Einstein-Gauss-Bonnet inflation

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- The inflation [1] was supposed to solve problems related with the hot big-bang model[2]

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[1] A.A. Starobinsky, *Relict Gravitation Radiation Spectrum and Initial State of the Universe* (In Russian), JETP Lett. **30** (1979) 682 [Pisma Zh. Eksp. Teor. Fiz. **30** (1979) 719–723]

[2] R. Brout, F. Englert, E. Gunzig, The Creation of the Universe as a Quantum Phenomenon, Annals Phys., **115**, 78 (1978).
A. A. Starobinsky, "A New Type of Isotropic Cosmological Models Without Singularity," Phys. Lett. B **91**, 99 (1980);
D. Kazanas, "Dynamics of the Universe and Spontaneous Symmetry Breaking," Astrophys. J., **241**, L59 (1980);
K. Sato, "First-order phase transition of a vacuum and the expansion of the Universe," MNRAS, **195**, 467 (1981);
A. H. Guth, "The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems," Phys. Rev. D **23**, 347 (1981);
A. D. Linde, " A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems," Phys. Lett. B **108**, 389 (1982);
A. Albrecht, P. J. Steinhardt, "Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking," Phys. Rev. Lett. **48**, 1220 (1982);
A. D. Linde, "Chaotic Inflation," Phys. Lett. B **129**, 177 (1983);
V. F. Mukhanov and G. V. Chibisov, "Quantum Fluctuation and Nonsingular Universe. (In Russian)," JETP Lett. **33**, 532 (1981) [Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)];

- The inflationary stage preceded the Big Bang stage

- The inflationary stage is slow-roll on the quasi de Sitter solution, $|\dot{H}| \ll H^2$

- Stability/unstability of de Sitter solutions

- We consider the model with the Gauss-Bonnet term multiplied to a function of the scalar field ϕ:

$$S = \int d^4x \frac{\sqrt{-g}}{2} \left[FR - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) - \xi(\phi)\mathcal{G} \right], \quad (1)$$

where the functions $V(\phi)$, and $\xi(\phi)$ are differentiable ones, R is the Ricci scalar, F is a constat and

$$\mathcal{G} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4R_{\mu\nu} R^{\mu\nu} + R^2$$

is the Gauss-Bonnet term. We assume that $F(\phi) > 0$ and $V(\phi) > 0$ during inflation.

In the spatially flat Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = \, - \, dt^2 + a^2(t) \left(dx^2 + dy^2 + dz^2\right),$$

one obtains the following system of evolution equations [3]:

$$6H^2 \left(F - 4H\xi_{,\phi}\dot{\phi}\right) = \dot{\phi}^2 + 2V - 6HF_{,\phi}\dot{\phi}, \tag{2}$$

$$2\dot{H} \left(F - 4H\xi_{,\phi}\dot{\phi}\right) = \, - \, \dot{\phi}^2 + 4H^2 \left(\ddot{\xi} - H\xi_{,\phi}\dot{\phi}\right) - \ddot{F} + HF_{,\phi}\dot{\phi}, \tag{3}$$

$$\ddot{\phi} + 3H\dot{\phi} = 3\left(\dot{H} + 2H^2\right) F_{,\phi} - V_{,\phi} - 12\xi_{,\phi}H^2 \left(\dot{H} + H^2\right), \tag{4}$$

where $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor, dots denote the derivatives with respect to the cosmic time t and $A_{,\phi} \equiv dA/d\phi$ for any function $A(\phi)$.

[3] C. van de Bruck and C. Longden, Phys. Rev. D **93**, 063519 (2016)[arXiv:1512.04768].

Effective potential

It is interesting to get the second order correction of inflationary parameter in obtained model. The most convenient way for consideration of inflationary parameter is application of effective potential approach.

