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Exact 4pt amplitudes in various fishnet theories

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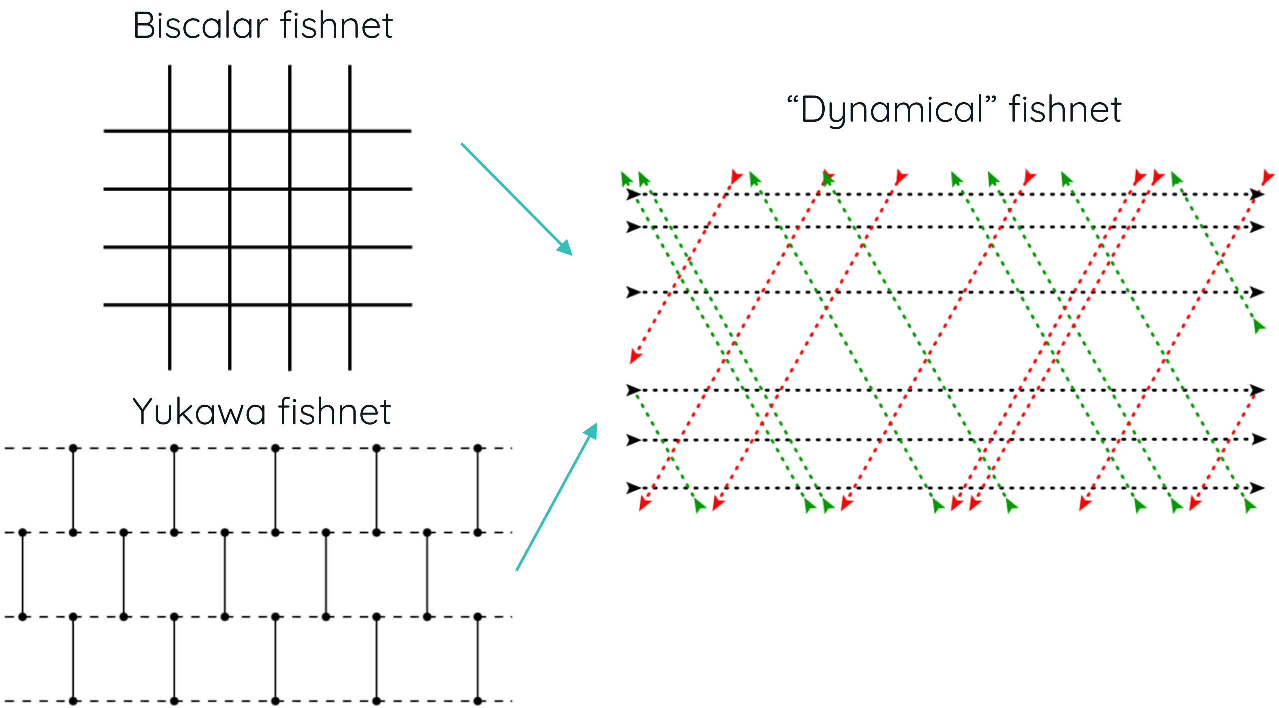
Introduction

- Fishnet graphs are conformally invariant Feynman integrals with regular structure = integrable lattice model
- There are many fishnet theories:
  - Scalar fishnets → biscalar theory
  - Dynamical fishnets → gamma-deformed SYM, chiral-0 CFT

Zamolodchikov'80

Kazakov, Korchemsky, Gudrogan, Gromov, Preti, Caetano et al

Examples of fishnet graphs



Introduction

- Fishnet graphs are conformally invariant Feynman integrals with regular structure = integrable lattice model

Zamolodchikov'80

- Similar graphs appear different SYM(like) theories
- There are many fishnet theories:
  - Scalar fishnets → biscalar theory
  - Dynamical fishnets → beta/gamma-deformed SYM

Kazakov, Korchemsky, Gudrogan, Gromov, Preti, Caetano et al

Gamma-deformed Lagrangian

Kazakov, Gurdogan

Gamma-deformed SYM:

$$\mathcal{L} = N_c \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}D^{\mu}\phi_i^{\dagger}D_{\mu}\phi^i + i\bar{\psi}_{\dot{\alpha}\ A}D^{\dot{\alpha}\alpha}\psi_{\alpha}^{A}\right] + \mathcal{L}_{\mathrm{int}}$$

