beyond jet-bundles). In the usual PDE theory an anlog is the *Vinogradov* approach.

- Full-scale generalization of BV beyond jet-bundles setup is not straitforward
  - BV is desgined for Lagrangian systems (defined on jet-bundles)
  - How (BV) Lagrangian can be encoded in on-shell terms? Even for non-gauge PDEs this is an open problem. Can be seen as an invariant version of the invers problem of the calculus of variations.

- It turns out that a bridge between BV formalism and the invariant geometrical approach to PDEs becomes manifest using the *Alexandrov-Kontsevich-Schwartz-Zaboronsky 1994* (AKSZ)-like framework. This was originally proposed as an elegant BV formulation of topological models. Somewhat similar approach (in terms of free differential algebras (FDA)) was independently developped by *M.Vasiliev* in the context of higher-spin theories. It is also worth mentioning FDA approach to supergravity by *D'Auria, Fre,...*.

Lagrangians and locality

Ignore locality and consider field-histories as finite-dim space with coordinates ϕ^i. Functions $E_i(\phi)$ define a stationary surf. $E_i = 0$.

Assuming regularity pick independent T^α among E_i ($T^\alpha = 0$ define the same surface). Then the following action:

$$S = k_{\alpha\beta} T^\alpha T^\beta$$

define equivalent EOMs. If we disregard locality, global geometry issues and irregular situations all equations are Lagrangian. In the local setting "invers problem of the calculus of variations".

Lagrangians are needed to define quantum theory and the Lagrangian formalism is extremely useful: Noether theorem, consistent interactions, and nearly all QFT methods are essentially lagrangian.

Standard setup: jet-bundle apporoach

PDEs and jet-bundles

Fiber-bundle $\mathcal{F} \to X$ (global aspects are not discussed):
base space (independent variables or space-time coordinates): x^a, $a = 1, \ldots, n$.
Fiber: (dependent variables or fields ϕ^i)

Jet-bundle:
A point of J^k is a pair $(p, [\sigma])$, where $[\sigma]$ is an equivalence class of sections $\sigma : X \to \mathcal{F}$ such that their partial derivatives at p coincide to order k:

$$\partial_{a_1} \ldots \partial_{a_l} \phi(x))|_p = (\partial_{a_1} \ldots \partial_{a_l} \phi'(x))|_p, \quad l \leqslant k$$

One can use x^i, and values of derivatives as coordinates:

$J^0(\mathcal{F}): \quad x^a, \phi^i, \qquad J^1(\mathcal{F}): \quad x^a, \phi^i, \phi^i_a\,, \qquad J^2(\mathcal{F}) \quad x^a, \phi^i, \phi^i_a, \phi^i_{ab}\,, \qquad \ldots$

Projections:

$$\ldots \to J^N(\mathcal{F}) \to J^{N-1}(\mathcal{F}) \to \ldots \to J^1(\mathcal{F}) \to J^0(\mathcal{F}) = \mathcal{F}$$

Useful to work with $\mathcal{J} := \mathcal{J}^\infty$.

A local function is the one that depends on only a finite number of the coordinates.
A local function $f = f(x, \phi, \phi_a, \phi_{ab} \ldots)$ can be evaluated on a section $\sigma : X \to \mathcal{F}$ as

$$f(\sigma) := f(x, \sigma^*(\phi^i), \partial_a \sigma^*(\phi^i), \ldots)$$

Total derivative: (imitates the action of standard partial derivative)

$$D_a := \frac{\partial}{\partial x^a} + \phi^i_a \frac{\partial}{\partial \phi^i} + \phi^i_{ab} \frac{\partial}{\partial \phi^i_a} + \dots$$

Main property:

$$\partial_a(f(\sigma)) = (D_a f)(\sigma)\,.$$

Total derivatives generate Cartan distribution.

Similarly one defines local forms.

Space-time differentials dx^a. Horizontal differential:

$$d_{\mathsf{h}} \equiv dx^a D_a\,, \qquad d_{\mathsf{h}}^2 = 0\,.$$

PDE theory in terms of jet-bundle

A system of partial differential equations (PDE) is a collection of local functions on $\mathcal{J}$

$$E_\mu[\phi, x]\,.$$

The equation manifold (stationary surface): $\mathcal{E} \subset \mathcal{J}$ singled out by: (prolonged equation)

$$D_{a_1} \dots D_{a_l} E_\mu = 0\,, \qquad l = 0, 1, 2, \dots$$

understood as the algebraic equations in $\mathcal{J}$.

