Geometric transitions in systems with spatial periodicity

Valery N. Marachevsky

Saint Petersburg State University,
St.Petersburg, Russia

International Conference on Quantum Field Theory,
High-Energy Physics, and Cosmology
Dubna, Russia

18-21 July 2022

In this talk

1. Casimir-Polder potential.
2. Lifshitz theory: gauge-invariant formalism.
3. Solution of diffraction grating problem. Casimir energy in the system of two gratings. Scattering method.
4. Lateral Casimir force experiment in the system of two diffraction gratings.
5. Casimir-Polder potential experiment in the system atom - diffraction grating.
6. Torque in the system of two rotated infinite gratings. Energy discontinuity (1D - 2D geometric transition).
7. Giant torque in the system of two rotated finite gratings.

Casimir-Polder potential

Consider propagation of an electromagnetic field from a dipole source at the point $\mathbf{r}' = (0, 0, L)$ characterized by electric dipole moment $\mathbf{d}$. In this case, components of the four-current density of the dipole source must be written in the form [V.N.Marachevsky and Yu.M.Pis'-mak, *Phys.Rev.D* **81**, 065005 (2010)]:

$$\rho(t, \mathbf{r}) = -d^k(t)\partial_k \delta^3(\mathbf{r} - \mathbf{r}') \,, \tag{1}$$

$$j^k(t, \mathbf{r}) = \partial_t d^k(t)\delta^3(\mathbf{r} - \mathbf{r}') \,. \tag{2}$$

One can evaluate atomic polarizability:

$$\alpha_{ij}(t_1 - t_2) = i\langle T(\hat{d}_i(t_1), \hat{d}_j(t_2))\rangle \,, \tag{3}$$

where $\hat{d}_i(t)$ in (3) are operators of electric dipole moment.

Casimir-Polder potential

We obtain

$$\hat{V}(t) = \int d^3\mathbf{r}\, \hat{J}_\mu(t,\mathbf{r})\hat{A}^\mu(t,\mathbf{r}) = -\int dt \;\; \hat{\mathbf{d}}(t)\hat{\mathbf{E}}(t,\mathbf{r}') \qquad (4)$$

with operators $\hat{d}^k(t)$ in the four-current density (1), (2). The Casimir-Polder potential is derived from (4) in the second order perturbation theory:

$$U(L) = -\int_0^\infty \frac{d\omega}{2\pi}\alpha^{ij}(i\omega)D^E_{ij}(i\omega,\mathbf{r}',\mathbf{r}'), \qquad (5)$$

where

$$D_{ij}(t-t',\mathbf{r},\mathbf{r}') = i\langle T(\hat{E}_i(t,\mathbf{r})\hat{E}_j(t',\mathbf{r}'))\rangle, \qquad (6)$$

$$D^E_{ij}(t-t',\mathbf{r},\mathbf{r}') = D_{ij}(t-t',\mathbf{r},\mathbf{r}') - D^{(0)}_{ij}(t-t',\mathbf{r},\mathbf{r}'). \qquad (7)$$

Casimir energy

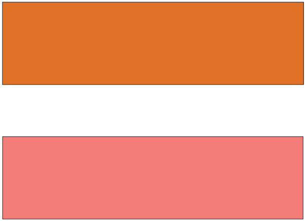
The Casimir energy of two perfectly conducting parallel plates separated by a distance a :

$$E = -\frac{S\pi^2}{720a^3} \,, \tag{8}$$

S is the plate area [H.B.G. Casimir, On the attraction between two perfectly conducting plates, *Proc. Kon. Ned. Akad. Wet. B* **51**, 793–795 (1948)] .

Lifshitz theory for two half-spaces

Consider two dielectric half-spaces $z \leq 0$, $z \geq a$ and the vacuum slit $0 < z < a$ between them.



Pressure P is equal to T_{zz} component of the energy-momentum tensor, it is expressed in terms of electric and magnetic Green functions of fluctuating electromagnetic field:

$$P = T_{zz}(z_0) = -\frac{i}{2}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\Big[D^E_{xx}(z_0,z_0) + D^E_{yy}(z_0,z_0) - D^E_{zz}(z_0,z_0) + D^H_{xx}(z_0,z_0) + D^H_{yy}(z_0,z_0) - D^H_{zz}(z_0,z_0)\Big].$$

It is convenient to derive local components of electric Green functions for a given $k_z = \sqrt{\omega^2 - k_x^2 - k_y^2}$ in a local basis $\mathbf{e}_r$, $\mathbf{e}_\theta$, $\mathbf{e}_z$. Cartesian components of electric Green functions can be expressed in terms of local components as follows:

$$D^E_{xx}(\omega, \mathbf{r}, \mathbf{r}') = \int \left(D^E_{rr}(\omega, k_r) \cos^2\theta + D^E_{\theta\theta}(\omega, k_r) \sin^2\theta \right) e^{i\mathbf{k}_\parallel(\mathbf{r}_\parallel - \mathbf{r}'_\parallel)} \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2},$$

$$D^E_{yy}(\omega, \mathbf{r}, \mathbf{r}') = \int \left(D^E_{rr}(\omega, k_r) \sin^2\theta + D^E_{\theta\theta}(\omega, k_r) \cos^2\theta \right) e^{i\mathbf{k}_\parallel(\mathbf{r}_\parallel - \mathbf{r}'_\parallel)} \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2},$$

$$D^E_{zz}(\omega, \mathbf{r}, \mathbf{r}') = \int D^E_{zz}(\omega, k_r) e^{i\mathbf{k}_\parallel(\mathbf{r}_\parallel - \mathbf{r}'_\parallel)} \frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}.$$

Local components of electric Green functions:

$$D^E_{rr}(\omega, k_r) = \frac{ik_z}{2\Delta_{TM}}\Big[e^{ik_z z}\big(r_{TM1}r_{TM2}e^{ik_z(2a-z_0)} - r_{TM2}e^{ik_z z_0}\big)+ e^{-ik_z z}\big(r_{TM1}r_{TM2}e^{ik_z(2a+z_0)} - r_{TM1}e^{ik_z(2a-z_0)}\big)\Big],$$

$$D^E_{\theta\theta}(\omega, k_r) = \frac{i\omega^2}{2k_z\Delta_{TE}}\Big[e^{ik_z z}\big(r_{TE1}r_{TE2}e^{ik_z(2a-z_0)} + r_{TE2}e^{ik_z z_0}\big)+ e^{-ik_z z}\big(r_{TE1}r_{TE2}e^{ik_z(2a+z_0)} + r_{TE1}e^{ik_z(2a-z_0)}\big)\Big],$$

$$D^E_{zz}(\omega, k_r) = \frac{ik_r^2}{2k_z\Delta_{TM}}\Big[e^{ik_z z}\big(r_{TM1}r_{TM2}e^{ik_z(2a-z_0)} + r_{TM2}e^{ik_z z_0}\big)+ e^{-ik_z z}\big(r_{TM1}r_{TM2}e^{ik_z(2a+z_0)} + r_{TM1}e^{ik_z(2a-z_0)}\big)\Big],$$

where $\Delta_{TM} \equiv 1 - r_{TM1}r_{TM2}e^{2ik_z a}$, $\Delta_{TE} \equiv 1 - r_{TE1}r_{TE2}e^{2ik_z a}$ with Fresnel coefficients r_{TM1}, r_{TM2}, r_{TE1}, r_{TE2}.
[V.N.Marachevsky and A.A.Sidelnikov, *Universe* **7**, 195 (2021).]

