Discrete symmetries in neutrino oscillations in a dense medium and an electromagnetic field

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Abstract

We study T-violation in the probabilities of neutrino flavor oscillations and spin rotation in the presence of dense matter and electromagnetic field. The model under study is a theory with Lorentz-violation. For such models the conditions of the CPT-theorem are not fulfilled and T-violation is not necessarily a consequence of the CP-violation. We obtain a sufficient condition of T violation, which indicates that T-violation can arise not only due to the presence of a nonzero CP-violating phase in the mixing matrix, but also due to simultaneous influence of matter and electromagnetic field. We obtain the probabilities of spin-flavor transitions in the three-flavor model taking into account neutrino diagonal magnetic moments and interaction with matter via neutral currents. Using the explicit form of the probabilities we show that the transition probabilities for left-handed neutrinos in matter composed of particles differs from the probabilities for right-handed antineutrinos in matter composed of antiparticles only in the sign of the T-violating term.

INTRODUCTION

The Standard Model of electroweak interactions based on the non-Abelian gauge symmetry of the interactions, the generation of particle masses due to the spontaneous symmetry breaking mechanism, and the ideology of particle generation mixing is universally recognized. Its predictions obtained in the framework of perturbation theory are in a very good agreement with the experimental data, and there is no serious reason, at least at the energies available at present, for its main propositions to be revised. However, in describing such an important and firmly experimentally established phenomenon as neutrino oscillations, a phenomenological, in fact, theory based on the pioneer works by B. Pontecorvo and Z. Maki et al. is used. This theory is well developed and is consistent with the experimental data. In this theory it is taken for granted that the mass and flavor states can be connected by a unitary transformation. This supposition is not correct.

In relativistic quantum field theory a particle is usually associated with an irreducible representation of the Poincaré group. The eigenvalue in this representation of the Casimir operator constructed from the translation operators squared is identified with the observed mass of the particle.
The existence of states which are superpositions of states with different masses contradicts the relativistic invariance of the theory, if the canonical momentum operator is identified with the translation operator. It obviously follows from the fact that the metric is defined on the hyperboloid in the momentum space determined by the value of the particle mass.

N.N. Bogolubov, A.A. Logunov, A.I. Oksak, and I.T. Todorov, *General Principlels of Quantum Field Theory* (Kluwer Academic Publishers, Dordrecht, 1990).

To overcome this difficulty it seems natural to associate some set of particles (a multiplet) with an irreducible representation of a wider group. A number of theorems

- L. Michel, Phys. Rev., **137**:2B (1965), B405-B408.
- S. Coleman, Phys. Rev., **138**:5B (1965), B1262-B1267.
- S. Weinberg, Phys. Rev., **139**:3B (1965) B597-B601.

indicate that the only reasonable extension of the symmetry group of the theory is the direct product of the Poincaré group and a group of internal symmetry.

However, in this point we meet the Jost theorem.

R. Jost, Helv. Phys. Acta, **39**:4 (1966), 369-375.

L. O'Raifeartaigh, Phys. Rev. Letters, **14**:14 (1965), 575-577; Phys. Rev., **161**:5 (1967), 1571-1575.

It seems that due to this theorem the result of extending the symmetry group of the theory will be trivial: the masses of the components of the multiplets will be equal. Just this circumstance is the most important reason of the failure of constructing the Fock space for flavor states.
However, it is possible to circumvent this result.

WAVE FUNCTIONS SPACES

It is well known (see, for example,

A.O. Barut, R. Raczka, Theory of Group Representations and Applications, World Scientific, Singapore, 1986.)

that the derivation algebra of the Poincaré algebra contains not only the translation operators P^{μ} and generators of the Lorentz group $M^{\mu\nu}$, but also the generator of dilatation D:

$$[P^{\mu}, D] = P^{\mu}, \quad [M^{\mu\nu}, D] = 0. \tag{2.1}$$

So it is possible to construct an external automorphism of this algebra, which leads to a scaling transformation of the translation generators. For irreducible representations the dilatation change the value of the Casimir operator which defines the mass of a particle. Hence, such a transformation allows to consider particles with different masses even in the case when the masses were initially equal.

