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Gravitational chiral anomaly and anomalous transport

"International Conference
on Quantum Field Theory,
High-Energy Physics, and
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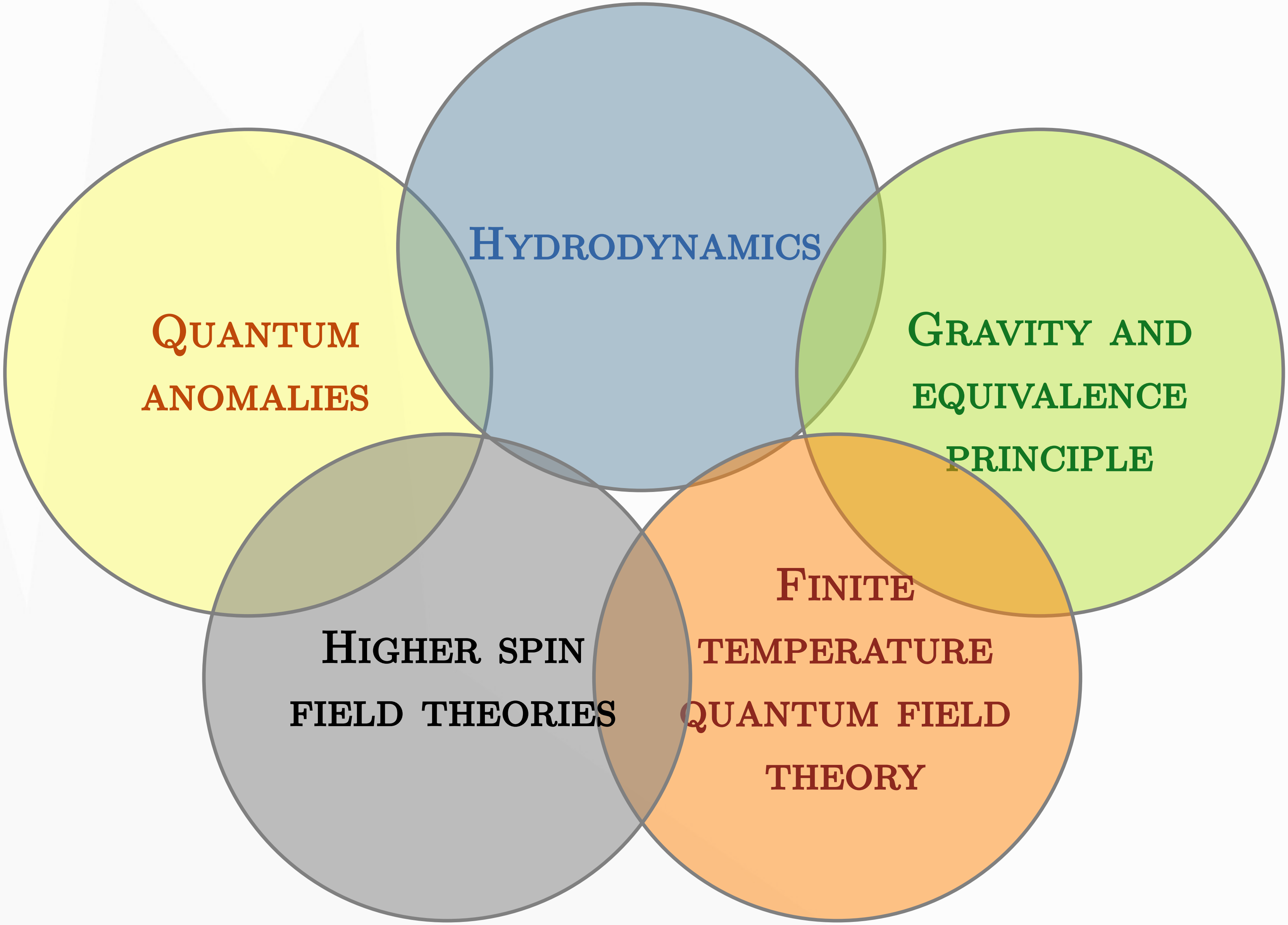
Part 1

Introduction

WHAT WILL BE DISCUSSED?



HYDRODYNAMICS

QUANTUM ANOMALIES

GRAVITY AND EQUIVALENCE PRINCIPLE

HIGHER SPIN FIELD THEORIES

FINITE TEMPERATURE QUANTUM FIELD THEORY

GRAVITY IN FLAT SPACETIME: CHESHIRE CAT GRIN

“Well! I've often seen a cat without a grin,' thought Alice 'but a grin without a cat! It's the most curious thing i ever saw in my life!”

— Lewis Carroll, Alice in Wonderland

GRAVITY IN FLAT SPACETIME: CHESHIRE CAT GRIN

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— Lewis Carroll, Alice in Wonderland

MANIFESTATION OF THE GAUGE CHIRAL ANOMALY

Accounting for quantum field effects in hydrodynamic equations **[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]**:

- Generalization of **[L.D. Landau, E.M. Lifshits, Hydrodynamics, vol. VI. Nauka, 1986]**

Conservation equations:

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} j_\lambda$$

+

Quantum chiral anomaly

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

+

Second law of thermodynamics

$$\partial_\mu s^\mu \geq 0$$

$$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta$$

CME: $j^\mu = C\mu_5 B^\mu$

and

CVE: $j^\mu_A = C\mu^2 \omega^\mu$

- In [Shi-Zheng Yang, et al. Symmetry, 2022] and [M. Buzzegoli, Lect.Notes Phys., 2021] it was shown that in the global equilibrium [F. Becattini. Acta Phys. Polon. B, 2016] :

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \quad , \quad \partial_\mu \frac{\mu}{T} = -\frac{E_\mu}{T}$$

for a non-dissipative fluid, only the **current conservation** condition can be applied instead of the entropy non-increase law.

MODERN DEVELOPMENT AND THE PROBLEM

A whole set of similar effects has been found at the intersection of quantum field theory and hydrodynamics [M. N. Chernodub et al. 2110.05471].

Experimental search is in progress:

- CME not yet found in ion collisions at RHIC [D.E. Kharzeev et al. 2205.00120].
- Condensed matter copies of the effects are found in semimetals [Qiang Li et al. Nature Phys. 12 (2016)].

What about the **gravitational chiral anomaly**?

- A connection between the thermal chiral effect $\mathbf{j}_A \sim T^2 \mathbf{\Omega}$ and the gravitational anomaly is predicted [K. Landsteiner et al. Phys. Rev. Lett., 107:021601, 2011].
- However, the anomaly grows **rapidly** with **spin** [M. J. Duff, 1982]:

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_S = \frac{(S - 2S^3)}{96\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

- For the thermal CVE – the linear dependence on spin [GP, O.V. Teryaev, V.I. Zakharov, Phys. Rev. D, 102(12):121702(R), 2020].

How does the **gravitational** chiral anomaly manifest itself in **hydrodynamics**? Can the analysis [D.T. Son and P. Surowka, PRL, 2009] be **generalized** to the case of **curved** space-time?

