Nonlocal condensates for OPE of vacuum correlators

Alexandr Pimikov[1]

based on arXive:2205.12948 [hep-ph]

[1] BLTB JINR, Dubna

International Conference on QFT, HEP, and Cosmology

Outline

- Motivation
- OPE and its role in QCD sum rules
- Taylor series and tensor expansion of gluon NonLocal Condensates (NLC)
  - two-gluon NLC
  - three-gluon NLC
  - four-gluon NLC
- Application of gluon NLCs to glueball related OPEs
- Conclusion

Motivation

Problem:

- How do the quarks and gluon bind into hadrons?
- Description of QCD processes at low and medium energies.

Nonperturbative QCD approaches:

- Light-Cone Sum Rules
- QCD Sum Rules

- Lattice QCD
- Bethe-Salpeter equation
- Flux tube model
- Holographic QCD, AdS/QCD
- Constituent models

Applications:

Structure of Hadrons

- pion DA, rho-meson DA, photon DA, EM pion FF and transition pion FF.

Spectrometry of Exotic Hadrons

- $0^{\pm-}$, $0^{\pm+}$Glueballs
- Pentaquarks with color-octet substructure

Determination of spectrum parameters from requirement of agreement between two ways to correlator as proposed by Shifman&VZ (1979)

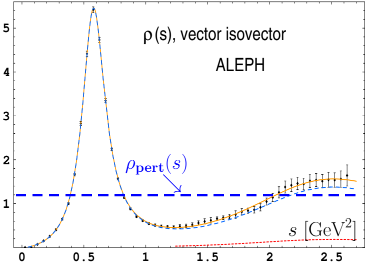
$$\Pi(Q^2) = i\int d^4x\, e^{iqx}\langle 0|TJ(0)J^\dagger(x)|0\rangle\,,$$

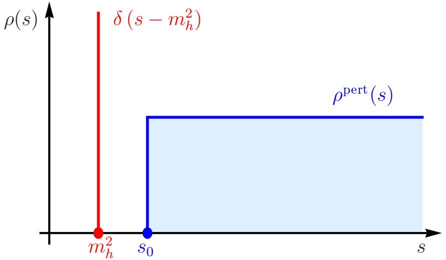
where current J_h describes the state $|h\rangle$: $\langle 0|J_h|h\rangle = f_h\,.$

- 1st way — Dispersion relation: decay constants f_h, masses m_h and others,

$$\Pi_{\text{had}}\left(Q^2\right) = \int_0^\infty \frac{\rho_{\text{had}}(s)\;ds}{s+Q^2} + \text{subtractions}\,.$$

  - model spectral density: $\rho_{\text{had}}(s) = f_h^2\,\delta\left(s-m_h^2\right) + \rho_{\text{pert}}(s)\,\theta\left(s-s_0\right)\,.$





QCD SR Approach

Determination of spectrum parameters from requirement of agreement between two ways for correlator:

- 1st way – Phenomenological way
  Dispersion relation: decay constants f_h and masses m_h,

$$\Pi_{\text{had}}\left(Q^2\right) = \int\limits_0^{\infty} \frac{\rho_{\text{had}}(s)\ ds}{s+Q^2} + \text{subtractions}\,.$$

  - model spectral density: $\rho_{\text{had}}(s) = f_h^2\,\delta\left(s-m_h^2\right) + \rho_{\text{pert}}(s)\,\theta\left(s-s_0\right)$.

- 2nd way – Theoretical way – Operator product expansion:

$$\Pi_{\text{OPE}}\left(Q^2\right) = \Pi_{\text{pert}}\left(Q^2\right) + \sum_n C_n \frac{\langle 0|:O_n:|0\rangle}{Q^{2n}}\,.$$

  - Condensates $\langle 0|:O_n:|0\rangle \equiv \langle O_n\rangle$ collect poorly known nonpert. information. O_n - local operators.
  - Coefficient function are calculable in QCD

QCD SR reads:

$$\Pi_{\text{had}}\left(Q^2, m_h, f_h\right) = \Pi_{\text{OPE}}\left(Q^2\right)\,.$$

Bound state condition:

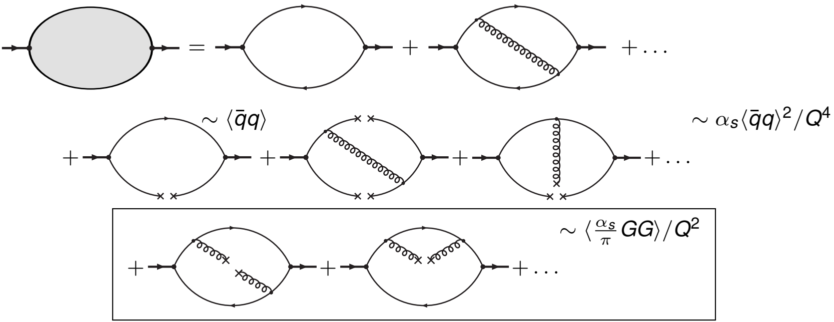
$$Im\Pi_{\text{had}} \sim \rho_{\text{had}}(s) > 0 \quad\rightarrow\quad Im\Pi_{\text{OPE}} > 0$$

- 2nd way — Operator product expansion:

$$\Pi_{\rm OPE}\left(Q^2\right) = \Pi_{\rm pert}\left(Q^2\right) + \sum_n C_n \frac{\langle 0|\,:O_n:\,|0\rangle}{Q^{2n}}\,.$$

Condensates $\langle 0|\,:O_n:\,|0\rangle \equiv \langle O_n\rangle$, where O_n - local scalar operators

  - Most of reliable OPE calculations are performed up to dimension-6 order. There are a lack of theoretical studies on the higher order calculations.