Such considerations conform to the spirit of the Standard Model, where the masses of all particles are generated due to the phenomenon of spontaneous symmetry breaking and are proportional to the vacuum expectation value of the Higgs field.

Let us choose the group $SU(3)$ as the group of internal symmetry. Such an assumption is quite reasonable because the experimental data indicate that there are three generations of fermions. And let us consider the direct product of the Poincaré group and the group $SU(3)$ as an extended symmetry group of the theory.

Let us consider an irreducible representation of this group which is constructed as the direct product of the Dirac representation of the Poincaré group and the fundamental representation of the $SU(3)$ group. The explicit form of the Lie algebra elements in the present case is obvious. We have the standard realization of the Poincaré group generators multiplied by 3×3 identity matrix $\mathbb{I}$

$$P_\mu = \mathrm{i}\partial_\mu \mathbb{I}, \quad M_{\mu\nu} = \mathrm{i}\Big((x_\mu \partial_\nu - x_\nu \partial_\mu) + (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/4\Big)\mathbb{I}, \qquad (2.2)$$

and the Hermitian generators X_k, $k = 1...8$ of the fundamental representation of the $SU(3)$ group defined by the Gell-Mann matrices

$$\begin{aligned}
X_1 &= \left(e^{(1)} \otimes e^{(2)}\right) + \left(e^{(2)} \otimes e^{(1)}\right), X_2 = \mathrm{i}\left(e^{(2)} \otimes e^{(1)}\right) - \mathrm{i}\left(e^{(1)} \otimes e^{(2)}\right), \\
X_3 &= \left(e^{(1)} \otimes e^{(1)}\right) - \left(e^{(2)} \otimes e^{(2)}\right), X_4 = \left(e^{(1)} \otimes e^{(3)}\right) + \left(e^{(3)} \otimes e^{(1)}\right), \\
X_5 &= \mathrm{i}\left(e^{(3)} \otimes e^{(1)}\right) - \mathrm{i}\left(e^{(1)} \otimes e^{(3)}\right), X_6 = \left(e^{(2)} \otimes e^{(3)}\right) + \left(e^{(3)} \otimes e^{(2)}\right), \\
X_7 &= \mathrm{i}\left(e^{(3)} \otimes e^{(2)}\right) - \mathrm{i}\left(e^{(2)} \otimes e^{(3)}\right), X_8 = \left(e^{(1)} \otimes e^{(1)}\right) + \left(e^{(2)} \otimes e^{(2)}\right) \\
&\qquad -2\left(e^{(3)} \otimes e^{(3)}\right).
\end{aligned} \tag{2.3}$$

Here the vectors $e^{(l)}$ which, for definiteness, can be chosen as follows

$$e^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad e^{(2)} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad e^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{2.4}$$

are a basis of a three-dimensional vector space over the field of complex numbers.

Let us consider the space $\mathcal{H}_{m,1/2}$ of solutions of the matrix Dirac equation

$$(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I} - m\mathbb{I})\,\Psi(x) = 0, \tag{2.5}$$

which describes a multiplet containing three particles. Introduce in this space the scalar product

$$(\Psi, \Phi) = \sum_{i=1}^{3} \int d\mathbf{x}\, \Psi_i^{\dagger}(\mathbf{x}, t)\, \Phi_i(\mathbf{x}, t) \tag{2.6}$$

with summation over the coordinates of the vectors $e_i^{(l)}$. Then one can consider this space as the direct sum of the spaces of irreducible unitary representations of the direct product of the Poincaré group and the $SU(3)$ group corresponding to positive and negative frequencies. The action of this group in $\mathcal{H}_{m,1/2}$ is determined by the generators $P_{\mu}, M_{\mu\nu}, X_k$. Obviously, in these representations all the components of the multiplet have equal masses.

Let us assume that the neutrinos, the charged leptons as well as the down- and up-type quarks are combined to four different multiplets and each multiplet corresponds to a different representations of the extended symmetry group of the theory.