Contents

Part 2

Hydrodynamics from the gravitational chiral anomaly

DECOMPOSITION OF THE TENSORS

The classification of hydrodynamic effects in curved space-time needs the **expansion of the tensors** into components defined in the rest frame $u_\mu = (1,0,0,0)$:

Thermal vorticity tensor
[M. Buzzegoli, E. Grossi,
F. Becattini, JHEP, 10:091, 2017]

$$\varpi_{\mu\nu} = -\frac{1}{2}(\nabla_\mu \beta_\nu - \nabla_\nu \beta_\mu)$$

acceleration: $\alpha_\mu = \varpi_{\mu\nu} u^\nu$

vorticity: $w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta} u^\nu \varpi^{\alpha\beta}$

Riemann tensor

$R_{\mu\nu\alpha\beta}$ ⇒ In **[L.D. Landau, E.M. Lifschits, Course of Theoretical Physics, Vol. 2]** it is shown that it can be decomposed into 3 three-dimensional tensors:

$A_{ik},\ B_{ik},\ C_{ik}$ ⇒

Covariant generalization

$$A_{\mu\nu} = u^\alpha u^\beta R_{\alpha\mu\beta\nu}$$

$$B_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\eta\rho} u^\alpha u^\beta R_{\beta\nu}{}^{\eta\rho}$$

$$C_{\mu\nu} = \frac{1}{4}\epsilon_{\alpha\mu\eta\rho}\epsilon_{\beta\nu\lambda\gamma} u^\alpha u^\beta R^{\eta\rho\lambda\gamma}$$

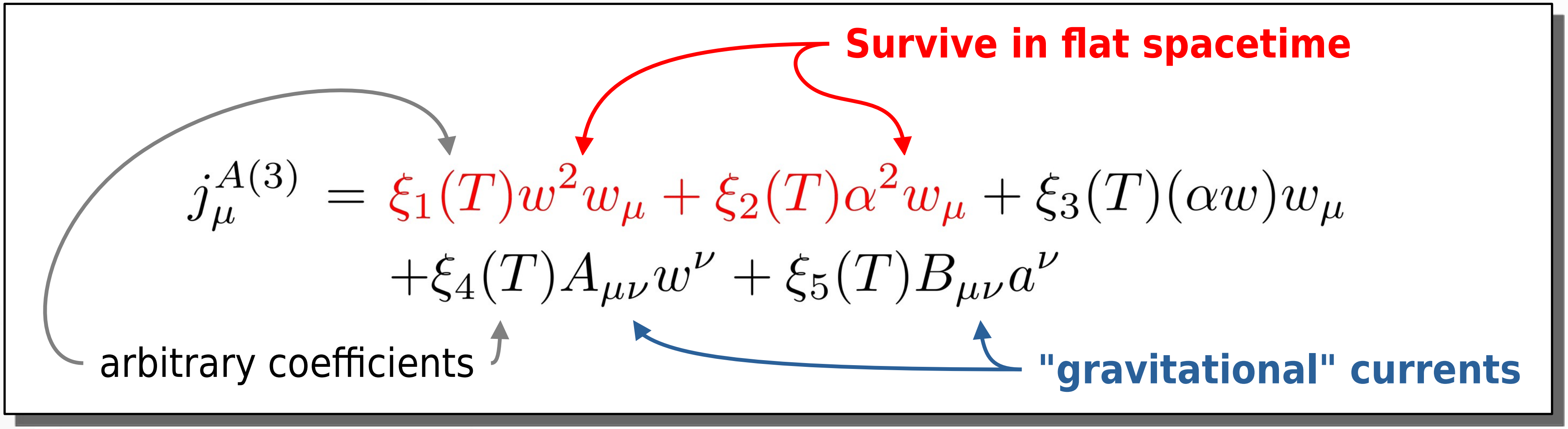
- Transform to $A_{ik},\ B_{ik},\ C_{ik}$ in the rest frame $u_\mu = (1,0,0,0)$.
- Have similar properties. In particular, we apply the condition (gravity is an *external* field):

$R_{\mu\nu} = 0$ ⇒ $A_{\mu\nu} = -C_{\mu\nu}\,, \quad A^\mu_\mu = 0\,, \quad B_{\mu\nu} = B_{\nu\mu}$ ⇒ 10 components

GRADIENT EXPANSION IN THE CURVED SPACETIME

The gravitational chiral anomaly has the **4th order** in gradients – it is to be related to the **3rd order** terms in gradient expansion of the axial current.

Hydrodynamic expansion of the axial current up to the **3rd order** in gradients:



Survive in flat spacetime

$$j_\mu^{A(3)} = \xi_1(T)w^2 w_\mu + \xi_2(T)\alpha^2 w_\mu + \xi_3(T)(\alpha w)w_\mu$$
$$+\xi_4(T)A_{\mu\nu}w^\nu + \xi_5(T)B_{\mu\nu}a^\nu$$

arbitrary coefficients

"gravitational" currents

Substitute to the **anomaly**: $\nabla_\mu j_A^\mu = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$

Global thermodynamic **equilibrium** [F. Becattini, Acta Phys. Polon. B, 47:1819, 2016.]

$$\nabla_\mu\beta_\nu + \nabla_\nu\beta_\mu = 0$$

in particular:

$\nabla_\mu T = T^2\alpha_\mu$ **Luttinger relation**

$\nabla_\mu w_\nu = T(-g_{\mu\nu}(w\alpha) + \alpha_\mu w_\nu) - T^{-1}B_{\nu\mu}$

CONSERVATION EQUATION: SYSTEM OF EQUATIONS

$$
\begin{aligned}
\nabla_\mu j^\mu_{A(3)} &= (\alpha w) w^2 \left(-3T\xi_1 + T^2\xi_1' + 2T\xi_3\right) \\
&\quad + (\alpha w)\alpha^2 \left(-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3'\right) \\
&\quad + A_{\mu\nu}\alpha^\mu w^\nu \left(T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3\right) \\
&\quad + B_{\mu\nu} w^\mu w^\nu \left(-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5\right) \\
&\quad + B_{\mu\nu}\alpha^\mu\alpha^\nu \left(T^2\xi_5' - T\xi_5 - T^{-1}\xi_3\right) \\
&\quad + A_{\mu\nu}B^{\mu\nu} \left(-T^{-1}\xi_4 + T^{-1}\xi_5\right) \\
&= 32\mathcal{N} A_{\mu\nu}B^{\mu\nu} .
\end{aligned}
$$

Independent pseudoscalars

Gravitational chiral anomaly

The coefficient in front of each pseudoscalar is to be **zero** → the **system of equations** for the unknown **coefficients** $\xi_n(T)$.

