Description of neutrino propagation processes in a highly inhomogeneous medium within the framework of the model of quantum spinor field system with singular potentials.

Yury M. Pismak

Department of Theoretical Physics,
Saint-Petersburg State University

International Conference on Quantum Field Theory, High-Energy Physics, and Cosmology.
BLTP JINR, Dubna, 18-21 July 2022

Formulation of model

The main idea of the proposed by Symanzik (K.Symanzik, Nucl. Phys. **B 190** , 1 (1981)) approach for modeling the interacting of quantum field with space-time inhomogeneities (defects) is to describe this system with the action functional of the form:

$$S(\varphi) = S_V(\varphi) + S_{def}(\varphi)$$

where

$$S_V(\varphi) = \int L(\varphi(x)) d^D x, \;\; S_{def}(\varphi) = \int_\Gamma L_{def}(\varphi(x)) d^{D'} x,$$

and Γ is a subspace of dimension $D' \leq D$ in D-dimensional space.

Formulation of model

From the basic principles of QED (gauge invariance, locality, renormalizability) it follows that for thin film without charges and currents, which shape is defined by equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with photon field $A_\mu(x)$ and spinor fields $\bar{\psi}(x), \psi(x)$ reads

$$S_{def}(\varphi) = S_\Phi(A) + S_\Phi(\bar{\psi}, \psi).$$

The action $S_\Phi(A)$ is a surface Chern-Simon action

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho} \partial_\lambda \Phi(x) A_\mu(x) F_{\nu\rho}(x) \delta(\Phi(x)) dx$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$, $\varepsilon^{\lambda\mu\nu\rho}$ denotes totally antisymmetric tensor ($\varepsilon^{0123} = 1$), a is a constant dimensionless parameter.

Formulation of model

The fermion defect action can be written as

$$S_\Phi(\bar{\psi},\psi)=\int\bar{\psi}(x)[\lambda+u^\mu\gamma_\mu+\gamma_5(\tau+v^\mu\gamma_\mu)+\omega^{\mu\nu}\sigma_{\mu\nu}]\psi(x)\delta(\Phi(x))dx$$

Here, γ_μ, $\mu=0,1,2,3$, are the Dirac matrices, $\gamma_5=i\gamma_0\gamma_1\gamma_3\gamma_3$, $\sigma_{\mu\nu}=i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)/2$, and λ, τ, u_μ,v_μ, $\omega^{\mu\nu}=-\omega^{\nu\mu}$, $\mu,\nu=0,1,2,3$ are 16 dimensionless parameters.
It is the most general form of gauge invariant action concentrated on the defect surface being invariant in respect to reparametrization of one and not having any parameters with negative dimensions.

Formulation of model

The full action of the model, which satisfies the requirement of locality, gauge invariance and renormalizability, has the form

$$S(\bar{\psi}, \psi, A) = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\hat{\partial} - m + e\hat{A})\psi \\ + S_{def}(A) + S_{def}(\bar{\psi}, \psi).$$

Due to the requirements of renormalizability interaction of the fields is described by standard contribution $e\bar{\psi}\hat{A}\psi$ to the QED action.

Formulation of model

For stationary defects $\partial_0 \Phi(x) = 0$.
For the plane $x_3 = l$

$$\Phi(x) = x_3 - l.$$

For the sphere with radius r_0 :

$$\Phi(x) = \sqrt{x_1^2 + x_2^2 + x_3^2} - r_0,\ \vec{\partial}\Phi(x) = \frac{\vec{x}}{|\vec{x}|} = \vec{n}(\vec{x})$$

For the cylinder of radius R placed along the x_3 axis :

$$\Phi(x) = x_1^2 + x_2^2 - R^2,\ \vec{\partial}\Phi(x) = (x_1, x_2, 0) = \vec{n}(x)R$$

The limit $a \to \infty$ corresponds to perfectly conducting surface with conditions $n_\mu \tilde{F}^{\mu\nu}|_S = 0$.

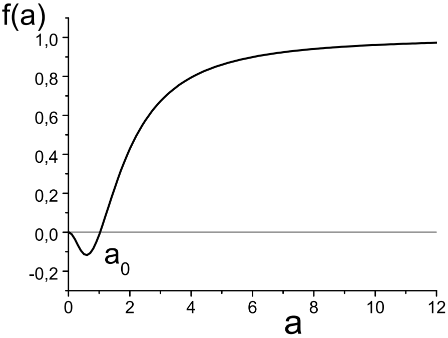
Casimir energy for two parallel planes

For the simplest case of two plane parallel infinite films the Casimir energy was calculated in V. N. Markov and Yu. M. Pis'mak, J. Phys. A 39:21, 6525 (2006); arXive:hep-th/0505218. If the defects are concentrated on planes $x_3 = 0$ and $x_3 = r$, the defect action has the form:

$$S_\Phi = S_{2P} = \frac{1}{2}\int (a_1\delta(x_3) + a_2\delta(x_3 - \mathrm{r}))\varepsilon^{3\mu\nu\rho}A_\mu(x)F_{\nu\rho}(x)dx.$$