Magnetic Green function can be evaluated from electric Green function as follows:

$$D^H_{il}(\omega, \mathbf{x}, \mathbf{x}') = \frac{1}{\omega^2} \epsilon_{ijk} \epsilon_{lmn} \frac{\partial}{\partial x^j} \frac{\partial}{\partial x'^m} D^E_{kn}(\omega, \mathbf{x}, \mathbf{x}').$$

Local components of magnetic Green functions:

$$D^H_{rr}(\omega, k_r) = \frac{ik_z}{2\Delta_{TE}} \Big[e^{ik_z z} \big(r_{TE1} r_{TE2} e^{ik_z(2a - z_0)} - r_{TE2} e^{ik_z z_0} \big) + e^{-ik_z z} \big(r_{TE1} r_{TE2} e^{ik_z(2a + z_0)} - r_{TE1} e^{ik_z(2a - z_0)} \big) \Big],$$

$$D^H_{\theta\theta}(\omega, k_r) = \frac{i\omega^2}{2k_z\Delta_{TM}} \Big[e^{ik_z z} \big(r_{TM1} r_{TM2} e^{ik_z(2a - z_0)} + r_{TM2} e^{ik_z z_0} \big) + e^{-ik_z z} \big(r_{TM1} r_{TM2} e^{ik_z(2a + z_0)} + r_{TM1} e^{ik_z(2a - z_0)} \big) \Big],$$

$$D^H_{zz}(\omega, k_r) = \frac{ik_r^2}{2k_z\Delta_{TE}} \Big[e^{ik_z z} \big(r_{TE1} r_{TE2} e^{ik_z(2a - z_0)} + r_{TE2} e^{ik_z z_0} \big) + e^{-ik_z z} \big(r_{TE1} r_{TE2} e^{ik_z(2a + z_0)} + r_{TE1} e^{ik_z(2a - z_0)} \big) \Big].$$

Lifshitz pressure between two dielectric half-spaces separated by a distance a at zero temperature [E.M.Lifshitz, The theory of molecular attractive forces between solids, *Zh. Eksp. Teor. Fiz.* **29**, 94–110 (1955)]:

$$P = -\frac{1}{2\pi^2} \int_0^\infty d\omega \int_0^\infty dk_r k_r \exp(-2\sqrt{\omega^2 + k_r^2} a) \sqrt{\omega^2 + k_r^2} \times \left(\frac{r_{TM1}(i\omega, k_r) r_{TM2}(i\omega, k_r)}{\Delta_{TM}(i\omega, k_r)} + \frac{r_{TE1}(i\omega, k_r) r_{TE2}(i\omega, k_r)}{\Delta_{TE}(i\omega, k_r)} \right). \quad (9)$$

Lifshitz energy on a unit surface:

$$\frac{E}{S} = \frac{1}{2} \iiint \frac{d\omega dk_x dk_y}{(2\pi)^3} \ln\Big[\Delta_{TM}(i\omega, k_r) \Delta_{TE}(i\omega, k_r)\Big], \quad (10)$$

here $\Delta_{TM} = 1 - r_{TM1} r_{TM2} e^{2ik_z a}$, $\Delta_{TE} = 1 - r_{TE1} r_{TE2} e^{2ik_z a}$, $k_r = \sqrt{k_x^2 + k_y^2}$, $k_z = \sqrt{\omega^2 - k_r^2}$.

The ground state energy of the bosonic system:

$$E = \sum_i \frac{\omega_i}{2}, \tag{11}$$

the sum is over all eigenfrequencies of the system.
To evaluate (11) the argument principle can be used:

$$\frac{1}{2\pi i} \oint \phi(\omega) \frac{d}{d\omega} \ln f(\omega) d\omega = \sum \phi(\omega_0) - \sum \phi(\omega_\infty), \tag{12}$$

where

$$\phi(\omega) = \omega/2$$

and

$$f(\omega) = \det(I - R_{2up}(\omega) R_{1down}(\omega)). \tag{13}$$

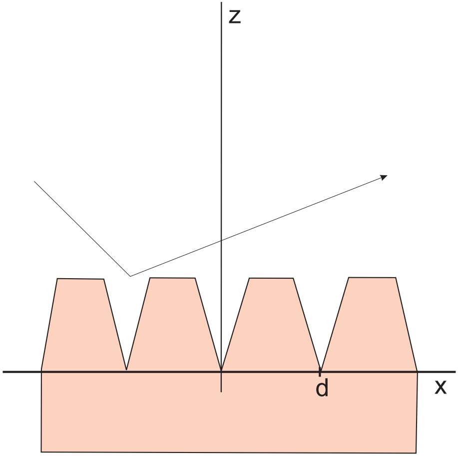
Scattering approach: flat boundaries

- K. Schram, On the macroscopic theory of retarded Van der Waals forces, *Phys. Lett. A* **43**, 282–284 (1973).
- Y.S. Barash and V.L. Ginzburg, Electromagnetic fluctuations in matter and molecular (Van-der-Waals) forces between them, *Sov. Phys. Usp.* **18**, 305–322 (1975).
- V.V. Nesterenko and I.G. Pirozhenko, Lifshitz formula by a spectral summation method, *Phys. Rev. D* **86**, 052503 (2012).
- S.Y. Buhmann. Dispersion Forces. Springer: Berlin/Heidelberg, Germany, 2012; Volumes I–II.
- V.N. Marachevsky and A.A. Sidelnikov, Green functions scattering in the Casimir effect, *Universe* **7**, 195 (2021).

Scattering approach: curved boundaries

- T. Emig, R.L. Jaffe, M. Kardar and A. Scardicchio, Casimir interaction between a plate and a cylinder, *Phys. Rev. Lett.* **96**, 080403 (2006).
- T. Emig, N. Graham, R.L. Jaffe and M. Kardar, Casimir forces between arbitrary compact objects, *Phys. Rev. Lett.* **99**, 170403 (2007).
- A. Lambrecht and V.N. Marachevsky, Casimir interaction of dielectric gratings, *Phys. Rev. Lett.* **101**, 160403 (2008).
- S.J. Rahi, T. Emig, N. Graham, R.L. Jaffe and M. Kardar, Scattering theory approach to electromagnetic Casimir forces, *Phys. Rev. D* **80**, 085021 (2009).
- A. Canaguier-Durand, P.A. Maia Neto, I. Cavero-Pelaez, A. Lambrecht and S. Reynaud, Casimir interaction between plane and spherical metallic surfaces, *Phys. Rev. Lett.* **102**, 230404 (2009).
- M. Bordag and I. Pirozhenko, Vacuum energy between a sphere and a plane at finite temperature, *Phys. Rev. D* **81**, 085023 (2010).