SOLUTION: KINEMATICAL VORTICAL EFFECT

The system of **equations** has the form:

$$\begin{cases} -3T\xi_1 + T^2\xi_1' + 2T\xi_3 = 0\,, \\ -3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3' = 0\,, \\ T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3 = 0\,, \\ -2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5 = 0\,, \\ T^2\xi_5' - T\xi_5 - T^{-1}\xi_3 = 0\,, \\ -T^{-1}\xi_4 + T^{-1}\xi_5 - 32\mathcal{N} = 0\,. \end{cases}$$

⇒

The **solution** looks like:

$$\begin{cases} \lambda_3 = 0 \\ \lambda_4 = -8\mathcal{N} - \dfrac{\lambda_1}{2} \\ \lambda_5 = 24\mathcal{N} - \dfrac{\lambda_1}{2} \\ \boxed{\dfrac{\lambda_1 - \lambda_2}{32} = \mathcal{N}} \end{cases}$$

(if there are no **dimensional** parameters other than temperature *T*)

$$\xi_1 = \lambda_1 T^3\,,\ \xi_2 = \lambda_2 T^3 \ldots$$

- $\lambda_3 = 0$ → conservation of current in flat space-time [GP, O.V. Teryaev, V.I. Zakharov, Phys. Rev. D, 105(4):L041701, 2022].
- $\dfrac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$ → fixes the relationship of the axial current in an **accelerated** and **vortical** flow with a **gravitational** chiral quantum anomaly.

GRAVITATIONAL ANOMALY INDUCED CURRENT

Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains a contribution** to the axial current induced by the gravitational chiral anomaly:

$$\begin{cases} j^A_\mu = \lambda_1(\omega_\nu\omega^\nu)\omega_\mu + \lambda_2(a_\nu a^\nu)w_\mu \qquad \Leftarrow \qquad R_{\mu\nu\alpha\beta} = 0 \\ \dfrac{\lambda_1 - \lambda_2}{32} = \mathcal{N} \qquad \Leftarrow \qquad \nabla_\mu j^\mu_A = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho} \end{cases}$$

- A new type of anomalous transport – the **Kinematical Vortical Effect (KVE)**.
- Does not explicitly depend on temperature and density → determined only by the **kinematics** of the flow.

KVE AND UNRUH EFFECT

- It is possible to distinguish **conserved** and **anomalous** parts of the current:

$$j^A_\mu = j^A_{\mu(\mathrm{conserv})} + j^A_{\mu(\mathrm{anom})}$$

Thermal vorticity tensor squared $\omega^2 - a^2 = -\frac{T^2}{2}\varpi_{\mu\nu}\varpi^{\mu\nu}$

$$j^A_{\mu(\mathrm{anom})} = 16\mathcal{N}\left\{(\omega^2 - a^2)\omega_\mu - A_{\mu\nu}\omega^\nu + B_{\mu\nu}a^\nu\right\} \Rightarrow \nabla^\mu j^A_{\mu(\mathrm{anom})} = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$j^A_{\mu(\mathrm{conserv})} = \frac{\lambda_1 + \lambda_2}{2}\left\{(\omega^2 + a^2)\omega_\mu - \frac{1}{2}A_{\mu\nu}\omega^\nu - \frac{1}{2}B_{\mu\nu}a^\nu\right\} \Rightarrow \nabla^\mu j^A_{\mu(\mathrm{conserv})} = 0$$

- Consider the term with acceleration from the anomalous part of the current:

$$j^A_{\mu(\mathrm{anom})} = -16\mathcal{N}a^2\omega_\mu$$

- **Unruh effect** [W.G. Unruh, 1976] – in an accelerated frame there is a thermal bath of particles with the **Unruh temperature**:

$$\boxed{T_U = |a|/(2\pi)}$$

Substitute $|a| \rightarrow 2\pi T_U$:

$$\boxed{j^A_{\mu(\mathrm{anom})} = 64\pi^2\mathcal{N}T_U^2\omega_\mu}$$

$$j^A_{\mu(\mathrm{anom})} = \frac{T_U^2}{6}\omega_\mu$$ for spin ½ → standard CVE

thermal CVE current is **proportional** to the **anomaly**!

- Match with [K. Landsteiner, et al. PRL, 2011] and [M. Stone, J. Kim. PRD, 2018], where thermal CVE $\mathbf{j}_A \sim T^2\mathbf{\Omega}$ is associated with the gravitational chiral anomaly!

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Part 3

Verification: spin 1/2

TRANSPORT COEFFICIENTS AND ANOMALY: SPIN 1/2

- In [GP, O.V. Teryaev, and V.I. Zakharov, JHEP, 02:146, 2019], [V. E. Ambrus, JHEP, 08:016, 2020], [A. Palermo, et al. JHEP 10 (2021) 077] and for ω^3 in [A. Vilenkin, Phys. Rev., D20:1807-1812, 1979] the following expression was obtained:

$$j_\mu^A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} \overbrace{- \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}}^{\text{KVE}}\right)\omega_\mu$$

- Comparing it with the well-known anomaly [L. Alvarez-Gaume, E. Witten, Nucl. Phys., B234:269, 1984]:

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

We see that the formula is **fulfilled**:

$$\boxed{\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}} \Rightarrow \left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)/32 = \frac{1}{384\pi^2}$$

Correspondence between gravity and hydrodynamics is shown!

- Keeping also the gravitational currents, we obtain:

$$j_\mu^{A(3)} = \left(-\frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}\right)\omega_\mu + \frac{1}{12\pi^2}B_{\mu\nu}a^\nu$$

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Part 4

Verification: spin 3/2

RARITA–SCHWINGER–ADLER MODEL OF SPIN 3/2

The **Rarita-Schwinger theory** – well-known theory of spin 3/2.
But this theory has a number of **pathologies.**

Generalized Hamiltonian dynamics: **Dirac bracket** instead of **Poisson bracket**

$$[F(\vec{x}),G(\vec{y})]_D = [F(\vec{x}),G(\vec{y})] - \int d^3w d^3z [F(\vec{x}),\chi^\dagger(\vec{w})] M^{-1}(\vec{w},\vec{z})[\chi(\vec{z}),G(\vec{y})]$$

$$M(\vec{x},\vec{y}) = [\chi(\vec{x}),\chi^\dagger(\vec{y})]$$

There is **singularity** in a Dirac bracket in weak gauge field limit for RS-theory!

Doesn't allow to construct perturbation theory!

Solved in [Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018] by introducing of interaction with additional spin ½ field:

$$\boxed{S = \int d^4x \, \big(-\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\partial_\nu\psi_\rho + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda\big)}$$

GRAVITATIONAL CHIRAL ANOMALY: METHOD OF CONFORMAL THREE-POINT FUNCTIONS

For a conformally symmetric theory, if

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu j^\mu_V = 0\,, \quad \partial_\mu j^\mu_A = 0\,,$$
$$T^\mu_\mu = 0\,, \quad T_{\mu\nu} = T_{\nu\mu}\,.$$