The idea of the proposed transformation, as has been previously mentioned, is based on the fact that it is possible to construct external automorphisms of this group in such a way that the spaces of the irreducible representations for different multiplets will not be identical.

A.E. Lobanov, Teor. Mat. Fiz. 192, 70 (2017).

A. E. Lobanov, Ann. Phys. 403, 82 (2019).

Indeed, in the space $\mathcal{H}_{m,1/2}$ the dilatations are determined by the operator

$$D = x_\mu \partial^\mu. \tag{2.7}$$

The operator of the Dirac equation

$$(\mathrm{i}\gamma^\mu \partial_\mu \mathbb{I} - m\mathbb{I})\, \Psi(x) = 0, \tag{2.8}$$

commutes with the generators of extended group.
However, for the generator of dilatation we have

$$[(\mathrm{i}\gamma^\mu \partial_\mu \mathbb{I} - m\mathbb{I})\, , D] = \mathrm{i}\gamma^\mu \partial_\mu \mathbb{I}. \tag{2.9}$$

That is the commutator is equal to zero on solutions for massless particles $m = 0$ only. Therefore, one can construct non-identical spaces $\mathcal{H}^{(i)}_{m,1/2}$, of irreducible representations for multiplets with non-zero masses.
It should be stressed that for massless fermions such a possibility is absent. However, in this case the theory possesses the scale invariance, and, as a consequence, the conformal invariance.

Let us associate the representation spaces for multiplets $\mathcal{H}^{(i)}_{m,1/2}$ including the neutrinos ($i = \nu$), the charged leptons ($i = e$) as well as the down- ($i = d$) and up- ($i = u$) type quarks with the space $\mathcal{H}_{m,1/2}$ using **a** unitary intertwining operators $\mathcal{K}^{(i)}$.

Then $\Psi^{(i)}(x)$ (the elements of the space $\mathcal{H}^{(i)}_{m,1/2}$) and $\Psi(x)$ (the elements of the space $\mathcal{H}_{m,1/2}$) will be connected by the unitary (with respect to the scalar product (2.6)) transformation

$$\Psi^{(i)}(x) = \mathcal{K}^{(i)}\Psi(x). \tag{2.10}$$

The intertwining operators can be constructed as follows.
For each multiplet introduce new basis vectors $n^{(l)}(i) \equiv n^{(l)}$, acting by a unitary matrix $\mathbb{V}^{(i)}$ on the old basis vectors $e^{(l)}$:

$$n_i^{(l)} = \sum_{j=1}^{3} V_{ij}^{(i)} e_j^{(l)}. \tag{2.11}$$

The vectors $n^{(l)}$ are normalized by the conditions (asterisk denotes the complex conjugation)

$$\sum_{i=1}^{3} n_i^{(l)} \overset{*}{n}_i^{(k)} = \delta_{kl}, \qquad \sum_{l=1}^{3} n_i^{(l)} \overset{*}{n}_j^{(l)} = \delta_{ij}. \tag{2.12}$$

The operators

$$\mathbb{P}_{(l)}^{(i)} = n^{(l)} \otimes \overset{*}{n}^{(l)}, \ \mathbb{P}_{(l)}^{(i)} \mathbb{P}_{(k)}^{(i)} = \delta_{kl} \mathbb{P}_{(l)}^{(i)}, \ \sum_{l=1,2,3} \mathbb{P}_{(l)}^{(i)} = \mathbb{I}. \tag{2.13}$$

are orthogonal projectors.

We can write the non-trivial intertwining operators in the form

$$\mathcal{K}^{(i)} = \sum_{l=1}^{3} \mathcal{D}^{(i)}_{(l)} \left(n^{(l)} \otimes e^{(l)} \right), \tag{2.14}$$

where

$$\mathcal{D}^{(i)}_{(l)} = \exp\left(\ln \mu^{(i)}_{l} (x_\nu \partial^\nu + 3/2) \right). \tag{2.15}$$

Here $\mu^{(i)}_l$ are positive numbers.

