Testing of the Realism Hypothesis (or Classicality) in the neutral B_s^0-mesons pairs

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International Conference on Quantum Field Theory, High-Energy Physics, and Cosmology

Dubna, Russia 21 July 2022

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Introduction

Since the creation of QM, some physicists attempted to reduce the probabilistic Quantum paradigm to the more familiar and deterministic Classical paradigm. On this way it's necessary to formulate the criteria of Classical paradigm. The well known sets of such criteria are Local Realism and Macroscopic Realism.

Tests of the Local Realism using the set of Wigner inequalities do not include any dependence on time.

Tests of the Macroscopic Realism using time-dependent Leggett–Garg inequalities require the technique of non-invasive (soft) measurements.

In the current talk we propose a time-dependent inequalities for tests of Hypothesis of Realism or Classicality. These tests do not require non-invasive measurements.

NSC and NSIT

The "No-signaling condition" (NSC) is written in the following form:

$$\sum_a w(a,\, b_\beta,\, \ldots \,|\, A,\, B,\, \ldots) = w(b_\beta,\, \ldots \,|\, B,\, \ldots),$$

where A is an observable selected for measurement, a is the measured value of the observable A, and $\sum_a$ sums all possible values of the observable A. The same notation is used for the observable B.

The "No-signaling in time" condition (NSIT) demands that the probability $w(q_j,\, q_i,\, \ldots \,|\, t_j,\, t_i,\, \ldots)$ of measurement of an observable Q at times t_i, $t_j > t_i$ and so on, does not depend on the state of the observable Q at time $t_k \neq \{t_i,\, t_j,\, \ldots\}$. Denoting $Q(t_i)$ as q_i, no-signaling in time condition may be written as follows:

$$\sum_{q_k} w(q_j,\, q_k,\, q_i,\, \ldots \,|\, t_j,\, t_k,\, t_i,\, \ldots) = w(q_j,\, q_i,\, \ldots \,|t_j,\, t_i,\, \ldots).$$

Hypothesis of Realism (Classicality)

1) At any time t_i a system is in a "real physical state" which exists impartially and independently of any observer. "Real physical states" are distinguished from each other by the values of observables that characterize the system under study. We do not suppose these values to be jointly measurable by any macroscopic device.

2) Observable physical states of the system are distinguished by the values of variables which can be jointly measurable in the system at time t_i.

3) For the considered system the NSIT condition and/or NSC are held.

4) The experimentalist has free will to plan, perform, and analyze the results of the experiments on the system.

The 2-time-dependent Wigner inequalities

$$
\begin{aligned}
& w\left(a_+^{(2)}, b_+^{(1)}, t\right) \le \\
\le\ & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right)\left[w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) + \right. \\
+\ & \left. w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\right] \cdot w\left(a_+^{(2)}, c_+^{(1)}, t_0\right) + \\
+\ & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\left[w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right) + \right. \\
+\ & \left. w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\right] \cdot w\left(a_-^{(2)}, c_+^{(1)}, t_0\right) + \\
+\ & w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t)\right)\left[w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) + \right. \\
+\ & \left. w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\right] \cdot w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + \\
+\ & w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t)\right)\left[w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t)\right) + \right. \\
+\ & \left. w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t)\right)\right] \cdot w\left(c_+^{(2)}, b_-^{(1)}, t_0\right).
\end{aligned}
$$

See N. Nikitin, V. Sotnikov, K. Toms, "Proposal of the experimental test of the time-dependent Wigner inequalities for neutral pseudoscalar particles", Phys. Rev. D 92, 016008 (2015).

An example of inequality violation – I

Consider a pair of neutral pseudoscalar mesons, which at time $t_0 = 0$ are in the Bell state

$$\left| \Psi^- \right\rangle = \frac{1}{\sqrt{2}} \left(\left| M^{(2)} \right\rangle \otimes \left| \bar{M}^{(1)} \right\rangle - \left| \bar{M}^{(2)} \right\rangle \otimes \left| M^{(1)} \right\rangle \right).$$

This state is anticorrelated by flavor of the pair, but is correlated by CP-parity (defined as $\left| M_1^{(i)} \right\rangle$ and $\left| M_2^{(i)} \right\rangle$) and mass/lifetime (defined as $\left| M_H^{(i)} \right\rangle$ and $\left| M_L^{(i)} \right\rangle$).

For B_s^0–mesons this state can be obtained from the decay

$$\Upsilon(5S) \to B_s^0 \, \bar{B}_s^0$$

with $B^0 \, \bar{B}^0$, $B^{0\,*} \, \bar{B}^0$, $B^{0\,*} \, \bar{B}^{0\,*}$, $B_s^{0\,*} \bar{B}_s^0$ and $B_s^{0\,*} B_s^{0\,*}$ background states.