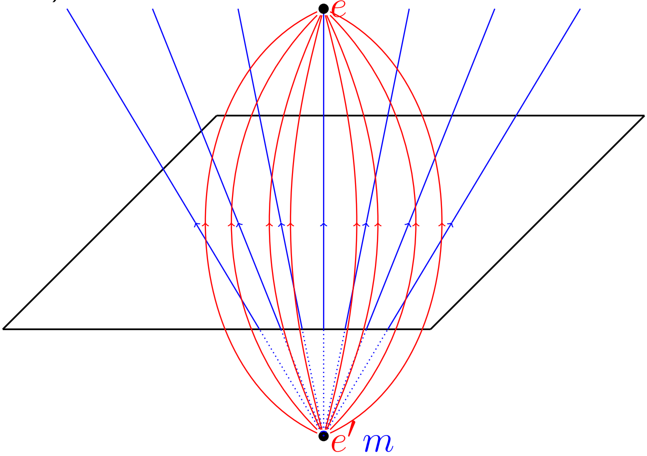
For this geometry, it is convenient to use notations like
$x = (x_0, x_1, x_2, x_3) = (\vec{x}, x_3)$, $\vec{x}^2 = x_0^2 - x_1^2 - x_2^2$, $|\vec{x}| = \sqrt{\vec{x}^2}$.

Casimir energy for two parallel planes



Function $f(a)$ determining the Casimir forces between two parallel planes. It is even (f(a)=f(-a)), has the minimum $f(a_m) = -0,11723$ at $a_m = \pm 0,5892$, and $f(a_0) = 0$ at $a_0 = 0$ and $a_0 = \pm 1,03246$.

For $x_3 > 0$ the magnet field (blue lines) coincides with the field of monopole in the point $(0, 0 - l)$ with magnet charge $m = ea/(1 + a^2)$. Red lines present the electric field, $e' = ea^2/(1 + a^2)$



Interaction of Dirac field with plane $x_3 = 0$

We will consider the material plane $x_3 = 0$ as a defect. In this case, in the Dirac part of the action

$$S(\overline{\psi}, \psi) = \int \overline{\psi}(x)(i\hat{\partial} - m + \Omega(x_3))\psi(x)dx,$$

the interaction of the spinor field with the plane is described with matrix $\Omega(x_3) = Q\delta(x_3)$. Since $\Omega(x_3)$ and $\delta(x_3)$ have the dimension of mass, the matrix Q is dimensionless. For homogeneous isotropic material plane in more general case, the matrix Q could be presented in the form:

$$Q = r_1 I + ir_2\gamma_5 + r_3\gamma_3 + r_4\gamma_5\gamma_3 +$$
$$+r_5\gamma_0 + r_6\gamma_5\gamma_0 + ir_7\gamma_0\gamma_3 + ir_8\gamma_1\gamma_2$$

with I - identity 4x4 matrix, γ_3, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$ are the Dirac matrices.

Modified Dirac equations

The movement of spinor particle in the field of defect $\Omega(x_3)$ is described by the Dirac equation

$$(i\hat{\partial} - m + \Omega(x_3))\psi(x) = 0.$$

It is one of the Euler-Lagrange equations, which is obtained by variational differentiating of the action over $\overline{\psi}(x)$. Taking the derivative over $\psi(x)$ we obtain the second equation

$$(\partial_\mu \overline{\psi}(x))\gamma^\mu + \overline{\psi}(x)(m - \Omega(x_3)) = 0.$$

The condition $\bar{\psi}(x) = \psi^*(x)\gamma_0$ fulfils if $\gamma_0\Omega^+(x) = \Omega(x)\gamma_0$. It is the case for real values of parameters $r_j,\ j = 1, ..., 8$.

Modified Dirac equations

We denote $\psi(x)$ the solution of the modified Dirac equation, and $\psi_-(x) = \psi(x)$ for $x_3 < 0$, $\psi_+(x) = \psi(x)$ for $x_3 > 0$. The spinors $\psi(x)_\pm$ for $x_3 \neq 0$ satisfy the free Dirac equation and boundary condition

$$\lim_{x^3 \to +0} \psi_+(x) = S \lim_{x^3 \to -0} \psi_-(x),$$

One can choose the regularization procedure for $\delta(x_3)$ in such a way that the matrix S is expressed in terms of Q as

$$S = \exp\{-i\gamma^3 Q\}.$$

Modified Dirac equations

We introduce the following notation. If M is 2×2 matrix

$$M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}, \ \vec{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},$$

then $M^{(\pm)}$ are the 4×4 matrixes

$$M^{(+)} = \begin{pmatrix} M_{11} & 0 & M_{12} & 0 \\ 0 & 0 & 0 & 0 \\ M_{21} & 0 & M_{22} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \ M^{(-)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_{11} & 0 & M_{12} \\ 0 & 0 & 0 & 0 \\ 0 & M_{21} & 0 & M_{22} \end{pmatrix}. \tag{1}$$

We denote also by τ_0 the unit 2×2 - matrix, and τ_1, τ_2, τ_3 the Pauli-matrices:

$$\tau_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \ \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Modified Dirac equations