  - We address gluon condensate part of OPE up to dimension-8 order
    10 condensates: $\langle GG\rangle,\ \langle GGG\rangle,\ \langle J^2\rangle,\ \langle GGGG\rangle,\ \cdots$

Local gluon condensates

We work in Fock–Schwinger (FS) gauge [1] with the gauge-fixing point $z_\mu = 0$:

$$(x_\mu - z_\mu)A^a_\mu(x) = 0\,,$$

In the FS gauge the Taylor expansion for the gluon field strength tensor $G_{\mu\nu}(x) \equiv g t^a G^a_{\mu\nu}(x)$ can be written in gauge–covariant form:

$$G_{\mu\nu}(x) \quad = \quad G_{\mu\nu}(0) + \frac{1}{1!} x_\alpha D_\alpha G_{\mu\nu}(0) + \frac{1}{2!} x_\beta x_\alpha D_\beta D_\alpha G_{\mu\nu}(0) + O(x^3)\,.$$

One condensate in dimension-4 and two condensates in dimension-6 ($G_{\mu\nu} \equiv g t^a G^a_{\mu\nu}(0)$):

$$G^4 = \langle \mathrm{Tr}\, G_{\mu\nu} G_{\mu\nu} \rangle\,, \quad G^6_1 = i\langle \mathrm{Tr}\, G_{\lambda\mu} G_{\mu\nu} G_{\nu\lambda} \rangle, G^6_2 = \langle \mathrm{Tr}\, J_\mu J_\mu \rangle\,,$$

We use the notation introduced in [Broadhurst'85,Grozin'86]:

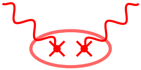
7 condensates in dimension-8

$$\begin{aligned}
G^8_1 &= \langle \mathrm{Tr}\, G_{\mu\nu} G_{\mu\nu} G_{\alpha\beta} G_{\alpha\beta} \rangle, \quad G^8_2 = \langle \mathrm{Tr}\, G_{\mu\nu} G_{\alpha\beta} G_{\mu\nu} G_{\alpha\beta} \rangle, \\
G^8_3 &= \langle \mathrm{Tr}\, G_{\mu\alpha} G_{\alpha\nu} G_{\nu\beta} G_{\beta\mu} \rangle, \quad G^8_4 = \langle \mathrm{Tr}\, G_{\mu\alpha} G_{\alpha\nu} G_{\mu\beta} G_{\beta\nu} \rangle, \\
G^8_5 &= i\langle \mathrm{Tr}\, J_\mu G_{\mu\nu} J_\nu \rangle, \qquad G^8_6 = i\langle \mathrm{Tr}\, J_\lambda [D_\lambda G_{\mu\nu}, G_{\mu\nu}] \rangle, \\
G^8_7 &= \langle \mathrm{Tr}\, J_\mu D^2 J_\mu \rangle\,, \quad G^8_{12} = G^8_1 - G^8_2\,, \; G^8_{34} = G^8_3 - G^8_4\,, \\
& G^8_{56} = 4G^8_5 - G^8_6\,, \quad G^8_{67} = 6G^8_7 - G^8_6\,.
\end{aligned}$$

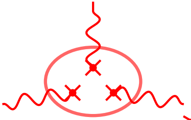
$$\begin{aligned}
G^4_1 &= 2\pi\langle \alpha_S G^2 \rangle\,, \; G^6_1 = -g^2\langle g G^3 \rangle/4\,, \; G^6_2 = g^2\langle J^2 \rangle/2\,, \\
G^8_{12} &= g^4\langle (f^{abc} G^b_{\mu\nu} G^c_{\alpha\beta})^2 \rangle/4\,, \; G^8_{34} = g^4\langle (f^{abc} G^b_{\mu\nu} G^c_{\nu\rho})^2 \rangle/4\,,
\end{aligned}$$

[1] The FS gauge is also known as a fixed-point gauge, radial gauge, and coordinate gauge.

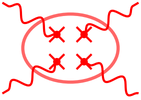
$$\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1)[x_1;x_2]G_{\mu_2\nu_2}(x_2)[x_2;x_1]\rangle =$$



$$i\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1)[x_1;x_2]G_{\mu_2\nu_2}(x_2)[x_2;x_3]G_{\mu_3\nu_3}(x_3)[x_3;x_1]\rangle =$$



$$\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1)[x_1;x_2]G_{\mu_2\nu_2}(x_2)[x_2;x_3]G_{\mu_3\nu_3}(x_3)[x_3;x_4]G_{\mu_4\nu_4}(x_4)[x_4;x_1]\rangle =$$



We performed tensor and Taylor expansion of NLCs at $x_i \to 0$ in Fock–Schwinger (FS) gauge and introduced scalar parametric function $M_k(x_1, \cdots, x_n)$:

$$\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1)\cdots G_{\mu_n\nu_n}(x_n)\rangle = \sum_k \Gamma^k_{\mu_1\nu_1\cdots\mu_n\nu_n}(x_1, \cdots, x_n) M_k(x_1, \cdots, x_n)$$

$\Gamma_{\mu_1\nu_1\cdots\mu_n\nu_n}$ – Lorentz rank-$(2n)$ tensor, e.g, $g_{\mu_1\mu_2}g_{\nu_1\nu_2}$, $x_{1\mu_1}x_{2\nu_1}x_{1\mu_2}x_{2\nu_2}$ (for $n=2$).

