On the impact of the EW corrections to the relation between the pole and running masses of top-quark: three different approaches to fixing VEV

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Outline

- PROBLEM: Despite serious theoretical studies *(Bednyakov, Denner, Dittmaier, Jegerlehner, Kalmykov, Kniehl, Martin, Pikelner, Veretin and others)*, the EW effects are still not taken into account in process of conversion from the pole to running mass of heavy quarks (primarily for top-quark) in the phenomenologically oriented papers (only QCD corrections)

- Some approaches to determining the VEV

  - Fleischer-Jegerlehner tadpole scheme
  - Scheme with the fixed VEV $v_0 \approx 246\ \mathrm{GeV}$
  - The VEV as minimum of the effective Higgs potential

- Comparison

- Outlook, open questions

Relation between the pole and running masses of t-quark

The pole mass M_t is defined as a pole of the renormalized quark propagator on the mass shell (ON-SHELL). It can be related to the bare mass $m_{t,0}$ via

$$m_{t,0} = Z_m^{\text{OS}} M_t,$$

where the mass renormalization constant Z_m^{OS} is expressed through the self-energy quark operator.

The running mass $\overline{m}_t(\mu)$ is defined within the $\overline{\text{MS}}$ subtraction scheme of UV divergences. Its relation with the bare mass:

$$m_{t,0} = Z_m^{\overline{\text{MS}}}\, \overline{m}_t(\mu)$$

In the Standard Model (SM) the ratio $M_t/\overline{m}_t(\mu)$ reads:

$$\frac{M_t}{\overline{m}_t(\mu)} = 1 + \delta^{\text{QCD}}(\mu) + \delta^{\text{EW}}(\mu) + \delta^{\text{QCD}\times\text{EW}}(\mu),$$

where $\delta^{\text{QCD}}(\mu) = \sum\limits_{n\geq 1} \delta_n^{\text{QCD}}(\mu)\left(\frac{\alpha_s(\mu)}{4\pi}\right)^n$, $\delta^{\text{EW}}(\mu) = \sum\limits_{n\geq 1} \delta_n^{\text{EW}}(\mu)\left(\frac{\alpha(\mu)}{4\pi\sin^2\theta_W(\mu)}\right)^n$,

$$\delta^{\text{QCD}\times\text{EW}}(\mu) = \sum_{n\geq 2}\sum_{k=1}^{n-1} \delta_{k,n-k}^{\text{QCD}\times\text{EW}}(\mu)\left(\frac{\alpha_s(\mu)}{4\pi}\right)^k \left(\frac{\alpha(\mu)}{4\pi\sin^2\theta_W(\mu)}\right)^{n-k},$$

$\alpha_s(\mu) = g_s^2(\mu)/4\pi, \alpha(\mu) = e^2(\mu)/4\pi,$ where g_s – is a gauge coupling of the $SU(3)_c$ group, e – of the $U(1)_{em}$, $\sin\theta_W$ – is a sine of the Weinberg angle, defined in the $\overline{\text{MS}}$-scheme.

One-loop level in QCD:

$$\delta_1^{\rm QCD}(\mu) = C_F\left(4 - 3\ln\frac{M_t^2}{\mu^2}\right), \quad \textit{[Tarrach (81)]}.$$

Next orders:

$\delta_2^{\rm QCD}(\mu)$ *[Gray, Broadhurst et al. (90); Avdeev, Kalmykov 9701308]*

$\delta_3^{\rm QCD}(\mu)$ *[Melnikov, Ritbergen 9912391; Chetyrkin, Steinhauser 9911434]*,

$\delta_4^{\rm QCD}(\mu)$ *[Marquard, Smirnov A., Smirnov V. et al. 1606.06754]*

Problem

Despite the existence of the EW corrections (sometimes dominant ones in the definite approaches due to the tadpole contributions with top-quark loop), only QCD corrections are taken into account in the ratio $M_t/\overline{m}_t(\mu)$ in phenomenologically oriented works where the transition $M_t \to \overline{m}_t(\mu)$ is employed and the value $\overline{m}_t(\overline{m}_t)$ is extracted afterwards.