$$\mathcal{L}_{\mathrm{int}} = N_c g\ \mathrm{Tr}[\frac{g}{4}\{\phi_i^{\dagger},\phi^i\}\{\phi_j^{\dagger},\phi^j\} - g\, e^{-i\epsilon^{ijk}\gamma_k}\phi_i^{\dagger}\phi_j^{\dagger}\phi^i\phi^j$$
$$- e^{-\frac{i}{2}\gamma_j^-}\ \bar{\psi}_j\phi^j\bar{\psi}_4 + e^{+\frac{i}{2}\gamma_j^-}\ \bar{\psi}_4\phi^j\bar{\psi}_j + i\epsilon_{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\psi^k\phi^i\psi^j$$
$$- e^{+\frac{i}{2}\gamma_j^-}\ \psi_4\phi_j^{\dagger}\psi_j + e^{-\frac{i}{2}\gamma_j^-}\ \psi_j\phi_j^{\dagger}\psi_4 + i\epsilon^{ijk}e^{\frac{i}{2}\epsilon_{jkm}\gamma_m^+}\bar{\psi}_k\phi_i^{\dagger}\bar{\psi}_j]$$

Why? $\mathrm{Tr}(\Phi_1[\Phi_2,\Phi_3]) \to \mathrm{Tr}(\Phi_1\Phi_2\Phi_3 e^{i\pi\beta} - \Phi_1\Phi_3\Phi_2 e^{-i\pi\beta})$

DS-limit: strong twist + weak coupling

General Lagrangian

Kazakov, Gurdogan

The Lagrangian of chiral CFT in DS-limit:

$$\mathcal{L}_{\phi\psi} = N_c \mathrm{Tr}\left(-\frac{1}{2}\partial^\mu \phi_j^\dagger \partial_\mu \phi^j + i\bar{\psi}_j^{\dot{\alpha}} (\tilde{\sigma}^\mu)_{\dot{\alpha}}^{\alpha} \partial_\mu \psi_\alpha^j\right) + \mathcal{L}_{\mathrm{int}}$$

Certain Yukawa and 4-scalar interactions survive only

$$\mathcal{L}_{\mathrm{int}} = N_c \,\mathrm{Tr}\Big[\xi_1^2\, \phi_2^\dagger \phi_3^\dagger \phi^2 \phi^3 + \xi_2^2\, \phi_3^\dagger \phi_1^\dagger \phi^3 \phi^1 + \xi_3^2\, \phi_1^\dagger \phi_2^\dagger \phi^1 \phi^2 + i\sqrt{\xi_2\xi_3}(\psi^3\phi^1\psi^2 + \bar{\psi}_3 \phi_1^\dagger \bar{\psi}_2) + i\sqrt{\xi_1\xi_3}(\psi^1\phi^2\psi^3 + \bar{\psi}_1\phi_2^\dagger\bar{\psi}_3) + i\sqrt{\xi_1\xi_2}(\psi^2\phi^3\psi^1 + \bar{\psi}_2\phi_3^\dagger\bar{\psi}_1)\Big].$$

Special cases:

$\xi_1 = \xi_2 = \xi_3$ - beta deformed SYM (SUSY!)

$\xi_1 = \xi_2 = 0, \xi_3 \equiv \xi \neq 0$ - biscalar fishnet theory

Biscalar CFT at any dimension

Gromov, Kazakov
Bork, I.R et al

Lagrangian of biscalar CFT:

$$\mathcal{L}_{main} = N_c \, \mathrm{tr} \left(\phi_1^* \partial^2 \phi_1 + \phi_2^* \partial^2 \phi_2 + (4\pi)^2 \xi^2 \, \phi_1^* \phi_2^* \phi_1 \phi_2 \right) ,$$

Lagrangian of biscalar CFT at arbitrary dimension: $\omega \in (0, D/2)$

$$\mathcal{L}_{main} = N_c \, \mathrm{Tr} \left(\varphi_1^* (\partial^2)^\omega \varphi_1 + \phi_2^* (\partial^2)^{D/2-\omega} \phi_2 + (4\pi)^{D/2} \xi^2 \, \varphi_1^* \phi_2^* \phi_1 \varphi_2 \right) ,$$

Lagrangians are not complete at the quantum level so they should be supplemented by a counter-terms:

$$\mathcal{L}_{dt} \sim \sum_{j=1} \left(\alpha_j(\xi) \mathrm{Tr}[\mathcal{O}_j] \mathrm{Tr}[\tilde{\mathcal{O}}_j] \right)$$

Fokken'15

Non-trivial beta functions → the conformal symmetry is broken

But fishnet theories are integrable and conformal at the fixed point!

