Influence of the magnetic field and rotation on heavy mesons in medium.

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Topics

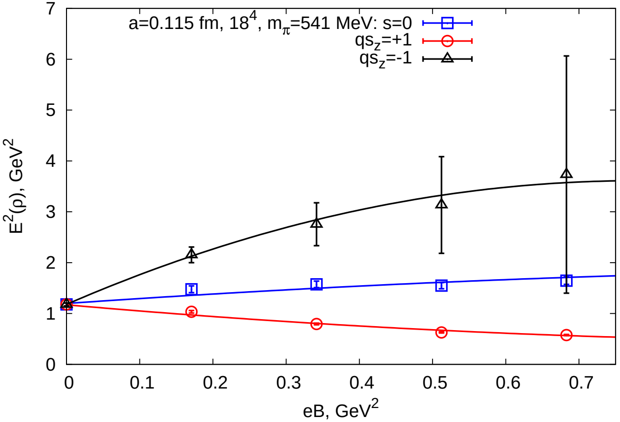
- Vector meson energy in magnetic field, absence of tachyonic mode.
- Polarizabilities in magnetic fields, longitudinal polarization.
- Dirac operator in rotating frame. Comparison of magnetic field and rotation.

Vector meson in large magnetic field naively may have negative energy squared: tachyonic mode.

Is it present in QCD?

Lattice calculations for ρ meson (Luschevskaya et. al., 2014): tachyonic mode is absent.
Now confirmed for K^* and charmonium.

Energy of $\rho^{\pm}$ meson for $s_z = 0, \pm 1$.



$$qs_z = -1: \quad E^2 = |eB| + g(eB) + m^2 - 4\pi m \beta_m (eB)^2 + \dots$$
$$qs_z = 0: \quad E^2 = |eB| + m^2 - 4\pi m \beta_m (eB)^2 + \dots$$
$$qs_z = +1: \quad E^2 = |eB| - g(eB) + m^2 - 4\pi m \beta_m (eB)^2 + \dots$$

Dirac operator with the magnetic field

Solve Dirac equation numerically on the lattice

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^\mu(\partial_\mu - iA_\mu)$$

in the external gauge field

$$A_{\mu\, ij} \to A_{\mu\, ij} + A^B_\mu \delta_{ij},$$

where

$$A^B_\mu(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}).$$

$$qB = \frac{2\pi k}{(aN_s)^2}, \quad k \in \mathbb{Z},$$

where $q = -1/3\, e$, aN_s is the lattice space extension.

Technique for energy calculation

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m_q}.$$

Calculate the correlation functions of $\rho^\pm$ on the lattice:

$$\langle \bar{\psi}_{d,u}(x)\gamma_i\psi_{u,d}(x)\bar{\psi}_{u,d}(y)\gamma_j\psi_{d,u}(y)\rangle_A = -Tr[\gamma_i D^{-1}_{u,d}(x,y)\gamma_j D^{-1}_{d,u}(y,x)],$$

$D_d^{-1} = D_u^{-1}$, where $x = (\mathbf{n}a, n_t a)$, $y = (\mathbf{n}'a, n_t' a), \mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}$

We obtain the correlation functions for different spin projections of the ρ meson on the magnetic field axis:

$$C(s_z=0)=\langle O_3(x)\bar{O}_3(y)\rangle_A$$
$$C(s_z=\pm 1)=\langle O_1(x)\bar{O}_1(y)\rangle_A+\langle O_2(x)\bar{O}_2(y)\rangle_A$$
$$\pm i(\langle O_1(x)\bar{O}_2(y)\rangle_A-\langle O_2(x)\bar{O}_1(y)\rangle_A)$$

where

$$O_1=\psi^{\dagger}_{d,u}(x)\gamma_1\psi_{u,d}(x),\quad O_2=\psi^{\dagger}_{d,u}(x)\gamma_2\psi_{u,d}(x),$$

$$O_3=\psi^{\dagger}_{d,u}(x)\gamma_3\psi_{u,d}(x).$$

for ρ^0:

$$\langle\bar{\psi}_d(x)\gamma_i\psi_d(x)\bar{\psi}_d(y)\gamma_j\psi_d(y)+\bar{\psi}_u(x)\gamma_i\psi_u(x)\bar{\psi}_u(y)\gamma_j\psi_u(y)\rangle_A=$$

$$-Tr[\gamma_i D_d^{-1}(x,y)\gamma_j D_d^{-1}(y,x)]-Tr[\gamma_i D_u^{-1}(x,y)\gamma_j D_u^{-1}(y,x)].$$