EUROPEAN ORGANISATION FOR NUCLEAR RESEARCH (CERN)

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17th December 2019

The result from data unfolded to parton level is compared with the NLO QCD predictions in two different renormalisation schemes. The top-quark running mass in the $\overline{\text{MS}}$ scheme yields the following value:

$$m_t(m_t) = 162.9 \pm 0.5\,(\text{stat}) \pm 1.0\,(\text{syst})\,{}^{+2.1}_{-1.2}\,(\text{theo})\ \text{GeV}.$$

The top-quark mass extracted in the pole-mass scheme yields

$$m_t^{\text{pole}} = 171.1 \pm 0.4\,(\text{stat}) \pm 0.9\,(\text{syst})\,{}^{+0.7}_{-0.3}\,(\text{theo})\ \text{GeV}$$

with a total uncertainty of $\Delta m_t^{\text{pole}} = {}^{+1.2}_{-1.1}$ GeV.

[9] The QCD relation between the two schemes is known to four loops, but here the series is truncated at two loops to match the precision of the $t\bar{t}$ + 1-jet cross section that was used to extract the mass in both schemes. The relationship between the two masses then takes the simple form:

$$m_t^{\text{pole}} = m_t(m_t)\left(1 + \frac{4}{3}\,\frac{\alpha_S\,(\mu = m_t)}{\pi}\right) + O(\alpha_S^2).$$

The pole mass result quoted in the text is obtained for $\alpha_S(163\text{ GeV}) \sim 0.116$.

EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH (CERN)

CERN-EP-2018-317
2019/05/02

CMS-TOP-17-001

Measurement of the $t\bar{t}$ production cross section, the top quark mass, and the strong coupling constant using dilepton events in pp collisions at $\sqrt{s} = 13$ TeV

Since the analysis is performed in the $\overline{\mathrm{MS}}$ scheme, the assumed m_t^{pole} of each PDF is converted into $m_t(m_t)$ using the RUNDEC [83, 84] code, according to the prescription by the corresponding PDF group.

[arXiv: 1812.10505]

But *RunDec* is a program written for QCD!

Fermilab-Pub-11/177-E

Determination of the pole and $\overline{\rm MS}$ masses of the top quark from the $t\bar{t}$ cross section

The D0 Collaboration

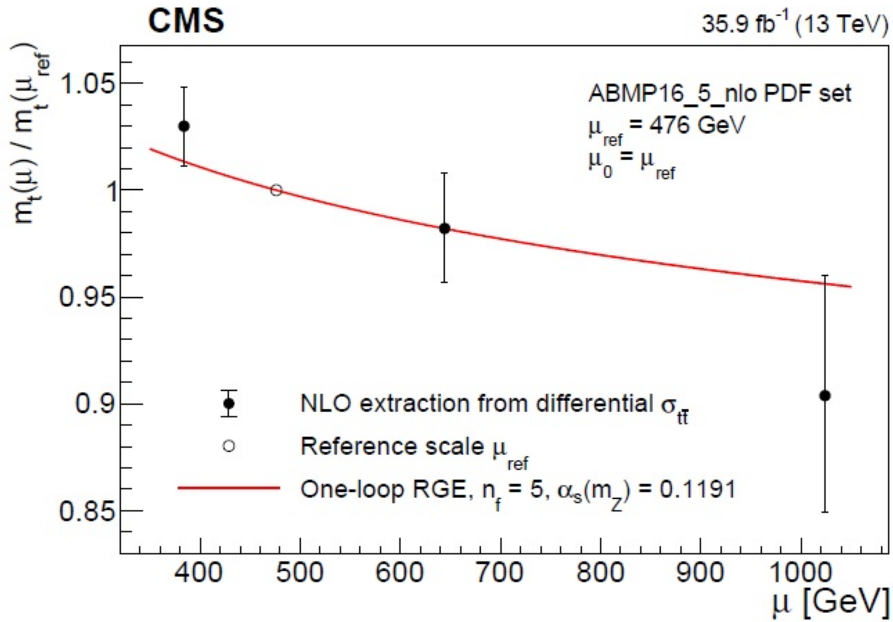
In case (*ii*) the cross section predictions use the pole-mass convention, and the value of $m_t^{\rm MC} = m_t^{\overline{\rm MS}}$ is converted to $m_t^{\rm pole}$ using the relationship at the three-loop level [5, 22]:

$$m_t^{\rm pole} = m_t^{\overline{\rm MS}}(m_t^{\overline{\rm MS}}) \left[1 + \frac{4}{3}\frac{\overline{\alpha}_s(m_t^{\overline{\rm MS}})}{\pi} + (-1.0414 N_L + 13.4434)\left(\frac{\overline{\alpha}_s(m_t^{\overline{\rm MS}})}{\pi}\right)^2 + (0.6527 N_L^2 - 26.655 N_L + 190.595)\left(\frac{\overline{\alpha}_s(m_t^{\overline{\rm MS}})}{\pi}\right)^3\right], \quad (3)$$

where $\overline{\alpha}_s$ is the strong coupling in the $\overline{\rm MS}$ scheme, and $N_L = 5$ is the number of light quark flavors.