The Dirac equations, which are the conditions that the state spaces are irreducible, are now written as follows:

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I}-\mathbb{M}^{(i)}\right)\Psi^{(i)}(x)=0, \tag{2.16}$$

where

$$\mathbb{M}^{(i)}=\sum_{l=1}^{3}m_{l}^{(i)}\mathbb{P}_{(l)}^{(i)},\quad m_{l}^{(i)}=m\mu_{l}^{(i)}. \tag{2.17}$$

are the mass matrices.

We define a basis in the spaces $\mathcal{H}^{(i)}_{m,1/2}$ in the form

$$\Psi^{(i)}_{q,\zeta,\mu^{(i)}_l}(x) = \mathcal{K}^{(i)}\Psi_{p,\zeta,l}(x) = \psi^{(i)}_{q,\zeta,\mu^{(i)}_l}(x)n^{(l)}, \tag{2.18}$$

where $\psi^{(i)}_{q,\zeta,\mu^{(i)}_l}(x)$ are the plane waves derived from the ordinary plane waves by a dilatation of the coordinates:

$$\psi^{(i)}_{q,\zeta,\mu_l}(x) = \frac{(\mu^{(i)}_l)^{3/2}}{\sqrt{2q^0}} u_{q,\zeta} e^{-\mathrm{i}\mu^{(i)}_l(qx)}. \tag{2.19}$$

Here $q^0 = \sqrt{\mathbf{q}^2 + m^2}$. The spinors $u_{q,\zeta}$ satisfy the equation

$$(\gamma^\mu q_\mu - m)u_{q,\zeta} = 0 \tag{2.20}$$

and are normalized by the condition

$$\bar{u}_{q,\zeta} u_{q,\zeta'} = 2m\delta_{\zeta,\zeta'}. \tag{2.21}$$

The generators of the Lorentz group do not change the form:

$$M^{(i)}_{\mu\nu} = \mathcal{K}^{(i)} M_{\mu\nu} \mathcal{K}^{(i)-1} = \mathrm{i}\Big((x_\mu \partial_\nu - x_\nu \partial_\mu) + (\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/4\Big)\mathbb{I}\,, \quad (2.22)$$

since the dilatation operators commute with the generators of rotations and boosts (see (2.1)). This fact is quite natural, because the results of the observations, in particular, of the study of particle oscillations cannot depend on the choice of the inertial reference frame used for measurement. For the translation generators, we have

$$P^{(i)}_\mu = \mathcal{K}^{(i)} P_\mu \mathcal{K}^{(i)-1} = \mathrm{i}\partial_\mu \mathbb{N}^{(i)}, \quad (2.23)$$

where

$$\mathbb{N}^{(i)} = \sum_{l=1}^{3} \frac{1}{\mu^{(i)}_l} \mathbb{P}^{(i)}_l = \mathbb{M}^{(i)-1}/m. \quad (2.24)$$

are proportional to the inverse mass matrices.

We emphasize once again that the eigenvalues of the Casimir operators constructed from the new generators $P_\mu^{(i)}, M_{\mu\nu}^{(i)}, X_k^{(i)}$ take the same values on the solutions of the modified Dirac equations as on the elements of the initial representation space $\mathcal{H}_{m,1/2}$.

In particular, $P^{(i)\mu} P_\mu^{(i)} \Psi^{(i)}(x) = m^2 \Psi^{(i)}(x)$, but now the parameter m is not the observed mass, it rather sets the scale of the multiplet masses. In other words, it represents the bare mass of the multiplet. The observed masses are determined by the action of the canonical momentum operator squared on the basis functions $\Psi^{(i)}_{q,\zeta,\mu_l^{(i)}}(x)$ and are equal to $m_l^{(i)} = m\mu_l^{(i)}$ for l-component of the multiplet.

Thus, all mass states can be characterized by the same observable with a continuous spectrum. And, therefore, it is possible to construct wave functions of flavor states in the form of linear combinations of wave functions of mass states.