$$\alpha_\pm^2 = \pm \frac{i\xi^2}{2} - \frac{\xi^4}{2} \mp \frac{3i\xi^6}{4} + \xi^8 \pm \frac{65i\xi^{10}}{48} - \frac{19\xi^{12}}{10} + O(\xi^{14}) \, .$$

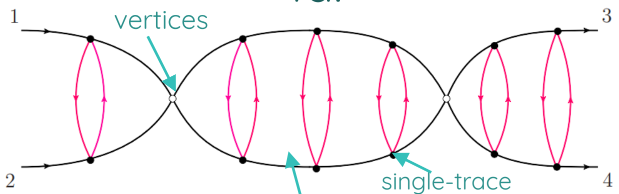
Fishnet CFT diagrams

$$\mathcal{A}_{\phi_1\phi_1\phi_1^\dagger\phi_1^\dagger} \sim \mathrm{Tr}(t^{a_1}t^{a_2})\mathrm{Tr}(t^{a_3}t^{a_4})A(z,\xi^2)$$

colour-ordered

Olivucci, Kazakov
Korchemsky
Bork, I.R et al

4d:

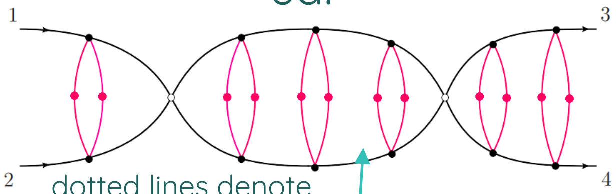


double-trace vertices

single-trace vertices

$\omega = 1$

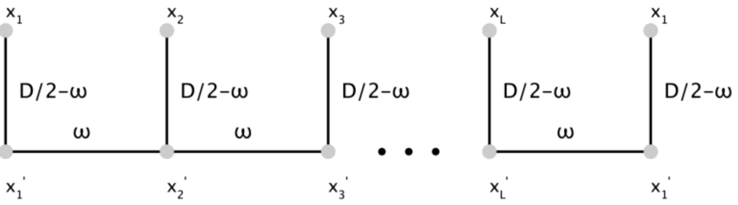
6d:



dotted lines denote doubled propagators

$\omega = 1$

Graph-building operator:



$$\Phi^{(l)}(x_i) = \mathcal{H}_L \Phi^{(l-2)}(x_i)$$

$$\mathcal{G}(u,v) = \sum_J C_{\Delta,J}\, g_{\Delta,J}(u,v)$$

J – spins, Δ – scaling dimension

4pt exact correlation functions and amplitudes

Korchemsky

Amplitude after LSZ reduction:

$$A(z,\xi) = \int_C \frac{dJ}{2i\sin(\pi J)} \int_{-\infty}^{+\infty} d\nu \frac{\mu(\nu,J)}{h(\nu,J)-\xi^4}\Omega_{\nu,J}(z) \qquad z = 1-2u/s$$

Kinematical part:

$$\Omega_{\nu,J}(z) = \frac{2^J}{\pi^2}\sinh^2(\pi\nu + i\pi J/2)\sum_{k=0}^{J}\frac{P_k(z)P_{J-k}(z)}{(J/2-k)^2+\nu^2}$$

Norm

$$\mu(\nu,J) = \frac{\nu^2(4\nu^2+(J+1)^2)(J+1)}{2^{J+4}\pi^7}\,.$$

Eigenvalue of the graph-building operator:

$$h(\nu,J)-\xi^4 = \left(\nu^2+\frac{J^2}{4}\right)\left(\nu^2+\frac{(J+2)^2}{4}\right)-\xi^4$$

Solutions:

$$\nu_i = \pm\frac{1}{2}\sqrt{\pm 2\left(\sqrt{J^2+2J+4\xi^4+1}+1\right)-J^2-2J}$$

$$J_i = -1 \pm \sqrt{\pm 4\sqrt{\xi^4-\nu^2}-4\nu^2+1}$$

Diagrams vs the exact amplitude

Korchemsky, I.R. and Bork

Single-trace contribution from Feynman diagrams:

$$A_+ \sim \xi^4(-H_0 - 1) + \xi^8(-H_{1,0,0} - 3\zeta_2 H_{-1} - 4\zeta_3 + 3) + \ldots$$

$$\delta = u/s$$

It is possible to reconstruct amplitude taking residues by scaling dimension and J, expanding it by **z=-1** and bootstraping

$$A_+^{(1)} = \left(-1 + \delta - \frac{\delta^2}{2} + \frac{\delta^3}{3} - \frac{\delta^4}{4} + \mathrm{O}(\delta^5)\right)\xi^4$$

$$A_+^{(3)} = \left(-3(\zeta(3) - 1) - \frac{\delta^2}{4} + \frac{\delta^3}{6} - \frac{11\delta^4}{96} + \mathrm{O}(\delta^5)\right)\xi^8$$

