Dense QCD in magnetic field

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with I.Ya. Aref'eva and P. Slepov

JHEP 07 (2021) 161; arXiv:2203.12539 [hep-th]

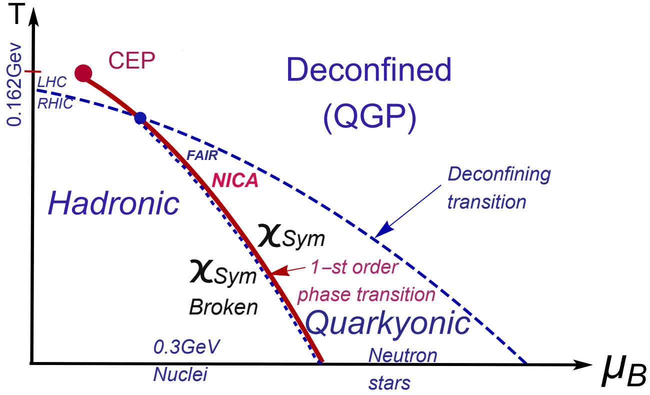
Overview

The Expected QCD Phase Diagram

Goal of Holographic QCD — describe QCD phase diagram

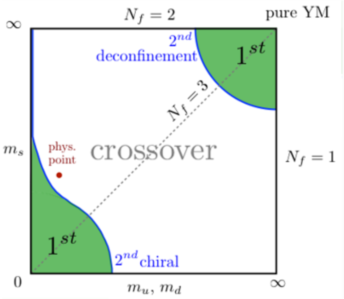
Requirements:

- reproduce the QCD results from perturbative theory at short distances
- reproduce Lattice QCD results at large distances (~ 1 fm) and **small** μ_B



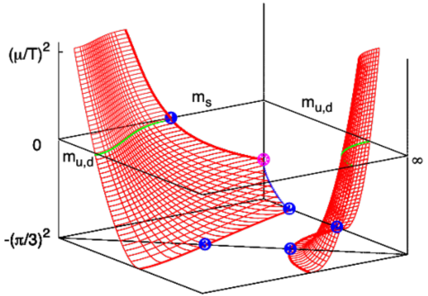
QCD Phase Diagram: Lattice

Phase diagram
on quark mass



Columbia plot
Brown et al., PRL (1990)

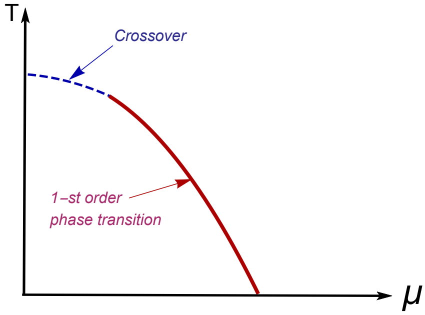
Main problem with $\mu \neq 0$
Imaginary chemical potential method



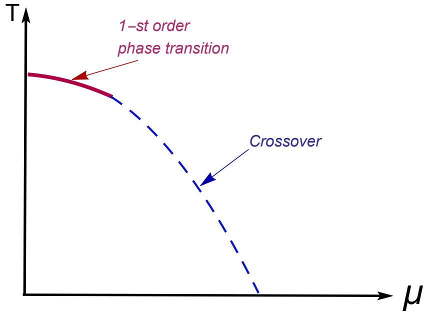
Philipsen, Pinke, PRD (2016)

“Light” and “Heavy” Quarks from Columbia Plot

Light quarks



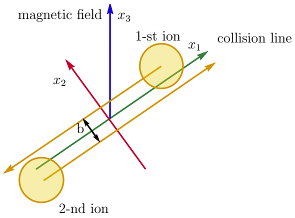
Heavy quarks

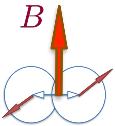


Anisotropy in QGP

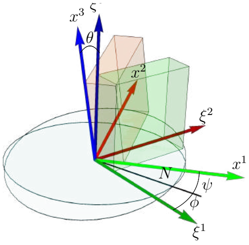
- Origins of QGP anisotropy
  - Primary anisotropy – longitudinal and 2 transversal directions in HIC
    - Multiplicity dependencies *ALICE* $\mathcal{M}(s) \sim s^{0.155(4)}$
    - *Aref'eva, Golubtsova, JHEP (2014)* $\mathcal{M}(s) \sim s^{1/(\nu+2)} \Rightarrow \nu = 4.5$
  - Secondary anisotropy – strong magnetic field eB $\sim 5 - 10\ m_\pi^2$

