Generalizing generalized supergravity

based on works
[2203.03372, 2011.11424, 2002.01915]
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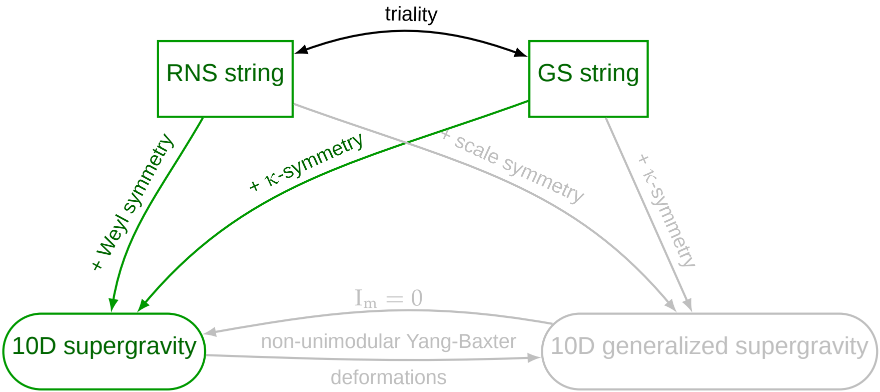
String theory and supergravity



From the RNS superstring to 10D SUGRA

RNS action (world-sheet SUSY):

$$S = \frac{1}{4\pi\alpha'}\int d^2\sigma\sqrt{g}\Big(G_{mn}\partial_I x^m \partial^I x^n + iB_{mn}\varepsilon^{IJ}\partial_I x^m \partial_J x^n + \alpha'\Phi R^{(2)}\Big). \qquad (1)$$

Weyl invariance $\langle T^\alpha{}_\alpha\rangle = 0$ (one-loop):

$$\begin{aligned}
\frac{1}{\alpha'}\beta^{1-\text{loop}}(\Phi) &= R - \frac{1}{12}H^2 + 4\nabla^m\nabla_m\Phi - 4(\nabla\Phi)^2 = 0\,,\\
\frac{1}{\alpha'}\beta^{1-\text{loop}}_{mn}(G) &= R_{mn} - \frac{1}{4}H_{mkl}H_n{}^{kl} + 2\nabla_m\nabla_n\Phi = 0\,, \qquad (2)\\
\frac{1}{\alpha'}\beta^{1-\text{loop}}_{mn}(B) &= \frac{1}{2}\nabla^k H_{kmn} - H_{kmn}\nabla^k\Phi = 0\,.
\end{aligned}$$

Weyl invariance of the RNS string $\Longrightarrow$ equations of 10D supergravity

[Callan (1985, 1986, 1989)]

From GS superstring to 10D SUGRA

Green-Schwarz superstring action (SUSY in target space)

$$S = \int d^2\sigma \sqrt{-G} - \int_\Sigma B\,, \qquad G = \det G_{IJ}\,, \tag{3}$$

$$G_{IJ} = E_I{}^a E_J{}^b \eta_{ab}\,, \qquad E_I{}^A = \partial_I z^M E_M{}^A(z)\,, \qquad z^M = (x^m,\, \theta^\mu)\,. \tag{4}$$

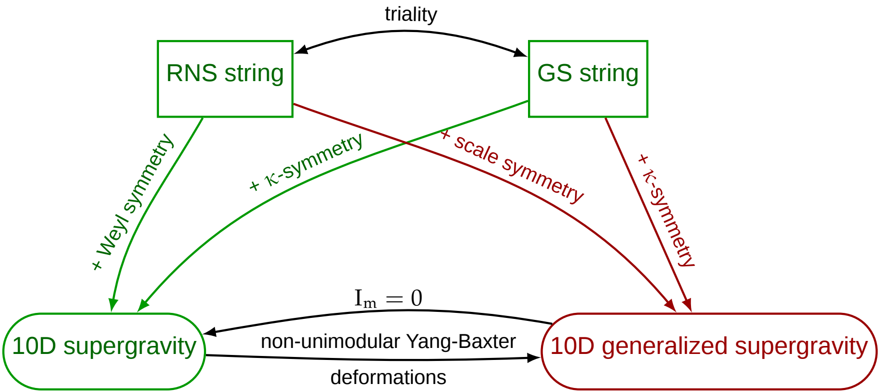
κ-symmetry (gauge away half of supersymmetries):