Constant contribution from Feynman diagrams:

$$A_- = 32i\pi^2\left[\xi^2 + \xi^6\left(\frac{3}{2} + \frac{\pi^2}{3}\right) + \xi^{10}\left(-\frac{49}{8} + \frac{\pi^2}{6} + \frac{2\pi^4}{45}\right) + O\left(\xi^{14}\right)\right]$$

from the exact amplitude:

$$A_- = 32i\pi^2\Bigg[\xi^2 + \left(\frac{3}{2} + \frac{\pi^2}{3}\right)\xi^6 + \left(-\frac{49}{8} + \frac{\pi^2}{6} + \frac{2\pi^4}{45}\right)\xi^{10} + \left(\frac{363}{16} - \frac{15\pi^2}{8} - \frac{\pi^4}{45} + \frac{\pi^6}{315}\right)\xi^{14} + O(\xi^{18})\Bigg].$$

Weak coupling in the Regge limit in 4d fishnets

Amplitude in Regge limit: Korchemsky'19

$z \to \infty$

$$A(z,\xi^2) = \int_{-\xi^2}^{\xi^2} d\nu \left[F(\nu, J_+)(z/2)^{J_+} - F(\nu, J_-)(z/2)^{J_-}\right] + (z \to -z)$$

Leading contribution from twist-2 operators: $J_2^{\pm} = -1 + \sqrt{1 - 4\nu^2 \pm 4\sqrt{\xi^4 - \nu^2}}\,,$

Result from the exact amplitude

$$A_{\mathrm{N^kLA}} = \frac{1}{\pi}\int_{-1}^{1} \frac{dx}{x^{k+1}} \sqrt{1-x^2}\, \mathrm{e}^{2Lx}\, a_{2k}(x, L)$$

$$a_0 = 1\,,$$

$$a_2 = 4x^2 - 2Lx + 1\,,$$

$$a_4 = 2L^2x^2 - 2L\left(2x^3 + x\right) + \frac{1}{3}\left(2x^2+1\right)\left(\pi^2x^2+3\right)$$

This result can be found from direct computations of Feynman diagrams

$$A_{LLO} \sim \frac{\log(z/2)^{2n+1}}{(2n+1)n!(n+1)!}$$

$$A_{\mathrm{NLA}} = \sum_{n\geq 0} L^{2n} \frac{(n-1)}{n!\,(n+1)!}\,,$$

$$A_{\mathrm{N^2LA}} = -\sum_{n\geq 0} L^{2n+1} \frac{(2n(n-1)(n+2) + \pi^2(n+1))}{(2n+1)n!\,(n+2)!}\,,$$

Agreement!

Strong coupling limit in biscalar theories

I.R., Bork et al

For the strong coupling limit it is useful to use the following amplitude:

$$A(z,1/\xi)=\frac{1}{2i}\int_C \frac{d(J/\xi)}{\sin(\pi J/\xi)}\int d\nu \frac{\mu(J/\xi,\nu)}{h(\nu,J/\xi)-1/\xi^4}\Omega_{\nu,\xi J}(z) \qquad 1/\xi\to\infty$$

$$\Omega_{\nu,J}(z)=\frac{2^J\sinh^2(\pi\nu+i\pi J/2)}{\pi^2(2\pi i)^2}\int_{[1,z]}\int_{[1,z]} dt_1 dt_2 \frac{(t_2^2-1)^J}{2^J(t_1-z)(t_2-z)^{J+1}}\Sigma(\mathcal{Z},\nu,J),$$

$$\Sigma(\mathcal{Z},\nu,J)=-\frac{i}{2\nu}\left(\Phi\left(\mathcal{Z},1,-\frac{J}{2}-i\nu\right)-\Phi\left(\mathcal{Z},1,-\frac{J}{2}+i\nu\right)+\right.$$
$$\left.+\mathcal{Z}^{1+J}\left(\Phi\left(\mathcal{Z},1,1+\frac{J}{2}+i\nu\right)-\Phi\left(\mathcal{Z},1,1+\frac{J}{2}-i\nu\right)\right)\right),$$

$$\mathcal{Z}\equiv\frac{(t_2^2-1)(t_2-z)}{(t_1^2-1)(t_1-z)},$$

$$\Phi(z,1,a)=\frac{1}{1-z}\frac{1}{a}+O(a^{-2})$$

$$\nu_1(J)=\frac{\sqrt{4-J^2}}{2\xi}-\frac{J}{2\sqrt{4-J^2}}+O(\xi),$$
$$\nu_2(J)=-\nu_1(J),$$
$$\nu_3(J)=\frac{i\sqrt{4+J^2}}{2\xi}+\frac{iJ}{2\sqrt{4+J^2}}+O(\xi),$$
$$\nu_4(J)=-\nu_3(J)$$