We consider one of the simplest background model, describing by the Werner state density matrix:

$$\hat{\rho}^{(W)} = x \left| \Psi^- \right\rangle \left\langle \Psi^- \right| + \frac{1}{4} (1 - x) \, \hat{1},$$

where $0 \leq x \leq 1$ is the purity parameter and $\hat{1}$ - the 4×4 identity matrix.

An example of inequality violation – II

All two-time-dependent Wigner inequalities can be reduced to the form

$$1 \leq R_N(t,\, t_0,\, ...),$$

where $N = \{1, ..., 8\}$ - the identification number of each inequality. These inequalities are violated when $R_N(t,\, t_0,\, ...) < 1$.

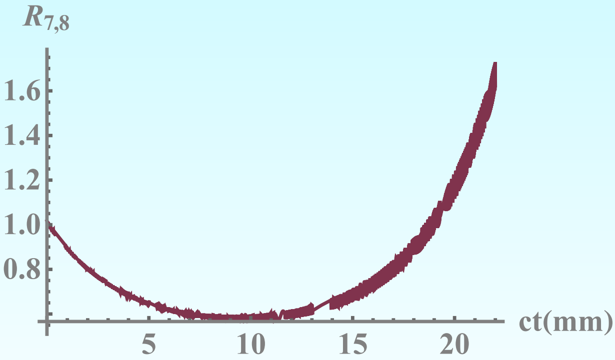
For B_s–mesons these Wigner inequalities are violated at $N = \{7,\, 8\}$:

$$\begin{array}{cc} a_+^{(i)} \to M_2^{(i)}; & a_-^{(i)} \to M_1^{(i)}; \\ b_+^{(i)} \to \bar{M}^{(i)}\,(M^{(i)}); & b_-^{(i)} \to M^{(i)}\,(\bar{M}^{(i)}); \\ c_+^{(i)} \to M_L^{(i)}; & c_-^{(i)} \to M_H^{(i)}. \end{array}$$

We denote the functions $R_N(t,\, t_0,\, ...)$ for the $B_s^0 \bar{B}_s^0$ Bell state $|\,\Psi^-\,\rangle$ at t_0 and $\tilde{R}_N(t,\, t_0,\, x,\, ...)$ – for the $B_s^0 \bar{B}_s^0$ Werner state at t_0. Note, that $\tilde{R}_N(t,\, t_0,\, x = 1,\, ...) = R_N(t,\, t_0,\, ...)$.

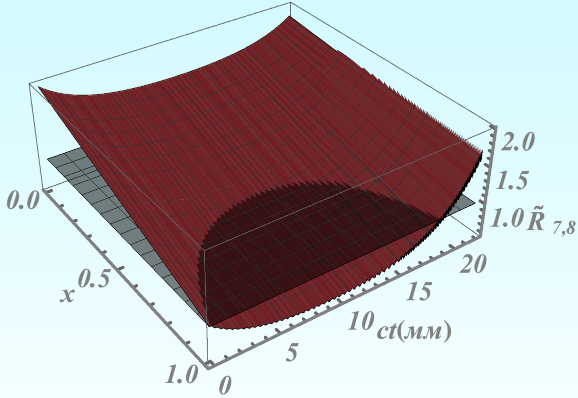
An example of inequality violation – III

The $R_{7,8}(t,\ t_0 = 0,\ ...)$ **functions for** $B_s^0\bar{B}_s^0$**–pairs at condition** $\left|\frac{q}{p}\right| = 1$**.**



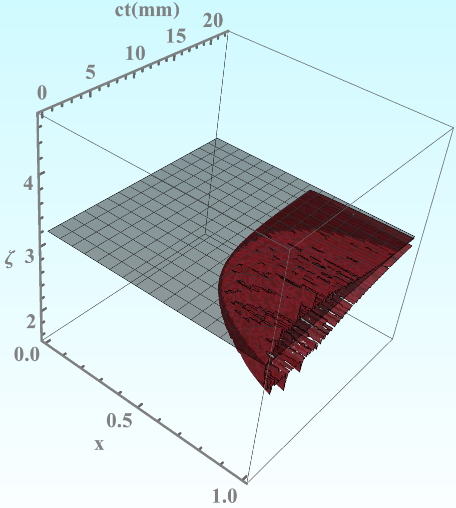
An example of inequality violation – IV

The $\tilde{R}_{7,8}(t,\ t_0 = 0,\ x,\ \ldots)$ functions for $B_s^0 \bar{B}_s^0$–pairs at condition $\left|\frac{q}{p}\right| = 1$.



An example of inequality violation – V

The region of $\tilde{R}_{7,8}(t,\ t_0 = 0,\ x,\ \zeta, ...) > 1$ for $B_s^0 \bar{B}_s^0$–pairs. We denote $\frac{q}{p} = e^{i\zeta}$.