It is proven in [J. Erdmenger.Nucl. Phys. B, 562:315–329, 1999], that the three-point function $\langle T\hat{T}_{\mu\nu}(x)\hat{T}_{\sigma\rho}(y)\hat{j}^A_\omega(z)\rangle_c$ has the **universal form**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^\omega_A(z)\rangle_c = \frac{1}{(x-z)^8(y-z)^8}$$
$$\times \mathscr{I}_T^{\mu\nu,\mu'\nu'}(x-z)\,\mathscr{I}_T^{\sigma\rho,\sigma'\rho'}(y-z)\,t^{TTA}_{\mu'\nu'\sigma'\rho'}{}^{\omega}(Z)$$

where the notations are introduced:

$$\mathscr{I}^T_{\mu\nu,\sigma\rho}(x) = \mathscr{E}^T_{\mu\nu,\alpha\beta} I^\alpha_\sigma(x) I^\beta_\rho(x)\,,$$
$$\mathscr{E}^T_{\mu\nu,\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{4}\eta_{\mu\nu}\eta_{\alpha\beta}\,,$$
$$t^{TTA}_{\mu\nu\sigma\rho\omega}(Z) = \frac{\mathscr{A}}{Z^6}\big(\mathscr{E}^T_{\mu\nu,\eta}{}^{\varepsilon}\,\mathscr{E}^T_{\sigma\rho,\kappa\varepsilon}\,\varepsilon_\omega{}^{\eta\kappa\lambda} Z_\lambda$$
$$-6\,\mathscr{E}^T_{\mu\nu,\eta\gamma}\mathscr{E}^T_{\sigma\rho,\kappa\delta}\,\varepsilon_\omega{}^{\eta\kappa\lambda} Z^\gamma Z^\delta Z_\lambda Z^{-2}\big)$$

“6” – consequence of $T^\mu_\mu = 0$

GRAVITATIONAL CHIRAL ANOMALY: METHOD OF CONFORMAL THREE-POINT FUNCTIONS

On the other hand, there is a **gravitational chiral anomaly**:

$$\langle \nabla_\mu j_5^\mu \rangle = a \frac{\pi^4}{384\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

It has been proven that:

$$\boxed{\mathscr{A} = a}$$

- **The anomaly can be determined by calculating the three-point correlator!**
- The anomaly can be calculated from the correlator in **flat space-time!**
- **No need** to explicitly find a one-loop three-point momentum **divergent** graphs: everything is done in the x-space!

GRAVITATIONAL CHIRAL ANOMALY: METHOD OF CONFORMAL THREE-POINT FUNCTIONS

We will consider the case of points on one 4-axis:

$$x_\mu = x\, e_\mu \,,\; y_\mu = y\, e_\mu \,,\; z_\mu = z\, e_\mu$$

Then the correlator should look like:

$$\boxed{\begin{aligned}
&\langle T\, \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}^{\omega}_{A}(z) \rangle = \mathscr{A} \Big(4(x-y)^5 \\
&\times (x-z)^3 (y-z)^3 \Big)^{-1} e_\vartheta \Big(\eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} \\
&+ \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 (e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} \\
&+ e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega}) \Big)
\end{aligned}}$$

GRAVITATIONAL ANOMALY IN RSA THEORY: DIAGRAMS

Let's decompose all the operators depending on the set of the fields:

$$\hat{T}^{\mu\nu} = \hat{T}^{\mu\nu}_{\bar{\psi}\psi} + \hat{T}^{\mu\nu}_{\bar{\lambda}\lambda} + \hat{T}^{\mu\nu}_{\bar{\psi}\lambda} + \hat{T}^{\mu\nu}_{\bar{\lambda}\psi}$$

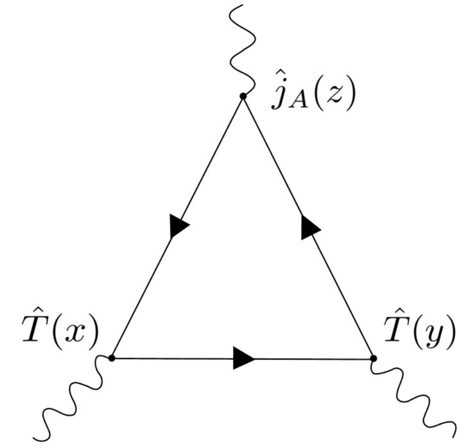
$$\hat{j}^{\mu}_{A} = \hat{j}^{\mu}_{A\bar{\psi}\psi} + \hat{j}^{\mu}_{A\bar{\lambda}\lambda}$$

Then the three-point function decomposes into **32 terms**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle_c = \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + (31\,\mathrm{terms})$$

However, many are equal to **zero** or depend on each other: there are only **4 independent** correlators.

A **typical** diagram
(different SETs in the vertices):



GRAVITATIONAL ANOMALY IN RSA THEORY: CALCULATION DETAILS

As a result, we have for the independent correlators:

$$\begin{aligned}
&\langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\psi}(x)\hat{T}^{\sigma\rho}_{\bar{\psi}\psi}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = \frac{1}{4\pi^6(x-y)^5(x-z)^4(y-z)^4}e_\vartheta(28e^2x^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega} - \varepsilon^{\sigma\vartheta\nu\omega}(2e^2e^\mu e^\rho(-14x^2\\
&+9z(x+y)+19xy-14y^2-9z^2)+(26x^2-3z(x+y)-49xy+26y^2+3z^2)\eta^{\mu\rho})+2e^2e^\nu(14x^2-19xy\\
&-9xz+14y^2-9yz+9z^2)(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})-38e^2xye^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-18e^2xze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+28e^2y^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}\\
&-18e^2yze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+18e^2z^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-26x^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-26x^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-(26x^2-3z(x+y)-49xy\\
&+26y^2+3z^2)\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+49xy\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+49xy\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}+3xz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+3xz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-26y^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}\\
&-26y^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}+3yz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+3yz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-3z^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-3z^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega})\,,
\end{aligned}$$

$$\begin{aligned}
&\langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\psi}(x)\hat{T}^{\sigma\rho}_{\bar{\psi}\lambda}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = \frac{1}{4\pi^6(x-y)^5(x-z)^4(y-z)^4}e_\vartheta(4\mathrm{e}^2x^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega} - \varepsilon^{\sigma\vartheta\nu\omega}(2\mathrm{e}^2e^\mu e^\rho(-2x^2\\
&+3z(7x+y)-17xy+7y^2-12z^2)+(10x^2+7xy-27xz-13y^2+19yz+4z^2)\eta^{\mu\rho})+2\mathrm{e}^2e^\nu(2x^2\\
&+17xy-21xz-7y^2-3yz+12z^2)(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})+34\mathrm{e}^2xye^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-42\mathrm{e}^2xze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}\\
&-14\mathrm{e}^2y^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-6\mathrm{e}^2yze^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+24\mathrm{e}^2z^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}-10x^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-10x^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-(10x^2+7xy\\
&-27xz-13y^2+19yz+4z^2)\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}-7xy\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-7xy\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}+27xz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+27xz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}\\
&+13y^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+13y^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-19yz\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-19yz\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-4z^2\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}-4z^2\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega})\,,
\end{aligned}$$

$$\langle T\hat{T}^{\mu\nu}_{\bar{\psi}\lambda}(x)\hat{T}^{\sigma\rho}_{\bar{\psi}\lambda}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = \frac{4e^2e_\vartheta(e^\nu(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})+e^\mu(e^\rho\varepsilon^{\sigma\vartheta\nu\omega}+e^\sigma\varepsilon^{\vartheta\nu\rho\omega}))}{\pi^6(x-y)^3(x-z)^4(y-z)^4}$$

$$\begin{aligned}
&\langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\lambda}(x)\hat{T}^{\sigma\rho}_{\bar{\lambda}\psi}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = \frac{5}{2\pi^6(x-y)^3(x-z)^4(y-z)^4}e_\vartheta(-2e^2e^\mu e^\rho\varepsilon^{\sigma\vartheta\nu\omega}-2e^2e^\nu(e^\rho\varepsilon^{\sigma\vartheta\mu\omega}+e^\sigma\varepsilon^{\vartheta\mu\rho\omega})\\
&-2e^2e^\mu e^\sigma\varepsilon^{\vartheta\nu\rho\omega}+\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega}+\eta^{\sigma\nu}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\sigma\mu}\varepsilon^{\vartheta\nu\rho\omega})\,.
\end{aligned}$$

Each term **differs** from what we need.

