QFTh, High-Energy Physics, and Cosmology, JINR 07/18-21/22

Speed of Sound Anomaly in a Hadon-Quark Bubbly Matter (NSs?)

Work in progress!

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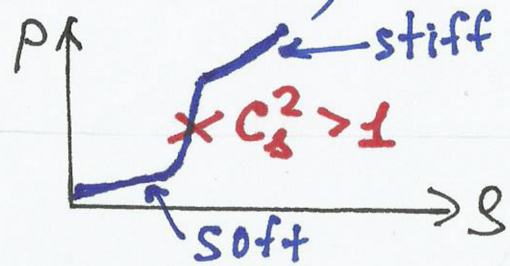
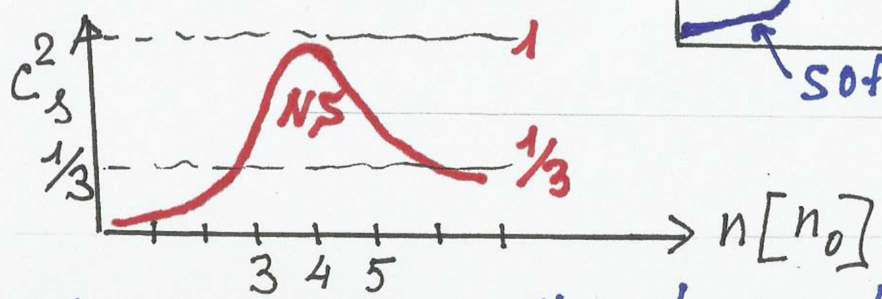
Inspiration/Motivation for this work:

(i) Discovery of $\sim 2 M_\odot$ neutron stars (NSs) with very stiff EoS (Equation of state)



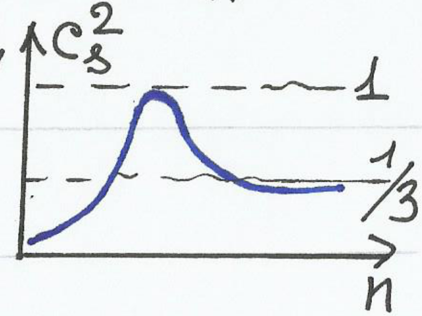
(ii) Breaking the sound barriere

$$c_s^2 = \frac{\partial P}{\partial \rho}$$



(iii) Invent a toy-model with such a puzzling behavior of c_s

- What does c_s^2 tell us? Matter part of the trace anomaly $\langle \Theta \rangle_{T,\mu} = \varepsilon - 3p \propto \left(\frac{1}{3} - c_s^2\right)$. $\left(\frac{\varepsilon - 3P}{T^4}\right)$ is a measure of interaction. Prior to $\simeq 2M_\odot$ discovery $c_s^2 = 1/3$ was conjectured to be a theoretical upper limit like $\eta/s = 1/4\pi$ (KSS). "Bump" implies extra degrees of freedom (quarks, hyperons).



- Multi-messenger signatures for quark matter in NSs. Quark core covered by normal nuclear matter. The EoS of NSs is currently higly uncertain, hundreds of models.
- What about quark matter droplets, lumps inside nuclear matter of NS?

- Quark matter (<u>QM</u>) <u>droplets</u>, seeds, lumps, pasta, embedded in hadronic (neutron) matter (<u>HM</u>). Cavitation, nucleation, spinodal decomposition. A lot of authors contributed:

  Glendenning, Pethick, Mishustin, Lugones, Kapusta, Shuryak, Blaschke, Maslov, Voskresensky, ...

  (→ picture slide)