$$\begin{aligned} &\delta_\kappa z^M E_M{}^a = 0\,, \\ &\delta_\kappa z^M E_M{}^{\alpha i} = \frac{1}{2}(1+\Gamma)^{\alpha i}{}_{\beta j}\kappa^{\beta j}, \quad \Gamma = \frac{1}{2\sqrt{-G}}\epsilon^{IJ}E_I{}^a E_J{}^b \gamma_{ab}\sigma^3 \end{aligned} \tag{5}$$

Bianchi identities + κ-symmetry $\Longleftarrow$ 10D supergravity

[Nilsson (1981)]

String theory and supergravity



From the GS superstring to generalized 10D SUGRA

More accurately: [Tseytlin, Wulff (2016)]

Bianchi identities + κ-symmetry ⟸ generalized 10D supergravity

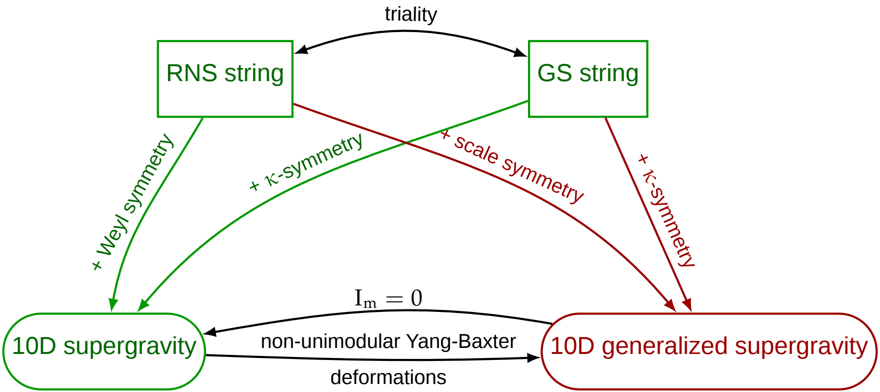
$$
\begin{aligned}
&R - \frac{1}{12} H^2 + 4\,\nabla^m X_m \;- 4\,X_m X^m = 0\,,\\
&R_{mn} - \frac{1}{4} H_{mkl} H_n{}^{kl} + \nabla_m X_n + \nabla_n X_m = 0\,,\\
&\frac{1}{2}\,\nabla_k H^{kmn} \;- H^{kmn} X_k - \nabla^m X^n \;+ \nabla^n X^m \;= 0\,,
\end{aligned}
\tag{6}
$$

- Instead of the dilaton the field:

$$
X_m = \partial_m \Phi + I_m - B_{mn} I^n \tag{7}
$$

- I^m — a Killing vector. $I^m = 0$ gives the usual supergravity.

String theory and supergravity



The actual discovery of generalized sugra

- GS string on $AdS_5 \times \mathbb{S}^5$ is integrable [Bena, Polchinski, Roiban (2004)]
- GS string is integrable on Yang-Baxter (η-)deformed background $AdS_5 \times \mathbb{S}^5$ [Vicedo, Delduc, Magro (2013)]

$$S = -\frac{(1+\eta^2)^2}{2(1-\eta^2)} \int d\tau d\sigma \, P_-^{ab} \, \mathrm{STr}\left[A_a . d \circ \frac{1}{1-\eta R_g \circ d}(A_b)\right] \tag{8}$$

- does not solve ordinary sugra equations [Arutyunov, Borsato, Frolov (2015)]
- solves equations of generalized sugra [Arutynov, Frolov, Hoare, Roiban, Tseytlin (2015)]

The membrane story

- κ-symmetry + Bianchi identities $\Longleftarrow$ equations of 11D supergravity
  [Bershoeff, Sezgin, Townsend (1987)]

$$S_{11} = \int d^{11}x\sqrt{G}\Big(R[G] - \frac{1}{48}F_{mnkl}F^{mnkl} + \dots\Big) \tag{9}$$

- While RNS string on gensugra backgrounds breaks Weyl symmetry to scale symmetry, the membrane has only scale symmetry. Nothing to break!