D_a are tangent to $\mathcal{E}$ and hence restricts to $\mathcal{E}$. So do the differentials d_{h} and d_{v}. $D_a|_{\mathcal{E}}$ determine a dim-n involutive distribution on $\mathcal{E}$ – Cartan distribution.

PDEs beyond jet-bundles

Definition: *[Vinogradov] PDE is a manifold $\mathcal{E}$ equipped with an involutive Cartan distribution $\mathcal{C} \subset T\mathcal{E}$.*
In plain terms: PDE is a stationary surface with an extra structure.

+ some regularity and general technical assumptions.

PDEs are isomorphic when the respective distributions are.

For $n = 0$ PDEs are just usual manifolds.

BV beyond jet-bundles (EOM level)

Nonlagrangian version of BV: forget about symplectic structure and keep d_{h}, BRST differential s, and ghost degree.

Barnich, M.G., Semikhatov, Tipunin 2004, *Lyakhovich, Sharapov, 2004...*

Gauge PDE: BV-BRST extension of the notion of PDE. Examples were in the literature (in the context of topological models or higher spin theories) the general concept appeared only in *Barnich, M.G. 2010* under the name of "parent formalism"

Idea: reformulate BV as an AKSZ sigma model. In the case of PDE the minimal equivalent formulation of this type has the equation manifold as a target space.

More refined and geometric definition of gauge PDE was in *M.G., Kotov, 2019*.

Q-manifolds

Def. Q-manifold (M, Q) is a $\mathbb{Z}$-graded supermanifold M equipped with the odd nilpotent vector field of degree 1, i.e.

$$Q^2 = 0\,, \qquad |Q| = 1\,, \quad \mathrm{gh}(Q) = 1$$

Example: Odd tangent bundle: $(T[1]X, d_X)$. If θ^a are coordinates on the fibres of $T[1]M$ in the basis $\frac{\partial}{\partial x^a}$:

$$d_X := \theta^a \frac{\partial}{\partial x^a}$$

Example: CE complex $(\mathfrak{g}[1], Q)$. If $\mathfrak{g}$ is a Lie algebra then $\mathfrak{g}[1]$ is equipped with Q structure. If c^α are (ghosts) i.e. coordinates on $\mathfrak{g}[1]$ in the basis e_α then

$$Q = \frac{1}{2} c^\alpha c^\beta U^\gamma_{\alpha\beta} \frac{\partial}{\partial c^\gamma}\,, \qquad [e_\alpha, e_\beta] = U^\gamma_{\alpha\beta} e_\gamma$$

Example: $(V[1](M), Q)$ where $V(M)$ Lie algebroid. Indeed generic Q of degree 1 locally reads as:

$$Q = c^\alpha R_\alpha - \frac{1}{2} c^\alpha c^\beta U^\gamma_{\alpha\beta}(z) \frac{\partial}{\partial c^\gamma}$$

R_α gives anchor, $U^\gamma_{\alpha\beta}$ bracket, $Q^2 = 0$ encodes compatibility.

Gauge PDE in $n = 0$ (trivial Cartan distribution) is a Q-manifold $(\mathcal{E}, Q)$ that is equivalent to a nonnegatively graded one.

If only ghost degree $0, 1$ variables are present then it is just a Lie algebroid.

Important feature: although this is an intrinsic definition ($\mathcal{E}$ is not embedded into some "jet space") there are infinitely many Q-manifolds representing the same gauge PDE.

Equivalence of Q-manifolds:

Idea: restrict to local analysis and suppose that (M, Q_M) can be represented as a product Q-manifold:

$$M = N \times T[1]V\,, \qquad Q_M = Q_N + d_{T[1]V} \quad V - \text{graded space}$$

then (M, Q_M) and (N, Q_N) are equivalent. Q-manifold of the form $(T[1]V, d_{T[1]V})$ is caled contractible. In coordinates:

$$Q_M = Q_N + v^\alpha \frac{\partial}{\partial w^\alpha}\,, \qquad Q_N = q^i(\phi)\frac{\partial}{\partial \phi^i}\,.$$

Often one can find a "minimal" equivalent Q-manifold (directly related to minimal models of L_∞ algebras).