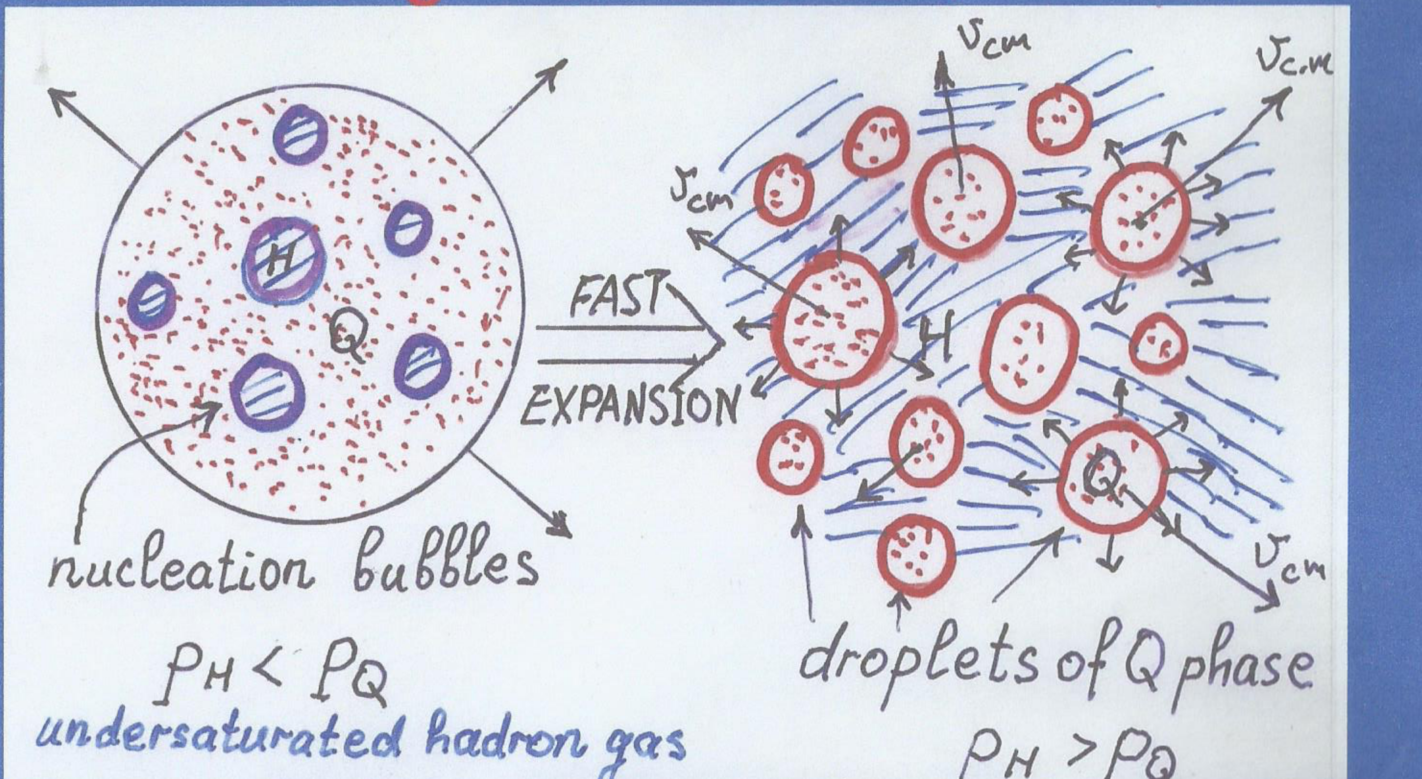
- How to describe the QM bubble surrounded by HM?

Lord Rayleigh gave the answer 105 years ago.
Rayleigh equation – cavitation damage of ship propellers $R\ddot{R} + \frac{3}{2}\dot{R}^2 = 0$, $\dot{R}(t) \sim (t_c - t)^{-3/5}$

collapse

(Somoluminescenting bubbles – visible to naked eye
FIAN, Kharzeev & Shuryak flash of dileptons)

Rapid expansion should lead to dynamical fragmentation of QGP



In the course of fast expansion the system enters spinodal instability when Q phase becomes unstable and splits into QGP droplets/hadron resonances

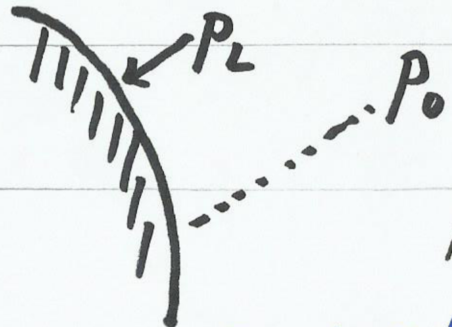
Csernai&Mishustin 1995, Mishustin 1999, Rafelski et al. 2000, Koch&Randrup, 2003

Extreme possibility - direct transition from quarks to hadrons without mixed phase

- $R\ddot{R} + \frac{3}{2}\dot{R}^2 = 0$ oversimplified, only inertia forces

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(P_L - P_0 - \underline{P(t)}\right)$$ Rayleigh-Plesset

$P(t) = p_m \sin\omega t$ acoustic wave



$$p_L = p_i - \frac{2\sigma}{R} - 4\eta\frac{\dot{R}}{R}$$

p_i – inside bubble, σ – surface tension

η – shear viscosity (water for Raileigh, NS matter for us)

(Almost) finally

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(p_i - p_0 - \frac{2\sigma}{R} - 4\eta\frac{\dot{R}}{R} - p_m \sin\omega t - \sigma_e \vec{B}^2 R\dot{R}\right)$$

σ_e conductivity

BK, ML PRD(21)

How to apply to NS and get c_s wiggly?

EoS of NS is needed!