    m_π – pion mass





Peripheral HIC



Twice Anisotropic Background

$$\mathcal{L} = R - \frac{f_1(\phi)}{4}\, F_{(1)}^2 - \frac{f_2(\phi)}{4}\, F_{(2)}^2 - \frac{f_B(\phi)}{4}\, F_{(B)}^2 - \frac{1}{2}\, \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

$$ds^2 = \frac{L^2}{z^2}\, \mathfrak{b}(z) \left[- g(z)\, dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right]$$

"Bottom-up approach"

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$$A_\mu^{(1)} = A_t(z) \delta_\mu^0$$

$$A_t(0) = \mu \qquad g(0) = 1$$

$$A_t(z_h) = 0 \qquad g(z_h) = 0 \qquad \phi(z_0) = 0$$

$$ds^2 = \frac{L^2}{z^2} \mathfrak{b}(z) \left[- g(z)\, dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2} \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)} \right]$$

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I.A., A.G. (2014), Giataganas (2013)

"Bottom-up approach"

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Dudal et al., (2019)

$$A_t(z_h) = 0 \qquad g(z_h) = 0 \qquad \phi(z_0) = 0 \ \rightarrow \ \sigma_{\text{string}}$$

$$ds^2 = \frac{L^2}{z^2}\,\mathfrak{b}(z)\left[- g(z)\ dt^2 + dx^2 + \left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_1^2 + e^{c_B z^2}\left(\frac{z}{L}\right)^{2-\frac{2}{\nu}} dy_2^2 + \frac{dz^2}{g(z)}\right]$$

I.A., A.G. (2014), Giataganas (2013) *Gürsoy, Järvinen et al., (2019)*

"Bottom-up approach"

Twice Anisotropic Background

$$\mathcal{L} = R - \frac{f_1(\phi)}{4} F^2_{(1)} - \frac{f_2(\phi)}{4} F^2_{(2)} - \frac{f_B(\phi)}{4} F^2_{(B)} - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)$$

$$A^{(1)}_\mu = A_t(z)\delta^0_\mu \qquad F^{(2)}_{\mu\nu} = q\ dy^1 \wedge dy^2 \qquad F^{(B)}_{\mu\nu} = q_B\ dx \wedge dy^1$$

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Dudal et al., (2019)

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I.A., A.G. (2014), Giataganas (2013) *Gürsoy, Järvinen et al., (2019)*

$$\mathfrak{b}(z) = e^{2\mathcal{A}(z)} \ \rightarrow \ \text{quarks mass}$$

"Bottom-up approach"

$\mathcal{A}(z) = -\,cz^2/4 \ \rightarrow$ heavy quarks background (b, t) *Andreev, Zakharov (2006)*

$\mathcal{A}(z) = -\,a\ln(bz^2+1) \ \rightarrow$ light quarks background (d, u) *Li, Yang, Yuan (2020)*

Thermodynamics in Holography

$$T = \left. \frac{|g'|}{4\pi} \right|_{z=z_h} \longrightarrow \text{QGP temperature (Maldacena)}$$

$$s = \frac{1}{4} \left. \sqrt{\prod_{i=1}^{3} g_{ii}} \right|_{z=z_h} \longrightarrow \text{muliplicity of process (Landau)}$$

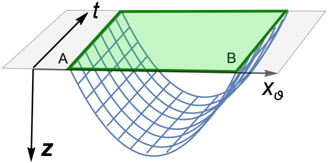
$$F = \int_{z_h}^{\infty} s\, dT \longrightarrow \text{background BH-BH phase transition}$$

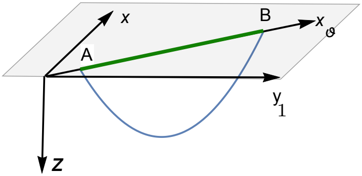
$$A_t = \mu - \rho z^2 + \dots \longrightarrow \text{chemical potential, density}$$

Temporal Wilson Loop

$$W[C_\vartheta] = e^{-S_{\vartheta,t}} \qquad \vec{n}: \qquad n_x = \cos\vartheta, \quad n_{y_1} = \sin\vartheta, \quad n_{y_2} = 0$$

