«Two-loop contribution to the pure Yang—Mills effective action»

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Yang—Mills theory

Yang and Mills (1954)
Trautman (1979)
Babelon and Viallet (1981)

Let G be a compact semisimple Lie group, and $\mathfrak{g}$ is its Lie algebra.

Then, t_a denote the corresponding generators, which satisfy

$$\mathrm{tr}\left(t^a t^b\right) = -2\delta^{ab}, \quad [t^a, t^b] = f^{abc} t^c \,.$$

killing form

structure constant

$$f^{abc} f^{aef} = f^{abf} f^{aec} - f^{acf} f^{aeb} \quad \text{and} \quad f^{abc} f^{abe} = c_2 \delta^{ce}$$

Yang-Mills connection components: $B_\mu(x) = B_\mu^a(x) t^a \,.$

Components of the field strength: $F_{\mu\nu}(x) = F_{\mu\nu}^a(x) t^a, \; F_{\mu\nu}^a = \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + f^{abc} B_\mu^b B_\nu^c \,.$

Classical action:

$$S[A] = \frac{1}{4g^2} \int_{\mathbb{R}^4} d^4x \, F_{\mu\nu}^a(x) F_{\mu\nu}^a(x) = \frac{W_{-1}}{4g^2}$$

coupling constant

Additional definitions

Faddeev and Popov (1967)
Faddeev and Slavnov (1991)
Faddeev (2009)

Covariant derivative: $D^{ab}_{x^\mu} h^{bc}(x) = \partial_{x^\mu} h^{ac}(x) + f^{abd} B^d_\mu(x) h^{bc}(x)\,.$

Laplace operators

and the Green's functions:

$$M_0^{ab} = -\,D_\mu^{ae} D_\mu^{eb}, \quad M_{1\,\mu\nu}^{\;ab} = M_0^{ab}\delta_{\mu\nu} - 2f^{acb}F^c_{\mu\nu}$$

$$M_0^{ab} G_0^{bc}(x,y) = \delta^{ac}\delta(x-y), \quad M_{1\,\mu\nu}^{\;ab} G_{1\,\nu\rho}^{\;bc}(x,y) = \delta^{ac}\delta_{\mu\rho}\delta(x-y)$$

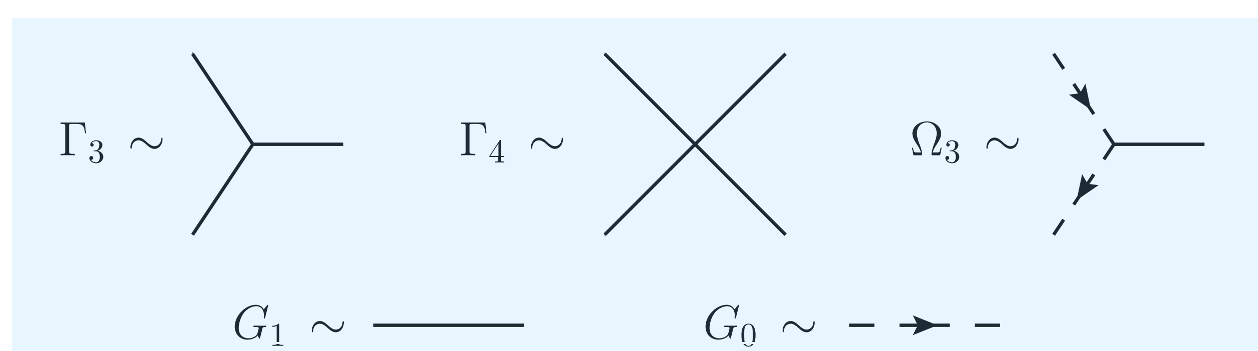
Vertices:

$$\Gamma_1 = -\int_{\mathbb{R}^4} d^4x\, \frac{\delta}{\delta J^a_\nu} D^{ab}_\mu F^b_{\mu\nu}, \qquad \Gamma_3 = \int_{\mathbb{R}^4} d^4x \left(D^{ae}_\mu \frac{\delta}{\delta J^e_\nu} \right) f^{abc} \frac{\delta}{\delta J^b_\mu} \frac{\delta}{\delta J^c_\nu},$$

$$\Gamma_4 = \frac{1}{4}\int_{\mathbb{R}^4} d^4x\, f^{abc} \frac{\delta}{\delta J^b_\mu} \frac{\delta}{\delta J^c_\nu} f^{aed} \frac{\delta}{\delta J^e_\mu} \frac{\delta}{\delta J^d_\nu}, \qquad \Omega_3 = \int_{\mathbb{R}^4} d^4x \left(D^{ab}_\mu \frac{\delta}{\delta b^b} \right) f^{aed} \frac{\delta}{\delta J^e_\mu} \frac{\delta}{\delta \bar{b}^d}\,.$$

ghost fields

Diagram tehcnique elements:



Background field method

DeWitt (1967)
Aref'eva, Slavnov, and Faddeev (1974)
't Hooft (1975)
Abbot (1982)
Jack and Osborn (1982)

The path integral formulation is written as

$$e^{-W[B]} = \int \mathscr{D}A\mathscr{D}c\mathscr{D}\bar{c}\, e^{-S[A]+S_{FP}[A,c,\bar{c}]},$$

in which the action can be presented after the shift $A_\mu = B_\mu + ga_\mu$ as

$$-S[B] - \frac{1}{g}\Gamma_1 - \frac{1}{2}\big(M_1 a, a\big) - \big(M_0 c, \bar{c}\big) + g\Gamma_3 + g\Omega_3 + g^2\Gamma_4\,.$$

Yang—Mills effective action

DeWitt (1967)
Aref'eva, Slavnov, and Faddeev (1974)
't Hooft (1975)
Abbot (1982)
Jack and Osborn (1982)

Auxiliary functional:

$$W[B] = \frac{1}{4g^2} W_{-1} + \left\{ \frac{1}{2} \ln\det\left(M_1/M_1|_{B=0}\right) - \ln\det\left(M_0/M_0|_{B=0}\right) \right\} + W_h[B],$$

where the multi-loop part is

$$W_h[B] = -\ln\left(\exp\left(-\Gamma_1/g - g\Gamma_3 - g^2\Gamma_4 + g\Omega_3 \right) Z[J,b,\bar{b}]\Big|_{J=b=\bar{b}=0} \right)\Bigg|_{only\ \mathrm{1PI}\ part}$$

and the generating functional is $Z[J,b,\bar{b}] = \exp(g_1 + g_0)$ with

$$g_1 = \frac{1}{2}\int_{\mathbb{R}^4} d^4x \int_{\mathbb{R}^4} d^4y\, J^a_\mu(x) G^{ab}_{1\mu\nu}(x,y) J^b_\nu(y) \ \text{and}\ g_0 = \int_{\mathbb{R}^4} d^4x \int_{\mathbb{R}^4} d^4y\, \bar{b}^a(x) G^{ab}_0(x,y) b^b(y)\,.$$

Effective action is equal to $W_{\text{eff}}[B] = W[B] - W[0]\,.$

Green's function expansion

Fock (1937)
DeWitt (1967)
Gilkey (1975)
Shore (1981)
Luscher (1982)

Let us consider a Laplace type operator $A^{ab}(x) = -IM_0^{cd}(x) - v^{ab}(x)$,

then we have the following decomposition

$$\left(A^{-1}\right)^{ab}(x,y) = R_0(x-y)a_0^{ab}(x,y) + R_1(x-y)a_1^{ab}(x,y) + R_2(x-y)a_2^{ab}(x,y) + \mathscr{P}\mathscr{S}^{ab}(x,y) + \ldots,$$

where

$$R_0(x) = \frac{1}{4\pi^2|x|^2},\ R_1(x) = -\frac{\ln(|x|^2\mu^2)}{16\pi^2},\ R_2(x) = \frac{|x|^2\left(\ln(|x|^2\mu^2)-1\right)}{64\pi^2},$$

and the coefficients a_n are from the following local heat kernel

$$\begin{cases} \left(\delta^{ac}\partial_\tau + A^{ac}(x)\right)K^{cb}(x,y;\tau) = 0; \\ K^{ab}(x,y;0) = \delta^{ab}\delta(x-y), \end{cases}$$

$$K^{ab}(x,y;\tau) = (4\pi\tau)^{-2}e^{-\frac{|x-y|^2}{4\tau}}\sum_{n=0}^{\infty}\tau^n a_n^{ab}(x,y),$$

where $\tau \to +0$.