- To analyze stability of de Sitter solutions in the Gauss-Bonnet gravity models with field non-minimally coupled with Ricci scalar [4] the effective potential was introduce. In the considering with model constant coupling field with Ricci scalar the effective potential can be presented in the form

$$V_{eff}(\phi) = \frac{\xi(\phi)}{3} - \frac{F^2}{4V(\phi)}. \tag{5}$$

[4] E. O. Pozdeeva, M. Sami, A. V. Toporensky and S. Y. Vernov, Phys. Rev. D **100** (2019) no.8, 083527, arXiv:1905.05085

• Note that the effective potential V_{eff} is not a unique function suitable to describe the stability of de Sitter solutions. For example, we can introduce the analog of effective potential

1. $\tilde{V}_{eff} = -(V_{eff})^{-1}$
2. analogically we have de Sitter solution if $\tilde{V}'_{eff}|_{\phi=\phi_0} = 0$,
3. if $\tilde{V}''_{eff}|_{\phi=\phi_0} > 0$, the de Sitter solution is stable,
4. if $\tilde{V}''_{eff}|_{\phi=\phi_0} < 0$ the de Sitter solution is unstable.

- The conditions are coincide because $\tilde{V}'_{eff} = \frac{V'_{eff}}{V^2_{eff}}$,

  $$\tilde{V}''_{eff} = \frac{{V_{eff}}''}{{V_{eff}}^2} - \frac{2\left({V_{eff}}'\right)^2}{{V_{eff}}^3}$$

  and in de Sitter solution we have: $\tilde{V}''_{eff} = \frac{{V_{eff}}''}{{V_{eff}}^2}$
- In the case of Einstein gravity ($U = 1$, $F = 0$) the alternative effective potential will coincide with potential $V(\phi)$.

In the slow-roll approximation, defined by the following conditions[5]:

$$\dot{\phi}^2 \ll V,\ |\ddot{\phi}| \ll 3H|\dot{\phi}|, \quad 4|\dot{\xi}|H \ll F,\ |\ddot{\xi}| \ll |\dot{\xi}|H,\ |\ddot{F}| \ll H|\dot{F}| \ll H^2F, \tag{6}$$

Eqs. (2)–(4) are:

$$3FH^2 \simeq V, \tag{7}$$

$$2F\dot{H} \simeq -\dot{\phi}^2 - 4H^3\xi_{,\phi}\dot{\phi} + HF_{,\phi}\dot{\phi}, \tag{8}$$

$$\dot{\phi} \simeq -\frac{V_{,\phi} + 12\xi_{,\phi}H^4 - 6H^2F_{,\phi}}{3H}. \tag{9}$$

[5] C. van de Bruck and C. Longden, Phys. Rev. D 93, 063519 (2016) [arXiv:1512.04768],

- The R^2 inflationary predictions [6] in the leading approximation in terms of inverse e-folding numbers $1/N$ for spectral index n_s and tensor-to-scalar ratio r:

$$n_s \simeq 1 - \frac{2}{N}, \quad r \simeq \frac{12}{N^2} \tag{10}$$

are in the best agreement with Planck 2018 [7] and BICEP/Keck 2021 data [8].

[6] A. A. Starobinsky, "Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations, Phys. Lett. **B117** 175 (1982).
A. Starobinsky, "The Perturbation Spectrum Evolving from a Nonsingular Initially de Sitter Cosmology and the Microwave Background Anisotropy," Sov. Astron. Lett. **9**, 302 (1983).

[7] Y. Akrami *et al.* [Planck], "Planck 2018 results. X. Constraints on inflation," [arXiv:1807.06211 [astro-ph.CO]].

[8] P. A. R. Ade *et al.* [BICEP and Keck], [arXiv:2110.00483 [astro-ph.CO]].

- The cosmological attractor models generalizes the prediction of R^2 Starobinsky inflation.
- The cosmological attractor models predict the same values of observable parameter n_{s} in the leading $1/N$ order approximation

$$n_s \simeq 1 - \frac{2}{N+N_0}, \tag{11}$$

- α–attractor models have additional constant C_α in r prediction

$$r \simeq \frac{12C_\alpha}{(N+N_0)^2}.$$

E-folding number formulation

- The analyze of slow-roll inflation in terms of the e-folding number representation with $A' = dA/dN$ is the most convenient.
- There exist two variant for interpretation of relation between time derivative and e-folding number derivative:
  1. $\frac{d}{dt} = H\frac{d}{dN_e}$ and
  2. $\frac{d}{dt} = -H\frac{d}{dN}$.