[arXiv: 1104.2887]

Recently the CMS Collaboration was discovered experimentally the running of the MS-scheme top quark mass for the first time [arXiv: 1909.09193]



The analysis was carried out within the one-loop QCD.

However, when extracting the pole (or Monte Carlo) mass of t-quark from data on cross sections $pp \to t\bar{t} + \ldots$, the EW effects are occasionally considered at least at one-loop level *(Czakon, Kühn, Mitov, Uwer and others)*

But!

During the determination of $\sigma(t\bar{t})$ through the $\overline{\mathrm{MS}}$-scheme parameters, there only the pure QCD is utilized in ratio $M_t/\overline{m}_t(\mu)$ *[Langenfeld, Moch et al. 0906.5273; Alekhin, Moch et al. 1207.0980; Fuster et al. 1704.00540; Catani et al. 2005.00557]*

Thus, the EW effects are neglected in the process of conversion $M_t \to \overline{m}_t(\mu)$ (although this is not always reasonable: see example below – the Fleischer-Jegerlehner tadpole scheme, where the $\mathcal{O}(\alpha)$ EW correction dominates the $\mathcal{O}(\alpha_s)$ QCD one).

Fleischer-Jegerlehner tadpole scheme *(Phys. Rev. D 23, (1981))*

The tadpole corrections are considered to preserve the gauge invariance of the relation between M_t and $\overline{m}_t(\mu)$ (or $\overline{m}_{t,0}$).

Scalar Higgs potential reads: $V(\phi) = -m_\phi^2\, \Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$.

The minimum is reached when

$$\Phi^\dagger\Phi = \frac{m_\phi^2}{2\lambda} = \frac{v^2}{2} \quad\Rightarrow\quad v = \frac{m_\phi}{\sqrt{\lambda}} \quad\Rightarrow\quad \gamma_v = \gamma_{m_\phi} - \frac{1}{2\lambda}\beta_\lambda$$

$$\gamma_v = \frac{\partial \ln v}{\partial \ln \mu^2}, \quad \gamma_{m_\phi} = \frac{\partial \ln m_\phi}{\partial \ln \mu^2}, \quad \beta_\lambda = \frac{\partial \lambda}{\partial \ln \mu^2}$$

In the FJ-scheme the running of $\overline{m}_t(\mu)$ is defined from the standard relation appearing after the spontaneous breaking symmetry in the Yukawa sector of the SM:

$$\overline{m}_t(\mu) = \frac{y_t(\mu)v(\mu)}{\sqrt{2}} \quad\Rightarrow\quad \gamma_t = \frac{\partial \ln \overline{m}_t}{\partial \ln \mu^2} = \gamma_{y_t} + \gamma_v.$$

$$\beta_g = \frac{\partial g}{\partial \ln \mu^2} = \frac{g^3}{16\pi^2}\left(-\frac{43}{12} + \frac{2}{3}n_G\right), \quad \beta_{g'} = \frac{\partial g'}{\partial \ln \mu^2} = \frac{g'^{\,3}}{16\pi^2}\left(\frac{1}{12} + \frac{10}{9}n_G\right),$$

$$\beta_{g_s} = \frac{\partial g_s}{\partial \ln \mu^2} = \frac{g_s^3}{16\pi^2}\left(-\frac{11}{2} + \frac{1}{3}n_f\right), \quad \gamma_{m_\phi} = \frac{\partial \ln m_\phi}{\partial \ln \mu^2} = \frac{1}{16\pi^2}\left(3\lambda + \frac{3}{2}y_t^2 - \frac{9}{8}g^2 - \frac{3}{8}g'^{\,2}\right),$$

$$\gamma_{y_t} = \frac{\partial \ln y_t}{\partial \ln \mu^2} = \frac{1}{16\pi^2}\left(-4g_s^2 + \frac{9}{4}y_t^2 - \frac{9}{8}g^2 - \frac{17}{24}g'^{\,2}\right),$$

$$\beta_\lambda = \frac{\partial \lambda}{\partial \ln \mu^2} = \frac{1}{16\pi^2}\left(12\lambda^2 + 6\lambda y_t^2 - 3y_t^4 - \frac{9}{2}\lambda g^2 - \frac{3}{2}\lambda g'^{\,2} + \frac{9}{16}g^4 + \frac{3}{8}g^2 g'^{\,2} + \frac{3}{16}g'^{\,4}\right).$$

for higher orders see e.g. *[Chetyrkin, Zoller, 1205.2892; Bednyakov, Pikelner, Velizhanin, 1303.4364]*