ℓ x – collision axes





Two special cases:

- $\vartheta = 0$ WL (longitudinal)
- $\vartheta = \pi/2$ WT (transversal)

$\ell \to \infty \qquad S \sim \sigma_{DW}\, \ell$

$\swarrow$

the string tension

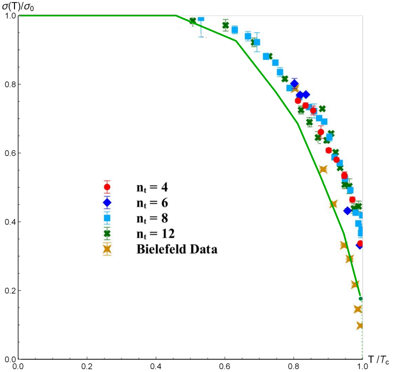
$$\sigma_{DW} = \frac{b(z) e^{\sqrt{\frac{2}{3}}\phi(z,z_0)}}{z^2} \sqrt{g(z)\left(z^{2-\frac{2}{\nu}}\sin^2(\vartheta) + \cos^2(\vartheta)\right)}\Bigg|_{z=z_{DW}}, \qquad \frac{\partial\sigma}{\partial z}\Bigg|_{z=z_{DW}} = 0$$

Aref'eva, K.R., Slepov, PLB **792** *(2019) 470 [arXiv:1808.05596]*

Our String Tension vs Lattice

- We take $z_0 = 10\exp(-z_h/4) + 0.1 \quad (\#)$

Aref'eva, K.R., Slepov
JHEP 06 (2021) 090
[arXiv:2009.05562]



- The green curve shows the string tension as function of temperature for $z_0 = (\#)$, $\mu = 0$, $\nu = 1$, $a = 4.046$, $b = 0.01613$, $c = 0.227$
- The dots with different decorations Lattice *Cardoso, Bicudo, PRD (2012)*
- The thin green dotted line shows the WL phase transition

Anisotropic Solution for "Light" Quarks

$$\mathcal{A}(z) = -\,a\ln(bz^2+1) \qquad f_1 = (1+bz^2)^a\, e^{-cz^2} z^{-2+\frac{2}{\nu}}$$

$$A_t = \mu\,\frac{e^{(2c-c_B)z^2/2} - e^{(2c-c_B)z_h^2/2}}{1-e^{(2c-c_B)z_h^2/2}} = \mu - \rho z^2 + \ldots \;\Rightarrow\; \rho = \frac{-\,\mu\,(2c-c_B)}{2\left(1-e^{(2c-c_B)z_h^2/2}\right)}$$

$$f_B = -\,\frac{2c_B\, z^{1-\nu/2}\, g}{q_B^2\, e^{cz^2/2}}\left(\frac{3cz}{2} + \frac{2}{\nu z} - c_B z - \frac{g'}{g}\right)$$

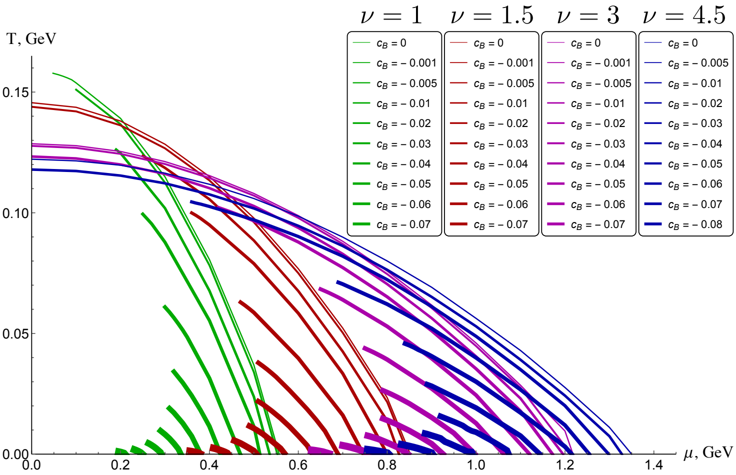
$$g = e^{c_B z^2}\left\{1 - \frac{I_1(z)}{I_1(z_h)} + \frac{\mu^2\,(2c-c_B)\,I_2(z)}{L^2\left(1-e^{(2c-c_B)z_h^2/2}\right)^2}\left(1 - \frac{I_1(z)}{I_1(z_h)}\,\frac{I_2(z_h)}{I_2(z)}\right)\right\}$$

