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Pair quarkonium production in Higgs boson decay

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Contents

1. Introduction
2. General formalism
3. Pair production of B_c mesons
4. Pair production of J/Ψ, Υ mesons
5. Numerical results

Recently, a search for rare Higgs decays into a pair of heavy vector quarkonia, $H \to VV$ ($V = J/\Psi$, Υ), has been firstly performed by the CMS Collaboration, and upper limits on the branching fractions have been measured to be

1. A.M.Sirunyan et al. [CMS Collaboration], Search for Higgs and Z-boson decays to J/Ψ Υ pairs in the four-muon final state in proton-proton collisions at $\sqrt{s} = 13$ TeV, Phys. Lett. B **797**, 134811 (2019).

$$B(H \to J/\Psi, J/\Psi) < 1.8 \cdot 10^{-3},$$
$$B(H \to \Upsilon(1S), \Upsilon(1S)) < 1.4 \cdot 10^{-3},$$

2. [CMS Collaboration], Search for Higgs boson decays into Z and J/Ψ and for Higgs and Z boson decays into J/Ψ or Υ pairs in pp collisions at $\sqrt{s} = 13.$ TeV, arXiv:2206.03525v1 [hep-ex] 7 Jun 2022.

$$B(H \to J/\Psi, J/\Psi) < 3.8 \cdot 10^{-4},$$
$$B(H \to \Upsilon(1S), \Upsilon(1S)) < 1.7 \cdot 10^{-3},$$

Theoretically, these processes have already been studied in the literature

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The aim of the present work is to give new analysis of pair quarkonium production in Higgs boson decays in the Standard Model:

1. We consider quark-gluon, quark-photon, photon-photon and Z-boson mechanisms of the pair charmonium and bottomonium production.
2. We take into account relativistic corrections connected with the relative motion of heavy quarks.

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Among the parameters of the Higgs sector, the coupling constants of the Higgs boson with various bosons g(HZZ), g(HWW), leptons $g(H\tau\tau)$, $g(H\mu\mu)$ and quarks g(Hcc), g(Hbb) stand out. They determine the decay processes of the Higgs boson into various particles. Due to its large mass and the presence of coupling constants with different particles, the Higgs boson has numerous decay channels. The decay channel of the Higgs boson into a pair of heavy quarks is also interesting because it creates the possibility of the production of bound states of heavy quarks. Rare exclusive decay processes of the Higgs boson into a pair of charmonium or bottomonium are of obvious interest both for studying the decay mechanisms and for studying the properties of bound states of quarks.

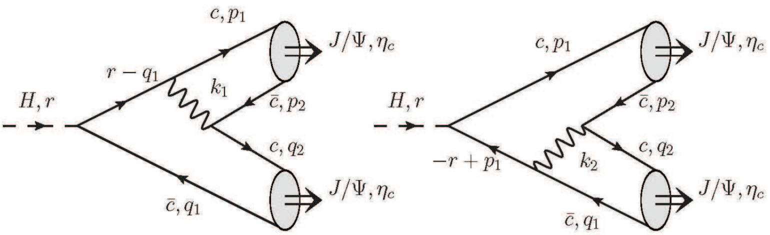

Figure: Quark-gluon mechanism of the pair charmonium production in the Higgs boson decay. Dashed line shows the Higgs boson and wavy line corresponds to the gluon.

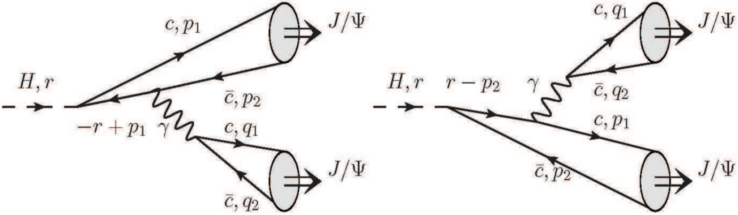

Figure: Quark-photon mechanism of the pair charmonium production in the Higgs boson decay. Dashed line shows the Higgs boson and wavy line corresponds to the photon.