In the context of gauge theories: w^α, v^α – are known as "generalized auxiliary fields" *Henneaux, 1990* (in the Lagrangian case).

Maps of Q-manifolds:

$$\phi : (M_1, Q_1) \to (M_2, Q_2), \quad \phi^* \circ Q_2 = Q_1 \circ \phi^*$$

Def. *[Kotov, Strobl]* Localy trivial bundle $\pi : E \to M$ of Q-manifolds is called Q-bundle if π is a Q-map. Section $\sigma : M \to E$ is called Q-section if it's a Q-map.

In general, $\pi : E \to M$ is not a locally trivial Q-budle.
Indeed, although locally $E \cong M \times F$ (product of manifolds) in general $Q \neq Q_F + Q_M$.

Notion of equivalence extends to Q-bundles.

PDE as a Q-bundle

Consider PDE $(E_X,\mathcal{C})$, E_X is a bundle $\pi_X : E_X \to X$ over space-time X, $\mathcal{C} \subset TE_X$ is a Cartan distribution generated by D_a, where x^a are local coordinates on X. π_X projects D_a to $\frac{\partial}{\partial x^a}$, i.e. $d\pi_X(D_a) = \frac{\partial}{\partial x^a}$.
Horizontal differential forms can be seen as functions on E_X extended to a bundle over $T[1]X$. Horizontal differential:

$$d_{\mathsf{h}} = \theta^a D_a \qquad (\theta^a \equiv dx^a)$$

defines a Q-structure on a Q-bundle $\pi : (E_{T[1]X}, d_{\mathsf{h}}) \to (T[1]X, d_X)$, where $d_X = \theta^a \frac{\partial}{\partial x^a}$

This Q-bundle $\pi : (E_{T[1]X}, d_{\mathsf{h}}) \to (T[1]X, d_X))$ encodes all the information about the starting point PDE $(E_X, \mathcal{C})$.

For instance, solutions are Q sections. If ψ^A are local coordinates on the fibres the section is parametrized by $\psi^A(x) = \sigma^*(\psi^A)$ Q-map condition $d_X \circ \sigma^* = \sigma^* \circ d_{\mathsf{h}}$ gives the usual coordinate form of the solution condition:

$$\frac{\partial}{\partial x^a}\psi^A(x) = \Gamma_a^A(\psi(x), x)\,, \qquad d_{\mathsf{h}} = \theta^a D_a = \theta^a\left(\frac{\partial}{\partial x^a} + \Gamma_a^A(\psi, x)\frac{\partial}{\partial \psi^A}\right)$$

also known as "unfolded" representation *M.Vasiliev*. In particular, fields of the unfolded form are coordinates on the equation manifold (stationary surface).

Note that Q-bundles originating from PDEs are quite special: $\mathbb{Z}$-grading (ghost degree) originates from just the space-time form degree (the only nonzero degree coordinates are θ^a).

Gauge PDEs

In terms of Q-bundles PDEs can be defined as Q-bundles over $T[1]X$ with horizontal $\mathbb{Z}$-grading. The extension to the case of gauge systems is surprisingly straitforward: just forget about horizontality

Def. Gauge pre-PDE is a Q-bundle $(E_{T[1]X}, Q)$ over $(T[1]X, \mathsf{d}_X)$

Equivalence of Q-manifolds exends to Q-bundles over $T[1]X$, giving the notion of equivalent reduction and equivalence of gauge pre-PDEs. Notion of gauge pre-PDE is too wide:

gauge PDE: equivalent to nonnegatively graded, realizable in term of a jet-bundle in a regular way.

Equations of motion and gauge symmetries

Solutions: $\sigma : T[1]X \to E_{T[1]X}$ is a solution if

$$d_X \circ \sigma^* = \sigma^* \circ Q$$

Gauge transformations:

$$\delta\sigma^* = d_X \circ \epsilon_\sigma^* + \epsilon_\sigma^* \circ Q,$$

Gauge parameter: $\epsilon_\sigma^* : \mathcal{C}^\infty(E_{T[1]X}) \to \mathcal{C}^\infty(T[1]X)$,

$$\mathrm{gh}(\epsilon_\sigma^*) = -1, \quad \epsilon_\sigma^*(fg) = \epsilon_\sigma^*(f)\sigma^*(g) \pm \sigma^*(f)\epsilon_\sigma^*(g)$$

Gauge for gauge symmetries . . .