Scalar functions $\quad M_k(x_1, \cdots, x_n) = c_0 + c_1 x_1^2 + c_2 x_1 \cdot x_2 + c_3 x_1 \cdot x_3 + \cdots$

are defined by vacuum condensates $c_i \sim \sum_j k_j \langle 0| : O_j : |0\rangle$ up to dimension-8 where O_j are scalar local operators, e.g., $\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}$, $\mathrm{Tr}\, G_{\mu\nu}G^{\mu\nu}G_{\alpha\beta}G^{\alpha\beta}$.

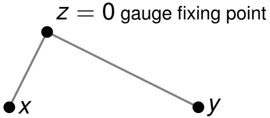
Two-gluon NLC expansion

Our result for two-gluon NLC is presented in new form and agrees with [Grozin'95]

$$\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x)[x;y]G_{\mu_2\nu_2}(y)[y;x]\rangle = \frac{1}{d(d-1)}\mathbb{A}(\mu_1,\nu_1)\mathbb{A}(\mu_2,\nu_2)\sum_{k=0}^{8}\Gamma_k(x,y)M_k(x,y)\,.$$

Wilson's line $[x;y]$ insures gauge invariance:

$$[x;y] = \mathbb{P}\mathrm{exp}\left\{ig\int_{P(x,y)} d\omega_\mu A_\mu(\omega)\right\} \to 1\,.$$



Operator $\mathbb{A}$ antisymmetrize $\mathbb{A}(\mu,\nu)T_{\mu,\nu} = T_{\mu,\nu} - T_{\nu,\mu}$
Master rank-4 tensors:

$$\begin{aligned}
\Gamma_0(x,y) &= g_{\mu_1\mu_2}g_{\nu_1\nu_2}/2\,,\\
\Gamma_1(x,y) &= (x-y)_{\mu_1}(x-y)_{\mu_2}g_{\nu_1\nu_2}(d+4)^{-1}\,,\\
\Gamma_2(x,y) &= (x_{\mu_1}y_{\mu_2} - y_{\mu_1}x_{\mu_2})g_{\nu_1\nu_2}\,,\;\; \Gamma_3(x,y) = (x_{\mu_1}y_{\mu_2} + y_{\mu_1}x_{\mu_2})g_{\nu_1\nu_2}\,,\\
\Gamma_4(x,y) &= \Delta\cdot\Gamma_0(x,y)\,,\;\; \Delta = x^2y^2 - (xy)^2\,,\\
\Gamma_5(x,y) &= (x^2y_{\mu_1}y_{\mu_2} + y^2x_{\mu_1}x_{\mu_2} - xy(x_{\mu_1}y_{\mu_2} + y_{\mu_1}x_{\mu_2}))g_{\nu_1\nu_2}\,,\\
\Gamma_6(x,y) &= x_{\mu_1}y_{\nu_1}x_{\mu_2}y_{\nu_2}\,,\\
\Gamma_7(x,y) &= (y^2)^2x_{\mu_1}x_{\mu_2}g_{\nu_1\nu_2}\,,\;\; \Gamma_8(x,y) = (x^2)^2y_{\mu_1}y_{\mu_2}g_{\nu_1\nu_2}\,.
\end{aligned}$$

Scalar functions for two-gluon NLC

$$M_0(x,y) = G_1^4 + \frac{(x-y)^2}{(d-2)(d+2)}\left(k_0^6 + \frac{k_1^6}{d+4} + \left(k_{0,1}^8 + \frac{k_{1,1}^8}{2(d+4)}\right)\frac{(x-y)^2}{4!}\right) + \ldots$$

$$M_i(x,y) = \frac{1}{(d-2)(d+2)}\left(k_i^{6,2} + \frac{k_{i,1}^8(x-y)^2 + 2k_{i,2}^8 xy}{4!}\right) + \ldots, \text{ for } i=1,2,$$

$$M_3(x,y) = \frac{k_{3,1}^8\,(x-y)^2}{4!(d-2)(d+2)} + \ldots,$$

$$M_i(x,y) = \frac{2k_{i,1}^8}{4!(d-3)(d-2)(d+1)(d+2)} + \ldots, \text{ for } i=4,5,6,$$

The symmetry concerning gluon field strength tensor exchange : $M_8(x,y) = M_7(y,x)$.
The dimension-6 coefficients k_i^6

i	k_i^6	$d=4$
0	$-(d+2)G_1^6$	$-6G_1^6$
1	$(d+4)\left[-(d-4)G_1^6 - (d-2)G_2^6\right]$	$-16G_2^6$
2	$-(d+2)G_1^6$	$-6G_1^6$

Dimension-8 coefficients of two-gluon NLC expansion

Coefficients in terms of local condensates.