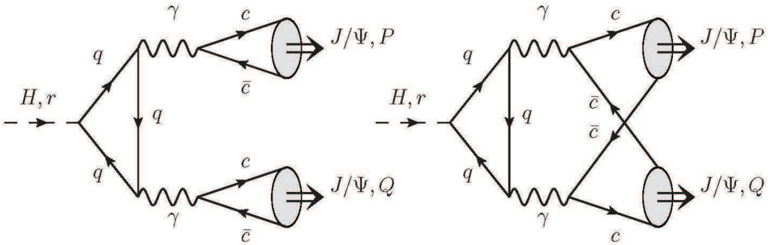

Figure: Quark loop mechanism of the pair charmonium production in the Higgs boson decay. Dashed line shows the Higgs boson and wavy line corresponds to the photon.

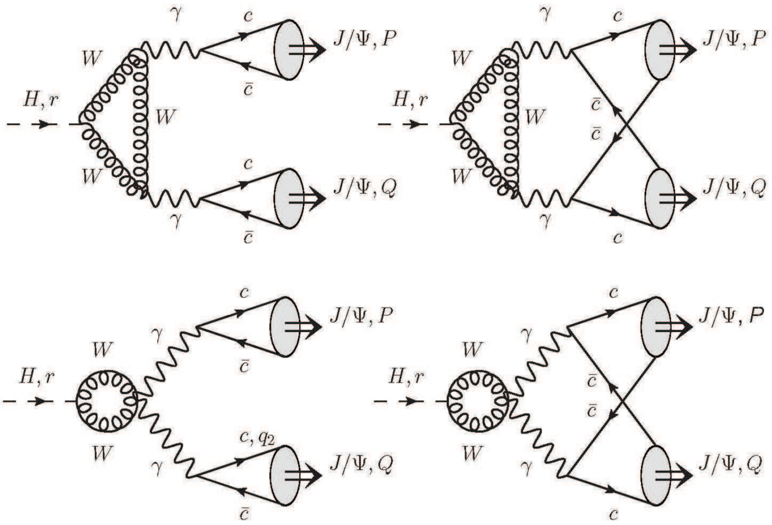

Figure: W-boson loop mechanism of the pair charmonium production in the Higgs boson decay. Dashed line shows the Higgs boson and wavy line corresponds to the photon.

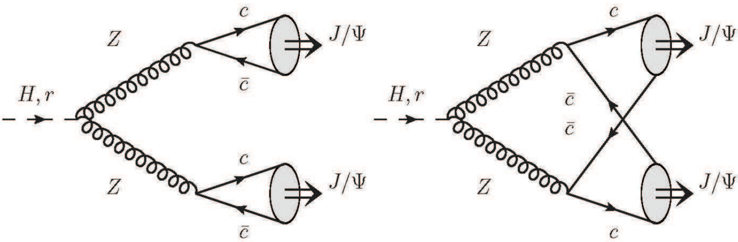

Figure: Z-boson mechanism of the pair charmonium production in the Higgs boson decay. Dashed line shows the Higgs boson and wavy line corresponds to the Z-boson.

Four-momenta of heavy quarks and antiquarks can be expressed in terms of relative and total four momenta as follows:

$$p_1 = \eta_1 P + p,\ p_2 = \eta_2 P - p,\ (p\cdot P) = 0,\ \eta_i = \frac{M_1^2 \pm m_1^2 \mp m_2^2}{2M_1^2},\quad (1)$$

$$q_1 = \rho_1 Q + q,\ q_2 = \rho_2 Q - q,\ (q\cdot Q) = 0,\ \rho_i = \frac{M_2^2 \pm m_1^2 \mp m_2^2}{2M_2^2}.$$

For pair production of quarkonia in the leading order of perturbation theory, it is necessary to obtain at the first stage two free quarks and two free antiquarks. Then they can form bound states with some probability at the next stage. In the quasipotential approach the decay amplitude can be presented as a convolution of a perturbative production amplitude of two c-quark and $\bar{c}$-antiquark pairs and the quasipotential wave functions of the final mesons:

$$\mathcal{M}(P,Q) = -i(\sqrt{2}G_F)^{\frac{1}{2}}\frac{2\pi}{3}M_{\mathcal{Q}\bar{\mathcal{Q}}}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\times \quad (2)$$