BV formulation (EOM level) as a gauge PDE

Fields Ψ^A (include genuine fields ϕ^i, ghosts c^α, antighosts π_μ, antifields $\mathcal{P}_a$, ...). Jet-bundle with coordinates $\Psi^A_{b_1\ldots}, x^a, \theta^a$
Horizontal differential: $d_{\mathsf{h}} = \theta^a D_a$
BV-BRST differential s:

$$\mathrm{gh}(s) = 1, \quad s^2 = 0, \quad [d_{\mathsf{h}}, s] = 0$$

BV jet-bundle as a Q-bundle over $T[1]X$ with $Q = d_{\mathsf{h}} + s$.

Formalism encodes BV as a particular case and hence all reasonable gauge theories. Justifies definition. Can be regarded as a BV beyond jet-bundles (at the level of equations of motion)

Example: Maxwell equation as a gauge PDE

Trivial bundle $T[1]X \times M$, Fiber coordinates:

$$C, \quad \mathrm{gh}(C) = 1\,, \qquad F_{a|b}, \quad F_{a|b_1 b_2}, \quad \ldots \quad F_{a|b_1 \ldots b_l} \quad \ldots \qquad \mathrm{gh}(F_{\ldots}) = 0$$

$F_{a|b_1 \ldots b_l}$ – irreducible tensors, symmetric in second group and traceless. Q-structure: *Stora, . . . , Brandt*

$$Qx^a = \theta^a, \quad Q\theta^a = 0, \quad QC = \frac{1}{2} F_{ab}\theta^a\theta^b, \quad QF_{a|b} = \theta^c F_{a|bc}, \quad \ldots$$

Equations of motion (promoting C, F to fields $\sigma^*(C) = A_a(x)\theta^A$, $\sigma^*(F_{\ldots}) = F_{\ldots}(x)$ *M.Vasiliev*

$$\partial_a A_b - \partial_b A_a = F_{a|b}\,, \qquad \partial_c F_{a|b} = F_{a|bc}\,, \quad \ldots$$

taking a trace of the 2nd gives $\eta^{bc}\partial_a F_{b|c} = 0$.

Reparametrization invariance and AKSZ sigma models

Suppose that $(E_{T[1]X}, Q)$ is a locally trivial Q-bundles. Restrict to local analysis. Then

$$(E_{T[1]X}, Q) = (T[1]X, d_X) \times (F, Q_F)$$

Gauge PDEs of this type are known as AKSZ sigma models.

In higher dimension: local triviality = reparmetrization invariance (in the context of BRST cohomology this was known as a posibility to eliminate d_{h} through change of variables, *Brandt, Dragon; Barnich, Brandt, Henneaux (1993))*)

In particular, any reparametrization-invariant gauge theory (e.g. gravity) can be locally represented as AKSZ sigma model *Barnich, M.G. 2010*

Example: zero-curvature equation

Take $E_{T[1]X} = (T[1]X, d_X) \times (\mathfrak{g}[1], Q)$, where $\mathfrak{g}$ is a Lie algbera and Q is a CE differential seen as a vector field. Let C^α denote coordinates on $\mathfrak{g}[1]$ then $QC^\alpha = -\frac{1}{2}U^\alpha_{\beta\gamma}C^\beta C^\gamma$. Denoting $\sigma^*(C^\alpha) = A^\alpha_a(x)\theta^a$ we get

$$d_X \circ \sigma^* = \sigma^* \circ Q \quad \Longrightarrow \quad dA + \frac{1}{2}[A, A] = 0$$

Gauge transformations:

$$\delta A = d\epsilon + [A, \epsilon]$$

Topological PDE (Nonlagrangian Chern-Simons). Finite-dim Q-bundle. Example known from *AKSZ*.

Lagrangian formalism beyond jet-bundles. Intrinsic action

Lagrangian induces presymplectic structure $\omega \in \bigwedge^{(n-1,2)}(\mathcal{E})$ on the equation manifold $\mathcal{E}$.