The end?

The membrane story

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Recall: generalized sugra comes from YB deformations.

Double field theory and supergravity

- Double field theory — O(10,10)-covariant formulation of 10D supergravity

$$S_{DFT} = \int d^{10}\mathbb{X}\, e^{-2d}\, \mathbb{Q}, \quad \mathbb{X}^M = \{x^m, \tilde{x}_m\} \tag{10}$$

- The most convenient here is the flux formulation

$$\begin{aligned}\mathbb{Q} = {} & \mathcal{H}^{AB}\mathcal{F}_A\mathcal{F}_B + \mathcal{F}_{ABC}\mathcal{F}_{DEF}\left(\frac{1}{4}\mathcal{H}^{AD}\eta^{BE}\eta^{CF} - \frac{1}{12}\mathcal{H}^{AD}\mathcal{H}^{BE}\mathcal{H}^{CF}\right)\\ & - \mathcal{F}_A\mathcal{F}^A - \frac{1}{6}\mathcal{F}_{ABC}\mathcal{F}^{ABC},\end{aligned} \tag{11}$$

Double field theory and supergravity

- $\mathcal{F}_{AB}{}^{C}$ — generalized anholonomicity coefficients (fluxes)

$$[E_A, E_B]_{\text{genLie}} = \mathcal{F}_{AB}{}^{C} E_C, \qquad E_A{}^{M} = \begin{pmatrix} e_a^m & 0 \\ -e_a^k B_{km} & e_m^a \end{pmatrix}, \qquad d = \varphi - \frac{1}{2}\log\det\|e_m{}^a\|. \tag{12}$$

- In supergravity parametrization:

$$\mathcal{F}_{abc} = -H_{abc}, \quad \mathcal{F}_{ab}{}^{c} = f_{ab}{}^{c}, \quad \mathcal{F}_{a} = 2e_a{}^{m}\nabla_m\varphi + f_{ab}{}^{b}, \qquad f_{ab}{}^{c} = -2e_a^m e_b^n \partial_{[m} e_{n]}^c \longleftarrow \text{anholonomicity coefficients} \tag{13}$$

- The DFT EoMs $\Longleftrightarrow$ 10D supergravity EoMs

Yang-Baxter deformation is a local O(d,d) rotation

- Given Killing vectors $k_a{}^m$ construct

$$\beta^{mn} = r^{ab} k_a{}^m k_b{}^n, \quad r^{ab} = \text{const} \tag{14}$$

- O(d,d) transform the fields

$$E'_M{}^A = O_M{}^N E_N{}^B, \qquad O_M{}^N = \begin{pmatrix} \delta_m{}^n & -\beta^{nm} \\ 0 & \delta_n{}^m \end{pmatrix} = \exp(\beta^{mn} T_{mn}), \tag{15}$$

- Deformation of fluxes is ruled by
(i)classical Yang-Baxter equation and (ii)unimodularity

$$\begin{aligned} \delta \mathcal{F}_{ABC} &\propto \text{CYBE} : f_{de}{}^{[a} r^{b|d|} r^{c]e} = 0 \\ \delta \mathcal{F}_a &= 2 I^m B_{mn} e_a{}^n, \\ \delta \mathcal{F}^a &= 2 I^a, \end{aligned} \tag{16}$$

[Bakhmatov, Borsato, Colgain, Kelekci, Sheikh-Jabbari]