$$\times Tr\left\{\Psi^{\mathcal{P},\mathcal{V}}(p,P)\Gamma_1^{\nu}(p,q,P,Q)\Psi^{\mathcal{P},\mathcal{V}}(q,Q)\gamma_\nu + \Psi^{\mathcal{P},\mathcal{V}}(q,Q)\Gamma_2^{\nu}(p,q,P,Q)\Psi^{\mathcal{P},\mathcal{V}}(p,P)\gamma_\nu\right\},$$

where $M_{\mathcal{Q}\bar{\mathcal{Q}}}$ is the mass of quarkonium, The relativistic wave functions of the bound quarks accounting for the transformation from the rest frame to the moving one with four momenta P, and Q, are

$$\Psi(p,P)=\frac{\Psi_0(\mathbf{p})}{\left[\frac{\epsilon(p)}{m}\frac{\epsilon(p)+m}{2m}\right]}\left[\frac{\hat v_1-1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}-\frac{\hat p}{2m}\right] \quad (3)$$

$$\gamma_5(\hat\varepsilon(P,S_z))(1+\hat v_1)\left[\frac{\hat v_1+1}{2}+\hat v_1\frac{\mathbf{p}^2}{2m(\epsilon(p)+m)}+\frac{\hat p}{2m}\right],$$

$$\Psi(q,Q)=\frac{\Psi_0(\mathbf{q})}{\left[\frac{\epsilon(q)}{m}\frac{\epsilon(q)+m}{2m}\right]}\left[\frac{\hat v_2-1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}+\frac{\hat q}{2m}\right] \quad (4)$$

$$\gamma_5(\hat\varepsilon(Q,S_z))(1+\hat v_2)\left[\frac{\hat v_2+1}{2}+\hat v_2\frac{\mathbf{q}^2}{2m(\epsilon(q)+m)}-\frac{\hat q}{2m}\right],$$

where the hat symbol means a contraction of the four–vector with the Dirac gamma matrices; $v_1=P/M_{\mathcal{Q}\bar{\mathcal{Q}}}$, $v_2=Q/M_{\mathcal{Q}\bar{\mathcal{Q}}}$; $\epsilon(p)=\sqrt{m^2+\mathbf{p}^2}$, m is $c(b)$-quark mass, and $M_{\mathcal{Q}\bar{\mathcal{Q}}}$ is the mass of charmonium (bottomonium) state. $\varepsilon^\lambda(P,S_z)$ is the polarization vector of the $J/\Psi(\Upsilon)$ meson.

When constructing the decay amplitudes with the production of a pair of S-wave B_c mesons, we introduce projection operators $\hat{\Pi}^{\mathcal{P},\mathcal{V}}$ for states with total spin $S=0,\ 1$ of the following form:

$$\hat{\Pi}^{\mathcal{P}}=[v_2(0)\bar{u}_1(0)]_{S=0}=\gamma_5\frac{1+\gamma^0}{2\sqrt{2}},\quad \hat{\Pi}^{\mathcal{V}}=[v_2(0)\bar{u}_1(0)]_{S=1}=\hat{\varepsilon}\frac{1+\gamma^0}{2\sqrt{2}}. \quad (5)$$

After that, the total amplitude of the Higgs boson decay in the leading order in strong coupling constant α_s can be represented in the form:

$$\mathcal{M}=\frac{4\pi}{3}M_{B_c}\int\frac{d\mathbf{p}}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}\mathrm{Tr}\{\mathcal{T}_{12}+\mathcal{T}_{34}\}, \quad (6)$$

$$\mathcal{T}_{12}=\Gamma_c\alpha_b\Psi^{\mathcal{P},\mathcal{V}}_{B_c}(p,P)\left[\frac{\hat{p}_1-\hat{r}+m_1}{(r-p_1)^2-m_1^2}\gamma_\mu+\gamma_\mu\frac{\hat{r}-\hat{q}_1+m_1}{(r-q_1)^2-m_1^2}\right]D^{\mu\nu}(k_2)\Psi^{\mathcal{P},\mathcal{V}}_{B_c}(q,Q)\gamma_\nu, \quad (7)$$

$$\mathcal{T}_{34}=\Gamma_b\alpha_c\Psi^{\mathcal{P},\mathcal{V}}_{B_c}(q,Q)\left[\frac{\hat{p}_2-\hat{r}+m_2}{(r-p_2)^2-m_2^2}\gamma_\mu+\gamma_\mu\frac{\hat{r}-\hat{q}_2+m_2}{(r-q_2)^2-m_2^2}\right]D^{\mu\nu}(k_1)\Psi^{\mathcal{P},\mathcal{V}}_{B_c}(p,P)\gamma_\nu, \quad (8)$$

where subscripts 12, 34 denote the contributions of amplitudes 1 and 2, 3 and 4 in Fig. 1