Correlation functions

$$G(\vec{p}, n_t) = \frac{1}{N^{3/2}} \sum_{\mathbf{n}\in\Lambda_3} \langle \psi^\dagger(\vec{n}, n_t)\Gamma_1\psi(\vec{n}, n_t)\psi^\dagger(\vec{0}, 0)\Gamma_2\psi(\vec{0}, 0)\rangle e^{-ia\mathbf{np}}$$

$p_i = 2\pi k_i/(aN),\ k_i = -N/2, ..., N/2.$
We obtain the masses from the asymptotic behaviour of correlators

$$\langle \psi^\dagger(\vec{0}, n_t)\Gamma_1\psi(\vec{0}, n_t)\psi^\dagger(\vec{0}, 0)\Gamma_2\psi(\vec{0}, 0)\rangle_A = \sum_k \langle 0|\hat{O}_1|k\rangle\langle k|\hat{O}_2^\dagger|0\rangle e^{-n_t E_k}.$$

The main contribution comes from $\langle 0|\hat{O}_1|k\rangle\langle k|\hat{O}_2^\dagger|0\rangle e^{-n_t E_0}$.
We set $\langle \mathbf{p}\rangle = 0$, so $E_0 = m_0$ because $E^2 - \mathbf{p}^2 = m^2$.

Influence of the magnetic field on the tensor polarization

The presence of a magnetic field creates a kind of anisotropy in space, which can lead to tensor polarization (alignment) of the vector meson and, after its decay, to dileptonic asymmetry in collisions of heavy ions.

Dileptonic asymmetry

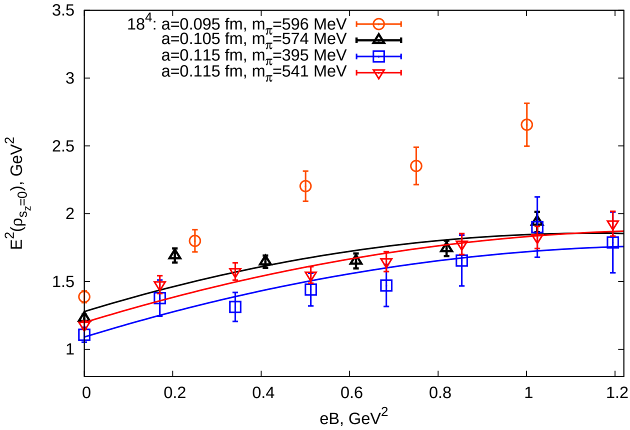
- is an important physical characteristic which can be utilized to disentangle contributions of some particles's decay channels;
- provide information about the evolution of quark-gluon plasma in non-central heavy-ion collisions.

The dilepton asymmetries in non-central heavy ion collisions depend on the energy behaviour of the vector mesons.

According to the vector dominance principle the vector mesons can directly convert to virtual photons.

In turn, the electromagnetic decay of virtual photons is one of the main sources of dilepton production in heavy-ion collisions.

Energy of the $\rho^{\pm}$ meson with $s_z = 0$ (Improved)

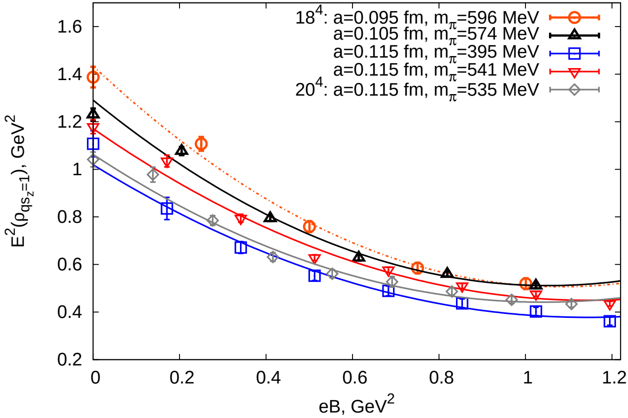


$$E^2 = |eB| + m^2 - 4\pi m\beta_m (eB)^2 + \ldots$$

The dipole magnetic polarizability of the $\rho^{\pm}$ meson for the spin projection $s_z = 0$.