Then, $\gamma_t = \frac{1}{16\pi^2}\left(-3\lambda + \frac{3}{4} y_t^2 - 4 g_s^2 - \frac{1}{3} g'^{\,2} - \frac{9}{32}\frac{g^4}{\lambda} - \frac{3}{16}\frac{g^2 g'^{\,2}}{\lambda} - \frac{3}{32}\frac{g'^{\,4}}{\lambda} + \frac{3}{2}\frac{y_t^4}{\lambda}\right).$

Accommodating the well-known relations

$$g = \frac{e}{\sin\theta_W},\quad g' = \frac{e}{\cos\theta_W},\quad M_W = \frac{gv}{2},\quad M_t = \frac{y_t v}{\sqrt{2}},\quad M_H^2 = 2\lambda v^2,\quad \cos\theta_W = \frac{M_W}{M_Z},$$

one can obtain that

$$\gamma_t = -4\,\frac{\alpha_s}{4\pi} + \frac{\alpha}{4\pi\sin^2\theta_W}\Bigg(\underbrace{\frac{1}{3} + \frac{3}{8}\frac{M_t^2}{M_W^2} - \frac{3}{8}\frac{M_H^2}{M_W^2} - \frac{1}{3}\frac{M_Z^2}{M_W^2} - \frac{3}{2}\frac{M_W^2}{M_H^2} - \frac{3}{4}\frac{M_Z^4}{M_W^2 M_H^2} + N_c\frac{M_t^4}{M_W^2 M_H^2}}_{\gamma_0^{\rm EW}\approx 25.80}\Bigg)$$

For $N_c = 3, n_G = 3, n_f = 6$ we have

$$\begin{aligned}\frac{\overline{m}_t(\mu)}{\overline{m}_t(\mu_0)} &= \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{4/7}\left(\frac{\alpha(\mu_0)}{\alpha(\mu)}\frac{\sin^2\theta_W(\mu)}{\sin^2\theta_W(\mu_0)}\right)^{6\gamma_0^{\rm EW}/19} \\ &= \left(1 - 7\,\frac{\alpha_s(\mu_0)}{4\pi}\ln\frac{\mu_0^2}{\mu^2}\right)^{-4/7}\left(1 - \frac{19}{6}\frac{\alpha(\mu_0)}{4\pi\sin^2\theta_W(\mu_0)}\ln\frac{\mu_0^2}{\mu^2}\right)^{6\gamma_0^{\rm EW}/19}.\end{aligned}$$

Using the analytic formula for the Passarino-Veltman function in real domain $M_t > M_H/2$, $M_t > M_Z/2$ and results *[Hempfling, Kniehl 9408313]*, one can get the following expression for the one-loop correction to $M_t/\overline{m}_t(\mu)$ in the FJ-scheme in terms of masses of particles of the SM:

$$\delta_1^{\rm EW}(\mu) = \delta_1^{\rm EW,\,(0)} + \delta_1^{\rm EW,\,(L)} \ln\frac{M_t^2}{\mu^2},$$