The trick in 10D

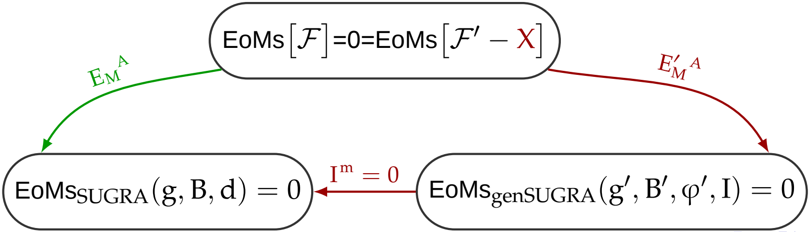
- Deform fluxes as if they would under non-unimodular YB

$$\begin{aligned} \mathcal{F}'_{ABC} &= \mathcal{F}_{ABC}, \\ \mathcal{F}'_{A} &= \mathcal{F}_{A} + X_{A}, \end{aligned} \tag{17}$$

- For $\mathcal{F}'_{ABC}$ to still define 10D supergravity fields $E'_M{}^A$ (Bianchi identities):

$$\mathcal{L}_X E'_A{}^M = 0, \quad \mathcal{L}_X d' = 0, \quad X_M X^M = 0. \tag{18}$$

- Equations for $E'_M{}^A$: 10D generalized supergravity



Generalization to 11 dimensions

In this form the procedure easily generalizes to 11 dimensions with replacements:

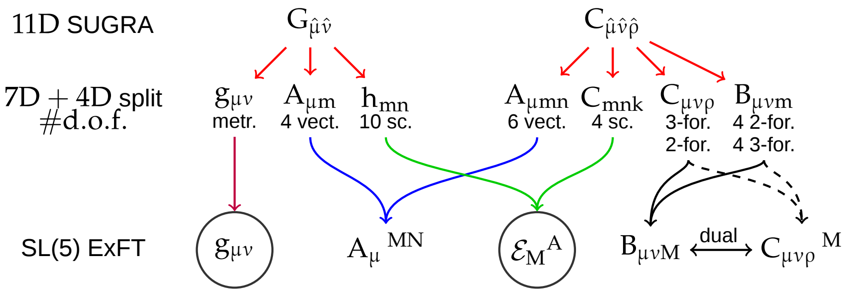
- DFT $\longrightarrow$ Exceptional FT — $E_{d(d)}$-covariant formulation of sugra
- field content $\{g_{mn}, B_{mn}, \varphi\} \longrightarrow \{g_{mn}, C_{mnk}\}$
- The deformation $X_M \longrightarrow X_{MNK}{}^L$ (no longer a generalized Killing vector)
- The Killing vector $I^m \longrightarrow J^{m,n}$

Exceptional field theory

- 11d sugra on $\mathbb{T}^d$ enjoys U-duality symmetry group $E_{d(d)}$
  [Cremmer, Julia (1979, 1981)]

$$E_{5(5)} = SO(5,5), \quad E_{4(4)} = SL(5), \quad E_{3(3)} = SL(3) \times SL(2). \tag{19}$$

- For E_d-covariance split is necessary



Flux formulation of the SL(5) theory

- Restrict to backgrounds $M_{11} = M_7 \times M_4$

$$g_{\mu\nu}(y^\mu, x^m) = e^{-2\phi(x^m)}\bar{g}_{\mu\nu}(y^\mu), \quad E_M{}^A(x) = e^{-\frac{\phi}{2}}\mathcal{E}_M{}^A(x). \tag{20}$$

- Generalized fluxes (anholonomicity)

$$\begin{aligned} [E_{AB}, E_C] &= F_{AB,C}{}^D F_D, \\ \mathcal{F}_{ABC}{}^D &= \frac{3}{2} Z_{ABC}{}^D - \frac{1}{2}\theta_{[AB}\delta_{C]}{}^D + \delta_{[A}{}^D Y_{B]C} \end{aligned} \tag{21}$$

- Flux Lagrangian of the theory (a subsector of 11D sugra)

$$\begin{aligned} \mathcal{L}' = {} & Y_{AB}Y^{AB} - \frac{1}{2}Y_A{}^A Y_B{}^B + 32\, Z^{ABC}Z_{ABC} + 32\, Z^{AB}{}_A Z_{BC}{}^C \\ & - \frac{7}{3}\,\theta_{AB}\theta^{AB} + \Lambda, \end{aligned} \tag{22}$$