V	m_π(MeV)	a(fm)	β_m(GeV^{-3})	$\chi^2/d.o.f.$
18^4	574 ± 7	0.105	0.03 ± 0.01	6.90
18^4	395 ± 6	0.115	0.028 ± 0.006	0.53
18^4	541 ± 3	0.115	0.027 ± 0.004	1.25

Energy of $\rho^{\pm}$ meson with $qs_z = +1$

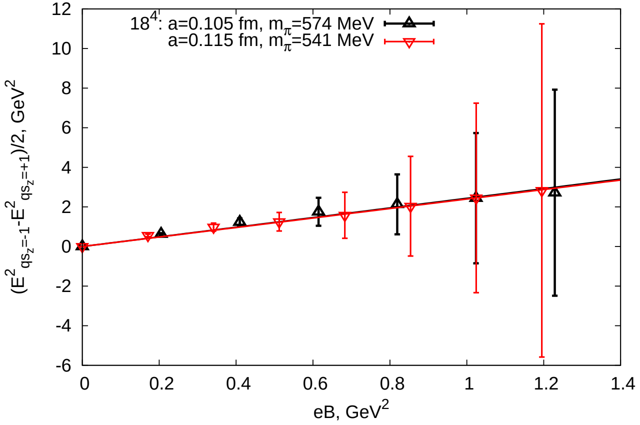


$$E^2 = |eB| - g(eB) + m^2 - 4\pi m \beta_m (eB)^2 + \dots$$

The dipole magnetic polarizability and g-factor of the $\rho^{\pm}$ meson for the spin projection $|s_z| = 1$.

V	m_π(MeV)	a(fm)	g-factor	β_m(GeV^{-3})	$\chi^2/d.o.f.$
18^4	574 ± 7	0.105	2.48 ± 0.19	-0.049 ± 0.010	2.66
18^4	541 ± 3	0.115	2.26 ± 0.14	-0.041 ± 0.006	2.32
20^4	535 ± 4	0.115	2.19 ± 0.12	-0.044 ± 0.006	1.48
18^4	395 ± 6	0.115	2.12 ± 0.13	-0.039 ± 0.006	1.49

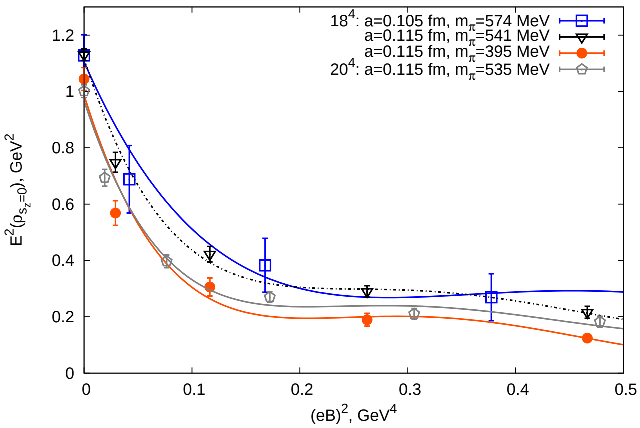
Energy and magnetic moment of $\rho^{\pm}$ meson



$$E^2(qs_z = -1) - E^2(qs_z = +1) = 2g(eB)$$

$g = 2.4 \pm 0.1$ for $a = 0.105$ fm and 2.40 ± 0.04 for $a = 0.115$ fm.

Energy of the ρ^0 meson



$$E^2 = m^2 - 4\pi m\beta_m(eB)^2 - 4\pi m\beta_m^{h2}(eB)^4 - 4\pi m\beta_m^{h4}(eB)^6 - 4\pi m\beta_m^{h6}(eB)^8$$

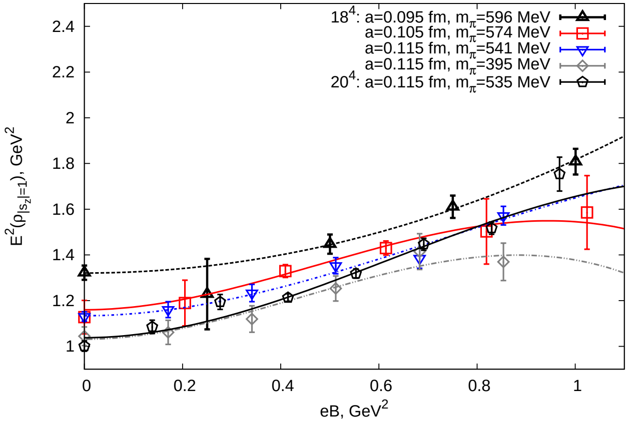
Determination of β_m value from different fits (new)

power of field	$(eB)^2$, GeV^4	β_m, GeV^{-3}	β_m^{2h}, GeV^{-7}	$\chi^2/d.o.f.$
2	[0 : 0.05]	1.28	–	–
4	[0 : 0.2]	0.86 ± 0.16	-3.12 ± 0.94	10.9
6	[0 : 0.4]	1.00 ± 0.18	-5.56 ± 1.64	7.5
8	[0 : 0.6]	1.09 ± 0.19	-7.38 ± 2.16	5.7