The main result

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The two-loop divergent part in $W_h[B] - W_h[0]$ is equal to $\eta W_{-1} + o(g^2)$, where

$$\eta = g^2\left((3d-3)\mathrm{I}_1 + \frac{(3d-4)}{2}\mathrm{I}_2 + \frac{(d+2)}{2}\mathrm{I}_3 + \frac{(2d-5)}{2d}\mathrm{I}_4 + \frac{(8-d)}{2}\mathrm{I}_5 + \frac{(d+2)}{2}\mathrm{I}_6 + \frac{(3d-4)}{2}\mathrm{I}_7 - \mathrm{I}_8 + \frac{3}{2}\mathrm{I}_9 + \frac{5}{2}\mathrm{I}_{10}\right).$$

$$\mathrm{I}_1 = c_2^2 \int_{\mathrm{B}_{1/\mu}} d^d x \left(\partial_{x^\mu} R_0(x)\right) R_0(x) \partial_{x^\mu} \left(\frac{|x|^2}{12d} R_1(x) + \frac{(d-24)}{12d} R_2(x) + \frac{(24-d)}{2^9 3 d \pi^2}|x|^2\right)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_2 = c_2^2 \int_{\mathrm{B}_{1/\mu}} d^d x \left(\partial_{x^\mu} R_0(x)\right)\left(\partial_{x^\mu} R_0(x)\right) \left(\frac{|x|^2}{12d} R_1(x) + \frac{(d-24)}{12d} R_2(x) + \frac{(24-d)}{2^9 3 d \pi^2}|x|^2\right)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_3 = -\frac{2c_2^2}{d} \int_{\mathrm{B}_{1/\mu}} d^d x \left(\partial_{x^\mu} R_0(x)\right)\left(\partial_{x^\mu} R_1(x)\right) R_1(x)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_4 = -2c_2^2 \int_{\mathrm{B}_{1/\mu}} d^d x\, R_0(x) \left(\partial_{x^\mu} R_1(x)\right)\left(\partial_{x^\mu} R_1(x)\right)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_5 = -\frac{c_2^2}{2d} \int_{\mathrm{B}_{1/\mu}} d^d x\, R_1(x) x^\mu \partial_{x^\mu} R_0^2(x)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_6 = \frac{c_2^2}{2d} \int_{\mathrm{B}_{1/\mu}} d^d x\, R_0^2(x) x^\mu \partial_{x^\mu} R_1(x)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_7 = \frac{c_2^2}{8d} \int_{\mathrm{B}_{1/\mu}} d^d x\, |x|^2 R_0^3(x)\Bigg|_{\text{reg.}}$$

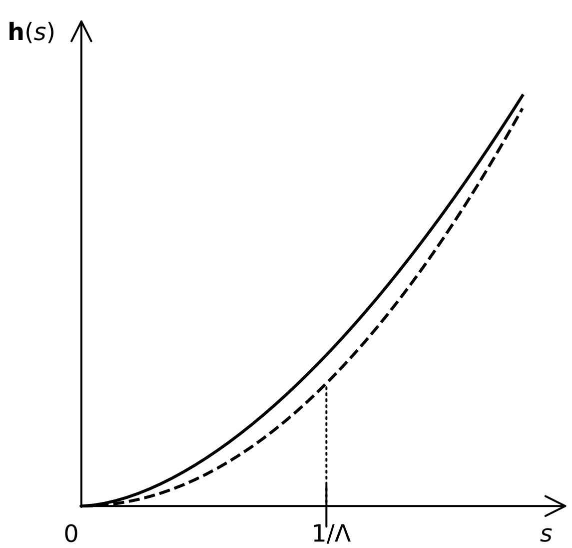
$$\mathrm{I}_8 = c_2^2 \int_{\mathrm{B}_{1/\mu}} d^d x \left(\partial_{x^\mu} R_0(x)\right) R_0(x) \partial_{x^\mu} \left(\frac{|x|^2}{12d} R_1(x) + \frac{1}{12} R_2(x) - \frac{|x|^2}{2^9 3 \pi^2}\right)\Bigg|_{\text{reg.}}$$