Strong coupling limit in 4d biscalar theory

I.R., Bork et al

In the high energy limit this integral can be evaluated by steepest descent method after residue by scaling dimension

$$\mathbf{L} \equiv \log\left(z + \sqrt{z^2-1}\right)$$

$z \to \infty$

$1/\xi \to \infty$

$$A(z,1/\xi) = \xi^{-1/2} \frac{4\pi\, \pi^{1/2}\, \mathbf{L} \exp\left(\frac{2}{\xi}\sqrt{\pi^2+\mathbf{L}^2}\right)}{i\sqrt{z^2-1}\,(\pi^2+\mathbf{L}^2)^{7/4} \sin\left(\frac{2\pi \mathbf{L}}{\xi\sqrt{\pi^2+\mathbf{L}^2}}\right)} + \dots$$

$$A(z,1/\xi) \sim \frac{z^{2/\xi-1}}{\log^{3/2}(z)} + \dots$$

$z \sim \pm 1$

$$A(z,1/\xi) = \frac{1}{\pi i} \int_C \frac{d(J/\xi)}{\sin(\pi J/\xi)} J\sqrt{4-J^2}(1+J/\xi) \exp\left(\frac{\pi\sqrt{4-J^2}}{\xi}\right)$$

$$A(1/\xi) = 8\frac{\exp(2\pi/\xi)}{2\pi^2 i}\sqrt{\xi} + \dots$$

$$A_{dt} \sim -64i\, \xi \exp(\pi/\xi)$$

4pt exact amplitudes in 6d fishnets

Korchemsky

6d amplitude:

$$A(z,\xi) = \int_C \frac{dJ}{2i\sin(\pi J)} \int_{-\infty}^{+\infty} d\nu \frac{\mu(\nu,J)}{h(\nu,J)-\xi^4}\Omega_{\nu,J}(z)$$

$$z = 1 - 2u/s$$

Kinematical part:

$$\Omega^{D=6}_{\nu,J}(z) = \frac{\partial}{\partial z}\Omega^{D=4}_{\nu,J}(z)$$

Eigenvalue of the graph-building operator:

$$h(\nu,J) - \xi^4 = \left(\nu^2 + \frac{J^2}{4}\right)\left(\nu^2 + \frac{(J+2)^2}{4}\right) - \xi^4$$

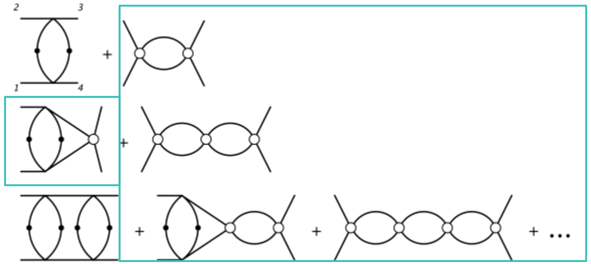
Norm:

$$\mu(\nu,J) = \frac{\nu^2(4\nu^2+(J+1)^2)(J+1)}{2^{J+4}\pi^7}\,.$$

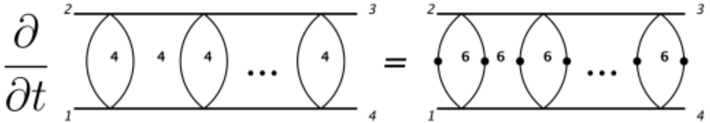
6d amplitude

I.R., Bork et al.

For 6d case perturbative expansion for single-trace contribution



From 4D integrals to 6D integrals

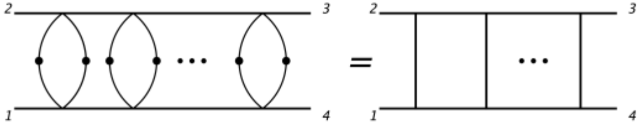


Easy to see it in alpha-representation

$$A_4^{D=6}(z,\xi) = \frac{\partial}{\partial z} A_4^{D=4}(z,\xi)$$

6D dotted bubble amplitude equals to 6D SYM planar boxes!

$$A_4^{\text{6d fishnet}} = 1/s\, A_4^{\text{6d scalar boxes}}$$



Non-renormalizable and non-conformal 6D (1,1) SYM contains CFT subsector?