  In the case of the first type formulation, the inflationary interval in the e-folding formulation is $-65 < N_e < 0$.
  In the case of the second type formulation, inflationary interval in the e-folding formulation is $0 < N < 65$.
  The second formation was applied in cosmological attractor approximation [9] and we follow to the second formulation with $N = -\ln\left(\frac{a}{a_{end}}\right)$.

[9] M. Galante, R. Kallosh, A. Linde and D. Roest, "Unity of Cosmological Inflation Attractors," Phys. Rev. Lett. **114** (2015) no.14, 141302 [arXiv:1412.3797 [hep-th]]. R. Kallosh and A. Linde, "Universality Class in Conformal Inflation," JCAP **1307**, 002 (2013) [arXiv:1306.5220 [hep-th]].

From Eqs. (7)–(9), we get the following leading-order equations:

$$\ln(H)' = 2W_{,\phi} V_{eff,\phi} \,, \tag{12}$$

$$\phi' = 4W V_{eff,\phi} \,, \tag{13}$$

where derivatives with respect to N_e are denoted by primes, $W \equiv V/F$ and the effective potential :

$$V_{eff}(\phi) = \frac{1}{3}\xi(\phi) - \frac{F^2(\phi)}{4V(\phi)}. \tag{14}$$

The slow-roll approximation (6) requires $|\epsilon_i| \ll 1$, $|\delta_i| \ll 1$, and $|\zeta_i| \ll 1$, where the slow-roll parameters are as follows:

$$\epsilon_1 = \frac{1}{2}\frac{(H^2)'}{H^2} \simeq \frac{1}{2}\frac{W'}{W}, \quad \epsilon_{i+1} = -\frac{\epsilon_i'}{\epsilon_i}, \quad i \geqslant 1, \tag{15}$$

$$\zeta_1 = -\frac{F'}{F}, \quad \zeta_{i+1} = -\frac{\zeta_i'}{\zeta_i}, \quad i \geqslant 1, \tag{16}$$

$$\delta_1 = -\frac{4H^2}{F}\xi' \simeq -\frac{4V}{3F^2}\xi', \quad \delta_{i+1} = -\frac{\delta_i'}{\delta_i}, \quad i \geqslant 1. \tag{17}$$

The relation between the tensor-to-scalar ratio r and square of the field derivative:

$$r = \frac{32W}{F}V_{eff}' = \frac{8}{F}\left(\phi'\right)^2. \tag{18}$$

The spectral index of scalar perturbations n_s can be presented via derivatives of the effective potential:

$$n_s = 1 + \frac{d}{dN}\ln\left(\frac{r}{\eta_0}\right) = 1 + \frac{d}{dN}\ln\left(\frac{F^2 r}{V}\right) = 1 + \frac{V_{eff}''}{V_{eff}'}. \tag{19}$$

The amplitude of the scalar perturbations :

$$A_s \simeq \frac{2H^2}{\pi^2 Fr} \simeq \frac{2W}{3\pi^2 Fr} = \frac{1}{48\pi^2 V_{eff}'}, \tag{20}$$

α-attractor generalization

- We consider the model in slow-roll regime using the e-folding number representation:

$$(\phi')^2 \simeq \frac{V'}{V}F + \frac{4\xi' V}{3F} = \frac{\left(H^2\right)'}{H^2}F + 4H^2\xi'. \tag{21}$$

- We present the first slow-roll parameters in terms of H^2, ξ:

$$\epsilon_1 = \frac{1}{2}\frac{(H^2)'}{H^2}, \delta_1 = -\frac{4H^2\xi'}{F}. \tag{22}$$