$$\mathrm{I}_9 = c_2^2 R_1^2(x)\Bigg|_{\text{reg.}}^{x=0}$$

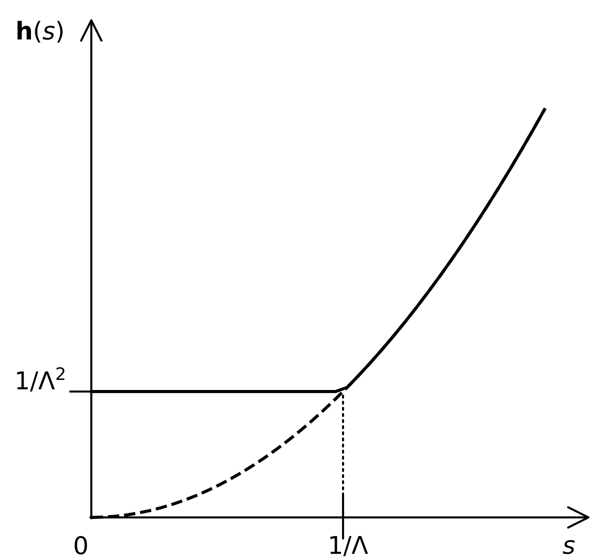
$$\mathrm{I}_{10} = c_2^2 R_0(x) R_2(x)\Bigg|_{\text{reg.}}^{x=0}$$

Examples

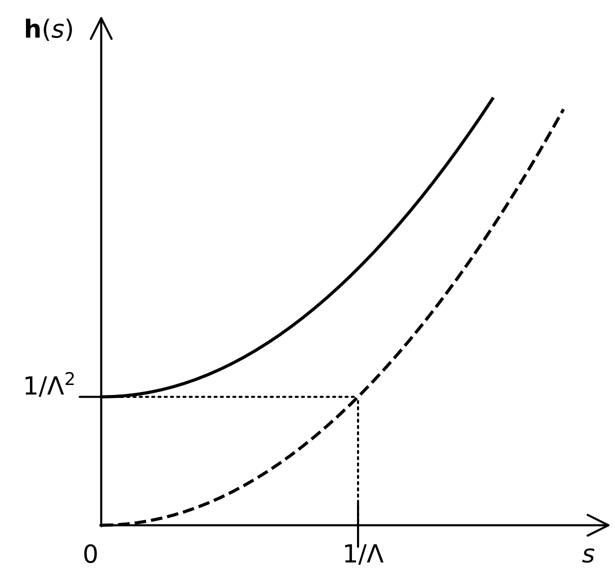
Integral	Dim. reg. $\frac{(4\pi)^4\mu^{2\varepsilon}}{c_2^2}\begin{pmatrix}\text{IR}\\ \text{part}\end{pmatrix}$	Cutoff-1 reg. $\frac{(4\pi)^4}{c_2^2}\begin{pmatrix}\text{IR}\\ \text{part}\end{pmatrix}$	Cutoff-2 reg. $\frac{(4\pi)^4}{c_2^2}\begin{pmatrix}\text{IR}\\ \text{part}\end{pmatrix}$	Cutoff-3 reg. $\frac{(4\pi)^4}{c_2^2}\begin{pmatrix}\text{IR}\\ \text{part}\end{pmatrix}$
I_1	$-1/\varepsilon^2 - 5/8\varepsilon$	$-L^2 - L/4$	$-L^2 + 5L/4$	$-L^2 - L/4$
I_2	$1/\varepsilon^2 + 5/8\varepsilon$	$L^2 + 5L/4$	$L^2 - 11L/36$	$L^2 + 5L(1/4 + \tilde{\alpha}/6)$
I_3	$-1/\varepsilon^2 - 1/4\varepsilon$	$-L^2$	$-L^2 + 3L/2$	$-L^2 + L/2$
I_4	$-2/\varepsilon$	$-4L$	$-4L$	$-4L$
I_5	$1/\varepsilon^2 + 1/4\varepsilon$	L^2	$L^2 - 3L/2$	L^2
I_6	$-1/4\varepsilon$	$-L/2$	$-L/2$	$-L/2$
I_7	$1/8\varepsilon$	$L/4$	$L/4$	$L/4$
I_8	$1/8\varepsilon$	$L/4$	$L/4$	$L/4$
I_9	$4/\varepsilon^2$	$4L^2$	$4L^2$	$4L^2$
I_{10}	0	$-2L$	$-2L$	$-2\tilde{\alpha}L$
η/g^2	$-2/\varepsilon$	$-9L/2$	$77L/18$	$L(2 - 5\tilde{\alpha}/3)$
c. t.	$-5/6\varepsilon$	not exist	$-25L/36$	0
Final	$-17/6\varepsilon$	—	$43L/12$	$L(2 - 5\tilde{\alpha}/3)$



Jack and Osborn (1981)
Bornsen and van de Ven (2003)



Ivanov and Kharuk (2020)



Shatashvili (1984)

Many thanks!