The values of the magnetic dipole polarizability of ρ^0 for the spin projection $s_z = 0$

V	a, fm	m_π, MeV	β_m, GeV^{-3}	β_m^{2h}, GeV^{-7}	$(eB)^2$	χ^2/n
18^4	0.105	574(7)	0.66 ± 0.16	-2.51 ± 0.98	[0 : 1.7]	1.04
18^4	0.115	541(3)	0.90 ± 0.16	-5.11 ± 1.59	[0 : 1.5]	2.46
20^4	0.115	535(4)	0.95 ± 0.15	-5.78 ± 1.60	[0 : 1.5]	2.63
18^4	0.115	395(6)	0.98 ± 0.30	-5.79 ± 2.74	[0 : 1.5]	3.32

Energy of the ρ^0 meson with $|s_z| = 1$



$$E^2 = m^2 - 4\pi m\beta_m(eB)^2 - 4\pi m\beta_m^{h1}(eB)^3$$

The values of the magnetic dipole polarizability of ρ^0 for the spin projection $|s_z| = 1$

V	a, fm	m_π, MeV	β_m, GeV^{-3}	β_m^{1h}, GeV^{-5}	eB	χ^2/n
18^4	0.105	574 ± 7	-0.10 ± 0.02	0.07 ± 0.02	[0 : 1.1]	0.47
18^4	0.115	541 ± 3	-0.07 ± 0.02	0.03 ± 0.03	[0 : 1.1]	0.87
20^4	0.115	535 ± 4	-0.10 ± 0.02	0.06 ± 0.03	[0 : 1.1]	1.54
18^4	0.115	395 ± 6	-0.11 ± 0.03	0.08 ± 0.03	[0 : 1.1]	0.65

Tensor magnetic polarizability

- The shape of the dilepton distribution is characterized by

$$\frac{d\sigma}{dM^2 d\cos\theta} = A(M^2)(1 + B\cos^2\theta)$$

$M^2 = (p_1 + p_2)^2$ is the energy of the lepton pair in their rest frame, θ is the angle between the momenta of the virtual photon and the lepton.

- The asymmetry coefficient B is defined by the polarization of the virtual photons produced in the collisions:

$$B = \frac{\gamma_\perp - \gamma_\parallel}{\gamma_\perp + \gamma_\parallel}$$

where the $\gamma_{\perp,\parallel}$ are the contributions of the transverse and longitudinal polarizations of the virtual intermediate photon.

For the vector particle the polarization tensor:

$$P_{ij} = \frac{3}{2}\langle s_i s_j + s_j s_i\rangle - 2\delta_{ij}.$$

If $w_{s_z=+1}$, $w_{s_z=-1}$ and $w_{s_z=0}$ are the probabilities that the ρ meson has a spin projection on the field direction equal to $+1$, -1 and 0 correspondingly, then

$$P_{33} = w_{s_z=+1} + w_{s_z=-1} - 2w_{s_z=0} = \frac{N_{s_z=+1} + N_{s_z=-1} - 2N_{s_z=0}}{N_{s_z=+1} + N_{s_z=-1} + N_{s_z=0}},$$

where $N_{s_z=+1}$, $N_{s_z=-1}$ and $N_{s_z=0}$ are the numbers of particles with different spin projections.

As $w_{s_z=+1} + w_{s_z=-1} + w_{s_z=0} = 1$, therefore, $P_{33} = 1 - 3w_{s_z=0}$ and $-2 \leq P_{33} \leq 1$.

The differential cross section for the decay of ρ meson to the lepton pair may be written as

$$\frac{d\sigma}{dM^2 d\cos\theta} = N(M^2)(1 + \frac{1}{4}P_{33}(3\cos^2\theta - 1)).$$

$$B = \frac{3P_{33}}{4 - P_{33}}.$$

For the transversely polarized ρ meson $B = 1$ and for the longitudinally polarized $B = -1$.