- The second slow-roll parameters are related with first slow-roll parameters: $\epsilon_2 = -\epsilon_1'/\epsilon_1, \quad \delta_2 = -\delta_1'/\delta_1$.
- The spectral index of scalar perturbations and the tensor-to-scalar ratio can be presented in terms e-folding numbers derivatives:

$$n_s = 1 - 2\epsilon_1 + \frac{r'}{r}, \tag{23}$$

$$r = 8|2\epsilon_1 - \delta_1| = 8\left(\frac{(H^2)'}{H^2} + \frac{4H^2\xi'}{F}\right) = \frac{8(\phi')^2}{F}. \tag{24}$$

Generalization of the α–attractors results

- Accordingly to inflationary parameters of α-attractor models without the Gauss-Bonnet term spectral index includes only logarithmic derivative of tensor-to-scalar ratio

$$\frac{r'}{r} = -\frac{2}{N+N_0}, \quad \text{and} \quad n_s \approx 1 + \frac{r'}{r}. \tag{25}$$

  in the leading order of $1/N$ approximation.
- The model without the Gauss-Bonnet term and exponential potential leading to cosmological-attractor prediction was considered in [10]
- We generalize this model to the Einstein-Gauss-Bonnet gravity.

[10] V. Mukhanov, Eur. Phys. J. C **73** (2013), 2486 [arXiv:1303.3925 [astro-ph.CO]].

Exponential form

To generalize cosmological attractor approximation to inflationary models with the Gauss-Bonnet term we compare (24) with (11):

$$\frac{r}{8} = \frac{(H^2)'}{H^2} + \frac{4H^2\xi'}{F} = \frac{3C_\alpha}{2(N+N_0)^2}. \tag{26}$$

For simplicity we suppose that all terms in this equation are proportional to $1/(N+N_0)^2$ and get the same approximation of slow-roll parameter ϵ_1 in leading $1/N$ order:

$$H^2 = H_0^2 \exp\left(-\frac{3C_\beta}{2(N+N_0)}\right), \quad \xi = \xi_0 \exp\left(\frac{3C_\beta}{2(N+N_0)}\right), \tag{27}$$

where C_β is a constant. We substitute (27) to (26) and get:

$$\frac{r}{8} = \frac{3C_\beta}{2(N+N_0)^2}\left(1 - \frac{4\xi_0 H_0^2}{F}\right), \tag{28}$$

fixing a relation between C_α and C_β:

$$C_\beta = \frac{C_\alpha}{1 - \frac{4\xi_0 H_0^2}{F}}, \quad H_0^2 \neq \frac{F}{4\xi_0}. \tag{29}$$

Accordingly (24) the derivative of field is related with e-folding number:

$$(\phi')^2 = \frac{3C_\alpha}{2(N+N_0)^2};\ \phi' = \frac{\omega_\phi\sqrt{\frac{3C_\alpha}{2}}}{N+N_0},\ \omega_\phi = \pm 1 \quad (30)$$

from here

$$\phi = \omega_\phi\sqrt{\frac{3C_\alpha}{2}}\ln\left(\frac{N+N_0}{N_\phi}\right), \quad N+N_0 = N_\phi\exp\left(\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right). \quad (31)$$

Using (9), (27) and (31) we construct family of the models with the Gauss-Bonnet interaction and potential with variable parameter C_α:

$$V = 3H_0^2\exp\left(-\frac{3}{2}\frac{C_\beta}{N_\phi}\exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\right), \quad (32)$$

$$\xi = \xi_0\exp\left(\frac{3}{2}\frac{C_\beta}{N_\phi}\exp\left(-\omega_\phi\sqrt{\frac{2}{3C_\alpha}}\phi\right)\right) \quad (33)$$

leading to appropriate inflationary scenarios. This model is generalization of the general relativity model obtained in [11]

[11] V. Mukhanov, Eur. Phys. J. C **73** (2013), 2486 [arXiv:1303.3925 [astro-ph.CO]].

