QED at strong and supercritical fields with heavy ions

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Outline of the talk

- Introduction
- QED at strong Coulomb field
- QED at supercritical Coulomb field
- Low-energy heavy-ion collisions
- How to observe the vacuum decay
- Conclusion

Introduction: tests of QED with atomic systems

Light atoms ($\alpha Z \ll 1$, weak fields):
Tests of QED to lowest orders in α and αZ.

Heavy few-electron ions ($\alpha Z \sim 1$, strong fields):
Tests of QED in nonperturbative in αZ regime.

Low-energy heavy-ion collisions at $Z_1 + Z_2 > 173$ (supercritical fields):
Tests of QED in supercritical regime.

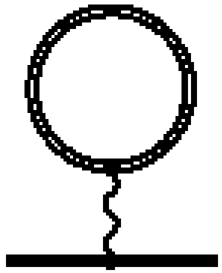
Binding energies in heavy few-electron ions

QED corrections

Calculations in the external field approximation $(M \to \infty)$

First-order QED corrections

P.J. Mohr, Ann. Phys., 1974

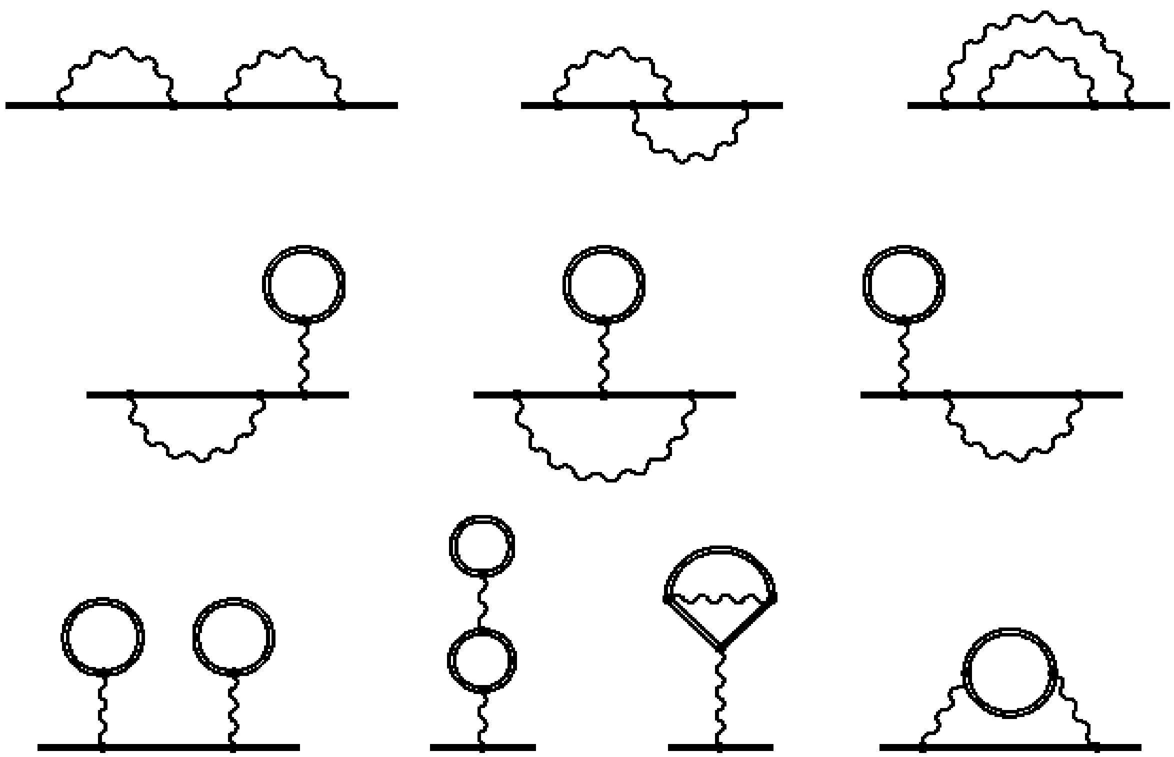


G. Soff and P.J. Mohr, PRA, 1988

N.L. Manakov et al., JETP, 1989

Binding energies in heavy few-electron ions

One-electron second-order QED corrections



Evaluation of the two-loop self-energy diagrams:

V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006.

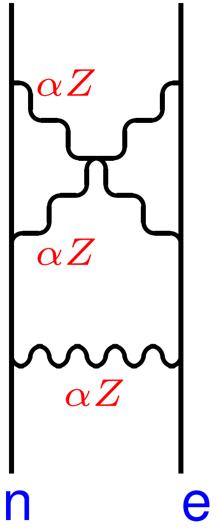
Binding energies in heavy few-electron ions

Nuclear recoil effect

Nonrelativistic theory: $m \to \mu = mM/(m+M)$.

Recoil effect in the full relativistic theory

A typical diagram:



Does a closed formula for the recoil correction to all orders in αZ exist?