Diffraction grating



O.M. Rayleigh, On the dynamical theory of gratings, *Proc. Roy. Soc. A* **79**, 399–415 (1907).

Rayleigh decomposition for 1D gratings.

Rayleigh expansion for an incident electromagnetic wave on a single grating

$$E_y(x,z) = I_p^{(e)} \exp(i\alpha_p x - i\beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(e)} \exp(i\alpha_n x + i\beta_n^{(1)} z),$$

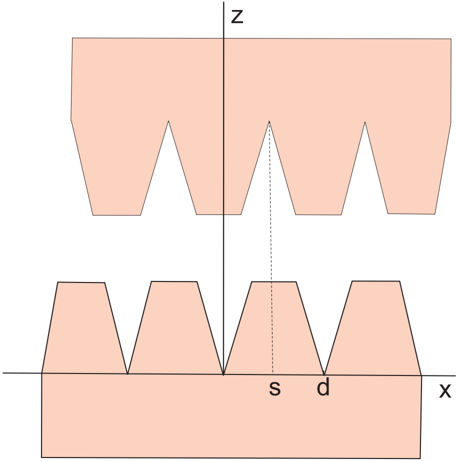
$$H_y(x,z) = I_p^{(h)} \exp(i\alpha_p x - i\beta_p^{(1)} z) + \sum_{n=-\infty}^{+\infty} R_{np}^{(h)} \exp(i\alpha_n x + i\beta_n^{(1)} z).$$

Here $\alpha_p = k_x + 2\pi p/d$ and $\beta_p^{(1)2} = \omega^2 - k_y^2 - \alpha_p^2$.
The reflection matrix is constructed as follows:

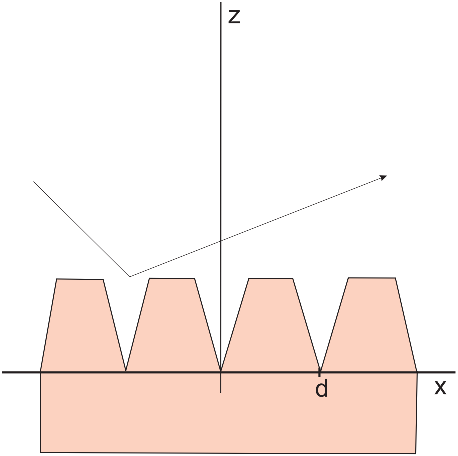
$$R_1(\omega) = \begin{pmatrix} R_{n_1 l_1}^{(e)}(I_p^{(e)} = \delta_{pl_1}, I_p^{(h)} = 0) & R_{n_2 l_2}^{(e)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl_2}) \\ R_{n_3 l_3}^{(h)}(I_p^{(e)} = \delta_{pl_3}, I_p^{(h)} = 0) & R_{n_4 l_4}^{(h)}(I_p^{(e)} = 0, I_p^{(h)} = \delta_{pl_4}) \end{pmatrix}.$$

Rayleigh expansion is exact outside gratings. The unknown coefficients can be determined from the exact solution of Maxwell equations.

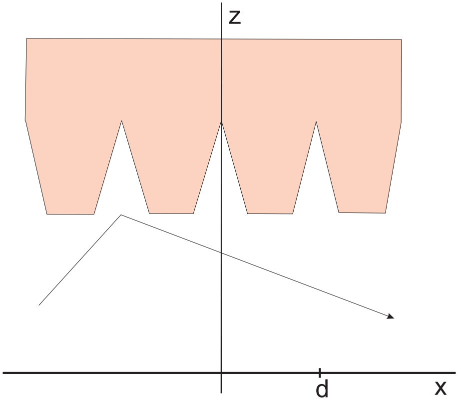
Two diffraction gratings



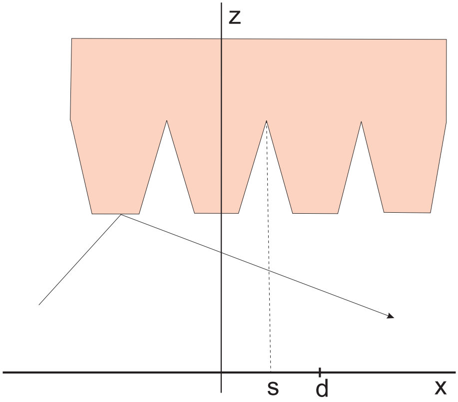
Reflection from the lower grating constituting R_{1down}



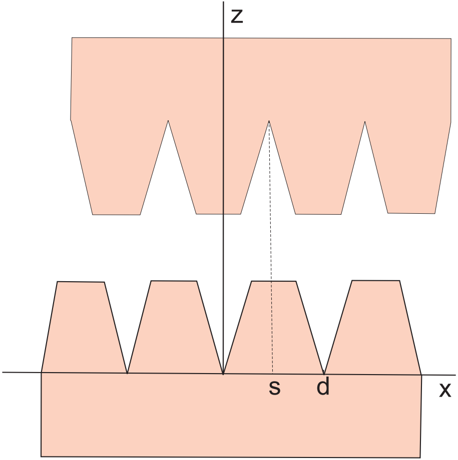
Reflection from the upper grating constituting R_{2up}



Reflection from the upper grating constituting R_{2up}



$R_{2up} R_{1down}$



Casimir energy of two gratings

$$E = \frac{1}{(2\pi)^3}\int_0^{+\infty} d\omega \int_{-\infty}^{+\infty} dk_y \int_{-\frac{\pi}{d}}^{\frac{\pi}{d}} dk_x \quad \ln\det\Big(I - R_{2up}R_{1down}\Big)$$

$$R_{2up}(i\omega, kx, ky) = Q^* K(i\omega) R_{2down}(i\omega, kx, -ky) K(i\omega) Q, \quad (14)$$