GRAVITATIONAL ANOMALY IN RSA THEORY: RESULT

Summing 9 correlators, we will obtain:

$$\langle T\,\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle_c = -19\Big(4\pi^6(x-y)^5 \times(x-z)^3(y-z)^3\Big)^{-1} e_\vartheta\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^\nu e^\rho\varepsilon^{\sigma\vartheta\mu\omega} +e^\mu e^\rho\varepsilon^{\sigma\vartheta\nu\omega}+e^\sigma e^\nu\varepsilon^{\vartheta\mu\rho\omega}+e^\sigma e^\mu\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

$$\Longleftrightarrow$$

Matches the form we want!

$$\langle T\,\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle = \mathscr{A}\Big(4(x-y)^5 \times(x-z)^3(y-z)^3\Big)^{-1} e_\vartheta\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^\nu e^\rho\varepsilon^{\sigma\vartheta\mu\omega} +e^\mu e^\rho\varepsilon^{\sigma\vartheta\nu\omega}+e^\sigma e^\nu\varepsilon^{\vartheta\mu\rho\omega}+e^\sigma e^\mu\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

(points on the same axis)

We can determine the factor in the anomaly:

$$\mathscr{A}_{RSA} = -19\mathscr{A}_{s=1/2} = -\frac{19}{\pi^6}$$

$$\langle\nabla_\mu \hat{j}^{\mu}_{A}\rangle_{RSA} = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

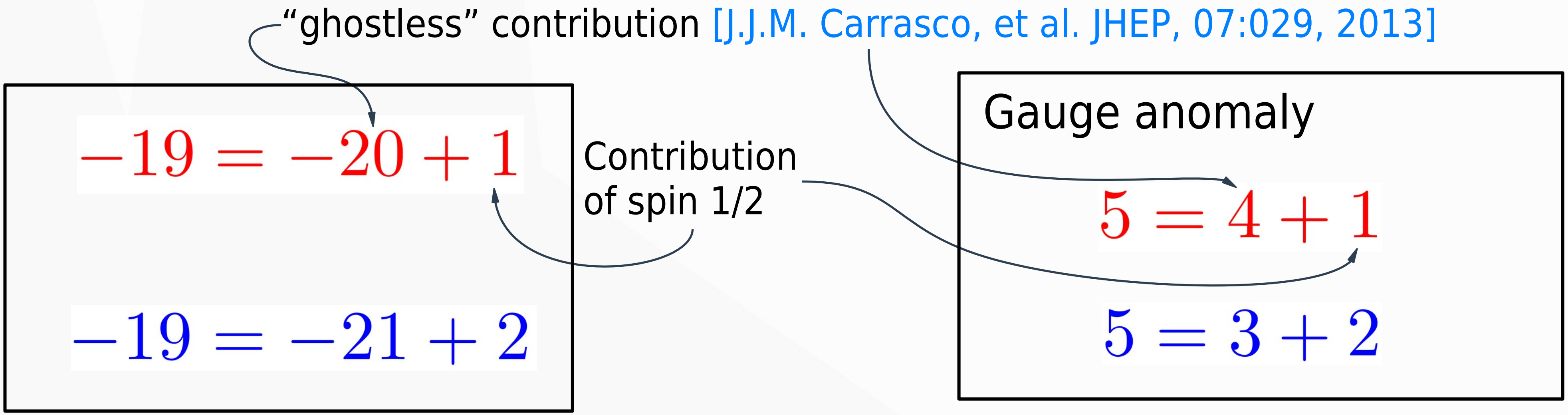
-19 times different from the anomaly for spin ½

GRAVITATIONAL ANOMALY IN RSA THEORY: RESULT

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_{RS} = \frac{-21}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]



“ghostless” contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

$$-19 = -21 + 2$$

Contribution of spin 1/2

Gauge anomaly

$$5 = 4 + 1$$

$$5 = 3 + 2$$

TRANSPORT COEFFICIENTS: DENSITY OPERATOR

Zubarev **density operator** as a basis for the description of the **vorticity** effects:

$$\hat{\rho} = \frac{1}{Z} \exp\Big\{ - \beta_\mu(x)\hat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\hat{J}^{\mu\nu}_x + \zeta\hat{Q} \Big\}$$

[M. Buzzegoli, E. Grossi, and F. Becattini. JHEP07, 119 (2018).]

Perturbation theory in vorticity gives **KVE**:

$$\langle \hat{j}_5^\lambda(x) \rangle \;=\; \ldots \; + \frac{\varpi_{\mu\nu}\varpi_{\rho\sigma}\varpi_{\alpha\beta}}{48|\beta|^3} \int_0^{|\beta|} d\tau_1 d\tau_2 d\tau_3 \langle T_\tau \, \hat{J}^{\mu\nu}_{-i\tau_1 u} \hat{J}^{\rho\sigma}_{-i\tau_2 u} \hat{J}^{\alpha\beta}_{-i\tau_3 u} \hat{j}_5^\lambda(0) \rangle_{\beta(x),c}$$

Let's again divide the operators depending on the set of fields:

$$\hat{T}^{\mu\nu} \;=\; \hat{T}^{\mu\nu}_{\bar{\psi}\psi} + \hat{T}^{\mu\nu}_{\bar{\lambda}\lambda} + \hat{T}^{\mu\nu}_{\bar{\psi}\lambda} + \hat{T}^{\mu\nu}_{\bar{\lambda}\psi}$$

Initially 5^8 correlators → only ~90 000 are nonzero

Typical **correlator** to be found (4-point):

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{1}{|\beta|^3} \int d\tau_x d\tau_y d\tau_z d^3x d^3y d^3z \times$$
$$x^i y^j z^k \langle T_\tau \hat{T}^{\alpha_1\alpha_2}(X) \hat{T}^{\alpha_3\alpha_4}(Y) \hat{T}^{\alpha_5\alpha_6}(Z) \hat{j}_A^\lambda(0) \rangle_{\beta(x),c}$$