We also introduce the tensor polarizability

$$\beta_t = \frac{\beta_{s_z=+1} + \beta_{s_z=-1} - 2\beta_{s_z=0}}{\beta_{s_z=+1} + \beta_{s_z=-1} + \beta_{s_z=0}},$$

which is the measure of the magnetic field effect on a vector meson, in particular for high magnetic fields and temperature $P \sim \beta_t$.

The tensor polarizability of the ρ^0 meson.

V	a(fm)	m_π(MeV)	β_t
18^4	0.105	574 ± 7	-3.3 ± 0.6
18^4	0.115	541 ± 3	-2.6 ± 0.2
20^4	0.115	535 ± 4	-2.8 ± 0.3
18^4	0.115	395 ± 6	-2.9 ± 0.5

The tensor polarizability of the $\rho^{\pm}$ meson.

V	a(fm)	m_π(MeV)	β_t
18^4	0.105	574 ± 7	2.3 ± 0.7
18^4	0.115	541 ± 3	2.5 ± 0.5
18^4	0.115	395 ± 6	2.7 ± 0.7

The large negative values of β_t suggest the dominating longitudinal polarization of the ρ^0-meson with respect to the magnetic field direction.

The dileptons are mainly emitted in the directions perpendicular to the magnetic field axis.

This is a convincing result, that the energy of the state with $s_z = 0$ decreases, and the energy of $|s_z| = 1$ increases.

The zero mass limit may be compared to the growth of the conductivity in magnetic field (talk of V.Braguta).

Rotation

- Consider propagator of heavy quark in rotating frame. This corresponds to the effect of rotation for quarks only.
- This is of interest because rotation acts differently on quarks and gluons (talk of A. Roenko)

Dirac operator with rotation and magnetic field

Dirac operator in Minkowski space:

$$\mathfrak{D} = i\hbar\gamma^\alpha D_\alpha - mc$$

$$\mathfrak{D} = \gamma^0 \mathcal{D}_0 + \boldsymbol{\gamma} \cdot \boldsymbol{\mathcal{D}} - mc,$$

where explicitly

$$\mathcal{D}_0 = \frac{i\hbar\partial}{c\,\partial t} + \frac{1}{c}\boldsymbol{\omega} \cdot \left(\frac{\hbar}{2}\boldsymbol{\Sigma} + \boldsymbol{r} \times \boldsymbol{\pi}\right),$$

$$\boldsymbol{\mathcal{D}} = -\boldsymbol{\pi}.$$

Here the generalized 3-momentum

$$\boldsymbol{\pi} = \boldsymbol{p} - q\boldsymbol{A} = -i\hbar\boldsymbol{\nabla} - \frac{q}{2}\boldsymbol{B} \times \boldsymbol{r}.$$

Dirac operator in discrete Euclidean space:

$$(D_W)_{x,y} = \frac{1}{2}\sum_{\mu}\gamma_\mu(\partial^f_{\mu,x,y} + \partial^b_{\mu,x,y}) + m\delta_{x,y} - \frac{ra}{2}\sum_{\mu,z}\partial^f_{\mu,x,z}\partial^b_{\mu,z,y}.$$

$$= \frac{m+4r}{a}\delta_{x,y} + \frac{1}{2a}\sum_{\mu}\left((\gamma_\mu - r)\delta_{x+\hat{\mu}a,y}U_{x,\mu} - (\gamma_\mu + r)\delta_{x-\hat{\mu}a,y}U^\dagger_{x-\hat{\mu}a,\mu}\right).$$

$$\partial^f_{\mu,x,y} = \frac{1}{a}\left(\delta_{x+\hat{\mu}a,y}U_\mu(x) - \delta_{x,y}\right), \quad \partial^b_{\mu,x,y} = \frac{1}{a}\left(\delta_{x,y} - \delta_{x-\hat{\mu}a,y}U^\dagger_\mu(x-\hat{\mu}a)\right).$$

$$U_\mu(x) = e^{-aA_\mu(x)}, \qquad A_\mu(x) = -ig\sum_{a=1}^{8} T_a A^a_\mu(x)$$

$$a \to 0: \qquad D = \gamma_\mu(\partial^\mu - A^\mu) + m.$$

Dirac operator in Euclidean space:

$$A_{\mu ij} = A^{gl}_{\mu ij} + A^{B}_{\mu}\delta_{ij}, \quad A^{B}_{\mu}(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}),$$

$$\frac{1}{2}\vec{w}\left(\frac{\hbar}{2}\Sigma + [\vec{r}, \left(-i\hbar\vec{\nabla} - \frac{q}{2}[\vec{B},\vec{r}]\right)]\right) = \frac{\hbar}{2c}(\vec{w},\Sigma) - i\frac{\hbar}{c}\vec{w}[\vec{r},\vec{\nabla}] - \frac{q}{2c}\vec{w}[\vec{B},\vec{r}].$$