Weak coupling in the Regge limit in 6d fishnets

I.R., Bork et al

LA-coefficients for 6d boxes and fishnets:

Coefficients from exact amplitude:

$$a_{(l)}^{LLA} = \frac{1}{l!(l+1)!},$$

$$a_{(l)}^{NLA} = \frac{2l(l-1)}{l!(l+1)!},$$

$$a_{(l)}^{NNLA} = \frac{2l(l-1)(l+2)+\pi^2(l+1)}{l!(l+2)!},$$

PT calculations of 6d boxes:

$$x\,\mathcal{B}^{(1)} = \underbrace{\frac{1}{2}L^2}_{LLA}+\dots$$

$$x\,\mathcal{B}^{(2)} = \underbrace{\frac{1}{12}L^4}_{LLA}+\underbrace{\frac{1}{3}L^3}_{NLA}+\underbrace{\frac{\pi^2}{3}L^2}_{NNLA}+\underbrace{\left(\frac{2\pi^2}{3}-2\zeta_3\right)L}_{NNNLA}+\dots$$

$$x\,\mathcal{B}^{(3)} = \underbrace{\frac{1}{144}L^6}_{LLA}+\underbrace{\left(\frac{1}{12}+\frac{\pi^2}{3}\right)L^5}_{NLA}+\underbrace{\left(\frac{1}{3}-\frac{\pi^2}{16}\right)L^4}_{NNLA}+\underbrace{\left(\frac{1}{3}+\frac{\pi^2}{2}-\frac{\zeta_3}{3}\right)L^3}_{NNNLA}+$$

Kazakov, Bork, Vlasenko'13

All are in perfect agreement with one, two and three loop computations!

Beta-deformed SYM

Kazakov, Gurdogan

The Lagrangian of chiral CFT in DS-limit:

$$\mathcal{L}_{\phi\psi} = N_c \mathrm{Tr}\left(-\frac{1}{2}\partial^\mu \phi_j^\dagger \partial_\mu \phi^j + i\bar{\psi}_j^{\dot{\alpha}} (\tilde{\sigma}^\mu)_{\dot{\alpha}}^{\alpha} \partial_\mu \psi_\alpha^j\right) + \mathcal{L}_{\text{int}}$$

Certain Yukawa and 4-scalar interactions survive only

$$\mathcal{L}_{\text{int}} = N_c \,\mathrm{Tr}\Big[\xi_1^2\, \phi_2^\dagger \phi_3^\dagger \phi^2 \phi^3 + \xi_2^2\, \phi_3^\dagger \phi_1^\dagger \phi^3 \phi^1 + \xi_3^2\, \phi_1^\dagger \phi_2^\dagger \phi^1 \phi^2 + i\sqrt{\xi_2 \xi_3}(\psi^3 \phi^1 \psi^2 + \bar{\psi}_3 \phi_1^\dagger \bar{\psi}_2)$$
$$+ i\sqrt{\xi_1 \xi_3}(\psi^1 \phi^2 \psi^3 + \bar{\psi}_1 \phi_2^\dagger \bar{\psi}_3) + i\sqrt{\xi_1 \xi_2}(\psi^2 \phi^3 \psi^1 + \bar{\psi}_2 \phi_3^\dagger \bar{\psi}_1)\Big].$$

Special cases:

$\xi_1 = \xi_2 = \xi_3$ — beta deformed SYM (SUSY!)

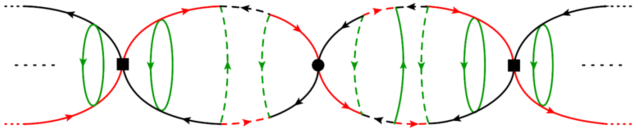
$$\mathcal{L}_{dt}^D/(4\pi)^{D/2} = \sum_i \sum_j \left(\alpha_{i,j}(\xi) \mathrm{tr}(O_{j,j+1}^{(i)}) \mathrm{tr}(\tilde{O}_{j,j+1}^{(i)})\right)$$