EXTERNAL CONDITIONS

Let us now consider the interaction of neutrino with a dense matter and electromagnetic field.
As it was proposed in the paper by Wolfenstein,

L. Wolfenstein, Phys. Rev. D 17, 2369 (1978)

if a matter density is high enough for considering the weak interaction of neutrino with the background fermions as coherent, it is possible to describe neutrino interaction with the matter by an effective potential. The origin of this effective potential is forward elastic scattering of neutrino on the matter fermions.
Interaction with the electromagnetic field is carried out due to magnetic and electric moments

K. Fujikawa and R. E. Shrock, Phys. Rev. Lett., 45, 963 (1980).

R. E. Shrock, Nucl. Phys. B 206, 359 (1982)

The relativistic equation describing the interaction of neutrinos with a dense medium and an electromagnetic field takes the form

$$\Big(\mathrm{i}\gamma^{\mu}\partial_{\mu}\mathbb{I} - \mathbb{M} - \frac{1}{2}\gamma^{\alpha}f_{\alpha}^{(e)}(1+\gamma^{5})\mathbb{P}^{(e)} - \frac{1}{2}\gamma_{\alpha}f^{\alpha(\mathrm{N})}(1+\gamma^{5})\,\mathbb{I} - \frac{\mathrm{i}}{2}\mu_{0}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M} - \frac{\mathrm{i}}{2}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M}_{h} - \frac{\mathrm{i}}{2}{}^{\star}\!F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M}_{ah}\Big)\,\Psi(x) = 0. \quad (3.1)$$

A. V. Chukhnova and A. E. Lobanov, Phys. Rev. D 101, 013003 (2020).

In this equation $F^{\mu\nu}$ is the electromagnetic field tensor, ${}^{\star}F^{\mu\nu} = -\frac{1}{2}e^{\mu\nu\rho\lambda}F_{\rho\lambda}$ is the dual electromagnetic field tensor. The interaction with magnetic and electric moments is taken into account by introducing the Hermitian matrices of transition moments $\mathbb{M}_h$ and $\mathbb{M}_{ah}$. The potentials

$$f_l^{\alpha(e)} = \sqrt{2}G_{\mathrm{F}}\left(j_l^{\alpha(e)} - \lambda_l^{\alpha(e)}\right) \tag{3.2}$$

determine the interaction of neutrino with the leptons by means of the charged currents and the potential

$$f_\alpha^{(\mathrm{N})} = \sqrt{2}G_{\mathrm{F}}\sum_{i=\nu,e,u,d}\sum_{l=1,2,3}\left(j_l^{\alpha(i)}\left(T^{(i)} - 2Q^{(i)}\sin^2\theta_{\mathrm{W}}\right) - \lambda_l^{\alpha(i)}T^{(i)}\right) \tag{3.3}$$

determines the contribution of the neutral currents.

Here

$$j_l^{\alpha(i)} = n_l^{(i)} \frac{p_l^{\alpha(i)}}{p_l^{0(i)}} = \{\bar{n}_l^{(i)} v_l^{0(i)}, \bar{n}_l^{(i)} \mathbf{v}_l^{(i)}\}, \tag{3.4}$$

are the currents, and

$$\lambda_l^{\alpha(i)} = n_l^{(i)} \frac{s_l^{\alpha(i)}}{p_l^{0(i)}} = \left\{ \bar{n}_l^{(i)} (\zeta_l^{(i)} \mathbf{v}_l^{(i)}), \bar{n}_l^{(i)} \left(\zeta_l^{(i)} + \frac{\mathbf{v}_l^{(i)} (\zeta_l^{(i)} \mathbf{v}_l^{(i)})}{1 + v_l^{0(i)}} \right) \right\}. \tag{3.5}$$

are the polarizations of the background fermions.
In this formulas $\bar{n}_l^{(i)}$ and $\zeta_l^{(i)}$ $(0 \leqslant |\zeta_l^{(i)}|^2 \leqslant 1)$ are the number density and the mean value of the polarization vector of the background fermions in the center-of-mass system of matter, consequently. In this reference frame the mean momentum of the fermions (i) is equal to zero. The 4-velocity of this reference frame denotes as $u_l^{\mu} = \{v_l^{0(i)}, \mathbf{v}_l^{(i)}\}$.