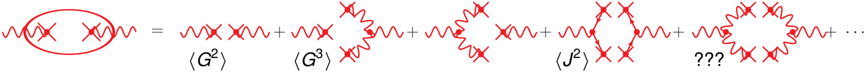
i,j	$k^8_{i,j}$	$d=4,\ J=0$	Selfdual
0,1	$2G^8_{12}+11G^8_{34}-3G^8_{56}$	$2G^8_{12}+11G^8_{34}$	$15G^8_{34}$
1,1	$2\left((d-6)G^8_{12}+(13d-48)G^8_{34}-6(d-3)G^8_{56}-(d-2)G^8_{67}\right)$	$-4\left(G^8_{12}-2G^8_{34}\right)$	0
1,2	$(d+4)\left(G^8_{12}-2G^8_{34}+G^8_{56}\right)$	$8\left(G^8_{12}-2G^8_{34}\right)$	0
2,1	$3G^8_{12}+24G^8_{34}-7G^8_{56}$	$3\left(G^8_{12}+8G^8_{34}\right)$	$30G^8_{34}$
2,2	$3G^8_{34}-G^8_{56}$	$3G^8_{34}$	$3G^8_{34}$
3,1	$-G^8_{12}+2G^8_{34}-G^8_{56}$	$2G^8_{34}-G^8_{12}$	0
4,1	$2(d-3)(d-2)G^8_{6}+(7-3d)dG^8_{12}+4(d+3)G^8_{34}$	$-4\left(5G^8_{12}-7G^8_{34}\right)$	$-12G^8_{34}$
5,1	$2(d-3)(d-2)G^8_{6}-((d-6)d+3)G^8_{12}-(d(d+6)-3)G^8_{34}$	$5G^8_{12}-37G^8_{34}$	$-27G^8_{34}$
6,1	$6\left((d-1)^2G^8_{34}+(d-3)(d-2)G^8_{6}+(d-4)G^8_{12}\right)$	$54G^8_{34}$	$54G^8_{34}$

The 3rd column: gluodynamics in space dimension $d=4$.

The last column: the vacuum gluon field strength tensor is (anti-)selfdual
$\tilde{G}^a_{\mu\nu}\equiv i\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}/2=\pm G^a_{\mu\nu}$.

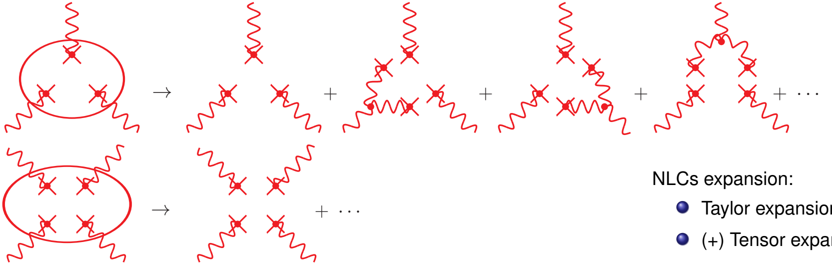
Diagrammatica and NLCs

- NLCs notation is best for OPE at high order.



Local condensates are inappropriate for graphical depiction at high order

- Dimension-6 term include also  $\sim \langle D_\alpha G_{\mu\nu} D_\beta G_{\rho\sigma} \rangle \sim \langle G^3 \rangle + \langle J^2 \rangle$
- Dimension-8 term is defined by 7 condensates: $\langle \mathrm{Tr}\, G_{\mu\nu} G_{\mu\nu} G_{\alpha\beta} G_{\alpha\beta} \rangle$, $\langle \mathrm{Tr}\, G_{\mu\nu} G_{\alpha\beta} G_{\mu\nu} G_{\alpha\beta} \rangle$, $\langle \mathrm{Tr}\, G_{\mu\alpha} G_{\alpha\nu} G_{\nu\beta} G_{\beta\mu} \rangle$, $\langle \mathrm{Tr}\, G_{\mu\alpha} G_{\alpha\nu} G_{\mu\beta} G_{\beta\nu} \rangle$, $\langle \mathrm{Tr}\, J_\mu G_{\mu\nu} J_\nu \rangle$, $\langle \mathrm{Tr}\, J_\lambda [D_\lambda G_{\mu\nu}, G_{\mu\nu}] \rangle$, $\langle \mathrm{Tr}\, J_\mu D^2 J_\mu \rangle$



NLCs expansion:

- Taylor expansion
- (+) Tensor expansion
- (+) Rearrangement of OPE

Usage of gluon NLC expansions

- Standard way – applying Taylor expansion and tensor expansion

$$G^a_{\mu\nu}(x) \quad = \quad G^a_{\mu\nu}(0) + \frac{1}{1!} x_\alpha D_\alpha G^a_{\mu\nu}(0) + \frac{1}{2!} x_\beta x_\alpha D_\beta D_\alpha G^a_{\mu\nu}(0) + \cdots .$$

  - In dimension-D order, intermediate expressions are rank-D tensor. In dimension-8 order – 17 tensor condensates, e.g., $\langle \mathrm{Tr}\, G_{\mu_1\mu_2} D_{\mu_3} G_{\mu_4\mu_5} D_{\mu_6} G_{\mu_7\mu_8} \rangle$.
  - Tensor expansion $\to$ 10 scalar local condensates. $G_{\alpha\beta} = i[D_\alpha, D_\beta]$

    $\langle \mathrm{Tr}\, D_{\mu_1} D_{\mu_2} D_{\mu_3} D_{\mu_4} D_{\mu_5} D_{\mu_6} D_{\mu_7} D_{\mu_8} \rangle = c_1 g_{\mu_1\mu_2} g_{\mu_3\mu_4} g_{\mu_5\mu_6} g_{\mu_7\mu_8} + \cdots$ 105 terms

- New way – using precalculated expansions of NLCs
  The result for vacuum correlator Π_D is the same in given dimension-D order:

$$\Pi_D = \boxed{\Pi_{\text{pert}} + \sum_{j=3}^{D} \Pi_j} = \boxed{\Pi_{\text{pert}} + \sum_{i=1}^{N_D} \Pi_{\text{NLC-}i}} ,$$

Standard New

Π_{pert} – the perturbative contribution,
Π_j – the nonperturbative contribution of dimension-j,
$\Pi_{\text{NLC}-i}$ – the i-th NLC contribution.
N_D – the number of NLC-based diagrams that contribute to dimension-D.