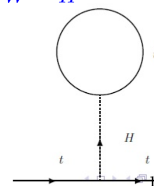
$$\begin{aligned}
\delta_1^{\rm EW,\,(0)} = & -\frac{23}{72} - \frac{M_t^2}{M_W^2} + \frac{25}{36}\frac{M_W^2}{M_t^2} + \frac{1}{2}\frac{M_W^2}{M_H^2} + \frac{1}{2}\frac{M_H^2}{M_W^2} - \frac{5}{9}\frac{M_Z^2}{M_t^2} + \frac{4}{9}\frac{M_Z^2}{M_W^2} + \frac{17}{72}\frac{M_Z^4}{M_W^2 M_t^2} \\
& + \frac{1}{4}\frac{M_Z^4}{M_W^2 M_H^2} - N_c\frac{M_t^4}{M_W^2 M_H^2} + \frac{3}{2}\frac{M_W^2}{M_H^2}\ln\frac{M_t^2}{M_W^2} + \frac{1}{16}\frac{M_H^4}{M_W^2 M_t^2}\ln\frac{M_t^2}{M_H^2} - \frac{1}{8}\frac{M_t^2}{M_W^2}\ln\frac{M_t^2}{M_t^2 - M_W^2} \\
& - \frac{1}{4}\frac{M_W^2}{M_t^2}\left(\frac{M_W^2}{M_t^2} - \frac{3}{2}\right)\ln\frac{M_W^2}{M_t^2 - M_W^2} + \left(\frac{3}{4}\frac{M_Z^4}{M_W^2 M_H^2} + \frac{2}{9}\frac{M_W^2 M_Z^2}{M_t^4} - \frac{5}{18}\frac{M_Z^4}{M_t^4} - \frac{3}{16}\frac{M_Z^4}{M_W^2 M_t^2}\right. \\
& \left. + \frac{17}{144}\frac{M_Z^6}{M_W^2 M_t^4}\right)\ln\frac{M_t^2}{M_Z^2} + \frac{M_Z^2}{M_t^2}\left(\frac{10}{9} + \frac{5}{9}\frac{M_Z^2}{M_t^2} - \frac{8}{9}\frac{M_W^2}{M_Z^2} - \frac{7}{72}\frac{M_Z^2}{M_W^2} - \frac{4}{9}\frac{M_W^2}{M_t^2}\right. \\
& \left. - \frac{17}{72}\frac{M_Z^4}{M_W^2 M_t^2}\right)\left(\frac{4M_t^2}{M_Z^2} - 1\right)^{1/2} \arccos\left(\frac{M_Z}{2M_t}\right) + \frac{1}{8}\frac{M_H^4}{M_W^2 M_t^2}\left(\frac{4M_t^2}{M_H^2} - 1\right)^{3/2} \arccos\left(\frac{M_H}{2M_t}\right),
\end{aligned}$$

$$\delta_1^{\rm EW,\,(L)} = \frac{1}{3} + \frac{3}{8}\frac{M_t^2}{M_W^2} - \frac{1}{3}\frac{M_Z^2}{M_W^2} - \frac{3}{8}\frac{M_H^2}{M_W^2} - \frac{3}{2}\frac{M_W^2}{M_H^2} - \frac{3}{4}\frac{M_Z^4}{M_W^2 M_H^2} + N_c\frac{M_t^4}{M_W^2 M_H^2}.$$

Terms proportional to N_c give the greatest contributions to $\delta_1^{\rm EW,\,(0)}$ and $\delta_1^{\rm EW,\,(L)}$ and arise due to the presented vacuum diagram.

Terms proportional to $1/M_H^2$ are manifestations of tadpoles.

$\arccos(x)$ appear because of application the OS-scheme.



Numerical values: the FJ-scheme

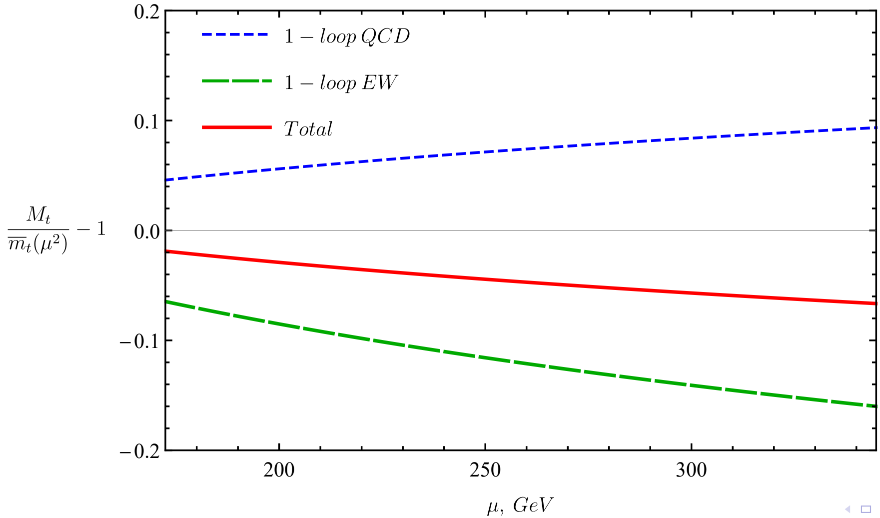
Using $M_t \approx 172.4$ GeV, $M_H \approx 125.10$ GeV, $M_W \approx 80.38$ GeV, $M_Z \approx 91.19$ GeV, $\alpha_s(M_Z^2) \approx 0.1179$, $\alpha^{-1}(M_Z^2) \approx 127.952$, $\sin^2\theta_W(M_Z^2) \approx 0.231$, we get