If all f^α and $F^{\mu\nu}$ are not dependent on the coordinates of event space, we can write solutions of evolution equation in the form of matrix exponentials, using the method developed in papers

A. E. Lobanov, Phys. Lett. B, **619**, 136 (2005).

E. V. Arbuzova, A. E. Lobanov, and E. M. Murchikova, Phys. Rev. D, **81**:4, 045001 (2010).

In this case, the wave functions of all mass states are also determined by the same observable with a continuous spectrum, which in vacuum makes sense of the neutrino 4-velocity u^μ up to a constant coefficient m.
This circumstance allows us to move to the quasi-classical approximation in a natural way:

$$x^\mu = \tau u^\mu$$

where τ is a neutrino proper time, which is related to the neutrino path length L by the formula

$$\tau = L/|\mathbf{u}|.$$

We can get the corresponding evolution equation, if we make the substitution

$$\gamma^{\mu}\partial_{\mu} \Rightarrow \gamma^{\mu}\left(\frac{\partial\tau}{\partial x^{\mu}}\right)\frac{d}{d\tau} = \gamma^{\mu}u_{\mu}\frac{d}{d\tau}. \tag{3.6}$$

It should be noted that this substitution is possible only when $u^{\mu} = \mathrm{const}$.

$$\left(\mathrm{i}\mathbb{I}\frac{d}{d\tau} - \mathcal{F}\right)\Psi(\tau) = 0, \tag{3.7}$$

where

$$\begin{aligned} \mathcal{F} = \mathbb{M} + \frac{1}{2}(f^{(e)}u)\mathbb{P}^{(e)} + \frac{1}{2}(f^{(\mathrm{N})}u)\mathbb{I} + \frac{1}{2}R_e\mathbb{P}^{(e)}\gamma^5\gamma^{\sigma}s_{\sigma}^{(e)}\gamma^{\mu}u_{\mu} \\ + \frac{1}{2}R_{\mathrm{N}}\mathbb{I}\gamma^5\gamma^{\sigma}s_{\sigma}^{(\mathrm{N})}\gamma^{\mu}u_{\mu} \\ - \mu_0\mathbb{M}\gamma^5\gamma^{\mu}\,{}^{\star}F_{\mu\nu}u^{\nu} - \mathbb{M}_h\gamma^5\gamma^{\mu}\,{}^{\star}F_{\mu\nu}u^{\nu} + \mathbb{M}_{ah}\gamma^5\gamma^{\mu}F_{\mu\nu}u^{\nu}. \end{aligned} \tag{3.8}$$

T-VIOLATION

A. V. Chukhnova and A. E. Lobanov, Phys. Rev. D 105, 073010 (2022).

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We study neutrino propagation in matter and electromagnetic field with constant characteristics. In this case, using the Backer-Campbell-Hausdorff formula we obtained the spin-flavor transition probabilities as the following formal expansion

$$W_{\alpha\to\beta} = \frac{1}{2}\sum_{n=0}^{\infty}\frac{(-\mathrm{i}\tau)^n}{n!}\mathrm{Tr}\left\{D_n\mathcal{P}_0^{(\beta)}(\gamma^\mu u_\mu + 1)\right\}, \qquad (4.1)$$

where

$$D_0 = \mathcal{P}_0^{(\alpha)},\ D_1 = [\mathcal{F}, \mathcal{P}_0^{(\alpha)}],\ D_2 = [\mathcal{F}, [\mathcal{F}, \mathcal{P}_0^{(\alpha)}]]... \qquad (4.2)$$

Here $\mathcal{P}_0^{\alpha}$ and $\mathcal{P}_0^{\beta}$ are projection operators on the states with definite polarization and flavor, which can be presented as products of a flavor projection operator and a polarization projection operator.

T-violation takes place, when there are nonzero terms of odd power in τ in the power expansion of probability.