The matrix S is presented in the form

$$S = S^{(+)} + S^{(-)}$$

with

$$S^{(\pm)} = e^{i\chi_\pm}\left(\varsigma_{0\pm}\tau_0^{(\pm)} + i\varsigma_{1\pm}\tau_1^{(\pm)} + \varsigma_{2\pm}\tau_2^{(\pm)} + \varsigma_{3\pm}\tau_3^{(\pm)}\right).$$

Here, $\chi_\pm$, $\varsigma_{j\pm}$, $0 \leq j \leq 3$ are real number which can be expressed in terms of parameters $r_1, \ldots, r_8$ of the model and

$$\varsigma_{0\pm}^2 + \varsigma_{1\pm}^2 - \varsigma_{2\pm}^2 - \varsigma_{3\pm}^2 = 1.$$

Free Dirac equations

The free Dirac equation in coordinate space reads

$$(i\hat{\partial} - m)\psi(x) = 0.$$

By substitution $\psi(x)$ in the form

$$\psi(x) = \frac{1}{(2\pi)^4}\int e^{-ipx}\psi(\bar{p})d\bar{p},\ \ \bar{p} = (p^0, p^1, p^2)$$

one obtains

$$(\hat{p} - m)\psi(\bar{p}) = 0.$$

For real $p_3 = \sqrt{p_0^2 - p_1^2 - p_2^2 - m^2}$ the considered spinor $\psi(x)$ describes the scattering state and by imaginary p_3 - the bound state.

Free Dirac equations

The general solution $\psi(\bar{p})$ of the Dirac equation can be presented as an arbitrary linear combination of linear independent spinors

$$\psi_1(\bar{p}) = \left\{ \begin{array}{c} 1 \\ 0 \\ \frac{-p_3}{m+p_0} \\ \frac{-p_1-ip_2}{m+p_0} \end{array} \right\}, \quad \psi_2(\bar{p}) = \left\{ \begin{array}{c} 0 \\ 1 \\ \frac{-p_1+ip_2}{m+p_0} \\ \frac{p_3}{m+p_0} \end{array} \right\}, \quad p_3 = \pm\sqrt{\bar{p}^2 - m^2}$$

for $p_0 > 0$ and

$$\psi_1'(\bar{p}) = \left\{ \begin{array}{c} \frac{p_1-ip_2}{m-p_0} \\ \frac{p_3}{p_0-m} \\ 0 \\ 1 \end{array} \right\}, \psi_2'(\bar{p}) = \left\{ \begin{array}{c} \frac{p_3}{m-p_0} \\ \frac{p_1+ip_2}{m-p_0} \\ 1 \\ 0 \end{array} \right\}.$$

for $p_0 < 0$.

Neutrino dynamics in free field approximation

To describe the processes called neutrino oscillations, one uses a model of system with three pairs of four-component spinor fields $\bar{\psi}_{\lambda j}(x), \psi_{\lambda j}(x)$, $\lambda = 1, 2, 3$, $j = 1, 2, 3, 4$ which free action functional reads as

$$S_0(\bar{\Psi}, \Psi, M) = \int \bar{\Psi}(x)(i\,\hat{\partial} + M)\Psi(x)dx =$$

$$= \sum_{\lambda,\lambda'=1}^{3} \sum_{j,j'=1}^{4} \int \bar{\psi}_{\lambda j}(x)(i\,\hat{\partial}_{jj'}\,\delta_{\lambda\lambda'} + M_{\lambda\lambda'}\delta_{jj'})\psi_{\lambda' j'}(x)dx$$

where $\delta_{\lambda\lambda'}, \delta_{jj'}$ are the Kronecker symbols. We used the notation $\Psi(x) = \{\psi_1(x), \psi_2(x), \psi_3(x)\}$, $\bar{\Psi}(x) = \{\bar{\psi}_1(x), \bar{\psi}_2(x), \bar{\psi}_3(x)\}$.

Neutrino dynamics in free field approximation

M is a Hermitian (3×3)–matrix with 3 eigenvalues m_μ and corresponding normalized 3-component eigenvectors e_μ:

$$\delta_{\lambda\lambda'} = 0, \text{ by } \lambda \neq \lambda', \; \delta_{\lambda\lambda} = 1, \; \sum_{\lambda'=1}^{3} M_{\lambda\lambda'} e_{\mu\lambda'} = m_\mu e_{\mu\lambda'},$$

$$(e^*_{\mu'} e_\mu) = \sum_{\lambda=1}^{3} e^*_{\mu'\lambda} e_{\mu\lambda} = \delta_{\mu\mu'}, \; (e^*_\lambda e_{\lambda'}) = \sum_{\mu=1}^{3} e^*_{\mu\lambda} e_{\mu\lambda'} = \delta_{\lambda\lambda'},$$

$$M_{\lambda\lambda'} = \sum_{\mu=1}^{3} m_\mu e_{\mu\lambda} e^*_{\mu\lambda'}, \; \lambda, \lambda'; \mu, \mu' = 1, 2, 3.$$

We assume that $0 \leq m_1 \leq m_2 \leq m_3$.