$$\delta_1^{\rm QCD}(\mu) = 5.33 - 4\ln\frac{M_t^2}{\mu^2},$$

$$\delta_1^{\rm EW}(\mu) = -24.26 + 25.80\ln\frac{M_t^2}{\mu^2}$$

where term $N_c \dfrac{M_t^4}{M_W^2 M_H^2}$ in $\delta_1^{\rm EW}$ is equal to

$$N_c \cdot \left(\frac{M_t}{M_W}\right)^2 \cdot \left(\frac{M_t}{M_H}\right)^2 \sim 3 \cdot 4.6 \cdot 1.9 = 26.22$$



As seen, the one-loop EW correction dominates the QCD effects at $M_t \le \mu \le 2M_t$ in FJ-scheme.

The scheme with the fixed VEV $v_0 = (\sqrt{2} G_F)^{-1/2} \approx 246$ GeV

This scheme was proposed and elaborated by *[Kniehl, Pikelner, Veretin, 1401.1844; 1503.02138; Jegerlehner, Kalmykov, Kniehl 1212.4319]*. In this scheme the $\overline{\rm MS}$-mass runs as the Yukawa coupling constant of the top quark: $\overline{m}_{t,Y}(\mu^2) = y_t(\mu^2) v_0/\sqrt{2}$.

$$\delta_{1,Y}^{\rm EW}(\mu) = \delta_{1,Y}^{\rm EW,\,(0)} + \delta_{1,Y}^{\rm EW,\,(L)} \ln \frac{M_t^2}{\mu^2},$$

$$\begin{aligned}
\delta_{1,Y}^{\rm EW,\,(0)} &= \frac{11}{36} - \frac{11}{8}\frac{M_t^2}{M_W^2} + \frac{25}{36}\frac{M_W^2}{M_t^2} + \frac{1}{16}\frac{M_H^2}{M_W^2} - \frac{5}{9}\frac{M_Z^2}{M_t^2} + \frac{109}{144}\frac{M_Z^2}{M_W^2} + \frac{17}{72}\frac{M_Z^4}{M_W^2 M_t^2} \\
&+ \left(\frac{3}{8} - \frac{3}{8}\frac{M_H^2}{M_W^2} + \frac{1}{16}\frac{M_H^4}{M_W^2 M_t^2}\right) \ln\frac{M_t^2}{M_H^2} - \frac{1}{8}\frac{M_t^2}{M_W^2} \ln\frac{M_t^2}{M_t^2 - M_W^2} + \frac{3}{8}\frac{M_Z^2}{M_Z^2 - M_W^2} \ln\frac{M_Z^2}{M_W^2} \\
&+ \frac{3}{8}\ln\frac{M_t^2}{M_W^2} - \frac{3}{8}\frac{M_W^2}{M_H^2 - M_W^2}\ln\frac{M_H^2}{M_W^2} - \frac{1}{4}\frac{M_W^2}{M_t^2}\left(\frac{M_W^2}{M_t^2} - \frac{3}{2}\right)\ln\frac{M_W^2}{M_t^2 - M_W^2} \\
&+ \left(\frac{3}{8}\frac{M_Z^2}{M_W^2} + \frac{2}{9}\frac{M_W^2 M_Z^2}{M_t^4} - \frac{5}{18}\frac{M_Z^4}{M_t^4} - \frac{3}{16}\frac{M_Z^4}{M_W^2 M_t^2} + \frac{17}{144}\frac{M_Z^6}{M_W^2 M_t^4}\right)\ln\frac{M_t^2}{M_Z^2} + \frac{M_Z^2}{M_t^2}\left(\frac{10}{9}\right. \\
&\left. + \frac{5}{9}\frac{M_Z^2}{M_t^2} - \frac{8}{9}\frac{M_W^2}{M_Z^2} - \frac{7}{72}\frac{M_Z^2}{M_W^2} - \frac{4}{9}\frac{M_W^2}{M_t^2} - \frac{17}{72}\frac{M_Z^4}{M_W^2 M_t^2}\right)\left(\frac{4M_t^2}{M_Z^2} - 1\right)^{1/2} \arccos\left(\frac{M_Z}{2M_t}\right) \\
&+ \frac{1}{8}\frac{M_H^4}{M_W^2 M_t^2}\left(\frac{4M_t^2}{M_H^2} - 1\right)^{3/2} \arccos\left(\frac{M_H}{2M_t}\right), \qquad \delta_{1,Y}^{\rm EW,\,(L)} = -\frac{5}{12} + \frac{9}{8}\frac{M_t^2}{M_W^2} - \frac{17}{24}\frac{M_Z^2}{M_W^2}.
\end{aligned}$$

Unlike the FJ case, the tadpole contributions are absent here, but the gauge independence is kept. However, the requirement $v_0 =$ const is set by "hands". The on-shell $\arccos(x)$ are the same.