Obtained model

Thus, using all restriction we get the following model of slow-roll inflation in terms of e-folding number:

$$\tilde{V} = V_0 \exp\left(-\frac{2N_0^2}{N+N_0}\right), \qquad \tilde{\xi} = \xi_0 \exp\left(\frac{2N_0^2}{N+N_0}\right), \tag{34}$$

$$\tag{35}$$

where $\xi_0 = \frac{3M_{Pl}^4}{4V_0} - \frac{(N_b+N_0)^2 \exp\left(-\frac{2N_0^2}{(N_b+N_0)}\right)}{32\pi^2 A_s N_0^2}$ and $A_s = 2.1 \times 10^{-9}$ is observation constraint. [12] Using all restriction to model constant we get the following expression for inflationary parameters

$$n_s = 1 - \frac{2}{N+N_0} - \frac{2N_0^2}{(N+N_0)^2}, \quad r = \frac{16N_0^2\left(3M_{Pl}^4 - 4V_0\xi_0\right)}{3M_{Pl}^4 (N+N_0)^2}.$$

[12] Y. Akrami *et al.* , *A&A* **641**, A10 (2020); arXiv:1807.06211

The observable values of n_s [13] $n_s = 0.965 \pm 0.04$, allows us to restrict values of N_0. Indeed, the parameter N_0 belongs to the following interval:

$$2 \quad \leqslant N_0 \leqslant 0.0199N - 0.510 + 0.0102\sqrt{195N^2 - 10000N + 2500}.$$

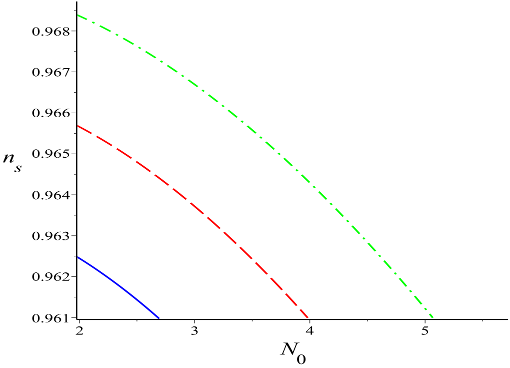


Figure: The inflationary parameter n_s as a function of N_0 for different numbers of e-foldings during inflation: $N = 55$ (blue solid curve), $N = 60$ (red dash curve) and $N = 65$ (green dash-dot curve).

[13] Y. Akrami *et al.* , *A&A* **641**, A10 (2020); arXiv:1807.06211

- We see in Fig. 1 that for any $55 \leqslant N \leqslant 65$ it is possible to find suitable values for N_0, in particular, the constraint $2 \leqslant N_0 \leqslant 5.06$ corresponds to $N = 65$.
- Note that in the case of $\xi_0 = 0$, one can get an approximation the inflationary parameters corresponding to the R^2 inflation [14] putting $C_\alpha = 1$ and, so, $N_0 = \sqrt{3}/2 \approx 0.87$.

[14] A.A. Starobinsky, Phys. Lett. B **91**, **99** (1980)
A.A. Starobinsky, Phys. Lett. B **117**, 175 (1982)
A.A. Starobinsky, Sov. Astron. Lett. **9**, 302 (1983).

The cosmological attractors models [15] lead to spectrum (11) in leading order approximation and at the same time allow two different relations between the tensor-to-scalar ratio and e-folding number: $r \sim (N + N_0)^{-2}$ and $r \sim (N + N_0)^{-1}$.

[15] Kallosh, R.; Linde, A. *J. Cosmol. Astropart. Phys.* **2013**, *07*, 002.
Galante, M.; Kallosh, R.; Linde, A.; Roest, D. *Phys. Rev. Lett.* **2015**, *114*, 141302
Roest, D. *J. Cosmol. Astropart. Phys.* **2014**, *01*, 007