Neutrino dynamics in free field approximation

Using the notations

$$\phi_\mu(x) = \sum_{\lambda=0}^{3} e^*_{\mu\lambda}\psi_\lambda(x), \;\; \bar{\phi}_\mu(x) = \sum_{\lambda=0}^{3} e_{\mu\lambda}\bar{\psi}_\lambda(x),$$

$$\mathbf{M}_{\mu\mu'} = \sum_{\lambda,\lambda'=1}^{3} e^*_{\mu\lambda} M_{\lambda\lambda'} e_{\lambda'\mu'} = m_\mu \delta_{\mu\mu'},$$

we can write the free action $S_0(\bar{\Psi}, \Psi, M)$ of the model in terms of the fields

$$\bar{\Phi}(x) = \{\bar{\phi}_1(x), \bar{\phi}_2(x), \bar{\phi}_3(x)\} \;, \Phi(x) = \{\phi_1(x), \phi_2(x), \phi_3(x)\}$$

as

$$S_0(\bar{\Psi}, \Psi, M) = S_0(\bar{\Phi}, \Phi, \mathbf{M}) = \bar{\Phi}(i\hat{\partial} - \mathbf{M})\Phi =$$
$$\sum_{\mu=1}^{3} \sum_{jj'=1}^{3} \int \bar{\phi}_{\mu j}(x)(i\,\hat{\partial}_{jj'} - m_\mu \delta_{jj'})\phi_{\mu j'}(x)$$

One says that the system is considered in a lepton (called also flavor) representation, if the fields $\bar{\Psi}(x), \Psi(x)$ and the non-diagonal mass matrix M are used for its description. In the so called mass representation the system states are characterized by the fields $\bar{\Phi}(x), \Phi(x)$ and diagonal mass matrix $\mathbf{M}$ (i.e., by the masses $m_\mu, \mu = 1, 2, 3$). For writing indices, we use the letter λ in the lepton representation and the letter μ in the mass one.

Neutrino dynamics in free field approximation

The considered system of spinor fields can be characterized by the local, independent from representation, bilinear function $G_\Gamma^{(\mu)}(x) = G_\Gamma^{(\lambda)}(x)$ defined by a (4×4) - matrix Γ as follows

$$G_\Gamma^{(\mu)}(x) = \bar{\Phi}(x)\Gamma\Phi(x) = \sum_{\mu=1}^{3} \bar{\phi}_\mu(x)\Gamma\phi_\mu(x) =$$

$$\sum_{\lambda,\,\lambda'=1}^{3} \sum_{\mu=1}^{3} e^*_{\mu\lambda} e_{\mu\lambda'}(\bar{\psi}_\lambda(x)\Gamma\psi_{\lambda'}(x)) = \sum_{\lambda,\,\lambda'=1}^{3} \delta_{\lambda\,\lambda'}(\bar{\psi}_\lambda(x)\Gamma\psi_{\lambda'}(x)) =$$

$$= \sum_{\lambda}^{3} \bar{\psi}_\lambda(x)\Gamma\psi_\lambda(x) = \bar{\Psi}(x)\Gamma\Psi(x) = G_\Gamma^{(\lambda)}(x).$$

However, properties of the components $\bar{\phi}_\mu(x)\Gamma\phi_\mu(x)$, $\bar{\psi}_\lambda(x)\Gamma\psi_\lambda(x)$ of $G_\Gamma^{(\mu)}(x)$ and $G_\Gamma^{(\lambda)}(x)$ appears to be essentially different.

Neutrino dynamics in free field approximation

In the stationarity point of $S_0(\bar{\Phi}, \Phi, \mathbf{M})$ the fields $\bar{\phi}_\mu(x), \phi_\mu(x)$ satisfy the Dirac equations

$$(i\,\hat{\partial} - m_\mu)\phi_\mu(x) = 0, \;\; i\,\partial_\nu \bar{\phi}_\mu(x)\gamma^\nu + m_\mu \bar{\phi}_\mu(x) = 0, \;\; \mu = 1, 2, 3.$$

If one chooses $\bar{\phi}_\mu(x), \phi_\mu(x)$ as theirs plane wave solutions

$$\phi_\mu(x) = e^{-ip_\mu x}\chi_\mu(p_\mu), \quad \bar{\phi}_\mu(x) = e^{ip_\mu x}\bar{\chi}_\mu(p_\mu),$$
$$(\hat{p}_\mu - m_\mu)\chi_\mu(p_\mu) = 0, \;\; \bar{\chi}_\mu(p_\mu)(\hat{p}_\mu - m_\mu) = 0, \quad p_\mu = \{p_\mu^0, p_\mu^1, p_\mu^2, p_\mu^3\},$$
$$p_\mu^2 = p_\mu^{0\,2} - p_\mu^{1\,2} - p_\mu^{2\,2} - p_\mu^{3\,2} = m_\mu^2,$$

then $\bar{\phi}_\mu(x)\Gamma\phi_\mu(x) = \bar{\chi}_\mu(p_\mu)\Gamma\chi_\mu(p_\mu)$ does not depend from the space-time point x.