Kijowski, Tulczyjew 1979, Crnkovic, Witten, 1987, Hydon 2005, Khavkine 2012, Alkalaev M.G. 2013, Sharapov 2016

Indeed, given a Lagrangian $\mathcal{L} \in \bigwedge^{n,0}(J^\infty(\mathcal{F}))$ define $\widehat{\chi} \in \bigwedge^{n-1,1}(J^\infty(\mathcal{F}))$:

$$d_{\mathsf{v}}\mathcal{L} = d_{\mathsf{v}}\phi^i \frac{\delta^{EL}\mathcal{L}}{\delta\phi^i} - d_{\mathsf{h}}\widehat{\chi}$$

Define $\chi = \widehat{\chi}|_{\mathcal{E}}$ and $\omega = d_{\mathsf{v}}\chi$

$$d_{\mathsf{v}}\omega = d_{\mathsf{h}}\omega = 0$$

Generic ω on $\mathcal{E}$ satisfying the above is called a compatible presymplectic structure on $\mathcal{E}$.

More generally, suppose PDE $\mathcal{E}$ is eqipped with a compatible ω. It follows $\omega = d(\chi + l)$ for some $\chi \in \bigwedge^{n-1,1}(\mathcal{E}), l \in \bigwedge^{n,0}(\mathcal{E})$. These define a natural action functional on sections of $\mathcal{E}$ called intrinsic action: *MG, 2016*

$$S^c[\sigma] = \int_X \sigma^*(\chi + l)$$

What this has to do with the PDE in question?

S^c doesn't depend on fields in the kernel of ω. Assuming regularity take a symplectic quotient. The resulting Lagrangian system is weaker, $\mathcal{E} \subset \mathcal{E}^c$. For a class of systems containing YM, Gravity etc. there exists ω such that S^c is equivalent to the standard Lagrangian.

Counterexample: systems with degree zero differential consequences, e.g. massive spin-2 system. *M.G. Gritsaenko 2021*

Example: scalar field

Lagrangian:

$$L = \frac{1}{2}\eta^{ab}\phi_a\phi_b - V(\phi)$$

$\mathcal{E}$ is coordinatized by $x^a, \phi, \phi_a, \phi_{ab}, \ldots$ with $\phi_{abc\ldots}$ traceless.

$$d_{\mathsf{h}} x^a = dx^a\,, \quad d_{\mathsf{h}}\phi = dx^a\phi_a,\,, \quad d_{\mathsf{h}}\phi_a = dx^b(\phi_{ab} - \frac{1}{n}\eta_{ab}\frac{\partial V}{\partial \phi})\,, \quad \ldots$$

The presymplectic potential and 2-form:

$$\chi = (dx)^{n-1}_a \phi^a d_{\mathsf{v}}\phi\,, \quad \omega = (dx)^{n-1}_a d_{\mathsf{v}}\phi^a d_{\mathsf{v}}\phi$$

The Hamiltonian:

$$\mathcal{H} = (dx)^n(\phi_a\phi^a - L|_{\mathcal{E}}) = \frac{1}{2}\phi^a\phi_a + V(\phi)$$

The intrinsic Larangian: *Schwinger*

$$\mathcal{L}^c = (dx)^n\left(\phi^a(\partial_a\phi - \frac{1}{2}\phi_a) - V(\phi)\right)$$

Metric gravity

Einstein-Hilbert action

$$S = \int d^n x \sqrt{-g}(R - 2\Lambda)$$

Coordinates on the stationary surf.: x^μ, $g_{\mu\nu}$, $\Gamma^\lambda{}_{\mu\nu}$ + independent derivatives of $\Gamma^\lambda{}_{\mu\nu}$. Presymplectic potential (schematically):

$$\chi = \sqrt{-g}(\Gamma^{\rho\mu\nu} - \text{“traces”})d_{\mathsf{V}} g_{\mu\nu}(dx)^{n-1}_\rho\,.$$

The intrinsic action concides with the familiar Palatini action:

$$S^C[g,\Gamma]] = \int d^n x \sqrt{-g}(\partial_\rho g_{\mu\nu}(\Gamma^{\rho\mu\nu} - \frac{1}{2}g^{\rho\mu}\Gamma_\lambda{}^{\lambda\nu} - \frac{1}{2}g^{\rho\nu}\Gamma_\lambda{}^{\lambda\mu} + \frac{1}{2}g^{\mu\nu}\Gamma_\lambda{}^{\lambda\rho} - \\ - \frac{1}{2}g^{\mu\nu}\Gamma^{\rho\lambda}{}_\lambda) - \Gamma^{\rho\mu\nu}\Gamma_{\nu\mu\rho} + \Gamma^{\nu\mu}{}_\mu \Gamma^\lambda{}_{\lambda\nu} - 2\Lambda)\,.$$