(L.N. Labzowsky, 1972; M.A. Braun, 1973)

Binding energies in heavy few-electron ions

Formula for the nuclear recoil effect to first order in m/M and to all orders in αZ *(V.M. Shabaev, Theor. Math. Phys., 1985)*:

$$\begin{aligned} \Delta E &= \Delta E_{\rm L} + \Delta E_{\rm H} \\ \Delta E_{\rm L} &= \frac{1}{2M} \langle a | [\vec{p}^2 - (\vec{D}(0) \cdot \vec{p} + \vec{p} \cdot \vec{D}(0))] | a \rangle \,, \\ \Delta E_{\rm H} &= \frac{i}{2\pi M} \int_{-\infty}^{\infty} d\omega \, \langle a | \Big(\vec{D}(\omega) - \frac{[\vec{p}, V_{\rm C}]}{\omega + i0} \Big) G(\omega + E_a) \Big(\vec{D}(\omega) + \frac{[\vec{p}, V_{\rm C}]}{\omega + i0} \Big) | a \rangle \,. \end{aligned}$$

Here $\vec{p}$ is the momentum operator, $G(\omega)$ is the Coulomb Green function, $D_m(\omega) = -4\pi\alpha Z \alpha_l D_{lm}(\omega)$, and $D_{ik}(\omega, r)$ is the transverse part of the photon propagator in the Coulomb gauge.

Extention to many-electron atoms: *V.M. Shabaev, Sov. J. Nucl. Phys., 1988.*

Numerical evaluation: *A.N. Artemyev, V.M. Shabaev, V.A. Yerokhin, PRA, 1995.*

Fully relativistic theory of the nuclear recoil effect

Simple formulation of the QED theory of the recoil effect in atoms

(V.M. Shabaev, PRA, 1998)

In the Schrödinger representation and the Coulomb gauge, the Hamiltonian of the whole system is

$$
\begin{aligned}
H \;=\; & \int d\vec{x}\,\psi^\dagger(\vec{x})[\vec{\alpha}\cdot(-i\vec{\nabla}_{\vec{x}} - e\vec{A}(\vec{x})) + \beta m]\psi(\vec{x}) \\
& + \frac{e^2}{8\pi}\int d\vec{x}d\vec{y}\,\frac{\rho_e(\vec{x})\rho_e(\vec{y})}{|\vec{x}-\vec{y}|} + \frac{1}{2}\int d\vec{x}[\vec{\mathcal{E}}_{\rm t}^2(\vec{x}) + \vec{\mathcal{H}}^2(\vec{x})] \\
& + \frac{e|e|Z}{4\pi}\int d\vec{x}\,\frac{\rho_e(\vec{x})}{|\vec{x}-\vec{X}_{\rm n}|} + \frac{1}{2M}[\vec{P}_{\rm n} - |e|Z\vec{A}(\vec{X}_{\rm n})]^2 \\
& - \vec{\mu}\cdot\vec{\mathcal{H}}(\vec{X}_{\rm n})\,,
\end{aligned}
$$

where the nucleus is considered as a nonrelativistic particle with mass M. The term $-\vec{\mu}\cdot\vec{\mathcal{H}}$ causes the hyperfine splitting of atomic levels and will be omitted.

Fully relativistic theory of the nuclear recoil effect

The total momentum of the system is given by

$$\vec{P} = \vec{P}_{\rm n} + \vec{P}_{\rm e} + \vec{P}_{\rm ph}\,,$$

where

$$\vec{P}_{\rm e} = \int d\vec{x}\,\psi^\dagger(\vec{x})(-i\vec{\nabla}_{\vec{x}})\psi(\vec{x})$$

is the electron-positron field momentum and

$$\vec{P}_{\rm ph} = \int d\vec{x}\,[\vec{\mathcal{E}}_{\rm t}(\vec{x}) \times \vec{\mathcal{H}}(\vec{x})]$$

is the electromagnetic field momentum. In the center-of-mass frame:

$$\vec{P}\Phi = (\vec{P}_{\rm n} + \vec{P}_{\rm e} + \vec{P}_{\rm ph})\Phi = 0\,.$$

Fully relativistic theory of the nuclear recoil effect

In the center-of-mass frame ($\vec{P}=0$), using

$$\vec{P}_{\rm n} = -\vec{P}_{\rm e} - \vec{P}_{\rm ph} = -\int d\vec{x}\, \psi^\dagger(\vec{x})(-i\vec{\nabla}_{\vec{x}})\psi(\vec{x}) - \int d\vec{x}\, [\vec{\mathcal{E}}_{\rm t}(\vec{x}) \times \vec{\mathcal{H}}(\vec{x})]$$

and replacing $\vec{X}_{\rm n} \to 0$, we get

$$\begin{aligned} H \;=\; & \int d\vec{x}\, \psi^\dagger(\vec{x})[\vec{\alpha}\cdot(-i\vec{\nabla}_{\vec{x}} - e\vec{A}(\vec{x})) + \beta m]\psi(\vec{x}) \\ & + \frac{e^2}{8\pi}\int d\vec{x}\, d\vec{y}\, \frac{\rho_e(\vec{x})\rho_e(\vec{y})}{|\vec{x}-\vec{y}|} + \frac{1}{2}\int d\vec{x}\, [\vec{\mathcal{E}}_{\rm t}^2(\vec{x}) + \vec{\mathcal{H}}^2(\vec{x})] \\ & + \frac{e|e|Z}{4\pi}\int d\vec{x}\, \frac{\rho_e(\vec{x})}{|\vec{x}|} + \frac{1}{2M}\Big[-\int d\vec{x}\, \psi^\dagger(\vec{x})(-i\vec{\nabla}_{\vec{x}})\psi(\vec{x}) \\ & - \int d\vec{x}\, [\vec{\mathcal{E}}_{\rm t}(\vec{x}) \times \vec{\mathcal{H}}(\vec{x})] - |e|Z\vec{A}(0)\Big]^2 . \end{aligned}$$