In the most simple case, when neutrino interacts with matter via neutral currents, and the transition moments are not taken into account, the evolution equation takes the form

$$\left(\mathrm{i}\gamma^\mu\partial_\mu\mathbb{I} - \mathbb{M} - \frac{1}{2}\gamma_\alpha f^{\alpha(\mathrm{N})}(1+\gamma^5)\,\mathbb{I} - \frac{\mathrm{i}}{2}F^{\mu\nu}\sigma_{\mu\nu}\mathbb{M}_d\right)\Psi(x) = 0. \tag{4.3}$$

Quasi-classical equation:

$$\left(\mathrm{i}\mathbb{I}\frac{d}{d\tau} - \mathcal{F}\right)\Psi(\tau) = 0, \tag{4.4}$$

where

$$\mathcal{F} = \mathbb{M} + \frac{1}{2}(f^{(\mathrm{N})}u)\mathbb{I} + \frac{1}{2}R_{\mathrm{N}}\mathbb{I}\gamma^5\gamma^\sigma s_\sigma^{(\mathrm{N})}\gamma^\mu u_\mu - \mathbb{M}_d\gamma^5\gamma^\mu\,{}^\star\!F_{\mu\nu}u^\nu. \tag{4.5}$$

In Eq.(4.5)

$$s_\sigma^{(\mathrm{N})} = \frac{u_\sigma(f^{(\mathrm{N})}u) - f_\sigma^{(\mathrm{N})}}{\sqrt{(f^{(\mathrm{N})}u)^2 - (f^{(\mathrm{N})})^2}}, \qquad R_{\mathrm{N}} = \sqrt{(f^{(\mathrm{N})}u)^2 - (f^{(\mathrm{N})})^2}. \tag{4.6}$$

In this case, the solution of Eq. (4.5) is

$$\Psi(\tau) = \frac{1}{\sqrt{2u_0}} U(\tau)\Psi_0. \tag{4.7}$$

The density matrix

$$\begin{aligned}\rho_\alpha(\tau) &= \frac{1}{4u^0} U(\tau)\big(\gamma^\mu u_\mu + 1\big)\left(1 - \gamma^5\gamma_\nu s_0^\nu\right)\mathbb{P}_0^{(\alpha)}\bar{U}(\tau) \\ &= \frac{1}{2u^0} U(\tau)\big(\gamma^\mu u_\mu + 1\big)\mathcal{P}_0^{(\alpha)}\bar{U}(\tau).\end{aligned} \tag{4.8}$$

Here the evolution operator $U(\tau)$ takes the form

$$U(\tau) = \frac{1}{2}\sum_i \sum_{\zeta_i=\pm 1} \exp\left\{-\mathrm{i}\tau\left(m_i + \frac{1}{2}(fu) - \frac{1}{2}\zeta_i R_i\right)\right\}\mathbb{P}_i\left(1 - \zeta_i\gamma^5\gamma_\mu s_i^\mu\right). \tag{4.9}$$

Here

$$\begin{aligned} s_i^\mu &= \left(u^\mu(fu) - f^\mu - 2\mu_i\, {}^\star F^\mu{}_\nu u^\nu\right)/R_i, \\ R_i &= \sqrt{(fu)^2 - f^2 + 4\mu_i^2(u_\alpha\, {}^\star F^{\alpha\mu}\, {}^\star F_{\mu\nu} u^\nu) - 4\mu_i(f^\mu\, {}^\star F_{\mu\nu} u^\nu)}. \end{aligned} \tag{4.10}$$

Spin-flavor transition probability

$$W_{\alpha\to\beta} = \mathrm{Tr}\left\{\rho_\alpha(\tau)\rho^\dagger_\beta(\tau=0)\right\}. \tag{4.11}$$