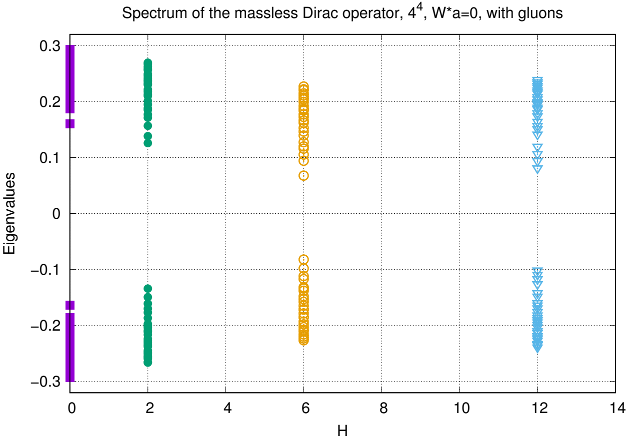
$\hbar = 1$, $c = 1$, so we add the following terms:

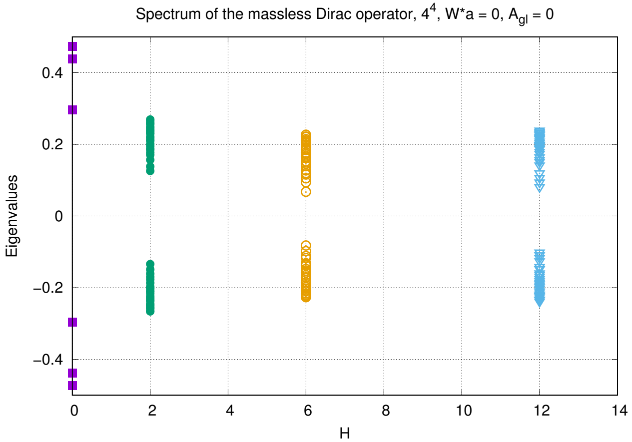
$$\frac{1}{2}w_i\Sigma^i.$$

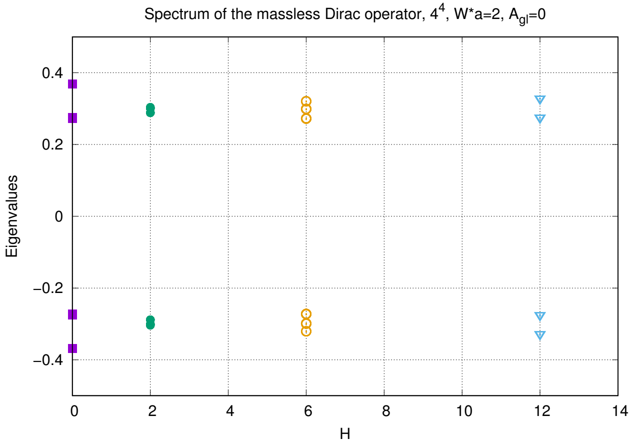
$$-i\vec{w}[\vec{r},\vec{\nabla}] = -iw_i e_{ijk}x_j\partial_k$$

$$-\frac{q}{2}\vec{w}[\vec{B},\vec{r}] = -\frac{q}{2}w_iA^i = \frac{qB}{2}(w_1x_2 - w_2x_1).$$

Spectrum of the massless Dirac operator




Spectrum of the massless Dirac operator, 4^4, W*a = 0, A_{gl} = 0
Eigenvalues
0.4
0.2
0
−0.2
−0.4
0
2
4
6
8
10
12
14
H


Spectrum of the massless Dirac operator, 4^4, W*a=2, A_gl=0
Eigenvalues
H
0.4
0.2
0
−0.2
−0.4
0
2
4
6
8
10
12
14

Conclusions

- For the heavy quarks the tachionic mode is absent.
- For the neutral ρ meson prevales the longitudinal polarization
- We modify the Dirac operator for the rotation frame, the spectrum. calculations are in progress.

Thank you for your attention!