Fully relativistic theory of the nuclear recoil effect

To zeroth order in α and to first order in m/M (but to all orders in αZ), the relativistic nuclear recoil operator is given by

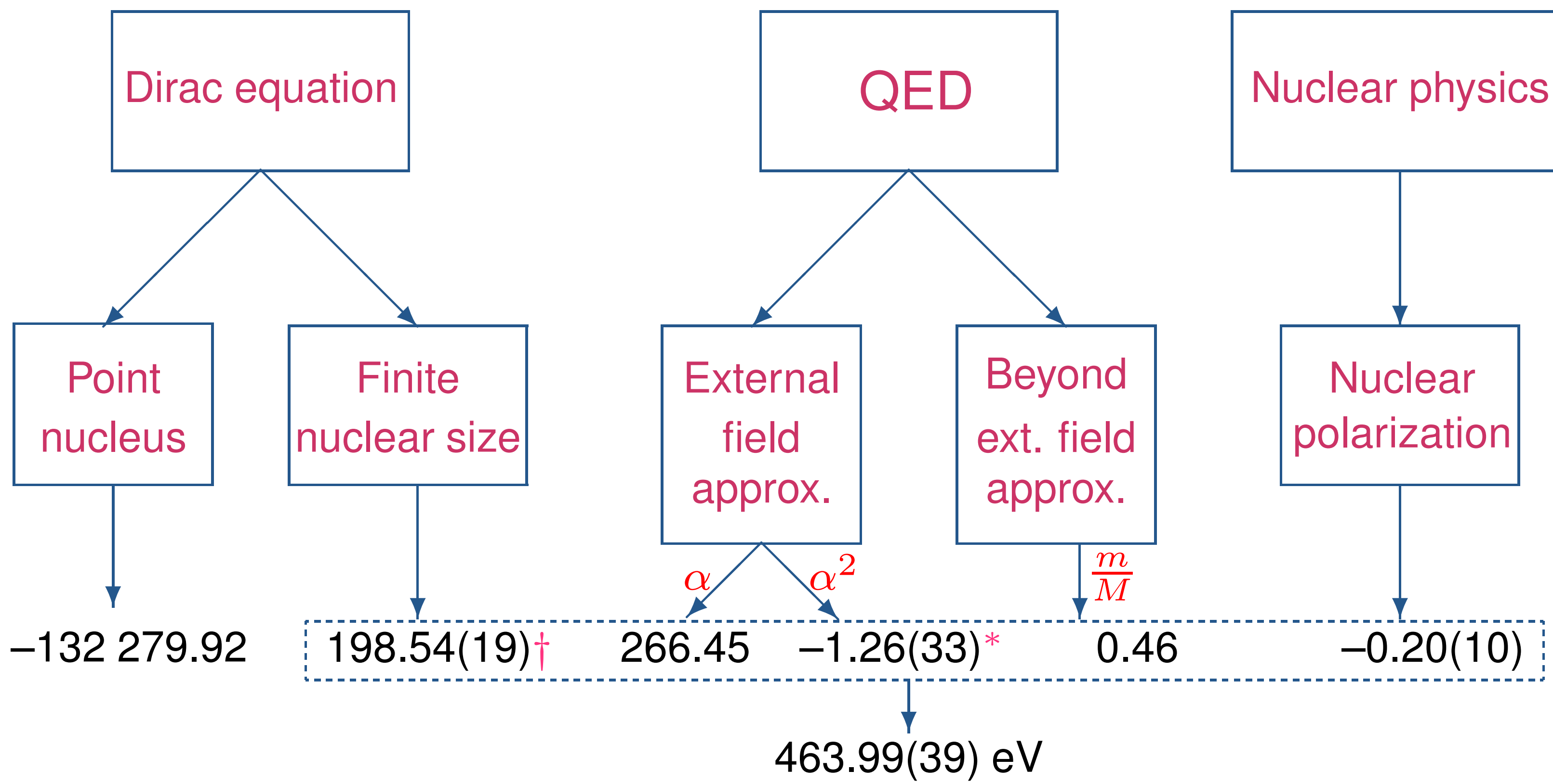
$$
\begin{aligned}
H_M \;=\; & \frac{1}{2M}\int d\vec{x}\,\psi^\dagger(\vec{x})(-i\vec{\nabla}_{\vec{x}})\psi(\vec{x})\int d\vec{y}\,\psi^\dagger(\vec{y})(-i\vec{\nabla}_{\vec{y}})\psi(\vec{y}) \\
& -\frac{eZ}{M}\int d\vec{x}\,\psi^\dagger(\vec{x})(-i\vec{\nabla}_{\vec{x}})\psi(\vec{x})\vec{A}(0)+\frac{e^2Z^2}{2M}\vec{A}^2(0)\,.
\end{aligned}
$$

To find the nuclear recoil effect for a state a, one should evaluate

$$
\Delta E_a = \langle\Phi_a|H_M|\Phi_a\rangle\,.
$$

The calculation can be performed by adding the Hamiltonian H_M to the standard QED Hamiltonian in the Furry picture. This results in appearing new lines and vertices in the Feynman rules.