- The EoS of NS matter is highly uncertain despite a great number of efforts/papers.
  Intermediate-density range interpolation !?
- A possible breakthrough

E. Annala, T. Gorda, A. Kurkela, J. Nättilä, and A. Vuorinen
Nature Physics 16 (2020) 907; arXiv 1903.09121

Ensemble of all viable EoS-s analyzed

Piece-wise polytropic form $\gamma \equiv d(\ln p)/d(\ln \varepsilon)$

Conclusion

$\gamma > 2$ HM

$\gamma < 1.75$ QM

piecewise polytropic interpolation $p_i(n) = k_i n^{\gamma_i}$

- Approximations: 1) polytrope with $\gamma < 1.75$

2) linear approximation $P_i = P_0\left(\frac{R}{R_0}\right)^{-3\gamma} \simeq P_0\left(1 - 3\gamma\frac{x}{R_0}\right)$

$x = R - R_0$

3) $\sigma \sim (1-10)\,MeV/fm^2$ and $\vec{B}^2$ dropped overcomed by η

- Equation in terms of $z = r - r_0$

$$\ddot{z} + \frac{4\eta}{\pi\rho R_0}\dot{z} + \omega_0^2 z = -b\,p_m \sin\omega t$$

$$\omega_0^2 = \frac{3\gamma P_0}{\rho R_0^2}, \quad b = \frac{4\pi R_0}{\rho}, \quad g \equiv \frac{4\eta}{\pi\rho R_0}$$

trivial eq-n $\quad y'' + ay' + by = c\sin\omega x$

Speed of sound and compressibility

$$\left.\begin{aligned} c_s^2 &= \frac{\partial p}{\partial \rho} \\ \varkappa &= -\frac{1}{V}\frac{\partial V}{\partial p} \end{aligned}\right\} \quad \varkappa = -\frac{\rho}{M}\frac{\partial (M/\rho)}{\partial p} = -\rho\,\frac{\partial(1/\rho)}{\partial \rho}\cdot\frac{\partial \rho}{\partial p} = \frac{1}{\rho}\,\frac{1}{c_s^2} \Rightarrow c_s = \frac{1}{\sqrt{\varkappa\rho}}$$

Single bubble compressibility

$$\varkappa = \frac{1}{\gamma P_0}\,\frac{1}{(1-\Omega^2)+i\delta\Omega}, \quad \Omega \equiv \frac{\omega}{\omega_0}.$$

$$\delta = g/\omega_0 = \frac{4\eta}{\pi\rho R_0\omega_0}$$ damping decrement

"Complex" speed of sound $k = k_1 + ik_2 = \frac{\omega}{c} - i\alpha$

dissipation

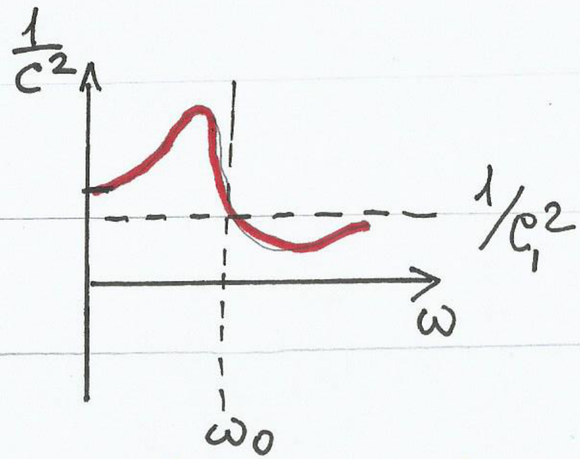
$$c_s^{ph} = \frac{\omega}{\mathrm{Re}\,k} = \frac{1}{\sqrt{\rho}\,\mathrm{Re}\sqrt{\varkappa}}$$

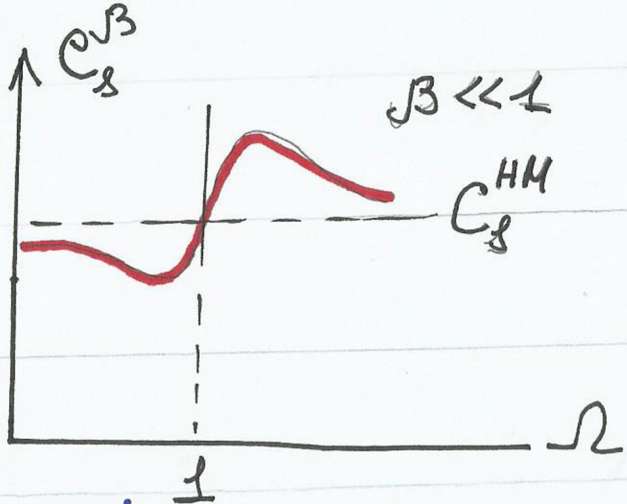
attenuation

- Bubbly matter $\varkappa = (1-\beta)\varkappa_1 + \beta\varkappa_2$,

  $\varkappa_1 \sim HM$, $\varkappa_2 \sim QM$, β volume consentration QM

- $$\frac{1}{c_s^2} = \underset{HM}{\frac{1}{c_{s1}^2}} + \frac{\beta}{\gamma P_0}\,\frac{1-\Omega^2}{(1-\Omega^2)^2+(\delta\Omega)^2}$$





- η shear viscosity value, not $\eta/s \sim 1/4\pi$

  $[\eta] = m^3 = g/cm\cdot s$ $\quad \eta \sim (10^{10} - 10^{20})\ g/cm\cdot s$?

  Yakovlev, Kolomeitsev & Voskresensky, Shuryak