Amplitude in beta-deformed SYM

Amplitude:

$$A(z,\xi) = \int_C \frac{dJ}{2i\sin(\pi J)} \int_{-\infty}^{+\infty} d\nu \frac{\mu(\nu, J)}{h(\nu, J) - \xi^4} \Omega_{\nu,J}(z)$$

Kazakov, Olivucci, Preti



$$\mathcal{A}_{\phi_1\phi_2\phi_1^\dagger\phi_2^\dagger} \sim \mathrm{Tr}(t^{a_1}t^{a_2})\mathrm{Tr}(t^{a_3}t^{a_4})A(z,\xi^2)$$

Eigenvalues of graph-building operator

$$\left(\frac{J^2}{4}+\nu^2\right)\left(\frac{(J+2)^2}{4}+\nu^2\right) - \xi^4 + \\ + \frac{8(J(J+2)-4\nu^2)\xi^4}{16\nu^2 + 8\nu^2(2+J(J+2)+2\xi^2) + J(J+2)(J(J+2)-4\xi^2)} = 0$$

additional fermionic contribution

Solutions:

$$\nu_1 = -\frac{1}{2}i(J+2) + \frac{i\xi^2}{4}\left(1+\sqrt{\frac{16}{J+2} - \frac{16}{J+1} + 1}\right) + \ldots$$

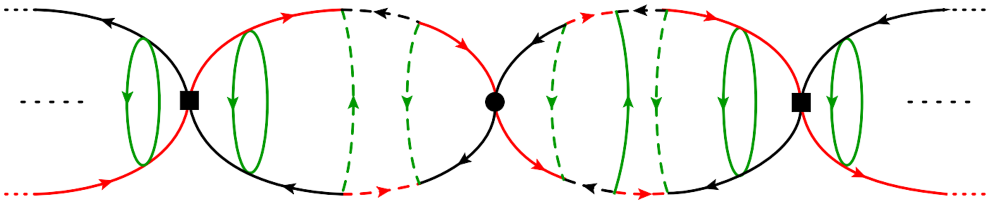
$$\nu_2 = -\frac{1}{2}i(J+2) + \frac{i\xi^2}{4}\left(1-\sqrt{\frac{16}{J+2} - \frac{16}{J+1} + 1}\right) + \ldots$$

$$\nu_3 = -\frac{iJ}{2} + \frac{i\xi^2}{4}\left(1-\sqrt{\frac{16}{J+1} - \frac{16}{J} + 1}\right) + \ldots$$

$$\nu_4 = -\frac{iJ}{2} + \frac{i\xi^2}{4}\left(1+\sqrt{\frac{16}{J+1} - \frac{16}{J} + 1}\right) + \ldots$$

Fishnet beta-deformed SYM CFT diagrams

Fishnet diagrams for beta-deformed:



Fixed point: $\alpha^2(\xi) = \sqrt{3}\xi^2 + O(\xi^4)$

One-loop amplitude from Feynman diagram:

$$A^{(1)}(z) \sim \xi^4 \left(\frac{1}{2}\log^2\left(\frac{1-z}{2}\right) - 2\log\left(\frac{z-1}{2}\right) + \frac{\pi^2}{2} + i\pi \right) + (z \to -z)$$

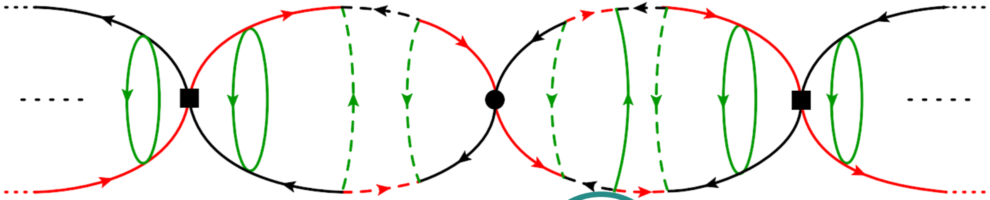
Result from the exact amplitude:

$$\mathrm{Re}[A(z \to -1)] = \xi^4 \left(\frac{\pi^2}{2} - \delta + \frac{3\delta^2}{8} + \mathrm{O}(\delta^3) \right)$$

Agreement!

Fishnet beta-deformed SYM CFT diagrams

Fishnet diagrams for beta-deformed:



Fixed point: $\alpha^2(\xi) = \sqrt{3}\xi^2 + O(\xi^4)$

Constant part of the exact amplitude:

$$A(\xi)\big|_{J=0} = i\pi\xi^2\left(3\xi^2 + \left(-\frac{58}{3} + \frac{\pi^2}{3}\right)\xi^4 + O\left(\xi^6\right)\right)$$

Agreement!