Relative momenta p, q of heavy quarks enter in the gluon propagators $D_{\mu\nu}(k_{1,2})$ and quark propagators as well as in relativistic wave functions. Accounting for the small ratio of relative quark momenta p and q to the mass of the Higgs boson M_H, we can simplify the inverse denominators of quark and gluon propagators as follows:

$$\frac{1}{(p_1+q_1)^2}=\frac{1}{\eta_1^2 M_H^2},\quad \frac{1}{(p_2+q_2)^2}=\frac{1}{\eta_2^2 M_H^2}, \tag{9}$$

$$\frac{1}{(r-q_1)^2-m_1^2}=\frac{1}{\eta_2 M_H^2},\quad \frac{1}{(-r-p_1)^2-m_1^2}=\frac{1}{\eta_2 M_H^2}, \tag{10}$$

$$\frac{1}{(r-p_2)^2-m_1^2}=\frac{1}{\eta_1 M_H^2},\quad \frac{1}{(-r-q_2)^2-m_1^2}=\frac{1}{\eta_1 M_H^2}. \tag{11}$$

In the case of B_c meson production of the same mass $\rho_1=\eta_1$, $\rho_2=\eta_2$. The formulas (9)-(11) mean that we completely neglect corrections of the form $|\mathbf{p}|/M_H$, $|\mathbf{q}|/M_H$. At the same time, we keep in the amplitudes (7), (8) the second-order correction for small ratios $|\mathbf{p}|/m_{1,2}$, $|\mathbf{q}|/m_{1,2}$ relative to the leading order result.

Relativistic amplitudes of the B_c meson pairs production:

$$\mathcal{M}_{\mathcal{P}\mathcal{P}}=\frac{32\pi}{3M_H^4}(\sqrt{2}G_F)^{\frac{1}{2}}M_{B_c}^2\left[\frac{\alpha_b r_1}{\eta_2^3}F_{1P}+\frac{\alpha_c r_2}{\eta_1^3}F_{2P}\right]|\tilde\Psi_{\mathcal{P}}(0)|^2, \quad (12)$$

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}=\frac{32\pi}{3M_H^4}(\sqrt{2}G_F)^{\frac{1}{2}}M_{B_c}^2\varepsilon_1^\lambda\varepsilon_2^\sigma\left[\frac{\alpha_b r_1}{\eta_2^3}F_{1V}^{\lambda\sigma}+\frac{\alpha_c r_2}{\eta_1^3}F_{2V}^{\lambda\sigma}\right]|\tilde\Psi_{\mathcal{V}}(0)|^2, \quad (13)$$

where ε_1^λ, ε_2^σ are the polarization vectors of spin 1 B_c mesons, $r_1=m_1/M_{B_c}$, $r_2=m_2/M_{B_c}$, the parameter $r_3=\frac{M_H}{M_{B_c}}$. The decay widths of the Higgs boson into a pair of pseudoscalar and vector B_c mesons:

$$\Gamma_{\mathcal{P}\mathcal{P}}=\frac{512\sqrt{2}\pi G_F|\tilde\Psi_{\mathcal{P}}(0)|^4\sqrt{\frac{r_3^2}{4}-1}}{9M_H^3r_3^7}\left[\frac{\alpha_b r_1}{\eta_2^3}F_{1P}+\frac{\alpha_c r_2}{\eta_1^3}F_{2P}\right]^2, \quad (14)$$

$$F_{1P}=-r_1-\eta_1+\frac{3}{2}r_3^2-\frac{1}{2}r_1r_3^2-\frac{1}{2}\eta_1r_3^2+\omega_{01}(-12r_2+2r_2r_3^2)+ \quad (15)$$