1s Lamb shift in H-like uranium, in eV



Experiment: 460.2(4.6) eV

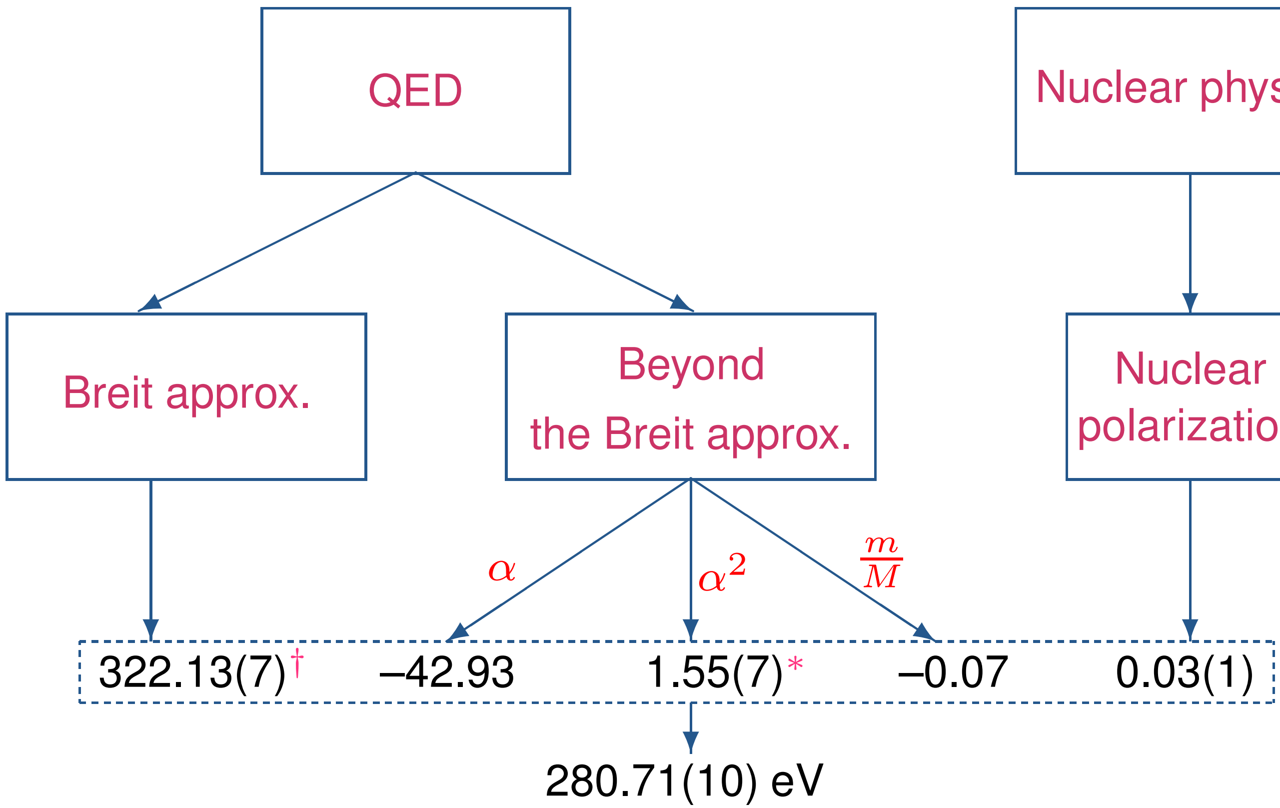
(A. Gumberidze, T. Stöhlker, D. Banas et al., PRL, 2005)

Test of QED: $\sim$ 2%

* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

$2p_{1/2}$-$2s$ *transition energy in Li-like uranium, in eV*



Experiment: 280.59(10) eV (J. Schweppe et al., PRL, 1991)
280.52(10) eV (C. Brandau et al., PRL, 2003)
280.645(15) eV (P. Beiersdorfer et al., PRL, 2005)