- Now we consider the effective potential of exponential form supposing that the tensor-to-scalar ration is $r \sim (N + N_0)^{-1}$.
- Such as $\xi = 3\,V_{eff} + \frac{24\,V'_{eff}}{r}$ the expressions for the slow-roll parameters ϵ_1 and δ_1 can be simplified as follows:

$$\epsilon_1 = \frac{1}{2}\left(\frac{r'}{r} - \frac{V''_{eff}}{V'_{eff}}\right), \quad \delta_1 = \left(\frac{r'}{r} - \frac{r}{8} - \frac{V''_{eff}}{V'_{eff}}\right) = 2\epsilon_1 - \frac{r}{8}, \qquad (36)$$

- We assume that in the case of

$$r = \frac{8r_0}{(N_e + N_0)} \qquad (37)$$

the upper values of parameter r_0 are rather small to save the slow-roll regime during inflation.

The choice of exponential effective potential $V_{eff} = C_{eff}\exp\left(-\frac{C_2}{N_e+N_0}\right)$ and tensor-to-scalar ratio $r = \frac{8r_0}{(N_e+N_0)}$ leads to the following model in terms of e-folding number:

$$V = \frac{r_0\,(N_e+N_0)}{4\,C_{eff}\,C_2\left(\exp\left(-\frac{C_2}{N_e}\right)\right)}, \quad \xi = \frac{3\,C_{eff}\exp\left(-\frac{C_2}{N_e+N_0}\right)\left(r_0\,(N_e+N_0)+C_2\right)}{r_0(N_e+N_0)} \tag{38}$$

Which leads to the following slow-roll parameters:

$$\epsilon_1 = \frac{1}{2(N_e+N_0)} - \frac{C_2}{2(N_e+N_0)^2}, \tag{39}$$

$$\epsilon_2 = \frac{-(N_e+N_0)+2\,C_2}{(N_e+N_0)\,(-(N_e+N_0)+C_2)} \tag{40}$$

$$\delta_1 = -\frac{r_0-1}{N_e+N_0} - \frac{C_2}{(N_e+N_0)^2}, \tag{41}$$

$$\delta_2 = \frac{(r_0-1)\,(N_e+N_0)+2\,C_2}{(N_e+N_0)\,((r_0-1)\,(N_e+N_0)+C_2)} \tag{42}$$

$C_2 = -(2\,N_0-1)\,N_0$

The start point of inflation N_b is related to the appropriate value of the spectral index:

$$n_s = 1 - \frac{2}{N_b + N_0} - \frac{(2N_0 - 1)N_0}{(N_b + N_0)^2}. \tag{43}$$

Let us present minimal values of δ_1 at key values of n_s:

1. if $n_s = 0.961$ then $\delta_1 \geq 1.7465$,
2. if $n_s = 0.965$ then $\delta_1 \geq 1.7186$,
3. if $n_s = 0.969$ then $\delta_1 \geq 1.6834$.

The saving of appropriate values of spectral index $n_s = 0.965 \pm 0.04$ leads to the divination of δ_1 from the slow-roll regime during inflation. Thus, the reconstruction of a minimally coupled model in EGB gravity leading to inflationary parameters of the cosmological attractor with $r \sim (N_e + N_0)^{-1}$ during the slow-roll regime is impossible.

Application of effective potential formulation

- In the the case then $F = M_{Pl}^2$ the effective potential of obtained model has the following form:

$$V_{eff} = C_{eff} \exp\left(-\frac{C_2}{N+N_0}\right), \tag{44}$$

where $C_{eff} = -\frac{3M_{Pl}^4 - 4V_0\xi_0}{12V_0}$, $C_2 = -\frac{9C_\alpha M_{Pl}^4}{2(3M_{Pl}^4 - 4V_0\xi_0)} = \frac{3C_\alpha M_{Pl}^4}{8V_0 C_{eff}}$.

- The application of relation between spectral index and effective potential leads to the second order correction

$$n_s = 1 - \frac{2}{N+N_0} + \frac{C_2}{(N+N_0)^2}, \tag{45}$$

where a constant $|C_2| \ll 60$.