$$\omega_{10}(2r_1+r_1r_3^2)+\omega_{10}\omega_{01}(6r_2-2r_1-\frac{3}{2}r_3^2-r_3^2r_2-r_3^2r_1),$$

$$\Gamma_{\mathcal{V}\mathcal{V}}=\frac{512\sqrt{2}\pi G_F|\tilde\Psi_{\mathcal{V}}(0)|^4\sqrt{\frac{r_3^2}{4}-1}}{9M_H^3r_3^7}\sum_{\lambda,\sigma}|\varepsilon_1^\lambda\varepsilon_2^\sigma\left[\frac{\alpha_b r_1}{\eta_2^3}F_{1V}^{\lambda\sigma}+\frac{\alpha_c r_2}{\eta_1^3}F_{2V}^{\lambda\sigma}\right]|^2, \quad (16)$$

$$F_{1V}^{\alpha\beta}=g_1v_1^\alpha v_2^\beta+g_2g^{\alpha\beta},\quad F_{2V}^{\alpha\beta}=\tilde g_1v_1^\alpha v_2^\beta+\tilde g_2g^{\alpha\beta}, \quad (17)$$

$$g_1=-1+\frac{1}{9}\omega_{10}\omega_{01},\quad g_2=-r_1-\eta_1+\frac{1}{2}r_3^2-\frac{4}{3}r_2\omega_{01}+2r_1\omega_{10}+\omega_{10}\omega_{01}(\frac{2}{3}r_2-\frac{2}{9}-\frac{1}{18}r_3^2).$$

General expressions for the decay rates (14), (16) contain numerous parameters. One part of the parameters, such as quark masses, the masses of B_c mesons are determined within the framework of quark models as a result of calculating the observed quantities. The parameters of quark models are found from the condition of the best agreement with experimental data. Another part of the relativistic parameters can also be found in the quark model as a result of calculating integrals with wave functions of quark bound states in the momentum representation.

In the case of S-states $\omega_{nk}^{P,V}$ are determined by the momentum integrals I_{nk} in the form:

$$I_{nk}^{P,V}=\int_0^\infty p^2R^{P,V}(p)\sqrt{\frac{(\epsilon_1(p)+m_1)(\epsilon_2(p)+m_2)}{2\epsilon_1(p)\cdot 2\epsilon_2(p)}}\left(\frac{\epsilon_1(p)-m_1}{\epsilon_1(p)+m_1}\right)^n\left(\frac{\epsilon_2(p)-m_2}{\epsilon_2(p)+m_2}\right)^k dp, \qquad (18)$$

$$\omega_{10}^{P,V}=\frac{I_{10}^{P,V}}{I_{00}^{P,V}},\ \omega_{01}^{P,V}=\frac{I_{01}^{P,V}}{I_{00}^{P,V}},\ \omega_{\frac12\frac12}^{P,V}=\frac{I_{\frac12\frac12}^{P,V}}{I_{00}^{P,V}},\ \omega_{20}^{P,V}=\frac{I_{20}^{P,V}}{I_{00}^{P,V}},\ \omega_{02}^{P,V}=\frac{I_{02}^{P,V}}{I_{00}^{P,V}},\ \omega_{11}^{P,V}=\frac{I_{11}^{P,V}}{I_{00}^{P,V}}, \qquad (19)$$

$$\tilde R(0)=\frac{\sqrt2}{\sqrt\pi}\int_0^\infty\sqrt{\frac{(\epsilon_1(p)+m_1)(\epsilon_2(p)+m_2)}{2\epsilon_1(p)\cdot 2\epsilon_2(p)}}p^2R(p)dp. \qquad (20)$$

Another source of relativistic corrections is related with the Hamiltonian of the heavy quark bound states which allows to calculate the bound state wave functions. The exact form of the bound state wave functions $\Psi^0_{B_c}(\mathbf{q})$ is important to obtain more reliable predictions for the decay widths.