Test of QED: $\sim$ 0.2%

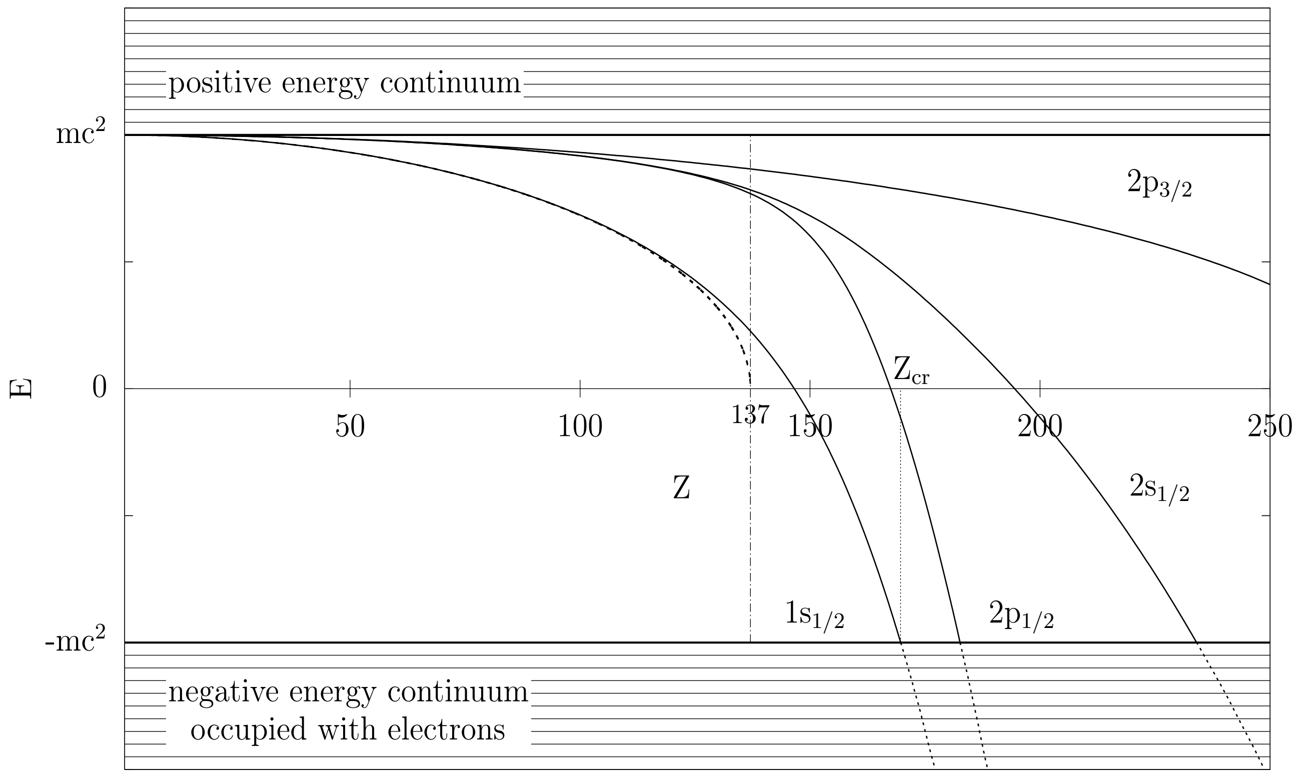
* V.A. Yerokhin, P. Indelicato, and V.M. Shabaev, PRL, 2006

† Y.S. Kozhedub, O.V. Andreev, V.M. Shabaev et al., PRA, 2008

QED at supercritical Coulomb field

Supercritical Coulomb field

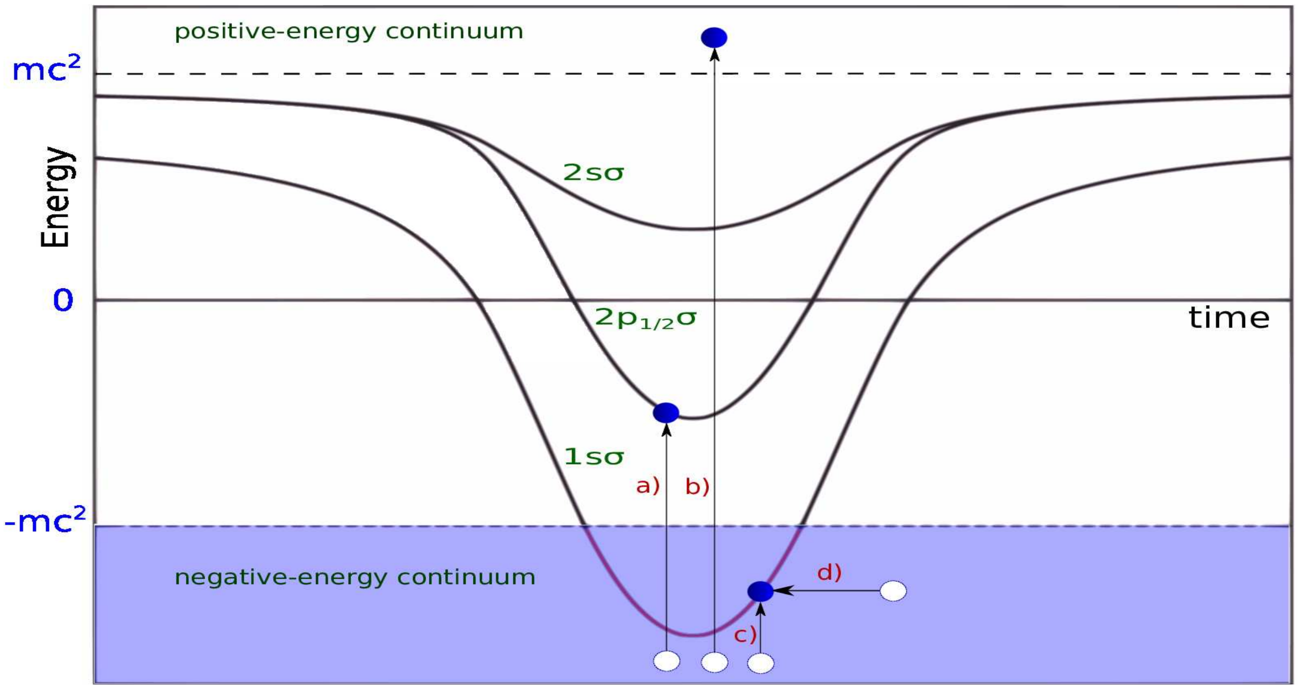
S.S. Gershtein, Ya.B. Zel'dovich, 1969; W. Pieper, W. Greiner, 1969



The $1s$ level dives into the negative-energy continuum at $Z_{\mathrm{crit}} \approx 173$.