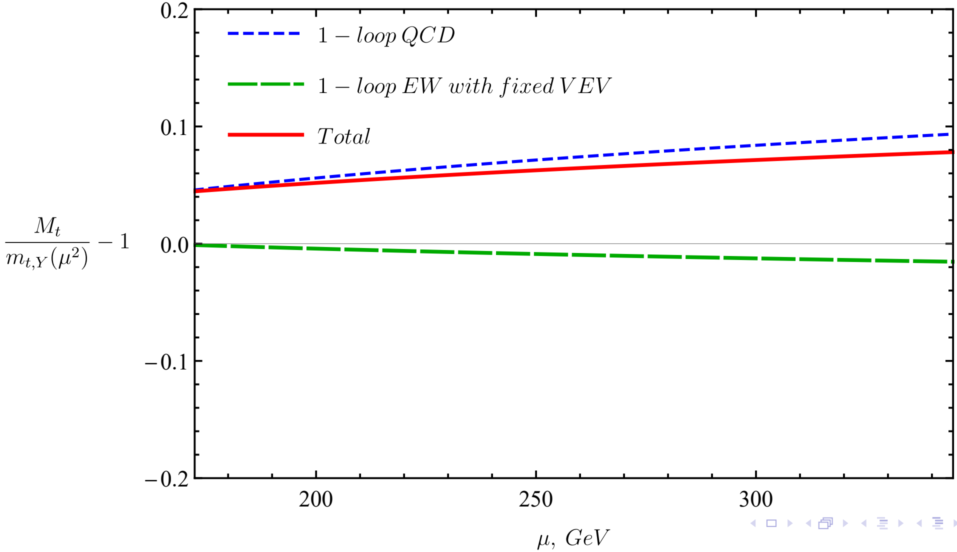
Numerical values: $v_0 \approx 246$ GeV

$$\delta_1^{\rm QCD}(\mu) = 5.33 - 4\ln\frac{M_t^2}{\mu^2},$$

$$\delta_{1,Y}^{\rm EW}(\mu) = -0.44 + 3.85\ln\frac{M_t^2}{\mu^2}$$

When $v_0 \approx 246$ GeV, then at the scale M_t the EW effect is much smaller than QCD one.

$$\alpha_s(M_t^2) = 0.108,\ \alpha^{-1}(M_t^2) = 127.58$$



The effective Higgs potential case

The effective Coleman-Weinberg potential obeys the following RG equation:

$$\left(\mu^2\frac{\partial}{\partial\mu^2}+\sum_i\beta_i\frac{\partial}{\partial g_i}-\gamma_\phi\phi\frac{\partial}{\partial\phi}\right)V_{eff}(\phi,\mu)=0$$

In one-loop approximation $V_{eff}(\phi,\mu)=V_0(\phi)+V_1(\phi,\mu)$, where $V_0(\phi)$ is a tree-level Higgs potential and $V_1(\phi,\mu)$ in the 't Hooft-Landau gauge $\xi=0$ reads *[Duncan, Philippe, Sher, (1985, 1989)]*:

$$\begin{aligned}V_1(\phi,\mu)=\frac{1}{16\pi^2}\Bigg(&\frac{3}{4}(\lambda\phi^2-m_\phi^2)^2\left(\ln\frac{\lambda\phi^2-m_\phi^2}{\mu^2}-\frac{3}{2}\right)+\frac{1}{4}(3\lambda\phi^2-m_\phi^2)^2\left(\ln\frac{3\lambda\phi^2-m_\phi^2}{\mu^2}-\frac{3}{2}\right)\\&-\frac{3}{4}y_t^4\phi^4\left(\ln\frac{y_t^2\phi^2}{2\mu^2}-\frac{3}{2}\right)+\frac{3}{32}g^4\phi^4\left(\ln\frac{g^2\phi^2}{4\mu^2}-\frac{3}{2}\right)+\frac{1}{16}g^4\phi^4+\frac{1}{32}(g^2+g'^{\,2})^2\phi^4\\&+\frac{3}{64}(g^2+g'^{\,2})^2\phi^4\left(\ln\frac{(g^2+g'^{\,2})\phi^2}{4\mu^2}-\frac{3}{2}\right)\Bigg).\end{aligned}$$