TRANSPORT COEFFICIENTS: CORRELATORS

The transport coefficients are expressed in terms of a combination of the correlators:

$$
\begin{aligned}
\xi_1 &= -\frac{1}{6}\Big(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} \\
&\quad + C^{01|01|02|3|221} - C^{01|01|01|3|222} - C^{01|02|02|3|211} \\
&\quad - C^{02|01|02|3|121} - C^{02|02|01|3|112}\Big)\,, \\
\xi_2 &= -\frac{1}{6}\Big(C^{02|00|00|3|111} + C^{00|02|00|3|111} + C^{00|00|02|3|111} \\
&\quad - C^{01|00|00|3|211} - C^{00|01|00|3|121} \\
&\quad - C^{00|00|01|3|112}\Big)\,.
\end{aligned}
$$

Direct calculation within the finite temperature QFT:

$$
\begin{aligned}
\xi_1 &= -\frac{T^3}{6}\Big(\frac{177}{80\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} \\
&\quad + \frac{177}{80\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2}\Big) = -\frac{53}{24\pi^2}T^3 \\
\xi_2 &= -\frac{T^3}{6}\Big(\frac{33}{40\pi^2} + \frac{53}{80\pi^2} + \frac{1}{2\pi^2} + \frac{3}{4\pi^2} \\
&\quad + \frac{47}{80\pi^2} + \frac{17}{40\pi^2}\Big) = -\frac{5}{8\pi^2}T^3
\end{aligned}
$$

Cubic gradients **(KVE)** in the **RSA** theory:

$$
j^{A(3)}_{\mu} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu
$$

TRANSPORT COEFFICIENTS AND ANOMALY: SPIN 3/2 INTERACTING WITH SPIN 1/2

The obtained formula for **cubic gradients** (KVE):

$$j_\mu^{A(3)} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu$$

Gravitational chiral **anomaly**:

$$\nabla_\mu j^\mu_A = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

⇒

$$\boxed{\frac{\lambda_1-\lambda_2}{32}=\mathcal{N}}$$

⇓

Direct **verification**:

$$\left(-\frac{53}{24\pi^2}+\frac{5}{8\pi^2}\right)\Big/32 = -\frac{19}{384\pi^2}$$

Coincidence of hydrodynamics and gravitational anomaly!

- For the RSA theory, the relationship between the transport coefficients in a **vortical accelerated fluid** and the **gravitational** chiral **anomaly** is shown: the factor **-19** from the anomaly is reproduced.

- Verification of the obtained formula in a very **nontrivial** case with higher spins and interaction.

Contents

Part 5

Conclusion

CONCLUSION

- The relationship between the hydrodynamic current in the third order of gradient expansion $(\omega_\nu \omega^\nu)\omega_\mu$ and $(a_\nu a^\nu)w_\mu$, the Kinematical Vortical Effect **(KVE)**, and the **gravitational chiral anomaly** has been proven:
  - The axial current in a flat space-time in a vortical and accelerated fluid turns out to be associated with a quantum violation of current conservation in a curved space-time.
- The obtained formula has been **verified** directly for **spin 1/2**.
- The obtained formula has been **verified** for **spin 3/2** using the **RSA** theory:
  - Cubic transport coefficients were derived using the statistical density operator expansion $\mathbf{-53/(24\pi^2)\omega^3}$ and $\mathbf{-5/(8\pi^2)a^2\omega}$.
  - The gravitational chiral anomaly was found by the method of conformal three-point functions: the factor in the anomaly is $\mathbf{-19/(384\pi^2)}$.
  - Correspondence between the KVE and the gravitational chiral anomaly is directly shown $\mathbf{[-53/(24\pi^2) + 5/(8\pi^2)] = -19/(384\pi^2)}$.

Additional slides

$$\begin{aligned}R_{\mu\nu\alpha\beta} \; = \; & u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\ & + \epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda{}_\beta - u_\beta B^\lambda{}_\alpha) \\ & + \epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda{}_\nu - u_\nu B^\lambda{}_\mu) \\ & + \epsilon_{\mu\nu\lambda\rho} \epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta} \, .\end{aligned}$$

$$\langle T^{\mu\nu} \rangle = - \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{W}}{\delta g_{\mu\nu}}$$

$$\begin{aligned}& \langle T \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}^\omega_A(z) \rangle_c = \frac{4}{\sqrt{-g(x)}\sqrt{-g(y)}\sqrt{-g(z)}} \\ & \times \frac{\delta}{\delta g_{\mu\nu}(x)} \frac{\delta}{\delta g_{\mu\nu}(y)} \Big(\sqrt{-g(z)} \langle \hat{j}^\omega_A(z) \rangle \Big) \, ,\end{aligned}$$

Method of conformal correlation functions

[GP, O.V. Teryaev, V.I. Zakharov. 2202.02168]

- There are various ways to calculate quantum anomalies.
- **One of the methods:**

  [J. Erdmenger, H. Osborn. Nucl. Phys. B, 483:431–474, 1997] → Gauge chiral anomaly
  [J. Erdmenger. Nucl. Phys. B, 562:315–329, 1999] → Gravitational chiral anomaly

- It is shown that three-point functions have a **universal form**. In particular, for the function with the currents:

$$\boxed{\begin{aligned}&\langle T\hat{j}_V^{\mu}(x)\hat{j}_V^{\nu}(y)\hat{j}_A^{\omega}(z)\rangle_c = \\ &= -4\mathscr{B}\frac{I^{\mu}_{\mu'}(x-z)I^{\nu}_{\nu'}(y-z)}{(x-z)^6(y-z)^6}\varepsilon^{\mu'\nu'\lambda\omega}\frac{Z_\lambda}{Z^4}\end{aligned}}$$

Designations introduced:

$$Z_\mu = \frac{(x-z)_\mu}{(x-z)^2} - \frac{(x-y)_\mu}{(x-y)^2}$$

$$I_{\mu\nu}(x) = \eta_{\mu\nu} - 2\frac{x_\mu x_\nu}{x^2}$$

It will be so for a conformally symmetric theory, if

$$\begin{gathered}\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu j_V^\mu = 0\,, \quad \partial_\mu j_A^\mu = 0\,, \\ T^\mu_\mu = 0\,, \quad T_{\mu\nu} = T_{\nu\mu}\,.\end{gathered}$$

Method of conformal correlation functions: gauge chiral anomaly

- Can this method be used to calculate anomalies in the RSA theory?

$$
\begin{aligned}
T^{\mu}_{\mu} = & \frac{1}{2}\varepsilon^{\lambda\mu\beta\rho}\left(\bar{\psi}_{\lambda}\gamma_5\gamma_{\mu}\partial_{\beta}\psi_{\rho} - \partial_{\beta}\bar{\psi}_{\lambda}\gamma_5\gamma_{\mu}\psi_{\rho}\right) \\
& + i\partial_{\eta}\left[(\bar{\psi}\gamma)\psi^{\eta} - \bar{\psi}^{\eta}(\gamma\psi)\right] \\
& + \frac{i}{2}\left[\bar{\lambda}(\gamma\partial)\lambda - (\partial\bar{\lambda}\gamma)\lambda\right] \\
& + im\left[(\bar{\psi}\gamma)\lambda - \bar{\lambda}(\gamma\psi)\right].
\end{aligned}
$$