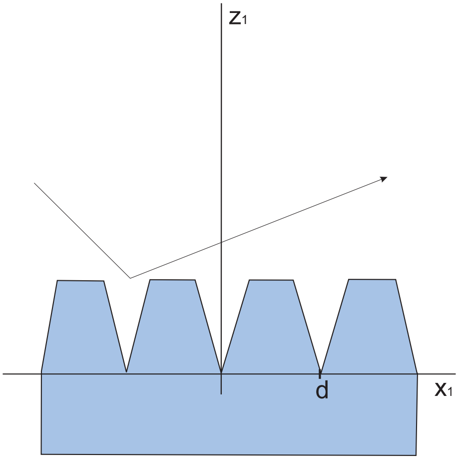
$$K(i\omega) = \begin{pmatrix} G_1 & 0 \\ 0 & G_1 \end{pmatrix}, \quad (15)$$

with matrix elements $e^{-L\sqrt{\omega^2+k_y^2+(k_x+\frac{2\pi p}{d})^2}}, p = -N \dots N$ on the main diagonal of a matrix G_1,

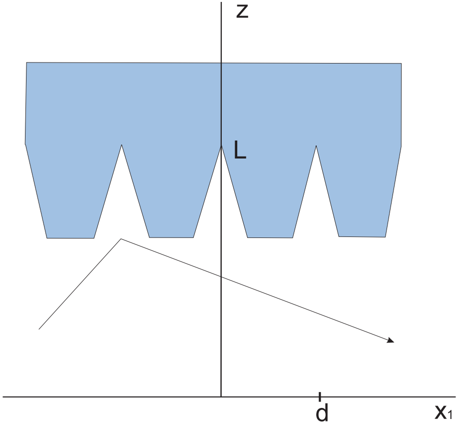
$$Q = \begin{pmatrix} G_2 & 0 \\ 0 & G_2 \end{pmatrix}, \quad (16)$$

with matrix elements $e^{2\pi ims/d}, p = -N \dots N$ on the main diagonal of a matrix G_2. [A.Lambrecht and V.N.Marachevsky, *Phys.Rev.Lett.* **101**, 160403 (2008); *Int. J. Mod. Phys. A* **24**, 1789–1795 (2009).].

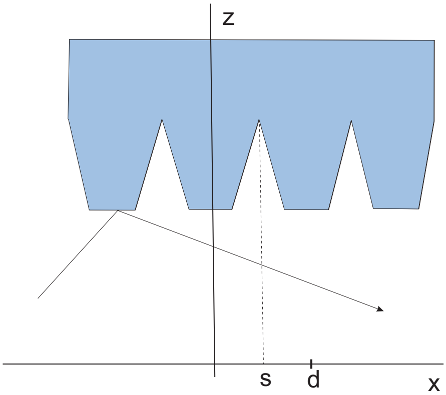
Reflection R_{2down}.



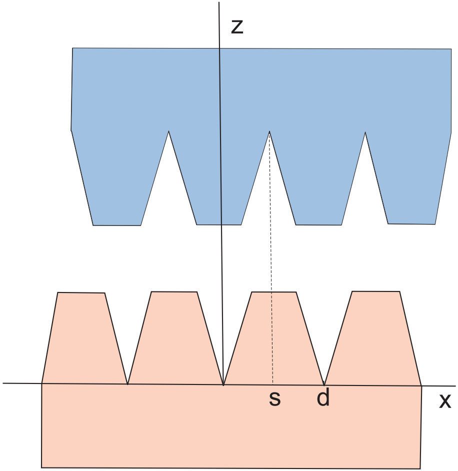
Change of coordinates $z = -z_1 + L$, $y = -y_1$ *in the solution.*



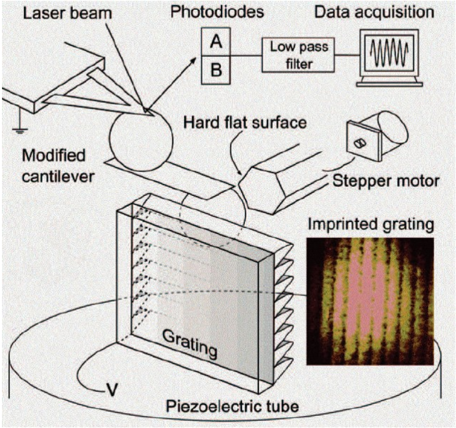
Lateral change of coordinates $x = x_1 - s$ in the solution. Reflection from the upper grating constituting R_{2up}.



$R_{2up}R_{1down}$



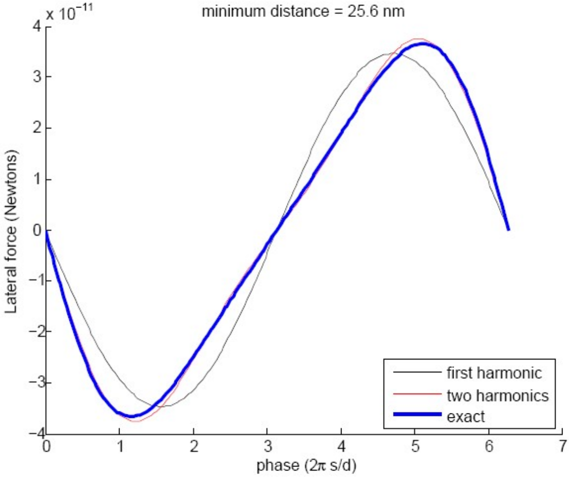
Lateral Casimir force experiment



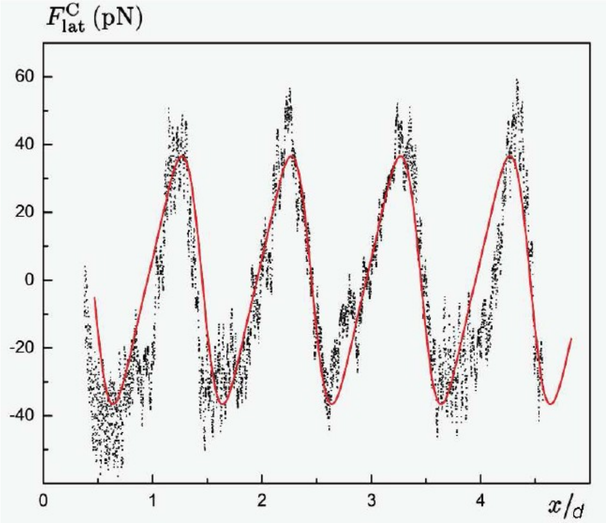
H.-C.Chiu, G.L.Klimchitskaya, V.N.Marachevsky, V.M.Mostepanenko and U.Mohideen, *Phys.Rev.B* **80**, 121402(R) (2009); *Phys.Rev.B* **81**, 115417 (2010).

Lateral Casimir force experiment

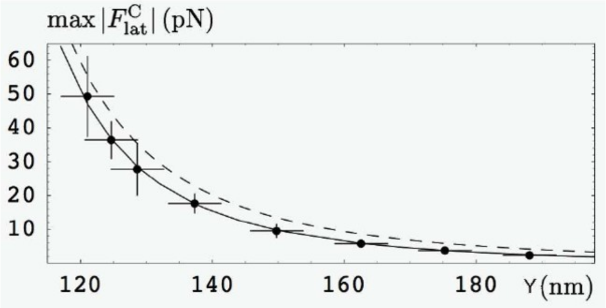
Consider gold sinusoidal corrugations with amplitudes A1= 85.4 nm, A2=13.7 nm, diameter of the sphere 2R=194.8 micrometers.