- Also using effective potential formulation we calculate amplitude of scalar perturbation

$$A_s = \frac{(N+N_0)^2}{48\pi^2 C_{eff} C_2} \exp\left(\frac{C_2}{N+N_0}\right). \tag{46}$$

Application of effective potential

- To construct a set of inflationary models with the same function $n_s(N_e)$ we put the condition that V'_{eff} does not change.
- It also guarantees that the parameter A_s does not change.
- To get the same function $\phi(N_e)$ in the slow-roll approximation we add the condition that the function W does not change. In other words, we consider the model with a double differentiable function $f(\phi)$

$$F = M_{Pl}^2 f(\phi),\; V = V_0 f(\phi)\exp\left(-\omega_0\exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right), \quad (47)$$

$$\xi(\phi) = \left(\xi_0 + \frac{3M_{Pl}^4}{4V_0}(f(\phi)-1)\right)\exp\left(\omega_0\exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right), \quad (48)$$

- Note that we do not fix the parameter $r(N_e)$:

$$r(N_e) = \frac{12C_\alpha}{f\cdot(N_e+N_0)^2}, \quad (49)$$

- The observation data gives restrictions on the function f. Other restrictions on this function can be obtained from the condition that the slow-roll approximation should be satisfied during inflation. We

The case of an exponential function F

- We consider exponential function $f(\phi)$

$$f(\phi) = f_0 \exp\left(\beta\omega_0 \exp\left(-\sqrt{\frac{2}{3C_\alpha}}\frac{\phi}{M_{Pl}}\right)\right) \quad \text{where } \beta \text{is a constant}$$

- Using the relation between e-folding number and field we formulate considering model in terms of e-folding number:

$$F = M_{Pl}^2 f_0 \exp\left(\frac{2N_0^2\beta}{N_e + N_0}\right), \; V = f_0 V_0 \exp\left(\frac{2N_0^2(\beta-1)}{N_e + N_0}\right)$$

$$\xi = \frac{\left(3M_{Pl}^4 f_0 \exp\left(\frac{2\beta N_0^2}{N_e+N_0}\right) - 3M_{Pl}^4 + 4\xi_0 V_0\right)\exp\left(\frac{2N_0^2}{N_e+N_0}\right)}{4V_0}.$$

- the corresponding tensor-to-scalar ratio can be presented such as

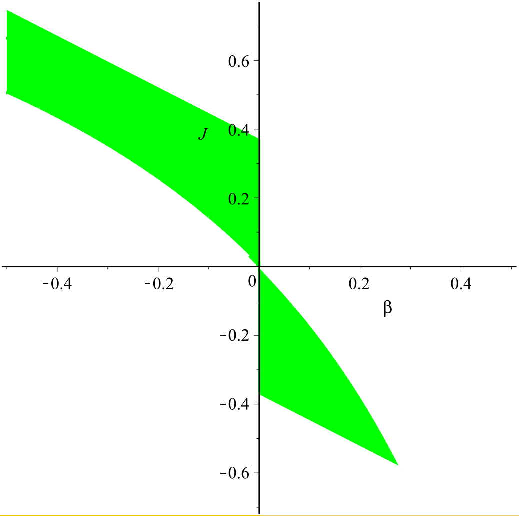
$$r = \frac{16N_0^2\left(3M_{Pl}^4 - 4V_0\xi_0\right)}{3M_{Pl}^4 f_0 (N_e + N_0)^2}\exp\left(-\frac{2N_0^2\beta}{N_e + N_0}\right)$$

- To fix f_0 we assume that at the end of inflation $F = M_{Pl}^2$, therefore, $f_0 = \exp\left(-2N_0\beta\right)$.
- From explicate form of the model slow-roll parameters
  1. the condition $|\beta| \leqslant 1/2$ is necessary to get $|\zeta_1| < 1$ during inflation
  2. at $N_e = 0$, we get $\delta_1(0) = \frac{8J}{3} + 2\beta$, $\delta_2(0) = \frac{2}{N_0} - \frac{2\beta(4J-3)}{4J+3\beta}$, where $J \equiv V_0\xi_0/M_{Pl}^4$.
  3. Let us consider the case $N_0 = 2$ in detail. We get