  - Using NLCs expansion – rearranging the contributions by collecting terms arising from one NLC and a specific hard part of the diagram

Two-gluon $0^{\pm+}$ glueballs

QCD SR for glueballs were first considered in [Novikov&SVZ'79] by the correlator

$$\Pi^P(q) = i \int d^4x\, e^{iqx} \langle T\{J_2^P(0) J_2^{P\dagger}(x)\}\rangle$$

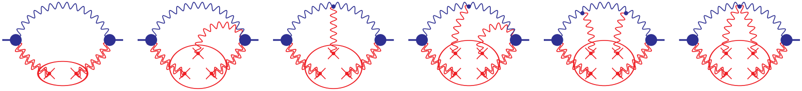
of the two-gluon current for the scalar 0^{++} and pseudo-scalar 0^{-+} glueball states:

$$J_2^P(x) = \alpha_S \delta^{a_1 a_2} T^P_{\mu\nu} G^{a_1}_{\mu_1\nu_1}(x) G^{a_2}_{\mu_2\nu_2}(x)\,,$$

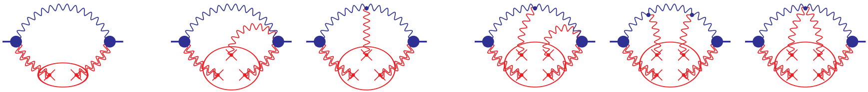
where $T^+_{\mu\nu} \equiv g_{\mu_1\mu_2} g_{\nu_1\nu_2}$, and $T^-_{\mu\nu} \equiv i\epsilon_{\mu_1\nu_1\mu_2\nu_2}/2$ specify the parity $P = \pm$.
The OPE of the correlator:

$$\Pi^P(q) = \Pi^P_{\text{LO}} + \sum_{i=0}^{5} \Pi^P_{\text{NLC-}i} + \cdots = \Pi^P_{\text{LO}} + \Pi^P_4 + \Pi^P_6 + \Pi^P_8 + \cdots\,,$$

Π^P_{LO} – the leading order perturbative contibution,
$\Pi^P_{\text{NLC-}i}$ – one of the six NLC groups of OPE contributions
Each group $\Pi^P_{\text{NLC-}i}$ is depicted by one diagram



Results for Two-gluon $0^{\pm+}$ glueballs



$$\Pi^P(q) = \Pi^P_{\rm LO} + \Pi^P_4 + \Pi^P_6 + \Pi^P_8 + \cdots = \Pi^P_{\rm LO} + \sum_{i=0}^{5} \Pi^P_{\text{NLC-}i} + \cdots ,$$

$$\Pi_4^{\pm} = \pm 4\alpha_S \langle \alpha_S G^2 \rangle ,$$

$$\Pi_6^{\pm} = \frac{8\alpha_S^2}{Q^2} \left(\pm\langle gG^3 \rangle + \frac{1}{3}\langle J^2 \rangle \right) ,$$

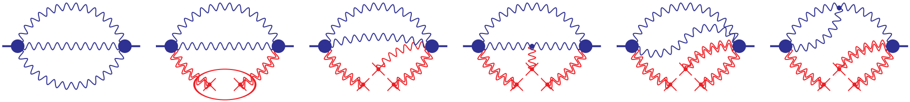
$$\Pi_8^{\pm} = \frac{2\alpha_S}{\pi Q^4} \left(2G_{34}^8 - G_{12}^8 \pm 12 G_{34}^8 \pm (4i\langle \mathrm{Tr}\, J_\mu G_{\mu\nu} J_\nu \rangle - i\langle \mathrm{Tr}\, J_\lambda [D_\lambda G_{\mu\nu}, G_{\mu\nu}] \rangle) \right) .$$

- Reproduced results [Novikov&SVZ'79]
- Additional contributions are obtained (blue color)
  - dimension-6 four-quark condensate $\langle J^2 \rangle \sim \langle \bar{q}\gamma_\mu q \bar{q}\gamma_\mu q \rangle$
  - dimension-8 mixed quark-gluon condensates

The correlator used in QCD SR [Latorre et. al. 1987, Hao et. al. 2005]:

$$\tilde{\Pi}^{\pm}(q) \quad = \quad i\int d^4x\, e^{iqx}\langle T\{J_3^{\pm}(x)J_3^{\dagger\pm}(0)\}\rangle = \tilde{\Pi}_{\rm LO}(Q^2) + \sum_{k=1}^{5}\tilde{\Pi}_k^{\pm}(Q^2) + \cdots\,,$$

where the subscript k numerates the NLC-based terms $\tilde{\Pi}_k^{\pm}$.