Lateral Casimir force experiment



Lateral Casimir force experiment



Maximum values of the measured lateral Casimir force are shown as crosses. Solid and dashed lines are predictions of the exact theory and the Proximity Force Approximation based on Lifshitz theory for two dielectric half-spaces.

Normal Casimir force experiments with gratings

- H.B. Chan, Y. Bao, J. Zou, R.A. Cirelli, F. Klemens, W. Mansfield and C. Pai, Measurement of the Casimir force between a gold sphere and a silicon surface with nanoscale trench arrays, *Phys. Rev. Lett.* **101**, 030401 (2008).
- Y. Bao, R. Guérout, J. Lussange, A. Lambrecht, R.A. Cirelli, F. Klemens, W.M. Mansfield, C.S. Pai and H.B. Chan, Casimir force on a surface with shallow nanoscale corrugations: Geometry and finite conductivity effects, *Phys. Rev. Lett.* **105**, 250402 (2010).
- A.W. Rodriguez, F. Capasso and S.G. Johnson, The Casimir effect in microstructured geometries, *Nature Photon.* **5**, 211-221 (2011).
- F. Intravaia, S. Koev, I.W. Jung, A.A. Talin, P.S. Davids, R.S. Decca, V.A. Aksyuk, D.A.R. Dalvit and D. López, Strong Casimir force reduction through metallic surface nanostructuring, *Nat. Commun.* **4**, 2515 (2013).

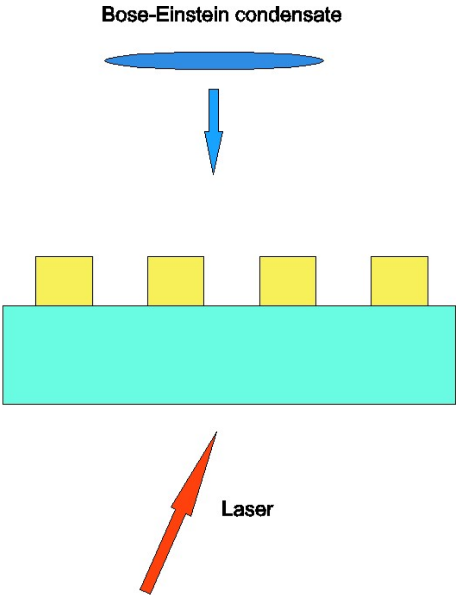
Normal Casimir force experiments with gratings

- L. Tang, M. Wang, C.Y. Ng, M. Nikolic, C.T. Chan, A.W. Rodriguez and H.B. Chan, Measurement of non-monotonic Casimir forces between silicon nanostructures, *Nature Photon.* **11**, 97–101 (2017).
- M. Wang, L. Tang, C.Y. Ng, R. Messina, B. Guizal, J.A. Crosse, M. Antezza, C.T. Chan and H.B. Chan, Strong geometry dependence of the Casimir force between interpenetrated rectangular gratings, *Nat. Commun.* **12**, 600 (2021).

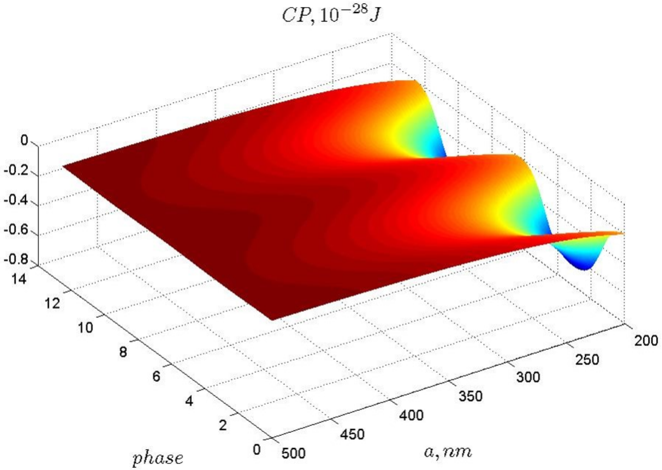
Casimir-Polder potential atom - grating experiment

- H. Bender, C. Stehle, C. Zimmermann, S. Slama, J. Fiedler, S. Scheel, S.Y. Buhmann and V.N. Marachevsky, Probing atom-surface interactions by diffraction of Bose-Einstein condensates, *Phys. Rev. X* **4**, 011029 (2014).
- S.Y.Buhmann, V.N. Marachevsky and S.Scheel, Impact of anisotropy on the interaction of an atom with a one-dimensional nano-grating, *Int. J. Mod. Phys. A* **31**, 1641029 (2016).

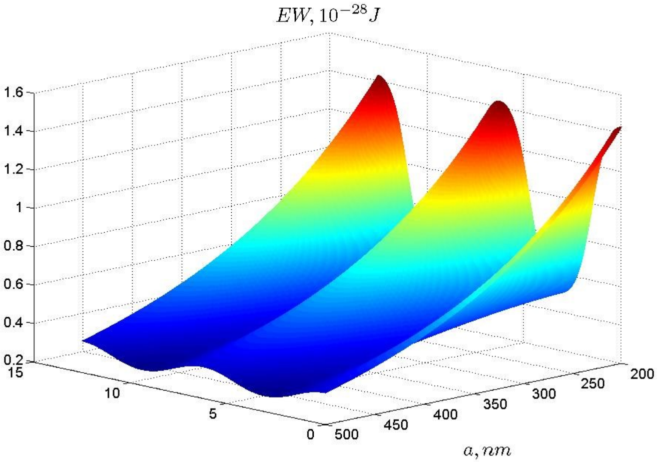
Casimir-Polder potential atom - grating experiment



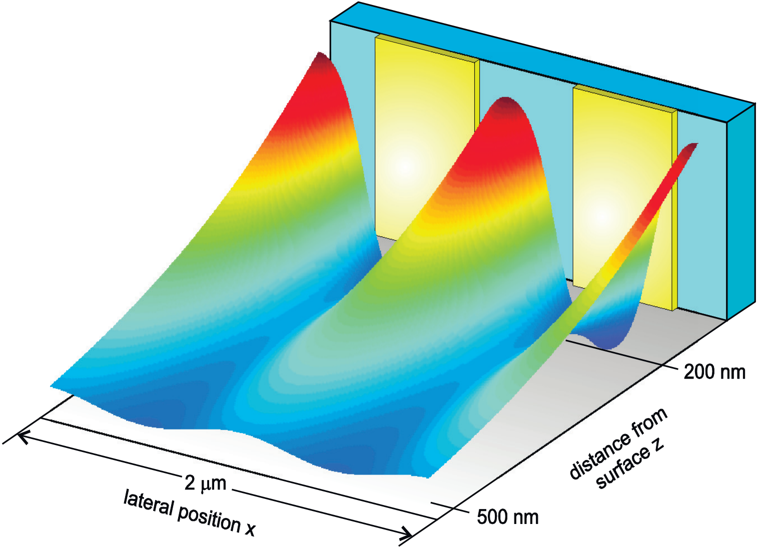
Casimir-Polder potential atom - grating experiment