$$-\frac{1}{2} \leqslant \frac{4}{3}J + 3\beta \leqslant \frac{1}{2}, \tag{50}$$

$$-2 \leqslant \frac{2\beta(3-4J)}{4J+3\beta} \leqslant 0. \tag{51}$$

Also, we have the conditions $|\beta| \leqslant 1/2$. So, it follows from inequalities (50) that $|J| \leqslant 3/4$. Note that $J = 3/4$ is excluded from expression for the effective potential). In Fig. 2, the green domain corresponds to the values of parameters J and β that satisfy inequalities (50) and (51). At $\beta = 0$, we get the initial model with a constant F.



Substituting the chosen values of the constants into formulas, we obtain

$$A_s = \frac{V_0(N_b+2)^2}{32\pi^2 M_{Pl}^4(3-4J)} \exp\left(-\frac{8}{(N_b+2)}\right), \tag{52}$$

$$r = \frac{64(3-4J)}{3(N_b+2)^2} \exp\left(\frac{4\beta N_b}{(N_b+2)}\right). \tag{53}$$

The values of the inflationary parameter r and the corresponding values of V_0 and ξ_0 for $N_b = 60$ are presented in Table 1. For any values of these parameters, $n_s = 928/961 \simeq 0.96566$ and $A_s = 2.1 \cdot 10^{-9}$. One can see that the parameter r increases with growth of J and all values of r, but one, do not contradict the observation data.

Table: Model parameters and the corresponding values of r for the exponential function F.

β	J	V_0/M_{Pl}^4	ξ_0	r
-0.5	0.72	$2.3556 \cdot 10^{-11}$	$3.0565 \cdot 10^{10}$	0.00009614
-0.5	0.5	$1.9630 \cdot 10^{-10}$	$2.5471 \cdot 10^{9}$	0.0008011
-0.3	0.5	$1.9630 \cdot 10^{-10}$	$2.5471 \cdot 10^{9}$	0.001737
-0.1	0.45	$2.3556 \cdot 10^{-10}$	$1.9103 \cdot 10^{9}$	0.004522
-0.1	0.2	$4.31863 \cdot 10^{-10}$	$4.6311 \cdot 10^{8}$	0.00829
0	0.2	$4.3186 \cdot 10^{-10}$	$4.6311 \cdot 10^{8}$	0.0122
0.1	-0.2	$7.4595 \cdot 10^{-10}$	$-2.6812 \cdot 10^{8}$	0.03106
0.1	-0.4	$9.0299 \cdot 10^{-10}$	$-4.4297 \cdot 10^{8}$	0.03760
0.2	-0.4	$9.0299 \cdot 10^{-10}$	$-4.4297 \cdot 10^{8}$	0.0554
0.25	-0.45	$9.4225 \cdot 10^{-10}$	$-4.7758 \cdot 10^{8}$	0.07011

- The effective potential can be applied to generation of inflationary scenarios with another non-minimal coupling of field with Ricci scalar.
- We consider another form of nonminimal couplings that tends to a constant at small values of the field:

$$F = M_{Pl}^2 f(\phi), \qquad f(\phi) = \frac{1+\tilde{f}(\phi)}{1+\tilde{f}(\phi_{end})}, \tag{54}$$

and get restriction to models constants to save slow-roll regime during inflation and get appropriate value of tensor-to-scalar ratio.

Conclusion

1. We introduce effective potential reformulation.
2. The effective potential approach allows us to reproduce inflationary parameters in exact form using calculations in terms of e-folding numbers directly.
3. We reformulate the problem of the slow-roll regime in Einstein-Gauss-Bonnet gravity in terms of e-folding numbers
4. We construct model with variable values of parameters which leads to the α-attractor approximation for inflationary parameters in leading order approximation in Einstein-Gauss-Bonnet gravity.
5. And use this model and effective potential to construct appropriate inflationary scenarios in Einstein-Gauss-Bonnet gravity with field non-minimally coupled with gravity.

Thank you for the attention