Strong coupling for beta-deformed SYM CFT

$z \to \infty$

$$A = \frac{-i\xi^{-\frac{3}{2}}}{\sqrt{z^2-1}} \frac{\pi^{3/2} e^{\frac{2\sqrt{L^2+\pi^2}}{\xi}} \csc\left(\frac{2\pi L}{\sqrt{L^2+\pi^2}\,\xi}\right)}{L\left(L^2+\pi^2\right)^{5/4}\left(6L^2+2\pi^2\right)} \Rightarrow A \sim \frac{z^{2/\xi-1}}{\log(z)^{11/2}}$$

$z \sim \pm 1$

$$A(z=\pm 1,\xi) = \frac{1}{i\pi}\int d(J/\xi) \frac{\left(J^2-4\right)^2 2^{\frac{J}{\xi}-2} e^{\frac{\pi\sqrt{4-J^2}}{\xi}} (\xi+J)\csc\left(\frac{\pi J}{\xi}\right)}{J\left(J^2+2\right)} \Rightarrow A \sim \frac{\sqrt{\xi}}{i\pi^2} e^{\frac{2\pi}{\xi}}$$

Constant part at strong coupling

$$A_{dt} \sim \frac{i\sqrt{3}\xi}{\pi} e^{\frac{2\pi}{\xi}}$$

6d CFT (?)

6D Lagrangian?

$$\mathcal{L}_{kin} = N_c \, \mathrm{Tr}\left[\phi_j^\dagger(\partial^2)^\omega\phi_j + \varphi_j^\dagger(\partial^2)^{\frac{D}{2}-\omega}\varphi_j + i\bar{\psi}_j\hat{\partial}^{\frac{D-\omega-1}{2}}\psi^j + i\bar{\chi}_j\hat{\partial}^{\frac{D-\omega+1}{2}}\chi^j\right]$$

$$\mathcal{L}_{int} = (2\pi)^{D/2-\omega} i\xi N_c \, \mathrm{Tr}\left[\psi^3\phi^1\chi^2 + \bar{\psi}_3\phi_1^\dagger\bar{\chi}_2 + \psi^3\varphi^1\psi^2 + \bar{\psi}_3\varphi_1^\dagger\bar{\psi}_2 + \text{permutations}\right] +$$

$$+ (2\pi)^{D/2}\xi^2 N_c \, \mathrm{Tr}\left[\phi_1^\dagger\varphi_2^\dagger\phi^1\varphi^2 + \phi_2^\dagger\varphi_3^\dagger\phi^2\varphi^3 + \phi_3^\dagger\varphi_1^\dagger\phi^3\varphi^1\right]$$

$$\Omega^{(6)}_{\nu,J}(z) = \partial_z\Omega^{(4)}_{\nu,J}(z)$$

Conclutions

- We calculated 4pt colour-ordered scattering amplitudes in weak and strong coupling in the different kinematical limits for biscalar fishnets in DS-limits for 6d and 4d (also to beta SYM)
- We checked it by direct PT-series calculations
- We found correspondence between series of 4pt Feynman diagrams in 4D/6D theories
- 6D generalization of beta SYM

Further development

Questions:

- Gravity dual theories and minimal volume calculations?
- Higher number of external fields?
- What is the parent theory for 6D theories?
- Can 6d fishnet “beta-deformed” SYM be derived from some 6d SYM?

Thanks for your attention!

Weak coupling limit of 4d amplitude

Expansion around z=-1:

I.R., Bork et al

$$\Omega_{\nu,J}(z=-1) = i2^J \frac{\sinh^2(\pi\nu + i\pi J/2)}{2\pi^2\,\nu}\left(\Psi\left(-1-\frac{J}{2}-i\nu\right)-\Psi\left(\frac{J}{2}-i\nu\right)-\Psi\left(-1-\frac{J}{2}+i\nu\right)+\Psi\left(\frac{J}{2}+i\nu\right)\right)$$

$$y = 2u/s$$

$$\Omega_{\nu,J}(-1+y) = \sum_{n=0} y^n\,\Omega^{(n)}_{\nu,J}(-1)$$

$$\mathrm{Re}A_4^{D=4}(-1+y,\xi) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} c^{(k,l)}\,(\xi^4)^{l+1}y^k,$$

$$c^{(k,l)} = \sum_{J=1}^{\infty}(-1)^J\sum_n \text{Rational function}^{(n)}(J)\times S_n(J)$$