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$$H = H_0 + \Delta U_1 + \Delta U_2, \quad H_0 = \sqrt{\mathbf{p}^2 + m_1^2} + \sqrt{\mathbf{p}^2 + m_2^2} - \frac{4\tilde{\alpha}_s}{3r} + (Ar + B), \tag{21}$$

$$\Delta U_1(r) = -\frac{\alpha_s^2}{3\pi r}\left[2\beta_0 \ln(\mu r) + a_1 + 2\gamma_E \beta_0\right], \quad a_1 = \frac{31}{3} - \frac{10}{9} n_f, \quad \beta_0 = 11 - \frac{2}{3} n_f, \tag{22}$$

$$\Delta U_2(r) = -\frac{2\alpha_s}{3m_1 m_2 r}\left[\mathbf{p}^2 + \frac{\mathbf{r}(\mathbf{r}\mathbf{p})\mathbf{p}}{r^2}\right] + \frac{2\pi\alpha_s}{3}\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)\delta(\mathbf{r}) + \frac{4\alpha_s}{3r^3}\left(\frac{1}{2m_1^2} + \frac{1}{m_1 m_2}\right)(\mathbf{S}_1\mathbf{L}) + \tag{23}$$

$$+\frac{4\alpha_s}{3r^3}\left(\frac{1}{2m_2^2} + \frac{1}{m_1 m_2}\right)(\mathbf{S}_2\mathbf{L}) + \frac{32\pi\alpha_s}{9m_1 m_2}(\mathbf{S}_1\mathbf{S}_2)\delta(\mathbf{r}) + \frac{4\alpha_s}{3m_1 m_2 r^3}\left[\frac{3(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})}{r^2} - (\mathbf{S}_1\mathbf{S}_2)\right] -$$

$$-\frac{\alpha_s^2(m_1 + m_2)}{m_1 m_2 r^2}\left[1 - \frac{4m_1 m_2}{9(m_1 + m_2)^2}\right],$$

$$\Delta V^{hfs}_{conf}(r) = f_V \frac{A}{8r}\left\{\frac{1}{m_1^2} + \frac{1}{m_2^2} + \frac{16}{3m_1 m_2}(\mathbf{S}_1\mathbf{S}_2) + \frac{4}{3m_1 m_2}\left[\frac{3(\mathbf{S}_1\mathbf{r})(\mathbf{S}_2\mathbf{r})}{r^2} - (\mathbf{S}_1\mathbf{S}_2)\right]\right\}, \tag{24}$$

Table: Numerical values of the relativistic parameters

$n^{2S+1}L_J$	M_{B_c}, GeV	$\tilde{R}(0)$, $\tilde{R}'(0)$	ω_{10}, $\tilde{\omega}_{10}$	ω_{01}, $\tilde{\omega}_{01}$	$\omega_{\frac{1}{2}\frac{1}{2}}$, $\tilde{\omega}_{\frac{1}{2}\frac{1}{2}}$	ω_{20}, $\tilde{\omega}_{20}$	ω_{02}, $\tilde{\omega}_{02}$	ω_{11}, $\tilde{\omega}_{11}$
1^1S_0	6.275	0.886	0.0728	0.0089	0.0254	0.0073	0.0001	0.0009
1^3S_1	6.317	0.750	0.0703	0.0086	0.0245	0.0069	0.0001	0.0009
2^3P_2	6.757	0.371	0.1020	0.0129	0.0362	0.0121	0.0002	0.0016
2^1P_1	6.736	0.531	0.1035	0.0131	0.0368	0.0124	0.0002	0.0016
2^3P_1	6.726	0.319	0.0998	0.0126	0.0354	0.0116	0.0002	0.0015
2^3P_0	6.688	0.281	0.0981	0.0123	0.0347	0.0113	0.0002	0.0014

Table: Numerical results for the decay widths

Final state $B_{\bar{b}c}B_{b\bar{c}}$	Nonrelativistic decay width $\Gamma_{nr}\cdot 10^{14}$ in GeV	Relativistic decay width $\Gamma_{rel}\cdot 10^{14}$ in GeV
$1^1S_0 + 1^1S_0$	125	45
$1^3S_1 + 1^3S_1$	125	20
$1^1S_0 + 2^3P_1$	0.32	0.12
$1^1S_0 + 2^1P_1$	0.77	0.59
$1^3S_1 + 2^3P_0$	9.81	1.45
$1^3S_1 + 2^3P_1$	0.87	0.16
$1^3S_1 + 2^1P_1$	0.16	0.10