Neutrino dynamics in free field approximation

For the similar quantity in the flavor representation one obtains

$$\bar{\psi}_\lambda(x)\Gamma\psi_\lambda(x) = \sum_{\mu,\,\mu'}^{3} e^*_{\mu\lambda} e_{\mu'\lambda} \bar{\phi}_\mu(x)\Gamma\phi_{\mu'}(x) =$$

$$= \sum_{\mu=1}^{3} e^*_{\mu\lambda} e_{\mu\lambda} \bar{\chi}_\mu(p_\mu)\Gamma\chi_\mu(p_\mu) +$$

$$+ \sum_{\mu\neq\mu'=1}^{3} e^*_{\mu\lambda} e_{\mu'\lambda} e^{i(p_\mu - p_{\mu'})x} \bar{\chi}_\mu(p_\mu)\Gamma\chi_{\mu'}(p_{\mu'}).$$

The dependence on the point x of this expression is determined by the factors $e^{i(p_\mu - p_{\mu'})x}$.

Neutrino dynamics in free field approximation

If space parts $\vec{p}_\mu = \{p^1_\mu, p^2_\mu, p^3_\mu\}$ of the 4-moments $p_\mu = \{p^0_\mu, p^1_\mu, p^2_\mu, p^3_\mu\}$ coincide by $\mu = 1, 2, 3$: $\vec{p}_1 = \vec{p}_2 = \vec{p}_3 = \{p^1, p^2, p^3\} = \vec{p}$, then $\exp\{i(p_\mu - p_{\mu'})x\} = \exp\{i(p^0_\mu - p^0_{\mu'})x^0\}$ and $\bar{\psi}_\lambda(x)\Gamma\psi_\lambda(x)$ does not depend on the space coordinates of $x = \{x^0, \vec{x}\} = \{x^0, x^1, x^2, x^3\}$. For given $\vec{p}, m_\mu$, the moment component p^0_μ is defined as $p^0_\mu = \sqrt{m^2_\mu + \vec{p}^{\,2}}$ and $\exp\{i(p_\mu - p_{\mu'})x\}$ is periodic function of the time coordinate x^0 with period $T_{\mu\mu'} = 2\pi/\left|\sqrt{m^2_\mu + \vec{p}^{\,2}} - \sqrt{m^2_{\mu'} + \vec{p}^{\,2}}\right|$. Thus, the function $\bar{\psi}_\lambda(x)\Gamma\psi_\lambda(x)$ describes an evolution of system which is characterized by 3 periods T_{12}, T_{13}, T_{23}. It is an example of a typical process called neutrino oscillations within the flavor description of system.

Neutrino dynamics in free field approximation

If $m_1 \leq m_2 \leq m_3$, and $m_{\mu'} \leq m_\mu$ then

$$p^0_\mu - p^0_{\mu'} = \sqrt{m^2_\mu + \vec{p}^{\,2}} - \sqrt{m^2_{\mu'} + \vec{p}^{\,2}} =$$

$$= (m_\mu - m_{\mu'})\left(1 - \frac{|\vec{p}\,|^2}{2m_\mu m_{\mu'}}\right) + \mathcal{O}\left(\frac{|\vec{p}\,|^4}{m^2_\mu m^2_{\mu'}}\right),$$

for small $|\vec{p}\,|^2/(m_\mu m_{\mu'})$ and

$$p^0_\mu - p^0_{\mu'} = \frac{m^2_\mu - m^2_{\mu'}}{2|\vec{p}\,|}\left(1 - \frac{m^2_\mu + m^2_{\mu'}}{4|\vec{p}\,|^2}\right) + \mathcal{O}\left(\frac{m^2_\mu m^2_{\mu'}}{|\vec{p}\,|^4}\right)$$

for small $(m_\mu m_{\mu'})/|\vec{p}\,|^2$. The free field approximation of the action functional enables to describe the propagation of neutrino in vacuum.

Interaction neutrino with matter

For processes in which the influence of the material environment is significant, it was proposed to represent this in the model by an additional potential in the Hamiltonian. In this way, models with constant and adiabatically varying density of the matter were constructed and studied by S.P. Mikheev, A.Yu. Smirnov and L. Wolfenstein. It was shown that the effective masses of neutrino is changed by its interaction with material media. This can cause resonance effects in the processes of neutrino oscillations (MSW resonance), which significantly change their characteristics.

Interaction neutrino with matter

The problem of modeling the interaction of neutrinos with external media remain actual at the present time. Many the methods are developed modeling the interactions of neutrino and matter with constant and adiabatically distributed density.
However, little attention has been paid to the study of boundary effects and phenomena generated by the strong inhomogeneous medium, for modeling of which it is necessary to take into account the interaction of neutrinos with singular density distribution concentrated in $d' < 4$ - dimensional subspace of the Minkowski space-time.