M.G., Gritsaenko 2021

BV formalism beyond jet-bundles

Presymplectic structures on gauge PDEs

Def. Compatible presymplectic structure on gauge PDE $(E_{T[1]X}, Q)$ is a vertical 2-form ω on $E_{T[1]X}$ satisfying:

$$d\omega = 0\,, \qquad L_Q\omega = 0\,, \qquad \mathrm{gh}(\omega) = n-1$$

Here $n = \dim X$ and vertical forms are understood as equivalence classes

Defines "Hamiltonian" (or, better, covariant BRST charge) via

$$i_Q\omega = d\mathcal{H}\,, \qquad \mathrm{gh}(\mathcal{H}) = n$$

ω is directly related to the BV symplectic structure $\overset{n}{\omega}$ extended as $\omega = \overset{n}{\omega} + \overset{n-1}{\omega} + \ldots \overset{0}{\omega}$ to be a cocycle of $d_{\mathsf{h}} + s$, i.e. $L_{d_{\mathsf{h}}+s}\omega = 0$.

Intrinsic BV action

ω defines action functional on the space of sections of $(E_{T[1]X}, Q, \omega)$

$$S[\sigma] = \int_{T[1]X} (\sigma^*(\chi)(d_X) - \sigma^*(\mathcal{H}))$$

where χ is a presymplectic potential, i.e. $\omega = d\chi$. $\chi \to \chi + d\rho$ adds boundray term.

BV-like extension (just like in AKSZ). Supersection $\hat{\sigma}$:

$$S^{BV}[\hat{\sigma}] = \int_{T[1]X} (\hat{\sigma}^*(\chi)(d_X) - \hat{\sigma}^*(\mathcal{H}))$$

If e.g. $\text{gh}(C) = 1$ then $\sigma^*(C) = A_a(x)\theta^a$ while $\hat{\sigma}^*(C) = \overset{0}{C}{}^a + A_a\theta^a + \overset{2}{\xi}_{ab}\theta^a\theta^b + \ldots$, In coordinates:

$$S^{BV}[\psi] = \int d^n x d^n \theta (\chi_A(\psi(x,\theta)\theta^c \frac{\partial}{\partial x^c}\psi^A(x,\theta) - \mathcal{H}(\psi^A(x,\theta))$$

Interpretation? What this has to do with the gauge PDE in question? *Alkalaev, MG 2013, MG 2016, MG, Kotov, ...*

Idea: assume ω regular and take a symplectic quotient. Does not always work in a naive way in interesting cases.

Refined idea: locally, sections are fiber-valued functions, take:

$$Smaps(T[1]X, F) = Smaps(X, M)\,, \qquad M = Smaps(\mathbb{R}^n[1], F)$$

M is finite-dimensional provided F is. Natural lift of ω to M

$$\omega^M = \int d^n\theta\, \omega_{AB}(\psi(\theta))d\psi^A(\theta) \wedge d\psi^B(\theta)\,, \qquad \mathrm{gh}(\omega^M) = -1$$

Now assume that ω^M is regular and take a symplectic quotient. We have arrived at BV formulation! With BV symplectic structure $\omega^M(dx)^n$ and BV master action S^{BV}!

M.G. Kotov, 2020; Dneprov, M.G. to appear

Example: Maxwell

Recall: $E_{T[1]X} = T[1]X \times M$, Fiber coordinates:

$$C, \quad \mathrm{gh}(C) = 1\,, \quad F^{a|b}, \quad F^{a|b_1 b_2}, \quad \ldots \quad F^{a|b_1 \ldots b_l} \quad \ldots \quad \mathrm{gh}(F^{\cdots}) = 0$$

$$Qx^a = \theta^a, \quad Q\theta^a = 0, \quad QC = \frac{1}{2} F^{a|b}\theta_a\theta_b, \quad QF^{a|b} = \theta_c F^{a|bc}, \quad \ldots$$

indexes rised/lowered with Minkowski metric.