$$I_1(z) = \int_0^z \left(1+b\xi^2\right)^{3a} e^{-\frac{3}{2}c_B\xi^2}\,\xi^{1+\frac{2}{\nu}}\,d\xi, \quad I_2(z) = \int_0^z \left(1+b\xi^2\right)^{3a} e^{(c-2c_B)\xi^2}\,\xi^{1+\frac{2}{\nu}}\,d\xi$$

$$\phi = \int_{z_0}^{z}\sqrt{\frac{4(\nu-1)}{\nu^2\xi^2} - 2c_B\left(3-\frac{2}{\nu}\right) - 2c_B^2\xi^2 + \frac{12ab}{1+b\xi^2}\left(1+2\,\frac{1+ab\xi^2}{1+b\xi^2}\right)}\,d\xi, \quad z_0 \neq 0$$

Aref'eva, Ermakov, K.R., Slepov, submitted to EPJC [arXiv:2203.12539]

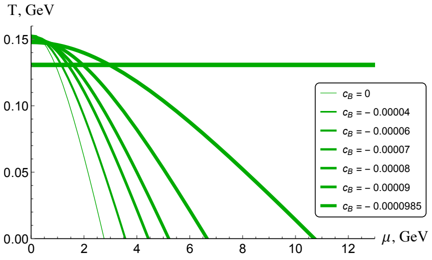
BH-BH Phase Transition:
"Light" Quarks in Magnetic Field



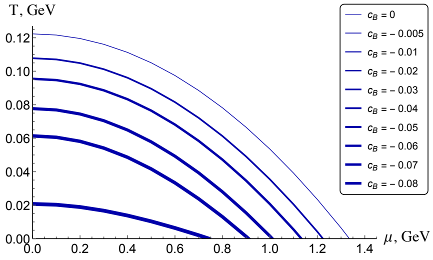
Aref'eva, Ermakov, K.R., Slepov, submitted to EPJC [arXiv:2203.12539]

Temporal Wilson loops for “Light” Quarks

$\nu = 1$



$\nu = 4.5$

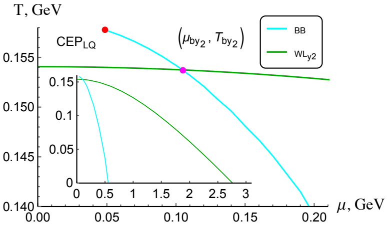


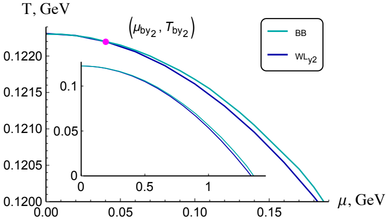
Aref’eva, Ermakov, K.R., Slepov, submitted to EPJC [arXiv:2203.12539]

Phase Transitions for "Light" Quarks

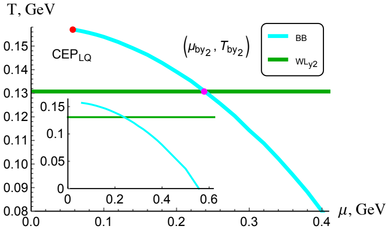
submitted to EPJC [arXiv:2203.12539]

$c_B = 0$

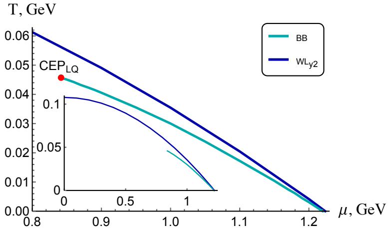




$c_B = -0.0000985$



$c_B = -0.03$



- We see the inverse magnetic catalysis

Anisotropic Solution for "Heavy" Quarks

$$\mathcal{A}(z) = -\,cz^2/4 \qquad f_1 = z^{-2+\frac{2}{\nu}}$$

$$A_t = \mu\,\frac{e^{(c-2c_B)z^2/4} - e^{(c-2c_B)z_h^2/4}}{1 - e^{(c-2c_B)z_h^2/4}} = \mu - \rho z^2 + \ldots \;\Rightarrow\; \rho = \frac{-\,\mu\,(c-2c_B)}{4\left(1 - e^{(c-2c_B)z_h^2/4}\right)}$$

$$f_B = -\,\frac{2c_B\, z^{1-\nu/2}\, g}{q_B^2\, e^{cz^2/2}}\left(\frac{3cz}{2} + \frac{2}{\nu z} - c_B z - \frac{g'}{g}\right)$$

$$g = e^{c_B z^2}\left\{1 - \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;\frac{3}{4}(2c_B-c)z^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;\frac{3}{4}(2c_B-c)z_h^2\right)} - \right.$$