Interaction neutrino with matter

A possible generalization of the QED functional $S_{def}(\bar{\psi}, \psi, A, f)$ for description of interaction of neutrino fields with singularly distributed medium could be proposed (Yu.Pismack and O.Shakhova, Symmetry 13, 2180 (2021)) for mass representation as

$$S_{def}(\bar{\Phi}, \Phi, \mathbf{L}, Q, f) = \int \bar{\Phi}(x)\mathbf{L}Q\Phi(x)\delta(f(x))dx = \sum_{\mu,\,\mu'=1}^{3} \int \mathbf{L}_{\mu\mu'}\bar{\phi}_{\mu}(x)Q\phi_{\mu'}(x)\delta(f(x))dx$$

Here the elements of hermitian (3×3)-matrix $\mathbf{L}$ and (4×4)-matrix Q are constant dimensionless parameters. The matrix Q is supposed to be presented as $Q = \sum_{j=1}^{16} r^j \Gamma_j$ with 16 complex numbers r^j and linear independent matrices Γ_j.

Since the dimension of product of two spinor fields is equal 3, by $d' < 3$ the defect action functional (DAF), must contain parameters with negative dimensions. It violates the renormalizability of the basic model. Hence, the only valid value for $d' < 4$ is $d' = 3$.

Interaction neutrino with matter

The solution of equation $f(x) = 0$ describes a region of Minkowski space filled with the interacting with neutrinos matter which properties are presented by the parameters r^k. We consider as extended material object the plane $x^3 = 0$. It corresponds to choosing $f(x) = x^3$. We put on the matrix Q the restriction $\gamma^0 Q \gamma^0 = Q^\dagger$ which is necessary for the scattering matrix unitarity.

The matrix Q is simplified, if there is a symmetry in the interaction of plane $x^3 = 0$ and spinor fields. If it is assumed that the material plane is isotropic and homogeneous, i.e., the DAF is invariant with respect to rotation about the x^3-axis and to translations along x^1, x^2 - directions, then Q has the form

$$Q = r^1 I + i\, r^2 \gamma^5 + r^3 \gamma^3 + r^4 \gamma^5 \gamma^3 + r^5 \gamma^0 + r^6 \gamma^5 \gamma^0 + r^7 \sigma^{03} + r^8 \sigma^{12}$$

where $r^k,\ k = 1, ..., 8$, are real numbers.

Interaction neutrino with matter

Although the action functional is Gaussian, the processes, which it describes, are nontrivial. We will study the scattering on the plane $x^3 = 0$ of particles, which are presented by the fields $\bar{\Phi}, \Phi$, by using the modified Dirac equations

$$\frac{\delta}{\delta\bar{\Phi}} S(\bar{\Phi}, \Phi, \mathbf{M}, \mathbf{L}, Q) = (i\hat{\partial} - \mathbf{M} + \mathbf{L} Q \delta(x^3))\Phi(x) = 0, \quad (2)$$

$$\frac{\delta}{\delta\Phi} S(\bar{\Phi}, \Phi, \mathbf{M}, \mathbf{L}, Q) = i\partial_\mu \bar{\Phi}(x)\gamma^\mu + \bar{\Phi}(\mathbf{M} + \mathbf{L} Q \delta(x^3)) = 0 \quad (3)$$

characterizing the point of stationarity of the functional $S(\bar{\Phi}, \Phi, \mathbf{M}, \mathbf{L}, Q)$. The ordinary way to do it is to find the solution of (2-3) and applying that to construct the currents of incident, reflected and transmitted particles.

Interaction neutrino with matter

If $\bar{\Phi}_+(x), \Phi_+(x)$ and $\bar{\Phi}_-(x), \Phi_-(x)$ denote solutions of (2–3) by $x^3 > 0$ and $x_3 < 0$ respectively, then they must satisfy the free Dirac equations

$$(i\,\hat{\partial} - \mathbf{M})\Phi_\pm(x) = 0,\ \ i\,\partial_\nu\bar{\Phi}_\pm(x)\gamma^\nu + \mathbf{M}\bar{\Phi}_\pm(x) = 0, \tag{4}$$

and conditions on the plane $x^3 = 0$:

$$\lim_{x^3\to+0}\Phi_+(x) = \Lambda S \lim_{x^3\to-0}\Phi_-(x) \tag{5}$$

with matrix S corresponding to the symmetry of considered interaction defined by the matrix Q, and a (3×3)-flavor matrix Λ.

Interaction neutrino with matter

The matrix S which must be expressed in terms of Q and used in such approach depends on choosing of regularization, but it is essential that both $S = S_1$ and $S = S_2$ obeys the requirement

$$S^{\dagger}\gamma^0\gamma^3 S = \gamma^0\gamma^3.$$

If the boundary condition is fulfilled and $\Lambda^{\dagger}\Lambda = 1$, then the above written equality ensures that

$$\lim_{x^3\to+0} \bar{\Phi}_{+}(x)\gamma^3\Phi_{+}(x) = \lim_{x^3\to-0} \bar{\Phi}_{-}(x)\gamma^3\Phi_{-}(x),$$

i.e, no additional current is created on the plane $x^3 = 0$ along the x^3 axis.