Low-energy heavy-ion collisions

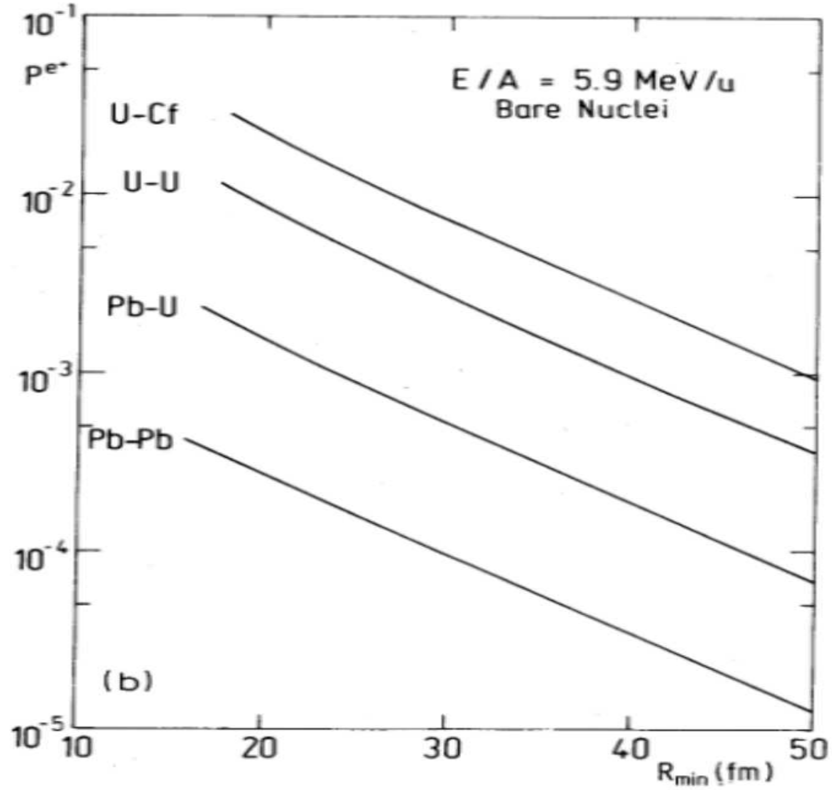
Creation of electron-positron pairs in low-energy heavy-ion collisions, with $Z_1 + Z_2 > 173$



Dynamical mechanism: a),b),c). Spontaneous mechanism (vacuum decay): d). The $1s$ state dives into the negative-energy continuum for about 10^{-21} sec.

Low-energy heavy-ion collisions

Positron production probability in 5.9 MeV/u collisions of bare nuclei as a function of distance of closest approach $R_{\min}$
(J. Reinhardt, B. Müller, and W. Greiner, Phys. Rev. A, 1981).



Conclusion by Frankfurt's group (2005):The vacuum decay could only be observed in collisions with nuclear sticking, in which the nuclei are bound to each other for some period of time by nuclear forces.