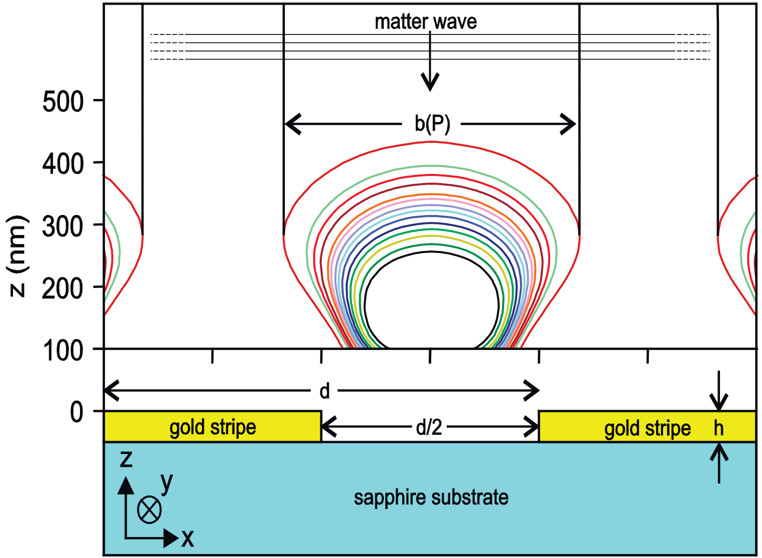
Casimir-Polder potential atom - grating experiment



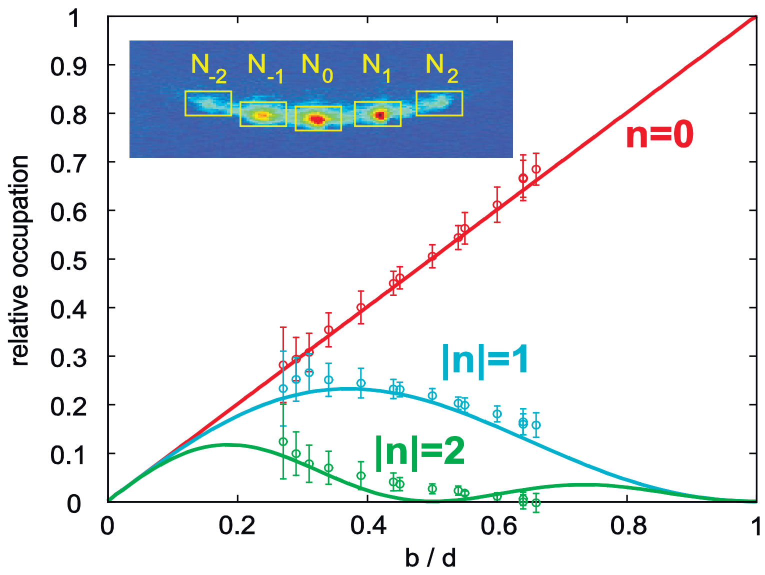
Casimir-Polder potential atom - grating experiment



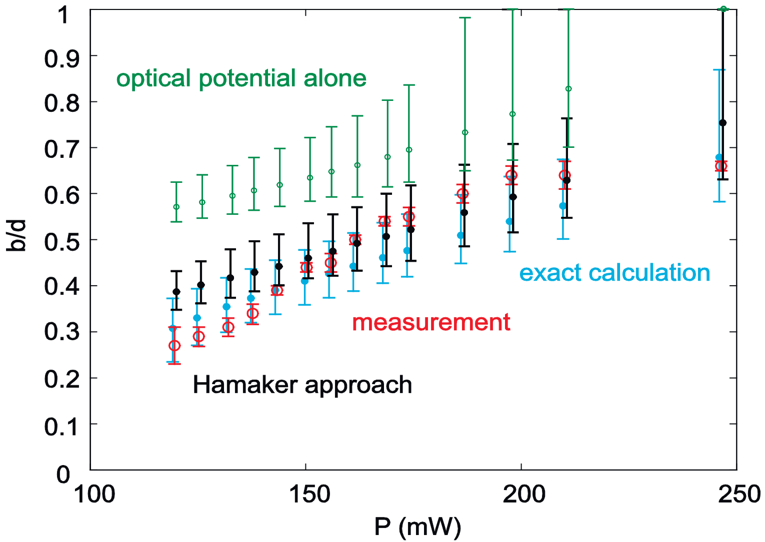
Casimir-Polder potential atom - grating experiment



Casimir-Polder potential atom - grating experiment



Casimir-Polder potential atom - grating experiment



- E. I. Kats, Van der Waals forces in non-isotropic systems, *Sov. Phys. JETP* **33**, 634 (1971).
- V. A. Parsegian and G. H.Weiss, Nonretarded van derWaals interaction between anisotropic long thin rods at all angles, *J. Chem. Phys.* **56**, 4393 (1972).
- Y. S. Barash, Moment of van der Waals forces between anisotropic bodies, *Radiophys. Quantum Electron.* **21**, 1138 (1978).
- S. J. van Enk, Casimir torque between dielectrics, *Phys. Rev. A* **52**, 2569 (1995).
- J.N. Munday, D. Iannuzzi, Y. Barash and F. Capasso, Torque on birefringent plates induced by quantum fluctuations, *Phys. Rev. A* **71**, 042102 (2005).

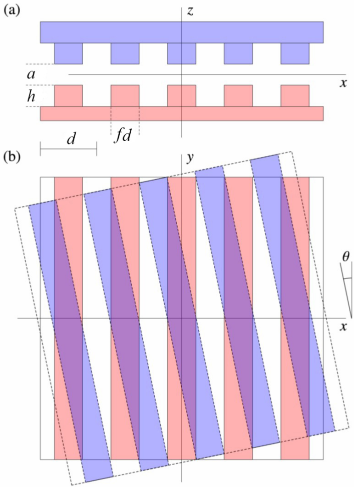
Torque in the Casimir effect

- R.B. Rodrigues, P.A. Maia Neto, A. Lambrecht and S. Reynaud, Vacuum-induced torque between corrugated metallic plates, *Europhys. Lett.* **76**, 822 (2006).
- T.G. Philbin and U. Leonhardt, Alternative calculation of the Casimir forces between birefringent plates, *Phys. Rev. A* **78**, 042107 (2008).
- X. Chen and J. C. H. Spence, On the measurement of the Casimir torque, *Phys. Status Solidi B* **248**, 2064 (2011).
- R. Guérout, C. Genet, A. Lambrecht and S. Reynaud, Casimir torque between nanostructured plates, *Europhys. Lett.* **111**, 44001 (2015).
- D. A. T. Somers and J. N. Munday, Casimir-Lifshitz Torque Enhancement by Retardation and Intervening Dielectrics, *Phys. Rev. Lett.* **119**, 183001 (2017).