Transmission coefficient

The characteristics of the scattering process depend essentially on the choice of parameters of the model, on polarization, energy and incidence angle of particles. For the particles moving orthogonal to the x_3-axes the transmission coefficient $K(k)$ has the form

$$K(k) = K_+(k)\sin(\vartheta)^2 + K_-(k)\cos(\vartheta)^2, \; K_\pm(k) = f_\pm(k)$$

where $k = \sqrt{\frac{p_0 - m}{p_0 + m}}$, p_0 is the energy, m is the mass of particle, and

$$f(k) = f(f_{max}, f_{ext}, k_{max}; k) = \frac{f_{max}}{1 + \frac{k_{max}}{f_{ext}^2}\left(\sqrt{\frac{k}{k_{max}}} - \sqrt{\frac{k_{max}}{k}}\right)^2},$$

$$f_\pm(k) = f(f_{max\pm}, f_{ext\pm}, k_{max\pm}; k).$$

Here, the parameters $f_{max}, f_{ext}, k_{max}$ are positive constants, $0 \leq f(k) \leq f_m$, $f_{max} = f(k_{max}) = \max f(k)$, $f_{ext} = |k_r - k_l|$, $f(k_l) = f(k_r) = f_m/2$.

Transmission coefficient

The function $f(q)$ can be presented as follows

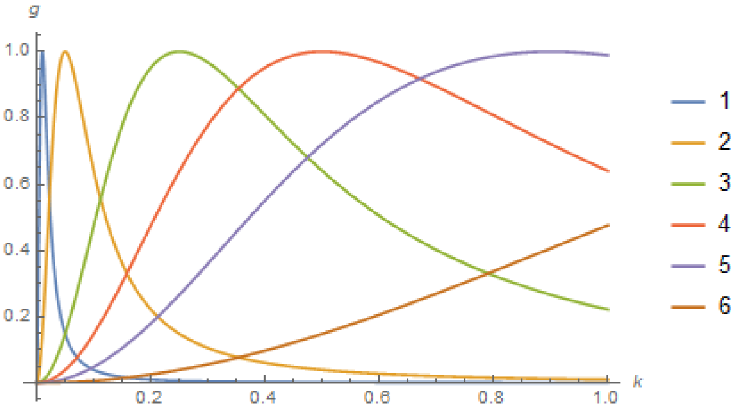
$$f(q) = \frac{f_{max} f_{ext}^2 q}{f_{ext}^2 q + (q_{max} - q)^2} = f_{max} g\left(\frac{q_{max}}{f_{ext}^2}; \sqrt{\frac{q}{q_{max}}}\right),$$

$$g(c;x) = \frac{1}{1 + c(x - x^{-1})^2}.$$

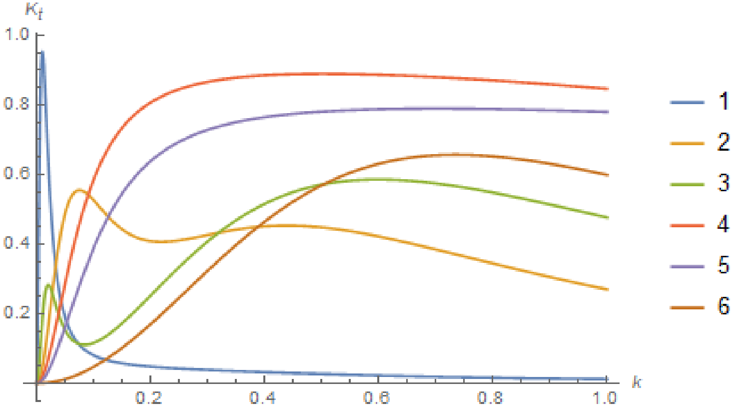
The features of scattering processes

The parameters of the model can be chosen so that the transmission coefficient is almost equal to unity at low particle energy and is almost zero for particles with high energy. One can choose the parameters and so that at high energies the particles almost completely pass through the plane, and at low energies they are almost completely reflected.

Transmission coefficient



The graphs of the function $g(1/4.k/c)$ by different values of parameters c: 1) c=0.01; 2)c =0.05; 3)c =0.25; 4)c =0.5; 5)c =0.9; 6)c = 2.5.



Transmission coefficients
$K_t((k)) = c_{1+} g(c_{2+}, k/c_(3+)) \cos(\vartheta)^2 + c_{1-} g(c_{2-}, k/c_{3-}) \sin(\vartheta)^2$
by different values of $c_{1\pm}$, $c_{1\pm}$, $c_{1\pm}$, ϑ.

Transmission coefficient

The parameters $c_{1\pm}, c_{2\pm}, c_{3\pm}, \vartheta$.