The effective VEV $v_{eff}(\mu)$ is defined from minimization of the $V_{eff}(\phi,\mu)$:

$$\left.\frac{\partial V_{eff}(\phi,\mu)}{\partial\phi}\right|_{\phi=v_{eff}}=0.$$

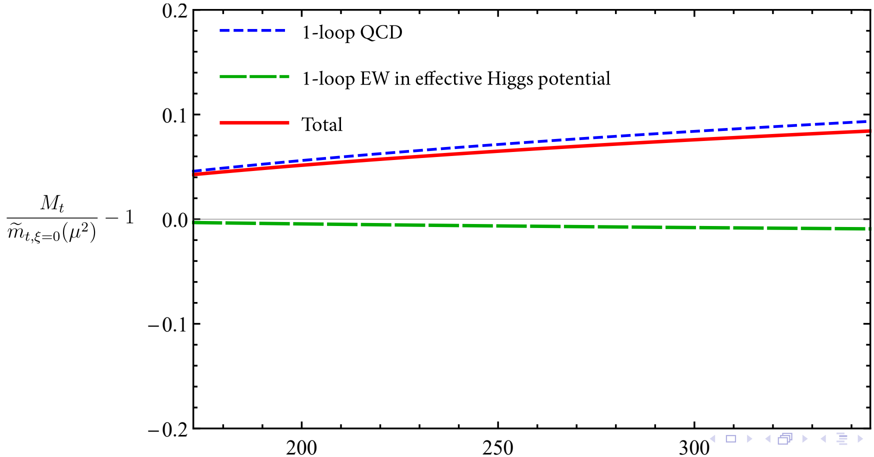
The relation between $v_{eff}(\mu)$ and $v(\mu)$, defined above in the FJ tadpole scheme, has the following form *[Martin, 1604.01134; Martin, Robertson, 1907.02500]*:

$$v_{eff}(\mu) = v(\mu)(1 + \Delta_1(\mu)),$$

$$\Delta_1(\mu) = \frac{\alpha(\mu)}{4\pi \sin^2\theta_W(\mu)} \left(3\frac{M_t^4}{M_W^2 M_H^2}\left(\ln\frac{M_t^2}{\mu^2} - 1\right) - \frac{3}{8}\frac{M_H^2}{M_W^2}\left(\ln\frac{M_H^2}{\mu^2} - 1\right) - \frac{3}{2}\frac{M_W^2}{M_H^2}\ln\frac{M_W^2}{\mu^2} - \frac{3}{4}\frac{M_Z^4}{M_W^2 M_H^2}\ln\frac{M_Z^2}{\mu^2} + \frac{1}{2}\frac{M_W^2}{M_H^2} + \frac{1}{4}\frac{M_Z^4}{M_W^2 M_H^2}\right).$$

In this scheme $\widetilde{m}_t(\mu) = \frac{y_t(\mu) v_{eff}(\mu)}{\sqrt{2}}$ and the relation between M_t and $\widetilde{m}_t(\mu)$

is a tadpole-free, but, generally speaking, is a gauge-dependent.



Conclusion

- There are many ways to determine the VEV and, as a consequence, the running mass of top-quark: FJ-scheme; scheme with the fixed $v_0 \approx 246$ GeV; approach with consideration of the effective SM potential

- Nowadays, none of these schemes are used in the experimentally-oriented works where the running mass of t-quark is extracted.

- The inclusion of the one-loop EW correction in the FJ-scheme significantly affects the behavior of the $M_t/\overline{m}_t(\mu)$ ratio – it exceeds the one-loop QCD effect and has the opposite sign.

- The incorporation into the analysis of the one-loop EW correction in the schemes with the fixed VEV and with the effective potential approach does not lead to a significant modification of the currently known ratio $M_t/\overline{m}_t(M_t)$. One could say that the values presented today for $\overline{m}_t(\overline{m}_t)$ are valid for these schemes. However, then the EW corrections to the cross sections must also be calculated in these schemes.

- In any case, accounting for the EW effects leads to additional uncertainties, which should be included in the treatment of data on the extraction of $\overline{m}_t$. This issue is relevant nowadays *[Dittmaier, Rzehak, 2203.07236; Mühlleitner, Schlenk, Spira, 2207.02524]*

Thank you for your attention!