Torque in the Casimir effect

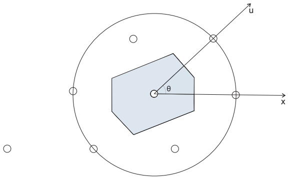
- D. A. T. Somers, J. L. Garrett, K. J. Palm, and J. N. Munday, Measurement of the Casimir torque, *Nature* **564**, 386 (2018).
- M.Antezza, H.B.Chan, B.Guizal, V.N.Marachevsky, R.Messina and M.Wang, Giant Casimir torque between rotated gratings and the $\theta = 0$ anomaly, *Phys.Rev.Lett.* **124**, 013903 (2020).

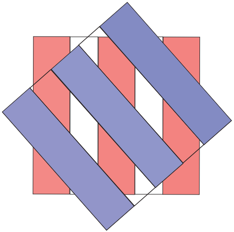
Rotated gratings



Consider the system of two Au rectangular gratings with parameters $d = 400$ nm, $f = 0.5$, $h = 200$ nm, $a = 100$ nm; the angle of rotation is θ.

Rotated gratings. Reciprocal lattice space.

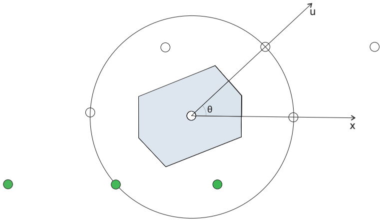


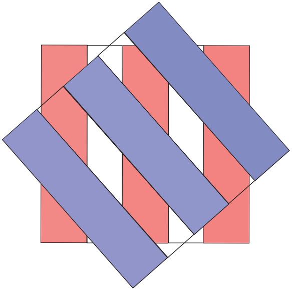


The vectors which are coupled by diffraction can be written as $\mathbf{k}_{nm} = \mathbf{k} + \frac{2\pi}{d}(n\mathbf{e}_x + m\mathbf{e}_u)$, where the vector $\mathbf{k}$ belongs to the first Brillouin zone.

Reflections from the lower grating constituting R_{1down}.

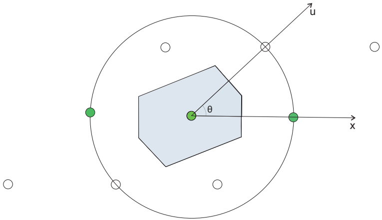
Reciprocal lattice vectors with $m = -1$ are highlighted green.

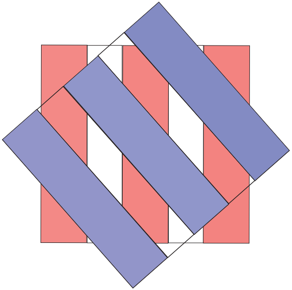




Reflections from the lower grating constituting R_{1down}

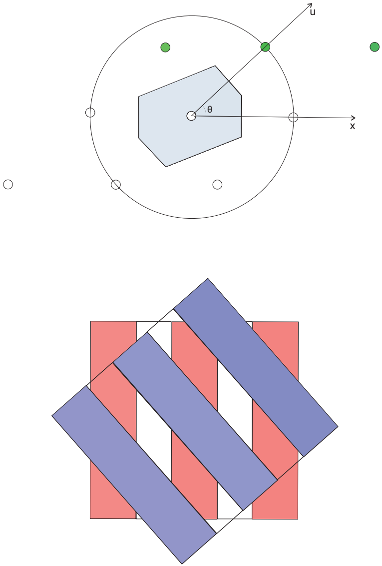
Reciprocal lattice vectors with $m = 0$ are highlighted green.





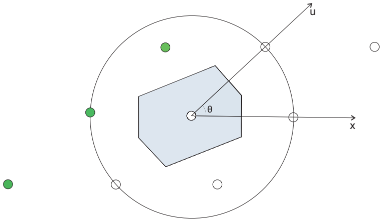
Reflections from the lower grating constituting R_{1down}

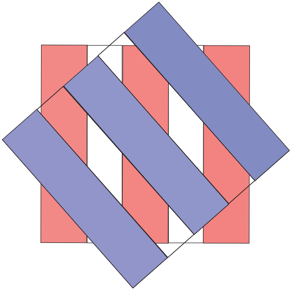
Reciprocal lattice vectors with $m = 1$ are highlighted green.



Reflections from the upper grating constituting R_{2up}

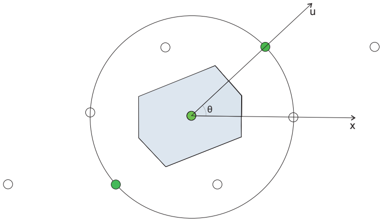
Reciprocal lattice vectors with $n = -1$ are highlighted green.

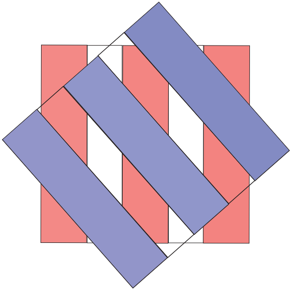




Reflections from the upper grating constituting R_{2up}

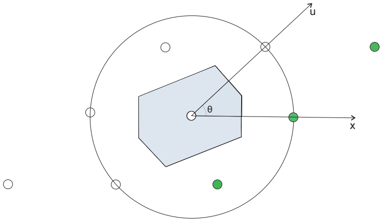
Reciprocal lattice vectors with $n = 0$ are highlighted green.

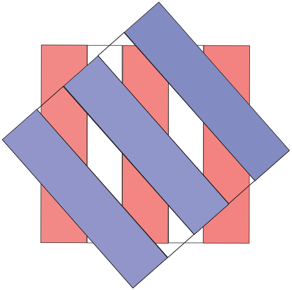




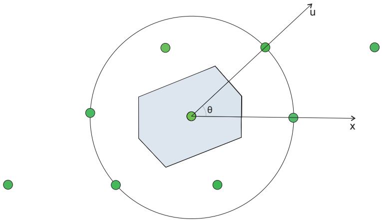
Reflections from the upper grating constituting R_{2up}

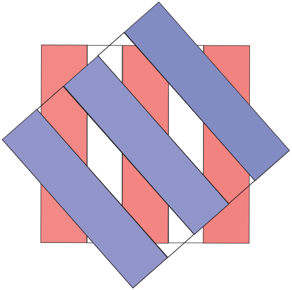
Reciprocal lattice vectors with $n = 1$ are highlighted green.