Five amplitudes of pair charmonium production presented in Fig. 1-Fig. 5 have the similar structure:

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(1)} = \frac{256\pi}{3M_H^4}(\sqrt{2}G_F)^{\frac{1}{2}}\, mM_{\mathcal{Q}\bar{\mathcal{Q}}}\alpha_s\varepsilon_1^{\lambda}\varepsilon_2^{\sigma}F_{1,\mathcal{V}\mathcal{V}}^{\lambda\sigma}|\tilde{\Psi}_{\mathcal{V}}(0)|^2, \quad (25)$$

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(2)} = \frac{288\pi}{M_H^2 M_{\mathcal{Q}\bar{\mathcal{Q}}}}(\sqrt{2}G_F)^{\frac{1}{2}}\, me_Q^2\alpha\varepsilon_1^{\lambda}\varepsilon_2^{\sigma}F_{2,\mathcal{V}\mathcal{V}}^{\lambda\sigma}|\tilde{\Psi}_{\mathcal{V}}(0)|^2, \quad (26)$$

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(3)} = \frac{2052\pi^2}{m_Q M_{\mathcal{Q}\bar{\mathcal{Q}}}}(\sqrt{2}G_F)^{\frac{1}{2}}\, e_q^2 e_Q^2\alpha^2\varepsilon_1^{\lambda}\varepsilon_2^{\sigma}F_{3,\mathcal{V}\mathcal{V}}^{\lambda\sigma}|\tilde{\Psi}_{\mathcal{V}}(0)|^2, \quad (27)$$

$$\mathcal{M}_{\mathcal{V}\mathcal{V}}{}^{(4)} = \frac{48\pi^2 M_Z M_W}{M_{\mathcal{Q}\bar{\mathcal{Q}}}^4}(\sqrt{2}G_F)^{\frac{1}{2}}\, e_q^2\alpha^2\cos\theta_W\varepsilon_1^{\lambda}\varepsilon_2^{\sigma}F_{3,\mathcal{V}\mathcal{V}}^{\lambda\sigma}|\tilde{\Psi}_{\mathcal{V}}(0)|^2, \quad (28)$$

$$F_{i,\mathcal{V}\mathcal{V}}^{\alpha\beta} = g_1^{(i)}v_1^{\alpha}v_2^{\beta} + g_2^{(i)}g^{\alpha\beta}, \quad g_1^{(1)} = -2 + \frac{2}{9}\omega_{10}^2, \quad (29)$$

$$g_2^{(1)} = -1 - 2r + r_3^2 + \frac{4}{3}r\omega_1 + \frac{1}{9}\omega_1^2 + \frac{2}{3}r\omega_1^2 - \frac{1}{9}r_3^2\omega_1^2,$$

$$g_1^{(2)} = 4 - \frac{4}{9}\omega_1^2,\; g_2^{(2)} = 2 + 4r - 2r_3^2 - \frac{8}{3}r\omega_1 - \frac{2}{9}\omega_1^2 - \frac{4}{3}r\omega_1^2 + \frac{2}{9}r_3^2\omega_1^2, \quad (30)$$

$$g_{1,Q}^{(3)} = -A_Q(t)(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2) + B_Q(t)(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2), \quad (31)$$

$$g_{2,Q}^{(3)} = A_Q(t)\left(-1 - \frac{2}{3}\omega_1 - \frac{1}{9}\omega_1^2 + \frac{1}{2}r_3^2 + \frac{1}{3}\omega_1 r_3^2 + \frac{1}{18}\omega_1^2 r_3^2\right),$$

$$g_1^{(4)} = -A_W(t)\left(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2\right) + B_W(t)\left(1 + \frac{2}{3}\omega_1 + \frac{1}{9}\omega_1^2\right), \quad (32)$$

$$g_2^{(4)} = A_W(t)\left(-1 - \frac{2}{3}\omega_1 - \frac{1}{9}\omega_1^2 + \frac{1}{2}r_3^2 + \frac{1}{3}\omega_1 r_3^2 + \frac{1}{18}\omega_1^2 r_3^2\right), \quad (33)$$