The SET trace is equal to zero just in the extended theory!

$$
\text{RS theory}: \quad T^{\mu}_{\mu} = i\partial_{\eta}\left[(\bar{\psi}\gamma)\psi^{\eta} - \bar{\psi}^{\eta}(\gamma\psi)\right] \neq 0\,,
$$

$$
\boxed{\text{RSA theory}: \quad T^{\mu}_{\mu} = 0\,.}
$$

[M. N. Chernodub, et al. 2110.05471],
[Yu Nakayama. Phys. Rept., 569:1–93, 2015]

- **We can use the described method for calculating anomalies!**

Method of conformal correlation functions: gauge chiral anomaly

Propagators in momentum representation, for example:

$$\langle T\psi^{\rho}_{a}(x)\bar{\psi}^{\sigma}_{b}(0)\rangle = \frac{i}{2(2\pi)^4}\int\frac{d^4p}{p^2}\left(\gamma^{\sigma}\not{p}\gamma^{\rho} - 2\left(\frac{1}{m^2}+\frac{2}{p^2}\right)p^{\sigma}p^{\rho}\not{p}\right)_{ab} e^{-ipx}$$

using the formula *(I am grateful to A. Pikelner and A. Bednyakov for the link)*:

$$\boxed{\begin{gathered}\int\frac{d^Dp\, e^{ipx}}{p^{2(\lambda+1-\alpha)}} = \frac{i2^{2\alpha}\pi^{\lambda+1}\Gamma(\alpha)}{x^{2\alpha}\Gamma(\lambda+1-\alpha)} \\ \lambda = 1-\varepsilon\,, \quad D = 2(\lambda+1)\,,\end{gathered}}$$

can be translated into a **coordinate representation** (need to find momenum integrals).

Method of conformal correlation functions: gauge chiral anomaly

As a result, we obtain for the **propagators** in the **coordinate representation**:

$$\langle T\psi_a^\rho(x)\bar{\psi}_b^\sigma(0)\rangle = \frac{i}{4\pi^2x^4}\Big[\gamma^\sigma \not{x}\gamma^\rho - 2\Big(1+\frac{4}{m^2x^2}\Big)$$

$$\times(\eta^{\rho\sigma}\not{x}+\gamma^\rho x^\sigma+\gamma^\sigma x^\rho)+8\left(1+\frac{6}{m^2x^2}\right)\frac{x^\rho x^\sigma \not{x}}{x^2}\Big]_{ab}$$

$$\langle T\lambda_a(x)\bar{\psi}_b^\sigma(0)\rangle = \frac{i}{2\pi^2 m x^4}\Big(\gamma^\sigma - \frac{4x^\sigma\not{x}}{x^2}\Big)_{ab},$$

$$\langle T\psi_a^\rho(x)\bar{\lambda}_b(0)\rangle = \frac{-i}{2\pi^2 m x^4}\Big(\gamma^\rho - \frac{4x^\rho\not{x}}{x^2}\Big)_{ab},$$

$$\langle T\lambda_a(x)\bar{\lambda}_b(0)\rangle = 0\,.$$

Method of conformal correlation functions: gauge chiral anomaly

As a result, we obtain:

$$\boxed{\langle T\hat{j}^{\mu}(x)\hat{j}^{\nu}(y)\hat{j}_A^{\omega}(z)\rangle_c = \frac{5e^2 e_{\vartheta}\varepsilon^{\vartheta\mu\nu\omega}}{\pi^6(x-y)^3(x-z)^3(y-z)^3}}$$

Comparing with the general form

$$\langle T\hat{j}_V^{\mu}(x)\hat{j}_V^{\nu}(y)\hat{j}_A^{\omega}(z)\rangle = \frac{4\mathscr{B}\, e^2 e_{\vartheta}\varepsilon^{\vartheta\mu\nu\omega}}{\pi^6(x-y)^3(x-z)^3(y-z)^3}$$

we obtain an anomaly factor:

$$\boxed{\langle\partial_{\mu}\hat{j}_A^{\mu}\rangle_{RSA} = -\frac{5}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}}$$

which confirms the result from [S.L. Adler.Phys. Rev. D, 97(4):045014, 2018]. Also checked for arbitrary *x*, *y*, *z* .

Method of conformal correlation functions: gravitational chiral anomaly

We will consider the case of points on one 4-axis:

$$x_\mu = x\, e_\mu \,, \; y_\mu = y\, e_\mu \,, \; z_\mu = z\, e_\mu$$

Then the correlator should look like:

$$\boxed{\begin{aligned}
&\langle T\, \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}^{\omega}_{A}(z) \rangle = \mathscr{A} \Big(4(x-y)^5 \\
&\times (x-z)^3 (y-z)^3 \Big)^{-1} e_\vartheta \Big(\eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} \\
&+ \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 (e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} \\
&+ e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega}) \Big)
\end{aligned}}$$

Method of conformal correlation functions: gravitational chiral anomaly

$\langle T\lambda_a(x)\bar{\lambda}_b(0)\rangle = 0$ The field λ is **nonpropagating**.

$$\begin{aligned}
&\langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\psi}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\lambda}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \\
&= \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\psi}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\lambda}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \\
&= \langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\psi}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\psi}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \\
&= \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\lambda}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\lambda}\psi}\hat{j}^A_{\bar{\psi}\psi}\rangle = \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle = 0
\end{aligned}$$

- **12** correlators **are zero**!

Method of conformal correlation functions: gravitational chiral anomaly

$$\langle T\lambda_a(x)\bar{\psi}^\sigma_b(0)\rangle = \frac{i}{2\pi^2 m x^4}\left(\gamma^\sigma - \frac{4x^\sigma \not{x}}{x^2}\right)_{ab}$$

➡ negative powers m

$$\hat{T}^{\mu\nu}_{\bar{\psi}\lambda} = \frac{i}{2} m\left(\bar{\psi}^\mu\gamma^\nu\lambda + \bar{\psi}^\nu\gamma^\mu\lambda\right)$$

➡ positive powers m

In the limit m → ∞, **11** more correlators **vanish**:

$$\boxed{\begin{aligned}
&\langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle, \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\lambda}\hat{j}^A_{\bar{\lambda}\lambda}\rangle, \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\lambda}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle,\\
&\langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\psi}\psi}\rangle, \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\psi}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle, \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\psi}\psi}\rangle,\\
&\langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}^A_{\bar{\lambda}\lambda}\rangle, \langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\lambda}\lambda}\hat{j}^A_{\bar{\psi}\psi}\rangle, \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\psi}\psi}\hat{j}^A_{\bar{\psi}\psi}\rangle,\\
&\langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\psi}\lambda}\hat{j}^A_{\bar{\psi}\psi}\rangle, \langle \hat{T}_{\bar{\lambda}\lambda}\hat{T}_{\bar{\lambda}\psi}\hat{j}^A_{\bar{\psi}\psi}\rangle \to 0 \quad (m \to \infty)
\end{aligned}}$$