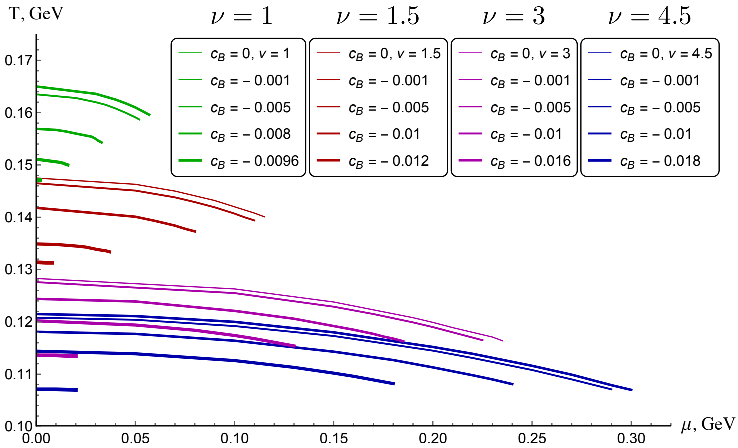
$$-\,\frac{\mu^2\,(2c_B-c)^{-\frac{1}{\nu}}}{4\left(1 - e^{(c-2c_B)\frac{z_h^2}{4}}\right)^2}\left(\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;\frac{3}{4}\,(2c_B-c)\,z^2\right)\right)\times$$

$$\left.\times\left[1 - \frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;\frac{3}{4}\,(2c_B-c)z^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;\frac{3}{4}\,(2c_B-c)z_h^2\right)}\,\frac{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;(2c_B-c)z_h^2\right)}{\Gamma\left(1+\frac{1}{\nu}\right) - \Gamma\left(1+\frac{1}{\nu}\;;(2c_B-c)z^2\right)}\right]\right\}$$

$$\phi = \int_{z_0}^{z}\frac{1}{\nu\xi}\sqrt{4\nu - 4 + \left(4\nu c_B + 3(3c-2c_B)\nu^2\right)\xi^2 + \left(\frac{3}{2}\,\nu^2 c^2 - 2c_B^2\right)\xi^4}\;d\xi, \qquad z_0 \neq 0$$

Aref'eva, K.R., Slepov, JHEP **07** *(2021) 161 [arXiv:2011.07023]*

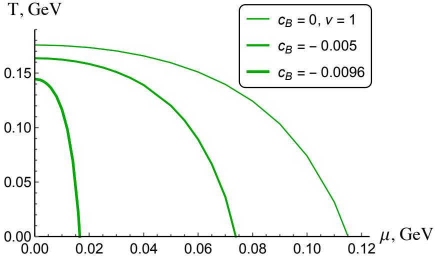
BH-BH Phase Transition: “Heavy” Quarks in Magnetic Field