GenYB deformation is a local E_d transformation

- Given Killing vectors $k_\alpha{}^m$ define

$$\Omega^{mnk} = \frac{1}{6}\rho^{\alpha\beta\gamma} k_\alpha{}^m k_\beta{}^n k_\gamma{}^k. \tag{23}$$

- SL(5) transform the fields

$$E'_M{}^A = O_M{}^N E_N{}^B, \qquad O = \begin{bmatrix} \delta_m{}^n & 0 \\ \frac{1}{3!}\varepsilon_{mpqr}\Omega^{pqr} & 1 \end{bmatrix} = \exp\left(\Omega^{mnk} T_{mnk}\right) \tag{24}$$

- Deformation of fluxes $\delta\mathcal{F}_{AB,C}{}^D \propto$ gCYBE $+$ gUni is ruled by (i)generalized CYBE and (ii)generalized unimodularity

$$\text{gCYBE} = 6\rho^{[\alpha_2|\alpha_7\beta_1}\rho^{|\alpha_3\alpha_4|\beta_2} f_{\beta_1\beta_2}{}^{|\alpha_5]} + \rho^{\beta_1\beta_2[\alpha_2}\rho^{\alpha_3\alpha_4\alpha_5]} f_{\beta_1\beta_2}{}^{\alpha_7},$$
$$\text{gUni} = \frac{1}{4} E^m{}_C E^n{}_A E^k{}_B E_l{}^E J^{lp} \varepsilon_{kmnp}, \quad J^{mn} = k_{\alpha_1}{}^m k_{\alpha_4}{}^n \rho^{\alpha_1\alpha_2\alpha_3} f_{\alpha_2\alpha_3}{}^{\alpha_4} \tag{25}$$

Apply the same trick for 11D bg's

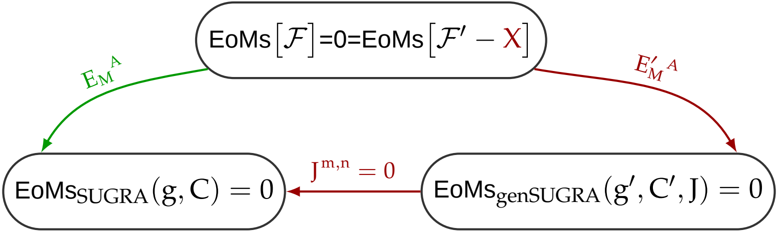
- Deform fluxes as if they would under non-unimodular genYB

$$\mathcal{F}'_{AB,C}{}^{D} = \mathcal{F}_{AB,C}{}^{D} + X_{ABC}{}^{D}, \tag{26}$$

- For $\mathcal{F}'_{AB,C}{}^{D}$ to still define 11D supergravity fields $E'_{M}{}^{A}$ (Bianchi identities):

$$\text{Bianchi}\left[\mathcal{F}'_{AB,C}{}^{D}\right] = 0 \tag{27}$$

- Equations for $E'_{M}{}^{A}$: 11D generalized supergravity



Constraints on J^{mn} from Bianchi

$$
\begin{aligned}
L_{e_a} J^{kl} + J^{nl}\partial_n \varphi\, e_a{}^k = 0, \qquad J^{mn}\partial_n \varphi = 0,\\
\nabla_m\left(e^{-\phi} I^{mn}\right) = 0, \qquad J^{m[n}J^{kl]} = 0,
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\nabla_{[m}Z_{n]} - \frac{1}{3}J^{kl}F_{mnkl} = 0,\\
\nabla_k\left(e^{-\varphi}J^{k[l}V^{p]}\right) = 0,\\
\nabla_k(J^{(pl)}V^k) - \nabla_k(V^{(p}J^{l)k}) = 0.
\end{aligned}
\tag{29}
$$