Casimir energy of two gratings depends on $R_{2up}R_{1down}$





Casimir energy and torque for two infinite rotated gratings

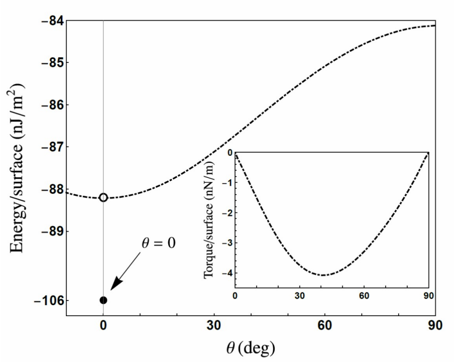
The Casimir energy of two infinite rotated gratings is defined by Rayleigh reflection coefficients contained in matrices R_{1down}, R_{2up} of the order $2(2N+1)^2$:

$$E(z,\theta) = \frac{1}{(2\pi)^3} \int_0^{+\infty} d\omega \int_{BZ} dk_x dk_y \ln \det\Big(I - R_{2up}(i\omega, k_x, k_y) R_{1down}(i\omega, k_x, k_y)\Big). \quad (17)$$

The Casimir torque:

$$\tau = -\frac{\partial E(z,\theta)}{\partial \theta}. \quad (18)$$

Torque for infinite rotated gratings. 1D-2D geometric transition.



M.Antezza, H.B.Chan, B.Guizal, V.N.Marachevsky, R.Messina and M.Wang, Giant Casimir torque between rotated gratings and the $\theta = 0$ anomaly, *Phys.Rev.Lett.* **124**, 013903 (2020).

Energy discontinuity at rotation angle $\theta = 0$

Consider wave vectors coupled by diffraction in reciprocal lattice space in 1D system (strictly for $\theta = 0$):

$$\mathbf{k}_n = \mathbf{k} + \frac{2\pi n}{d}\mathbf{e}_x, \tag{19}$$

the first Brillouin zone is $-\pi/d < k_x < \pi/d$, while k_y takes all real values.
Consider wave vectors coupled by diffraction in reciprocal lattice space in 2D system (for any finite θ):

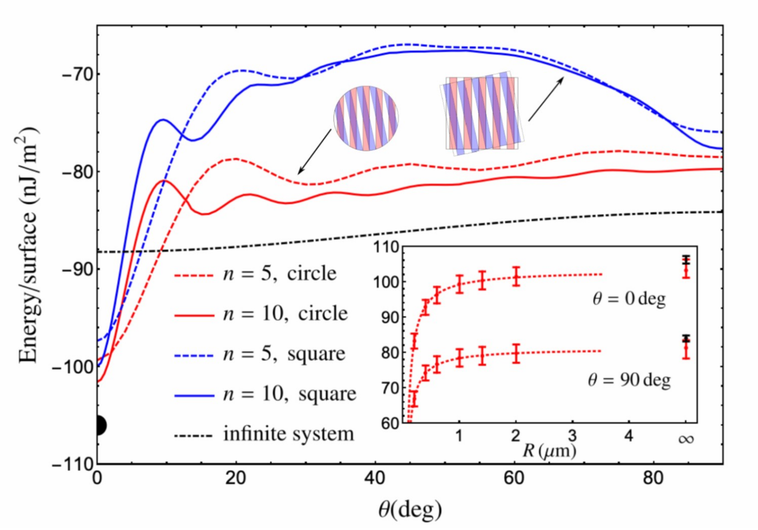
$$\mathbf{k}_{nm} = \mathbf{k} + \frac{2\pi}{d}(n\mathbf{e}_x + m\mathbf{e}_u) \tag{20}$$

While for two aligned gratings the y component of the total wave vector is strictly conserved in any scattering process, this conservation law is lost even for a small non-vanishing value of the rotation angle θ, since (see Eq.(20)) changing the value of the diffraction order m modifies the values of both x and y components of the wave vector.

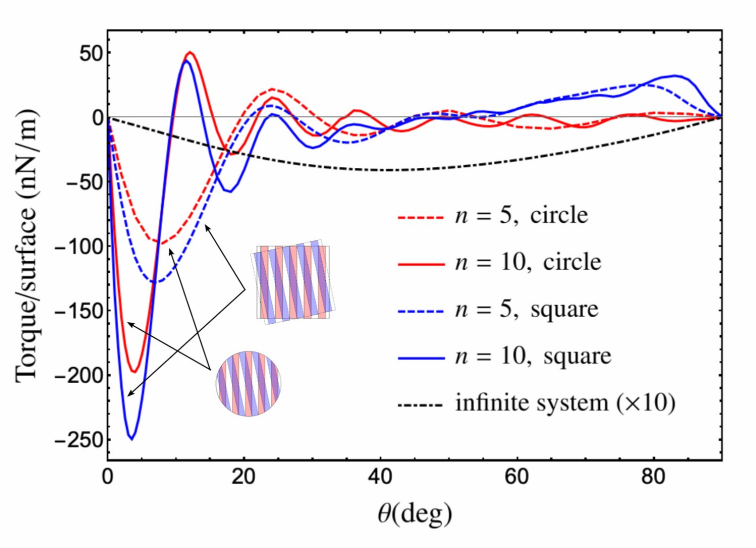
Energy discontinuity at rotation angle $\theta = 0$

The reason for the appearance of energy discontinuity at rotation angle $\theta = 0$ is breaking of conservation of the k_y component of the wave vector in reciprocal lattice space due to rotation of the system and, as a result, the fundamental change of the structure of reciprocal lattice space.

Casimir energy for finite rotated gratings



Casimir torque for finite rotated gratings



Conclusions

1. 1D-2D geometric transition (energy discontinuity at rotation angle $\theta = 0$) is found in the system of two infinite gratings with coinciding periods.
2. There is a conservation of k_y momentum in 1d system at rotation angle $\theta = 0$, breaking of k_y momentum conservation takes place in 2D system at any finite rotation angle θ.
   The reason for the appearance of energy discontinuity at rotation angle $\theta = 0$ is breaking of conservation of the k_y component of the wave vector in reciprocal lattice space due to rotation of the system and, as a result, the fundamental change of the structure of reciprocal lattice space.
3. Giant torque is found in the system of two finite rotated gratings. Torque is growing without bounds when the size of gratings increases.

Conclusions

4. The effect should be of strong interest due to a novel mechanism of symmetry breaking in the Brillouin zone which may be used to find analogous effects in various physical systems with spatial periodicity.

Thank you for attention !

Support from the project RSF №22 – 13 – 00151 is acknowledged.