Aref'eva, K.R., Slepov, JHEP **07** *(2021) 161 [arXiv:2011.07023]*

Temporal Wilson loops for "Heavy" Quarks

$\nu = 1$



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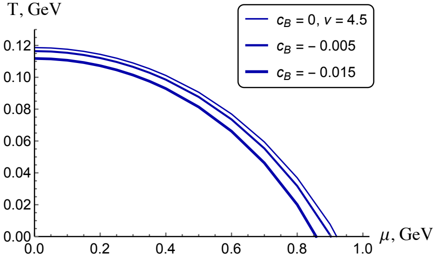
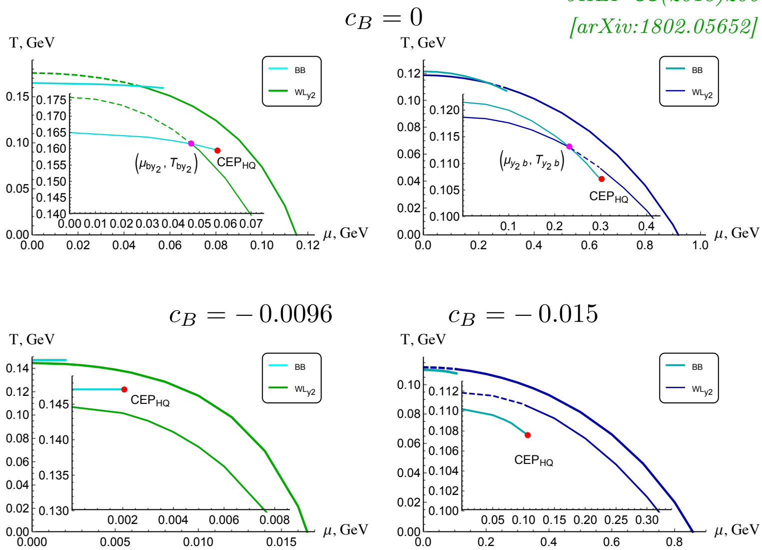


Aref'eva, K.R., Slepov, JHEP **07** *(2021) 161 [arXiv:2011.07023]*

Phase Transition for "Heavy" Quarks

JHEP **05** *(2018)206*
[arXiv:1802.05652]



- We see the inverse magnetic catalysis

Spatial Wilson Loops

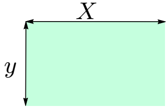
Nesting of the 2-dimensional world sheet in 5-dimensional space-time

$$X^0(\xi) = \text{const}$$

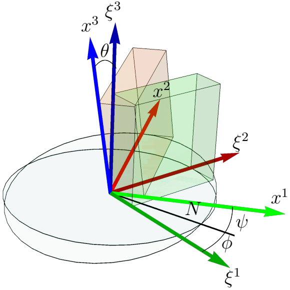
$$X^i(\xi) = \sum_{\alpha=1,2} a_{i\alpha}(\phi,\theta,\psi)\,\xi^\alpha, \qquad i = 1,2,3, \qquad \alpha = 1,2$$

$$X^4(\xi) = z(\xi^1)$$

- x^i – spatial coordinates
- $a_{ij}(\phi,\theta,\psi)$ – entries of the rotation matrix

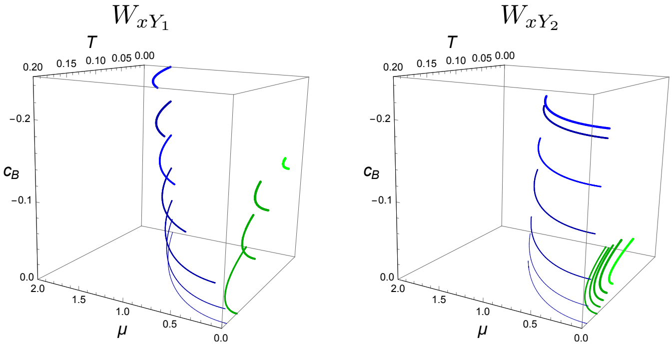


$y \times X$-rectangle



brick's orientation is equivalent to rectangle's orientation

Drag Forces Phase Transitions



Phase transition surfaces – surfaces stretched on blue/green curves

- Under variation of (T, μ, magnetic field) the spatial string tension undergoes the phase transition *TMP* **207***, 434 (2021) [arXiv:2012.05758]*
- The light green and blue lines represent to the end of the phase transitions
- Drag forces and energy losses undergo the phase transition as well

Pavel Slepov "Energy Loss in Strong Magnetic Field" poster, Thursday the 21-st

Conclusions

- We have an interplay between two types of phase transitions
  - Crossover
  - 1-st order phase transition
- Magnetic field unfluences the phase diagram for “heavy” and “light” quarks models
  - Inverse magnetic catalysis
  - External magnetic field suppresses BH-BH transition
  - For larger anisotropy $\nu > 1$ this suppression weakens

Conclusions

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- What to do next
  - Magnetic catalysis for “heavy” quarks model in magnetic field
  - Drag forces and energy losses for “light” quarks model
  - “Mixed” model corresponding to realistic quark masses
  - Other characteristics (susceptibility, transport coefficients, eta/s, direct-photon spectra, jet quenching, thermalization time, etc)

Thank you
for your attention