Method of conformal correlation functions: gravitational chiral anomaly

Of the 32 correlators, only **9** remain:

$$\begin{aligned}
&\langle T\hat{T}_{\mu\nu}(x)\hat{T}_{\sigma\rho}(y)\hat{j}^{A}_{\omega}(z)\rangle_c = \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle \\
&+\langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\lambda}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\lambda}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\psi}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle \\
&+\langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\psi}\lambda}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + \langle \hat{T}_{\bar{\psi}\lambda}\hat{T}_{\bar{\lambda}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + \langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle \\
&+\langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\psi}\lambda}\hat{j}^{A}_{\bar{\psi}\psi}\rangle + \langle \hat{T}_{\bar{\lambda}\psi}\hat{T}_{\bar{\lambda}\psi}\hat{j}^{A}_{\bar{\psi}\psi}\rangle
\end{aligned}$$

Of the 9, only **4** are independent:

$$\begin{aligned}
&\langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\psi}(x)\hat{T}^{\sigma\rho}_{\bar{\psi}\lambda}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = && \langle T\,\hat{T}^{\mu\nu}_{\bar{\lambda}\psi}(x)\hat{T}^{\sigma\rho}_{\bar{\psi}\lambda}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = \\
&= \langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\psi}(x)\hat{T}^{\sigma\rho}_{\bar{\lambda}\psi}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = && = \langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\lambda}(x)\hat{T}^{\sigma\rho}_{\bar{\lambda}\psi}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c\,, \\
&= -\langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\lambda}(y)\hat{T}^{\sigma\rho}_{\bar{\psi}\psi}(x)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = && \langle T\,\hat{T}^{\mu\nu}_{\bar{\lambda}\psi}(x)\hat{T}^{\sigma\rho}_{\bar{\lambda}\psi}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c = \\
&= -\langle T\,\hat{T}^{\mu\nu}_{\bar{\lambda}\psi}(y)\hat{T}^{\sigma\rho}_{\bar{\psi}\psi}(x)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c\,, && = \langle T\,\hat{T}^{\mu\nu}_{\bar{\psi}\lambda}(x)\hat{T}^{\sigma\rho}_{\bar{\psi}\lambda}(y)\hat{j}^{\omega}_{A\bar{\psi}\psi}(z)\rangle_c
\end{aligned}$$

Method of conformal correlation functions: gravitational chiral anomaly

A typical correlator has the form (2 terms - Wick's theorem):

Operator from the axial current

$$\langle T\hat{T}^{\mu\nu}_{\bar\psi\psi}(x)\hat{T}^{\sigma\rho}_{\bar\psi\psi}(y)\hat{j}^{\omega}_{A\bar\psi\psi}(z)\rangle_c = \lim_{\substack{x_1,x_2\to x\\ y_1,y_2\to y\\ z_1,z_2\to z}} \Bigg(-\operatorname{tr}\Big\{\mathcal{D}^{\mu\nu\eta_1\eta_2}_{(\bar\psi\psi)}(\partial^{x_1},\partial^{x_2})G^{\psi\bar\psi}_{\eta_2\eta_5}(x_2-z_1)\mathcal{J}^{\omega\eta_5\eta_6}_{A(\bar\psi\psi)}G^{\psi\bar\psi}_{\eta_6\eta_3}(z_2-y_1)$$
$$\times\mathcal{D}^{\sigma\rho\eta_3\eta_4}_{(\bar\psi\psi)}(\partial^{y_1},\partial^{y_2})G^{\psi\bar\psi}_{\eta_4\eta_1}(y_2-x_1)\Big\} - \operatorname{tr}\Big\{\mathcal{D}^{\mu\nu\eta_1\eta_2}_{(\bar\psi\psi)}(\partial^{x_1},\partial^{x_2})G^{\psi\bar\psi}_{\eta_2\eta_3}(x_2-y_1)$$
$$\times\mathcal{D}^{\sigma\rho\eta_3\eta_4}_{(\bar\psi\psi)}(\partial^{y_1},\partial^{y_2})G^{\psi\bar\psi}_{\eta_4\eta_5}(y_2-z_1)\mathcal{J}^{\omega\eta_5\eta_6}_{A(\bar\psi\psi)}G^{\psi\bar\psi}_{\eta_6\eta_1}(z_2-x_1)\Big\}\Bigg)$$

Operators from SET

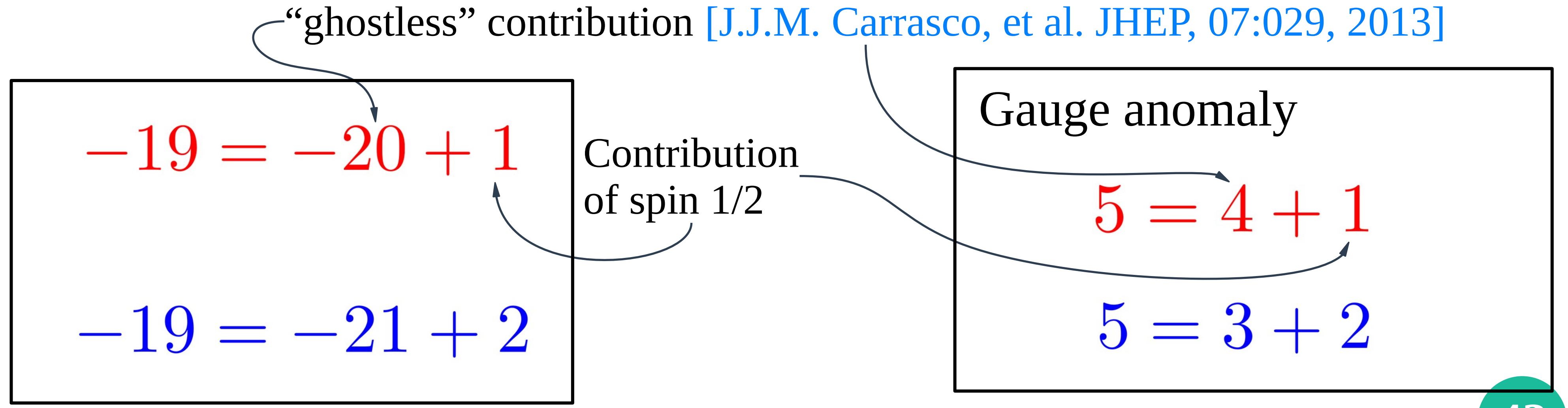
Green functions

Method of conformal correlation functions: gravitational chiral anomaly

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_{RS} = \frac{-21}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]



"ghostless" contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

$$-19 = -21 + 2$$

Contribution of spin 1/2

Gauge anomaly

$$5 = 4 + 1$$

$$5 = 3 + 2$$

Method of conformal correlation functions: gravitational chiral anomaly

General Rule will be:

Anomaly in RSA = RS-anomaly + 2 · (spin ½ anomaly)

Anomaly in RSA = (RS-anomaly “ghostless”) + 1 · (spin ½ anomaly)

Works both for the **gravitational** and the **gauge** chiral anoma