Transition probability without spin-flip

$$\begin{aligned} W_+ = \sum_i |\mathcal{U}_{\alpha i}|^2|\mathcal{U}_{\beta i}|^2(\cos^2\frac{R_i\tau}{2} + \sin^2\frac{R_i\tau}{2}(s_0 s_i)^2) + \\ + 2\sum_{i<j}(R_{ij\alpha\beta}\cos(m_j - m_i)\tau - I_{ij\alpha\beta}\sin(m_j - m_i)\tau) \\ \times(\cos\frac{R_i\tau}{2}\cos\frac{R_j\tau}{2} + \sin\frac{R_i\tau}{2}\sin\frac{R_j\tau}{2}(s_0 s_i)(s_0 s_j)) + \\ + 2\zeta_\alpha\sum_{i<j}(R_{ij\alpha\beta}\sin(m_j - m_i)\tau + I_{ij\alpha\beta}\cos(m_j - m_i)\tau) \\ \times(\sin\frac{R_i\tau}{2}\cos\frac{R_j\tau}{2}(s_0 s_i) - \cos\frac{R_i\tau}{2}\sin\frac{R_j\tau}{2}(s_0 s_j)). \end{aligned} \tag{4.12}$$

Transition probability with spin-flip

$$W_- = \sum_i |\mathcal{U}_{i\alpha}|^2 |\mathcal{U}_{i\beta}|^2 (1 - (s_0 s_i)^2) \sin^2 \frac{R_i}{2} +$$
$$+ 2 \sum_{i<j} (R_{ij\alpha\beta} \cos(m_j - m_i)\tau - I_{ij\alpha\beta} \sin(m_j - m_i)\tau)$$
$$\times \sin \frac{R_i}{2}\tau \sin \frac{R_j}{2}\tau (-(s_i s_j) - (s_i s_0)(s_j s_0))$$
$$+ 2\zeta_\alpha \sum_{i<j} (R_{ij\alpha\beta} \sin(m_j - m_i)\tau + I_{ij\alpha\beta} \cos(m_j - m_i)\tau)$$
$$\times \sin \frac{R_i}{2}\tau \sin \frac{R_j}{2}\tau e_{\mu\nu\rho\lambda} s_i^\mu s_j^\nu s_0^\rho u^\lambda. \quad (4.13)$$

The term, which violates T invariance, arises in the probabilities of transitions with the change of helicity W_{-}. This term is proportional to the value, which in the laboratory reference frame can be expressed in the three-dimensional form as follows

$$\zeta_{\alpha} e_{\mu\nu\rho\lambda} u^{\mu} s_0^{\nu} s_i^{\rho} s_j^{\lambda} = 2\zeta_{\alpha} \frac{\mu_j - \mu_i}{R_i R_j} \frac{1}{|\mathbf{u}|}([\mathbf{u} \times \mathbf{f}](u_0 \mathbf{B} - [\mathbf{u} \times \mathbf{E}])). \qquad (4.14)$$

Here B is the magnetic induction vector, **E** is the electric field strength, **f** is the spacial part of the 4-vector $f_{\mu}^{(N)}$, which is defined by the matter velocity and polarization. Obviously, the T -violating term vanishes when any two of the three vectors **u**, **f** and $(u^0 \mathbf{B} - [\mathbf{uE}])$ are collinear. That is, the probabilities are not T invariant, when the external conditions are characterized by two different preferred spacial directions. In particular, when we consider unpolarized matter at rest, the expressions for all the spin-flavor transition probabilities are T -invariant.

If we consider the interaction of neutrinos with charged antifermions, we can see that the transition probabilities for left-handed neutrino in ordinary matter and for right-handed antineutrino in matter composed of antiparticles are not similar anymore. To restore the initial expressions, one also needs to change the sign of the proper time τ. Thus, we obtain an interesting result. In the presence of electromagnetic field the probabilities obtained for transitions of left-handed neutrino in matter differ from those for right-handed antineutrino in matter composed of antiparticles only in the sign of the T-violating term.
That is, we reveal an extra source of T violation, which emerges due to collective effects and is not caused by complex entries in the mixing matrix. This source of T violation does not require introducing new symmetries, which imply either new particles to be included into the Standard Model or new properties of the presently known particles.

Thank you for your attention!