The 3-time-dependent Wigner inequalities

$$
\begin{aligned}
& w\left(a_+^{(2)}(t_2),\ b_+^{(1)}(t_1)\right) \leq \\
\leq\ & w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t_2)\right)\left[w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t_1)\right) + \right. \\
+\ & \left. w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t_1)\right)\right] \cdot w\left(a_+^{(2)}, c_+^{(1)}, t_0\right) + \\
+\ & w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t_2)\right)\left[w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t_1)\right) + \right. \\
+\ & \left. w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t_1)\right)\right] \cdot w\left(a_-^{(2)}, c_+^{(1)}, t_0\right) + \\
+\ & w\left(b_+^{(1)}(t_0) \to b_+^{(1)}(t_1)\right)\left[w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t_2)\right) + \right. \\
+\ & \left. w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t_2)\right)\right] \cdot w\left(c_+^{(2)}, b_+^{(1)}, t_0\right) + \\
+\ & w\left(b_-^{(1)}(t_0) \to b_+^{(1)}(t_1)\right)\left[w\left(a_+^{(2)}(t_0) \to a_+^{(2)}(t_2)\right) + \right. \\
+\ & \left. w\left(a_-^{(2)}(t_0) \to a_+^{(2)}(t_2)\right)\right] \cdot w\left(c_+^{(2)}, b_-^{(1)}, t_0\right).
\end{aligned}
$$

An example of inequality violation – VI

All types of three-time-dependent Wigner inequalities can be rewritten in the form

$$1 \leq A_N(t_2,\, t_1,\, t_0,\, ...),$$

where $N = \{1, ..., 8\}$ **- the identification number of each inequality. These inequalities are violated when** $A_N(t_2,\, t_1,\, t_0,\, ...) < 1$**.**

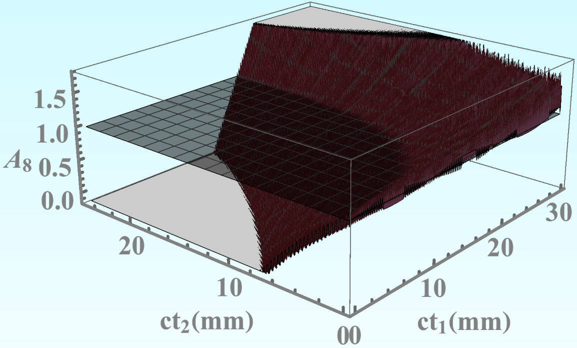
For B_s**–mesons these Wigner inequalities are violated at** $N = \{7,\, 8\}$**:**

$$\begin{array}{cc} a_+^{(i)} \to M_2^{(i)}; & a_-^{(i)} \to M_1^{(i)}; \\ b_+^{(i)} \to \bar{M}^{(i)}\,(M^{(i)}); & b_-^{(i)} \to M^{(i)}\,(\bar{M}^{(i)}); \\ c_+^{(i)} \to M_L^{(i)}; & c_-^{(i)} \to M_H^{(i)}. \end{array}$$

We define the functions $A_N(t_2,\, t_1,\, t_0,\, ...)$ **for the** $B_s^0 \bar{B}_s^0$ **paires at Bell state** $|\,\Psi^-\,\rangle$ **at** t_0 **and** $\tilde{A}_N(t_2,\, t_1,\, t_0,\, x,\, ...)$ **– for the** $B_s^0 \bar{B}_s^0$ **paires at the Werner state at** t_0**.**

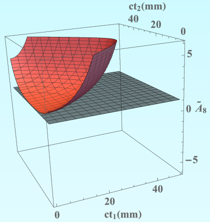
An example of inequality violation – VII

The $A_8(t_2,\ t_1,\ t_0 = 0,\ ...)$ functions for $B_s^0\bar{B}_s^0$–pairs at condition $\left|\frac{q}{p}\right| = 1$.

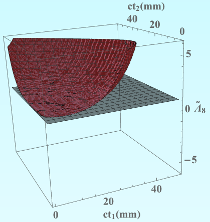


An example of inequality violation – VIII

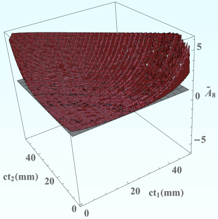
The $\tilde{A}_8(t_2,\, t_1,\, t_0 = 0,\, x,\, ...)$ **functions for** $B_s^0 \bar{B}_s^0$**–pairs at** $\left|\frac{q}{p}\right| = 1$**. The** x**– dependence of the function** $\tilde{A}_8(...,\, x,\, ...)$ **is linear.**



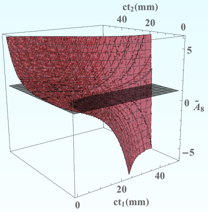
$x = 0.0$



$x = 0.3$



$x = 0.5$



$x = 0.7$

Conclusion

1) **We have presented the 2- and 3-time-dependent Wigner inequalities.**
2) **We have shown that 2- and 3-time-dependent Wigner inequalities are violated for both: the Bell state $|\Psi^-\rangle$ and the Werner state in the $B_s^0\bar{B}_s^0$–pairs.**

Acknowledgments

N. V. Nikitin is grateful for support under the Grant N. 22-22-00297 of the Russian Science Foundation.

Thank you!