Low-energy heavy-ion collisions

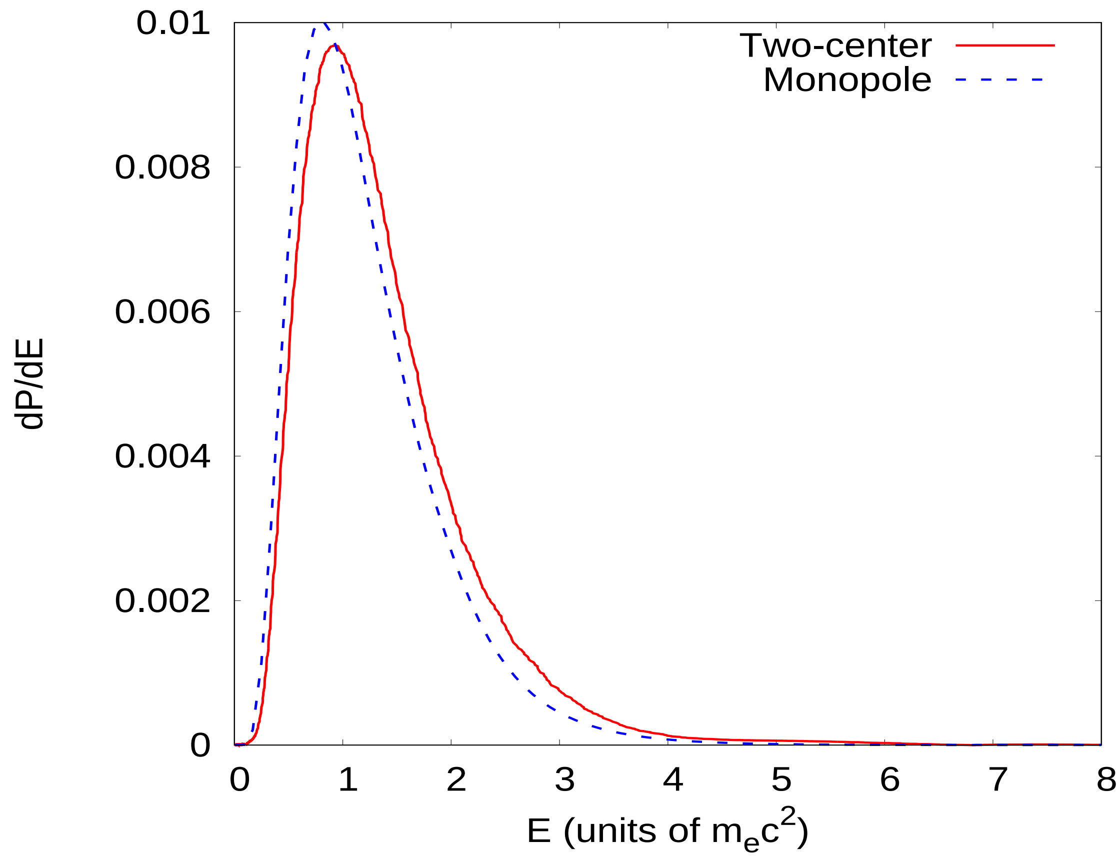
New methods for calculations of quantum dynamics of electron-positron field in low-energy heavy-ion collisions at subcritical and supercritical regimes have been developed:

- *I.I. Tupitsyn, Y.S. Kozhedub, V.M. Shabaev et al., Phys. Rev. A 82, 042701 (2010).*
- *I. I. Tupitsyn, Y. S. Kozhedub, V. M. Shabaev et al., Phys. Rev. A 85, 032712 (2012).*
- *G. B. Deyneka, I. A. Maltsev, I. I. Tupitsyn et al., Russ. J. of Phys. Chem. B 6, 224 (2012).*
- *G. B. Deyneka, I. A. Maltsev, I. I. Tupitsyn et al., Eur. Phys. J. D 67, 258 (2013).*
- *Y.S. Kozhedub, V.M. Shabaev, I.I. Tupitsyn et al., Phys. Rev. A 90, 042709 (2014).*
- *I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn et al., NIMB, 408, 97 (2017).*
- *R.V. Popov, A.I. Bondarev, Y.S. Kozhedub et al., Eur. Phys. J. D 72, 115 (2018).*
- *I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., Phys. Rev. A 98, 062709 (2018).*

Low-energy heavy-ion collisions

Pair creation beyond the monopole approximation

Positron energy spectrum for the U−U head-on collision at energy $E_{\rm cm}$ = 740 MeV *(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRA, 2018)*.



Low-energy heavy-ion collisions

Pair creation beyond the monopole approximation

U-U, $E_{\rm cm}$ = 740 MeV

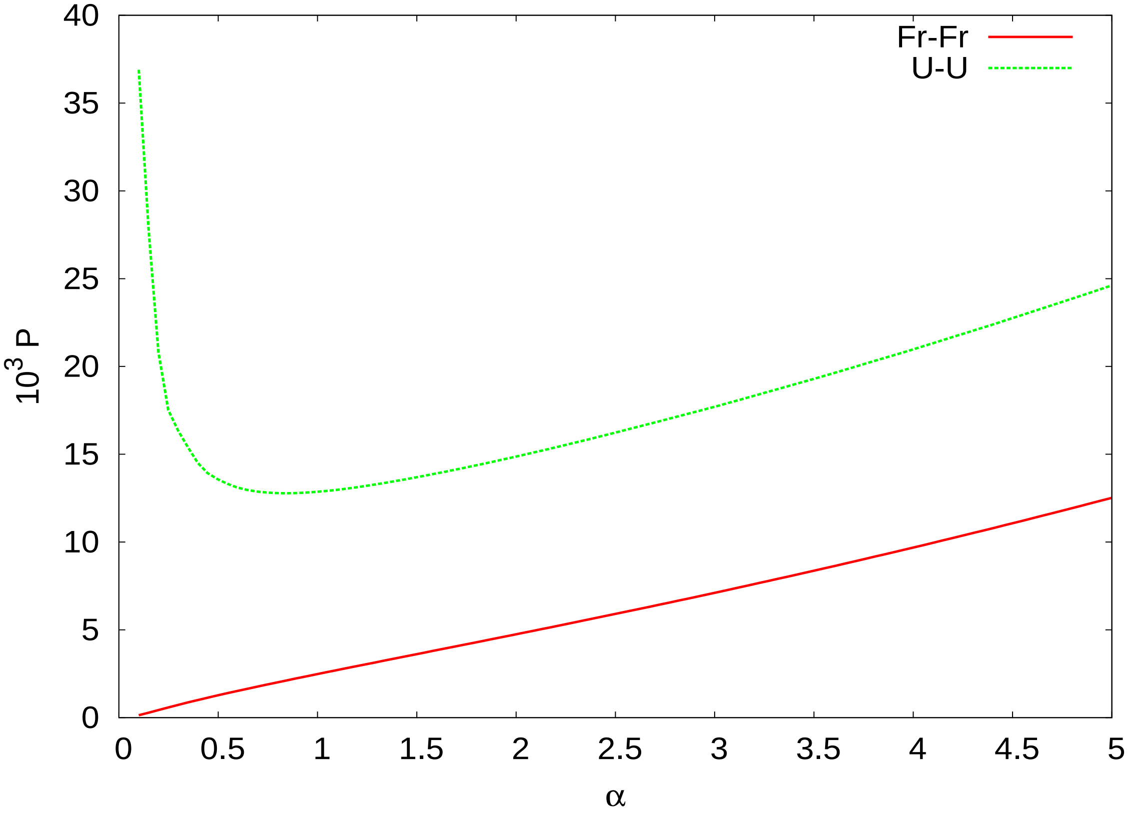
Expected number of created pairs as a function of the impact parameter b