The decay widths of the Higgs boson into a pair of pseudoscalar and vector charmonium states are determined by the following expressions:

$$\Gamma_{\mathcal{PP}} = \frac{2^{14}\sqrt{2}\pi\alpha_s^2 m^2 G_F|\tilde{\Psi}_{\mathcal{P}}(0)|^4\sqrt{\frac{r_3^2}{4}-1}}{9M_H^5 r_3^5} F_{\mathcal{PP}}^2, \quad (34)$$

$$\Gamma_{\mathcal{VV}} = \frac{2^{14}\sqrt{2}\pi\alpha_s^2 m^2 G_F|\tilde{\Psi}_{\mathcal{V}}(0)|^4\sqrt{\frac{r_3^2}{4}-1}}{9M_H^5 r_3^5} \sum_{\lambda,\sigma} |\varepsilon_1^\lambda \varepsilon_2^\sigma F_{\mathcal{VV}}^{\lambda\sigma}|^2, \quad (35)$$

$$F_{\mathcal{VV}}^{\lambda\sigma} = \Big[g_1^{(1)} + \frac{9}{16}r_3^2\frac{e_q^2\alpha}{\alpha_s}g_1^{(2)} + \sum_Q \frac{27\pi}{8}r_3^4\frac{e_Q^2 e_q^2\alpha^2 m_Q^2}{\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}}g_{1,Q}^{(3)} + \frac{9\pi e_q^2\alpha^2 r_3^4 M_Z M_W}{64\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}}g_1^{(4)} + \quad (36)$$

$$\frac{9M_H^4 M_W\alpha}{32M_Z^3 m M_{\mathcal{Q}\bar{\mathcal{Q}}}\alpha_s}\frac{\left(\frac{1}{2}-2q_c\sin^2\theta_W\right)^2}{\sin^2 2\theta_W\cos\theta_W}g_1^{(5)}\Big] v_1^\sigma v_2^\lambda + \Big[g_2^{(1)} + \frac{9}{16}r_3^2\frac{e_q^2\alpha}{\alpha_s}g_2^{(2)} + \sum_Q\frac{27\pi}{8}r_3^4\frac{e_q^2 e_Q^2\alpha^2 m_Q^2}{\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}}g_{2,Q}^{(3)} +$$

$$\frac{9\pi e_q^2\alpha^2 r_3^4 M_Z M_W}{64\alpha_s m M_{\mathcal{Q}\bar{\mathcal{Q}}}}\cos\theta_W g_2^{(4)} + \frac{9M_H^4 M_W\alpha}{32M_Z^3 m M_{\mathcal{Q}\bar{\mathcal{Q}}}\alpha_s}\frac{\left(\frac{1}{2}-2q_c\sin^2\theta_W\right)^2}{\sin^2 2\theta_W\cos\theta_W}g_2^{(5)}\Big] g^{\lambda\sigma},$$

In our work we use the unitary gauge for W-boson loop in which the one loop calculations are simplified and the dispersion theoretic method.

V.I. Borodulin, R.N. Rogalyov, S.R. Slabospitsky, CORE Version 2.1, IHEP 95-90, Protvino, 1995.

Preliminary result for pair charmonium production in Higgs boson decay:

$$\Gamma = 1.05 \cdot 10^{-12}\ GeV, \quad Br(H \to J/\Psi J/\Psi) = 3 \cdot 10^{-10}. \qquad (37)$$

$$\Gamma = 0.05 \cdot 10^{-12}\ GeV, \quad Br(H \to \Upsilon\Upsilon) = 0.16 \cdot 10^{-10}. \qquad (38)$$

Table: Preliminary results for the Higgs boson decay widths.

Contribution	$H \to J/\psi\ J/\psi$
Fig.1, $\mathcal{O}(\alpha_s)$	$0.06 \cdot 10^{-14}$
Fig.2, $\mathcal{O}(\alpha)$	$0.8 \cdot 10^{-12}$
Fig.3, $\mathcal{O}(\alpha^2)$	$0.02 \cdot 10^{-12}$
Fig.4, $\mathcal{O}(\alpha^2)$	$0.07 \cdot 10^{-12}$
Fig.5, $\mathcal{O}(\alpha)$	$0.002 \cdot 10^{-12}$

Thank you for attention!