The currents:

$$J_3^{+}(x) \quad = \quad g_s^3 f^{abc} G^a_{\mu\nu}(x)G^b_{\nu\rho}(x)G^c_{\rho\mu}(x)\,,\ \ \text{[Latorre et. al. 1987]},$$

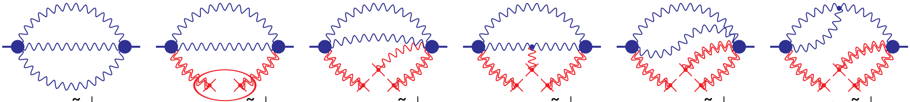
$$J_3^{-}(x) \quad = \quad g_s^3 f^{abc} \tilde{G}^a_{\mu\nu}(x)\tilde{G}^b_{\nu\rho}(x)\tilde{G}^c_{\rho\mu}(x)\,,\ \ \text{[Hao et. al. 2005]}\,,$$

where the dual tensor $\tilde{G}^a_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}G^a_{\alpha\beta}/2$.

We suggest another identical form for negative parity that is easier in use:

$$J_3^{-}(x) = g_s^3 f^{abc} G^a_{\mu\nu}(x)G^b_{\nu\rho}(x)\tilde{G}^c_{\rho\mu}(x) \sim \mathrm{Tr}\, G_{\mu\nu}(x)G_{\nu\rho}(x)\tilde{G}_{\rho\mu}(x)\,.$$

Revised OPE for Three-gluon $0^{\pm+}$ glueballs



$$\tilde{\Pi}_{LO}(Q^2) = -\frac{N_c^2 C_F}{5\cdot 8}\alpha_s^3 Q^8 \ln\frac{Q^2}{\mu^2},$$

$$\tilde{\Pi}_1^{\pm}(Q^2) = \pm 6\pi N_c \alpha_s^2 Q^4 \langle \alpha_s G^2\rangle - 6\pi N_f C_F \alpha_s^3 \langle \bar{q}q\rangle^2 Q^2 \ln\frac{Q^2}{\mu^2},$$

$$\tilde{\Pi}_2^{\pm}(Q^2) = \mp(9/4)N_c\alpha_s^2\langle g^3 G^3\rangle Q^2, \quad \text{agrees with [Hao'05]}(P=-1)$$

$$\tilde{\Pi}_3^{\pm}(Q^2) = \pm 5(9/4)N_c\alpha_s^2\langle g^3 G^3\rangle Q^2,$$

$$\tilde{\Pi}_4^{\pm}(Q^2) = \mp 9 N_c\alpha_s^2\langle g^3 G^3\rangle Q^2 \ln\frac{Q^2}{\mu^2}, \quad \text{agrees with [Hao'05]}(P=-1)$$

$$\tilde{\Pi}_5^{\pm}(Q^2) = -\tilde{\Pi}_4^{\pm}(Q^2),$$

where μ is the renormalization scale, N_c is the number of colors, N_f is the number of flavors and C_F is the Casimir operator in the fundamental representation.

Conclusions

- Expansions of gluon nonlocal condensates obtained in terms of scalar local condensates up to dimension-8 order.
  - two-gluon NLC
  - three-gluon NLC
  - four-gluon NLC
- New way to calculate OPE for vacuum correlators is suggested
  - It allows to bypassing Taylor and tensor expansions in fixed order OPE
  - Expansions can be used in modeling nonlocal condensates for OPE summation
- Examples of gluon NLCs application
  - New contributions to OPE of QCD SR for 2-gluon $0^{\pm+}$ glueballs at dimension-8 order
  - Revision of OPE of QCD SR for 3-gluon $0^{\pm+}$ at dimension-6 order

Thank you for your attention!

Pimikov Alexandr
pimikov@mail.ru

Back-up slides

Three-gluon condensate

NLC expansion up to dimension-8 given by the rank-6 Lorentz tensor:

$$i\langle \mathrm{Tr}\, G_{\mu_1\nu_1}(x_1)[x_1;x_2]G_{\mu_2\nu_2}(x_2)[x_2;x_3]G_{\mu_3\nu_3}(x_3)[x_3;x_1]\rangle$$
$$= \frac{\Gamma(d-2)}{\Gamma(d+1)}\left(\prod_j^3 \mathbb{A}(\mu_j,\nu_j)\right)\frac{1}{2}\sum_{i=0}^{7}\mathbb{A}\Gamma_i^{(abc)}M_i(x_a,x_b,x_c)+\ldots\,,$$

where the index (abc) could be one of the six permutations of (123) and is used to denote permutations of three sets: (μ_1, ν_1, x_1), (μ_2, ν_2, x_2), and (μ_3, ν_3, x_3).
The scalar functions are denoted by M_i. The dependence of the rank-6 master Lorentz tensor $\Gamma_i^{(abc)}$ on three coordinates x_1, x_2, x_3 is implied:

$$\Gamma_i^{(abc)}M_i(x_a,x_b,x_c)=T_{\mu_a\nu_a\mu_b\nu_b\mu_c\nu_c}(x_a,x_b,x_c)=T_{(abc)}\,,$$

where $T_{(abc)}$ is short for the rank-6 tensor.
The operator $\mathbb{A}$ is introduced to shorten the expression and respect asymmetry of the condensate with respect to permutations of the gluon field strength tensors.
The operator of anti-symmetrization:

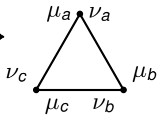
$$\mathbb{A}T_{(abc)} \;=\; T_{(123)}-T_{(132)}+T_{(231)}-T_{(213)}+T_{(312)}-T_{(321)}\,,$$

the operator of symmetrization $\mathbb{S}$:

$$\mathbb{S}T_{(abc)} \;=\; T_{(123)}+T_{(132)}+T_{(231)}+T_{(213)}+T_{(312)}+T_{(321)}\,,$$

Master rank-6 tensors for four-gluon NLC

$$\Gamma_0^{(abc)} = g_{\nu_c\mu_a} g_{\nu_a\mu_b} g_{\nu_b\mu_c}/3\,,$$



$$\Gamma_1^{(abc)} = \frac{1}{d+1}\left[(\mathbb{S}r_A^{(abc)})+(\mathbb{S}r_B^{(abc)}/2)\right](x_c)_\rho (x_a)_\sigma\,,$$

$$\Gamma_2^{(abc)} = \left[(x_c)_\rho (x_a)_\sigma - (x_c)_\sigma (x_a)_\rho\right] r_A^{(abc)}\,,$$

$$\Gamma_3^{(abc)} = (x_c)_\rho (x_a)_\sigma r_B^{(abc)}\,,$$

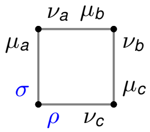
$$\Gamma_4^{(abc)} = \left[(\mathbb{A}r_A^{(abc)})-(\mathbb{S}r_B^{(abc)}/2)\right](x_c)_\rho (x_a)_\sigma\,,$$

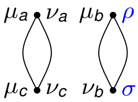
$$\Gamma_5^{(abc)} = \left[(x_c)_\rho (x_a)_\sigma + (x_c)_\sigma (x_a)_\rho\right] r_A^{(abc)}\,,$$

$$\Gamma_6^{(abc)} = (x_b)_\rho (x_b)_\sigma r_A^{(abc)}\,,$$

$$\Gamma_7^{(abc)} = \left[(x_a)_\rho (x_a)_\sigma + (x_b)_\rho (x_b)_\sigma + (x_c)_\rho (x_c)_\sigma\right] r_A^{(abc)}\,,$$

$$r_A^{(abc)} = g_{\nu_c\rho} g_{\sigma\mu_a} g_{\nu_a\mu_b} g_{\nu_b\mu_c}\,,\quad r_B^{(abc)} = g_{\mu_a\mu_c} g_{\nu_a\nu_c} g_{\mu_b\rho} g_{\nu_b\sigma}\,.$$





Scalar functions for three-gluon NLC

$$M_0(x,y,z) = G_1^6 + \frac{k_0^{8,3}}{8(d-3)(d+2)} \frac{(y-x)^2+(x-z)^2+(z-y)^2}{4} + \ldots\,,$$

$$M_i(x,y,z) = \frac{k_i^{8,3}}{8(d-3)(d+2)} + \ldots\,, \quad \text{for } i \geq 1\,,$$

The dimension-8 coefficients $k_i^{8,3}$ in terms of local condensates.

i	$k_i^{8,3}$	$d=4,\ J=0$	Selfdual
0	$4\left(G_{12}^8 - 4(d-2)G_{34}^8 + (d-3)G_{56}^8\right)$	$4\left(G_{12}^8 - 8G_{34}^8\right)$	$-24G_{34}^8$
1	$-2(d-3)(d-2)G_6^8 + (d-4)(d-1)G_{12}^8 + 4(d-1)G_{34}^8$	$12G_{34}^8$	$12G_{34}^8$
2	$4G_{12}^8 - 4(d+1)G_{34}^8$	$4\left(G_{12}^8 - 5G_{34}^8\right)$	$-12G_{34}^8$
3	$2(d-2)G_{12}^8 - 8G_{34}^8$	$4\left(G_{12}^8 - 2G_{34}^8\right)$	0
4	$(d-2)G_{12}^8 - 4G_{34}^8$	$2\left(G_{12}^8 - 2G_{34}^8\right)$	0
5	$-2(d-3)\left(G_{12}^8 - 2G_{34}^8 + G_{56}^8\right)$	$-2\left(G_{12}^8 - 2G_{34}^8\right)$	0
6	$-2(d-3)\left(G_{12}^8 - 2G_{34}^8 + G_{56}^8\right)$	$-2\left(G_{12}^8 - 2G_{34}^8\right)$	0
7	$-2\left(2(d-5)G_{34}^8 - (d-3)G_{56}^8 + G_{12}^8\right)$	$-2\left(G_{12}^8 - 2G_{34}^8\right)$	0

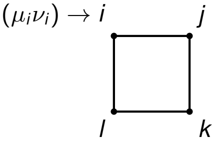
The 3rd column: gluodynamics in space dimension $d=4$.
The last column: the vacuum gluon field strength tensor is (anti-)selfdual $\tilde{G}_{\mu\nu}^a \equiv i\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}/2 = \pm G_{\mu\nu}^a$.