Presymplectic structure: *Alkalaev, M.G. 2013; A. Sharapov 2017*

$$\omega = (\theta)^{(n-2)}_{ab} dF^{a|b} dC\,,$$

indexes rised/lowered with Minkowski metric
Intrinsic action ($\sigma^*(C) = A_a(x)\theta^a, \sigma^*(F^{a|b}) = F^{a|b}(x)$):

$$S[\sigma] = \int d^n x (\partial_a A_b) F^{a|b} - \frac{1}{4}(F^{a|b})^2$$

Presymplectic structure on supermaps gives correct BV form!

$$\omega^M = d\overset{0}{C} \wedge \overset{2}{F}{}^{a|b}_{ab} + dA_a \wedge \overset{1}{F}{}^{a|b}_{b} + d\overset{0}{F}{}^{a|b} \wedge \overset{2}{C}_{ab}$$

Here:

$$\widehat{\sigma}^*(C) = \overset{0}{C}(x) + A_a(x)\theta^a + \frac{1}{2}\overset{2}{C}_{ab}(x)\theta^a\theta^b \ldots$$

$$\widehat{\sigma}^*(F_{a|b}) = \overset{0}{F}{}^{a|b}(x) + \overset{1}{F}{}^{a|b}_{c}(x)\theta^c + \frac{1}{2}\overset{2}{F}{}^{a|b}_{cd}(x)\theta^c\theta^d + \ldots$$

Formal path integral:

$$Z = \int_L exp(\frac{i}{\hbar}S_{BV})$$

where L comprise both usual gauge condition and a gauge condition for zero modes of ω^M. No need to take symplectic quotient explicitly! Analog of superfield formalism (known in AKSZ).

Example: Einstein gravity

Take $F = iso(1,3)[1]$ with coordinates e^a, ω^{ab} *Alkalaev, M.G. 2013*

$$\chi = \frac{1}{2}\epsilon_{abcd} d\omega^{ab} e^c e^d\,, \qquad \sigma = d\omega^{ab} de^c \epsilon_{abcd} e^d$$

"Hamiltonian"

$$\mathcal{H} = Q^A \chi_A = -\frac{1}{2}\omega^a_c \omega^{cb} \epsilon_{abcd} e^c e^d$$

Intrinsic action:

$$S^C = \int \chi_A \boldsymbol{d}\psi^A - \mathcal{H} = \int (\boldsymbol{d}\omega^{ab} + \omega^a{}_c \omega^{cb})\epsilon_{abcd} e^c e^d$$

Familiar Cartan-Weyl action for 4d GR. Generalization to $n > 2$ and $\Lambda \neq 0$ is straightforward. Reduces to usual AKSZ representation for 3d GR.

Just like in the case of scalar defines full BV formulation on the space of supermaps *MG, Kotov, 2020*

Conclusions

- Gauge PDEs – BV-BRST like extansions of standard PDEs. Rather flexible and invariant formalism (includes usual BV at EOM level as a particular case.

- Explicitly relates metric and frame-like formalism. Can be reagrded as a covariant hamiltonian formalism. In simple cases reproduces De Donder-Weyl formulations.

- Gauge PDEs as geometric objects. Well suited to work with diffeomorphims-invariant and topological models. Notion of equivalence.

- All the ingredients of the BV formulation are naturally encoded in the graded presympletic structure on the gaueg PDE.

- In the case of variational systems unifies Lagrangian and Hamiltonian BRST formalism, cf. BV/BFV approach of *Cattaneo et all.*

- Gives an invariant approach to study boundary values of gauge fields. In particular in the AdS/CFT correspondence context. *Bekaert, M.G. 2012*. In particular, Fefferman-Graham construction (and tractor calculus) can be seen as a certain gauge PDE. *M.G. 2012, M.G. Waldron 2011, Bekaert, M.G. Skvortsov 2017*

- Sucessful applications in constructing new models of HS theory, e.g. Type-B theory (AdS holographic dual to conformal spinor on the boundary) *M.G. Skvortsov 2018*

- Recent construction of Lagrangians for AdS_4 higher spin gravity in terms of presymplectic AKSZ. *Sharapov, Skvortsov 2020*

- Minimal presymplectic BV formulation of conformal gravity gives an alternative to *Kaku et all, 1977* frame like formulation. No artificial torsion-free constraint. *Dneprov, M.G., to appear*