1) $c_{1+} = 0.99, c_{2+} = 0.225, c_{3+} = 0.01, c_{1-} = 0.8, c_{2-} = 0.025, c_{3-} = 0.1, \cos(\vartheta)^2 = 0.95;$

2) $c_{1+} = 0.95, c_{2+} = 0.225, c_{3+} = 0.07, c_{1-} = 0.9, c_{2-} = 0.25, c_{3-} = 0.5, \cos(\vartheta)^2 = 0.55;$

3) $c_{1+} = 0.8, c_{2+} = 0.25, c_{3+} = 0.02, c_{1-} = 0.9, c_{2-} = 0.2, c_{3-} = 0.6, \cos(\vartheta)^2 = 0.35;$

4) $c_{1+} = 0.9, c_{2+} = 0.025, c_{3+} = 0.5, c_{1-} = 0.8, c_{2-} = 0.0025, c_{3-} = 0.7, \cos(\vartheta)^2 = 0.9;$

5) $c_{1+} = 0.8, c_{2+} = 0.025, c_{3+} = 0.7, c_{1-} = 0.7, c_{2-} = 0.0025, c_{3-} = 0.9, \cos(\vartheta)^2 = 0.9;$

6) $c_{1+} = 0.6, c_{2+} = 0.25, c_{3+} = 0.7, c_{1-} = 0.8, c_{2-} = 0.25, c_{3-} = 0.8, \cos(\vartheta)^2 = 0.7.$

The example we have considered with different parameters of the model shows that the interaction of neutrinos with a plane can effect their filtration. The plane transmit particles in a narrow interval of low energies and reflects almost completely all other ones.

The essential feature of the filtration process of particles upon collision with a plane is the possibility of essentially different transmission coefficients for neutrinos of different masses.

In this case, due to filtration of their flux, the regimes of the neutrino oscillations before the plane and behind it can be strongly different. This phenomenon can be used to estimate the masses of neutrinos of various types in carrying out analysis of experimental data.

Although the filtering and MSW-resonance results are similar in many ways, their mechanics are not the same. The MSW effect is formed non-locally in space and time. It requires a certain volume of matter and a certain period of time, generally speaking, that are different for various substances. The filtration occurs instantaneously and locally at $x3 = 0$.

Neutrinos and astrophysics

One of the current theoretical problems in neutrino astro-physics is constructing numerical models of dynamics of supernova explosions. Many research teams have been working on this issue. Perhaps, taking into account the filtering mechanism in such models will be useful to achieve a better understanding of the features of the process of collapse of the super-heavy star core.

In general terms, a possible scenario of its evolution can be presented as follows. If, in the core of the star, its shell filters neutrinos by energies, they can be divided into two classes. Particles with energies from the narrow range of the low-energy region leave the core unhindered. These neutrinos can be called free. For all the others, which we will call bound, the core shell is impermeable.

Neutrinos and astrophysics

In the process of the star's evolution, its core is subjected to the pressure of the increasing gravitational forces. In it, neutrinos are born, the free ones are emitted, and the bound neutrinos are accumulated in the core.
This can go on until the main features of the interaction between the core shell and neutrinos changes, the class of free neutrinos expands, and the star will emit them with further contraction without a significant change in its structure.
In our model, this can be described by changing the function $K_t(k)$. For instance, if the plots of possible $K_t(k)$, is shown on Figure 2, then within the change $(1)->(4)$, a large fraction of the bound high-energy neutrinos become free, they will leave the star, essentially changing the intensity and spectrum of its neutrino emission.

If the main interaction features of neutrinos with the core shell are not changed and at least some of the bound neutrinos do not become free, then the enormous energy accumulated by them in the core destroys its shell sooner or later. After that, the core and the star are exploded.

Conclusions

In our work, we considered the problem of neutrino interaction with matter. Using our experience in constructing models of QED in singular background fields, we have proposed a quantum-field approach, which may be useful for the theoretical description of neutrino propagation in a highly inhomogeneous medium.
It assumes the taking into account the basic symmetry principles of modern physics of fundamental interactions that underlie the Standard Model and can, in principle, be generalized to describe the interaction of all lepton fields with external environment.
The main attention was paid to the problem of neutrino scattering on a material plane, considered as a simplest example of process in the space with strongly inhomogeneous distribution of substance.

Conclusions

In a general form, for a model with an off-diagonal unitary matrix Λ mixing Dirac fields in the mass representation, expressions for the reflected and transmitted waves are obtained. For them, in the model with a diagonal Λ, an explicit solution was obtained, which was used to analyze the oscillations of neutrinos in the case of their motion orthogonally to the plane $x^3 = 0$.

It is shown that the parameters that determine the properties of material of the plane can be chosen so that its effect on the neutrino flux is similar to a filter that transmits particles in a narrow interval of low energies and reflects almost completely all other ones.

As a result of the collision of neutrino with the plane, the parallel component of the momentum does not change, and the orthogonal one does not change in absolute value. Only the amplitudes of fields can change significantly.

There is great interest among experimentalists to determine neutrino masses directly. One can assume that the employment of 2D materials and special surface treatment techniques in the construction of neutrino detectors would enable one to efficiently use the filtering mechanism in experiments of such a kind.

The maximal number of parameters in the model, the properties of which we have studied in detail, was eight for the matrix Q and three for the diagonal matrix L.

Not all of them are included in our results, and the question arises whether it is possible to reduce the number of parameters in the model without limiting its area of applicability.

Conclusions

We considered versions of the model with four, two, and one parameters in the Q matrix, simplified for symmetry reasons, and found a difference in the properties of their predicted transmission coefficients.

This raises the question of whether one can confine oneself to using the simplest models to describe real neutrino oscillation data.

We suppose that the proposed method for modeling the processes of interaction of neutrinos with matter can be useful for theoretical studies and analysis of the obtained experimental data.

Thank you for your attention!