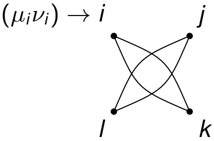
NLC expansion up to dimension-8 given by the rank-8 Lorentz tensors:

$$\langle \mathrm{Tr}(G_{\mu_1\nu_1}(x_1)[x_1;x_2]G_{\mu_2\nu_2}(x_2)[x_2;x_3]G_{\mu_3\nu_3}(x_3)[x_3;x_4]G_{\mu_4\nu_4}(x_4)[x_4;x_1])\rangle$$
$$= \frac{\Gamma(d-3)}{\Gamma(d+3)} \left(\prod_{n=1}^{4} \mathbb{A}(\mu_n,\nu_n)\right) \sum_{m=1}^{4} \Gamma_m M_m(x_1,x_2,x_3,x_4) + \ldots ,$$

where M_m are the scalar functions and Γ_m are the Lorentz tensors

$$\begin{aligned}
\Gamma_1 &= r_A^{(1234)} = \square , \qquad \Gamma_2 = r_A^{(1234)} + r_A^{(2314)} + r_A^{(3124)} = \square + \bowtie + ⧗ , \\
\Gamma_3 &= r_B^{(1234)} = \boxtimes , \qquad \Gamma_4 = r_B^{(1234)} + r_B^{(2314)} + r_B^{(3124)} = \boxtimes + ⧗ +)(, \\
r_A^{(ijkl)} &= g_{\nu_k\mu_l} g_{\nu_l\mu_i} g_{\nu_i\mu_j} g_{\nu_j\mu_k} \quad r_B^{(ijkl)} = g_{\mu_i\mu_k} g_{\nu_i\nu_k} g_{\mu_j\mu_l} g_{\nu_j\nu_l}/4 \,.
\end{aligned}$$





Four-gluon NLC scalar functions

$$
\begin{aligned}
M_m(x_1, x_2, x_3, x_4) \quad = \quad & k_m^{8,4} \\
& + a_m(x_1^2 + x_2^2 + x_3^2 + x_4^2) \\
& + b_m(x_1 \cdot x_2 + x_2 \cdot x_3 + x_3 \cdot x_4 + x_4 \cdot x_1) \\
& + c_m(x_1 \cdot x_3 + x_2 \cdot x_4) + \cdots .
\end{aligned}
$$

Expressions for the dimension-8 coefficients in terms of local condensates

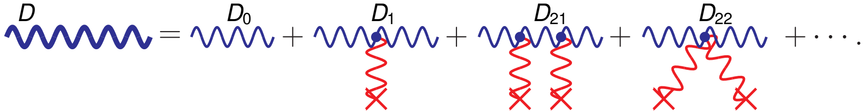
i	$k_i^{8,4}$	$d = 4,\ J = 0$	Selfdual
1	$(d+1)\left[(d+1)G_{34}^8 - G_{12}^8\right]$	$-5\left(G_{12}^8 - 5G_{34}^8\right)$	$15G_{34}^8$
2	$\left(d^2+3\right)G_4^8 + (1-d)G_1^8 - dG_2^8 + (1-d)G_3^8$	$-3G_1^8 - 4G_2^8 - 3G_3^8 + 19G_4^8$	$-3\left(G_1^8 + G_{34}^8\right)$
3	$-(d+1)\left[(d-2)G_{12}^8 - 4G_{34}^8\right]$	$-10\left(G_{12}^8 - 2G_{34}^8\right)$	0
4	$\left(d^2-d+2\right)G_1^8 - 4dG_3^8 - 4(d-1)G_4^8 + 2G_2^8$	$2\left(7G_1^8 + G_2^8 - 8G_3^8 - 6G_4^8\right)$	$9G_1^8 - 6G_{34}^8$

The 3rd column: gluodynamics in space dimension $d = 4$.
The last column: the vacuum gluon field strength tensor is (anti-)selfdual $\tilde{G}_{\mu\nu}^a \equiv i\epsilon_{\mu\nu\alpha\beta}G^{a\alpha\beta}/2 = \pm G_{\mu\nu}^a$.

Background field approach

The total gluon field is considered as a compound $\bar{A}^a_\mu = A^a_\mu + a^a_\mu$: the perturbative quantum gluon field a^a_μ and background field A^a_μ.
To keep the gluon propagator of the quantum field a^a_μ in Feynman gauge form, one should add the generalization gauge fixing term $(D_\mu a^a_\mu)^2/(-2)$ to the Lagrangian that causes modification of the interaction between background and quantum fields.
The gluon propagator

$$D_{\alpha\beta}(p) \;=\; i\int d^4x\, e^{ipx}\langle T\{a_\alpha(x)a_\beta(0)\}\rangle = D_{0\alpha\beta} + D_{1\alpha\beta} + D_{21\alpha\beta} + D_{22\alpha\beta} + \ldots .$$



For glueball and hybrid state currents, the correlator OPE could have terms:

$$i\int d^4x\; e^{ipx}\langle T\{\partial_\mu a_\alpha(x)\partial_\nu a_\beta(0)\}\rangle = \frac{g_{\alpha\beta}q_\mu q_\nu}{q^2} + \frac{2q_\mu q_\nu G_{\alpha\beta}}{q^4} - \frac{g_{\alpha\beta}q_\mu q_\rho G_{\nu\rho}}{q^4} + \ldots .$$

The first two terms can be easily obtained from propagator's expansion, while the third term is additional and related to the derivative of the quantum field $a_\beta(0)$.

Standard way problems

- To expand the product we need to expand each multiplier to corresponding order.
- Working in background field approach we need to remember that pert vertex is not equal vertex with background field
- For glueball and hybrid state QCD SR studies, one needs to calculate additional background field corrections. Using NLCs expansion get around this complication.