where

$$
Z_m = \partial_m \varphi - \frac{2}{3}\epsilon_{mnkl}J^{nk}V^l. \tag{30}
$$

Generalized equations

$$
\begin{aligned}
0 &= \mathcal{R}_{mn}[h_{(4)}] - 7\,\tilde\nabla_{(m}Z_{n)} - \frac{1}{3}h_{mn}(\nabla V) + 8(1+V^2)\Big(S_{mn}J^k{}_k - 2J^k{}_{(m}J_{n)k}\Big)\\
&\quad + 4V_mV_n\Big(J^{kl}J_{kl} - 2J^{kl}J_{lk}\Big) + 4V_kV_l\Big(4J_{(m}{}^kJ_{n)}{}^l - J^k{}_{(m}J^l{}_{n)} - 2S^{kl}S_{mn}\Big)\\
&\quad + 8V_kV_{(m}\Big(2J^l{}_{n)}J^k{}_l - 2S_{n)}{}^kJ^l{}_l + J^{kl}J_{n)l}\Big),\\
0 &= \frac{1}{7}e^{2\phi}\,\mathcal{R}[\bar g_{(7)}] + \frac{1}{6}\,(\nabla V)^2 + \tilde\nabla^m Z_m - 6Z_mZ^m - 2J^{mn}J_{mn} + \frac{4}{3}J_{mn}J^{nm},\\
0 &= \tilde\nabla^m F_{mnkl} - 6Z^mF_{mnkl} + 6\Big(2J^{pm}C_{m[nk}J_{l]p} - J^{pm}J_{p[n}C_{kl]m}\Big),\\
0 &= \mathcal{R}_{\mu\nu}[\bar g_{(7)}] - \frac{1}{7}\bar g_{\mu\nu}\mathcal{R}[\bar g_{(7)}],
\end{aligned}
\tag{31}
$$

where $S^{mn} = J^{(mn)}$, $F_{mnkl} = 4\partial_{[m}C_{nkl]}$ and

$$
\tilde\nabla_m = \nabla_m - \partial_m\varphi\,. \tag{32}
$$

Example of a solution: deformed $AdS_4 \times S^7$

Generate example solutions by nonunimodular genYB deformations.

deformation along $D \wedge M \wedge M$:

$$\Omega = \frac{4}{R^3}\rho_a \varepsilon^{abc} D \wedge M_{bd} \wedge M_c{}^d$$

of AdS_4 gives deformed background

$$\begin{aligned}
ds^2 &= \frac{R^2}{4z^2} K^{\frac{2}{3}} \left\{ dx_a dx^a + \frac{\rho_a x^a}{z^2} x^b dx_b dz + \left(1 - \frac{x_a x^a \rho_b x^b}{z^3}\right) dz^2 \right\} \\
&\quad + R^2 K^{-\frac{1}{3}} d\Omega^2_{(7)}, \\
F &= -\frac{3}{8}\frac{R^3}{z^4} K^2 \left(1 + \frac{1}{12}\frac{x_a x^a \rho_b \rho_c x^b x^c}{z^4}\right) dx^0 \wedge dx^1 \wedge dx^2 \wedge dz, \\
K^{-1} &= 1 + \frac{x_a x^a}{z^3}\rho_b x^b \left(1 - \frac{\rho_c x^c}{4z}\right) \\
\text{genCYBE} &= \rho^a \rho_a = 0.
\end{aligned} \tag{33}$$

Conclusions

- Common lore tells: there is no 11D generalized supergravity:
  1. no symmetry to break;
  2. no derivative of a scalar field to be replaced by J^{mn}/I^{m};
- Exceptional field theory approach provides a generalization of 11d supergravity
  1. new tensor $J^{m,n}$, with $J^{(m,n)}$ — a Killing tensor
  2. a split $11 = D + d$ is essential
  3. many contraints on $J^{m,n}$
- The theory is not void: example solutions have been provided
- The theory is 11-dimensional

Further research

- To call it the 11d generalized supergravity need to check κ-symmetry of the membrane
  1. $\mathrm{GL}(11)$ symmetry is broken to $\mathrm{GL}(7) \times \mathrm{GL}(4)$;
  2. the split could provide the desired field to organize $\mathrm{J}^{\mathrm{m,n}}$
- Is yes: enlarge the set of consistent vacua for the membrane;
- In any case:
  1. higher dimensional origin of 10d generalized supergravity;
  2. a setup for nonunimodular deformations and for dual field theories.

Thank you for your attention!

”Deformations open a way to the world of new knowledge”