(I.A. Maltsev, V.M. Shabaev, R.V. Popov et al., PRA, 2018) .

b (fm)	Monopole approximation	Two-center approach
0	1.29×10^{-2}	1.38×10^{-2}
10	7.26×10^{-3}	8.01×10^{-3}
20	2.75×10^{-3}	3.46×10^{-3}
30	1.04×10^{-3}	1.42×10^{-3}
40	4.12×10^{-4}	7.04×10^{-4}

The two-center result for $b = 0$ has been confirmed by a different method *(R.V. Popov, A.I. Bondarev, Y.S. Kozhedub et al., EPJD, 2018)* .

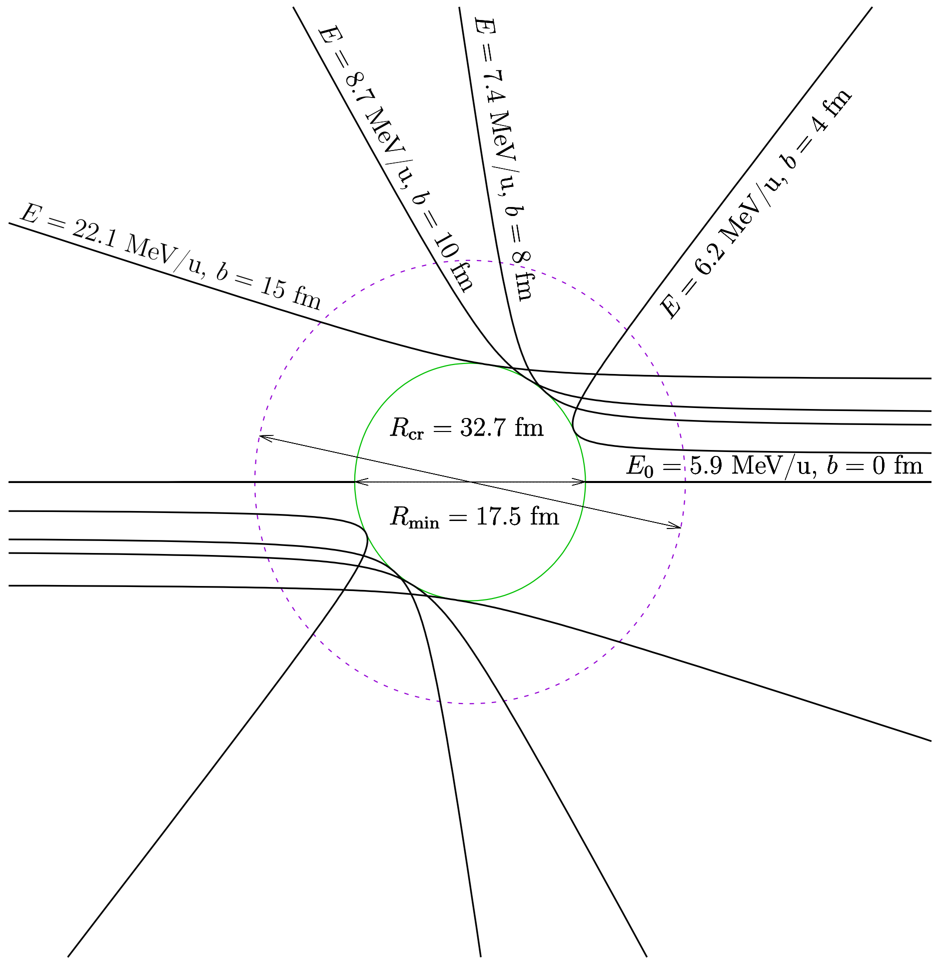
Low-energy heavy-ion collisions



Pair creation with artificial trajectories for the supercritical U−U and subcritical Fr−Fr head-on collisions at $E_{\rm cm} = 674.5$ and $E_{\rm cm} = 740$ MeV, respectively. The trajectory $R_\alpha(t)$ is defined by $\dot{R}_\alpha(t) = \alpha \dot{R}(t)$, where $R(t)$ is the classical Rutherford trajectory *(I.A. Maltsev, V.M. Shabaev, I.I. Tupitsyn et al., PRA, 2015)*.

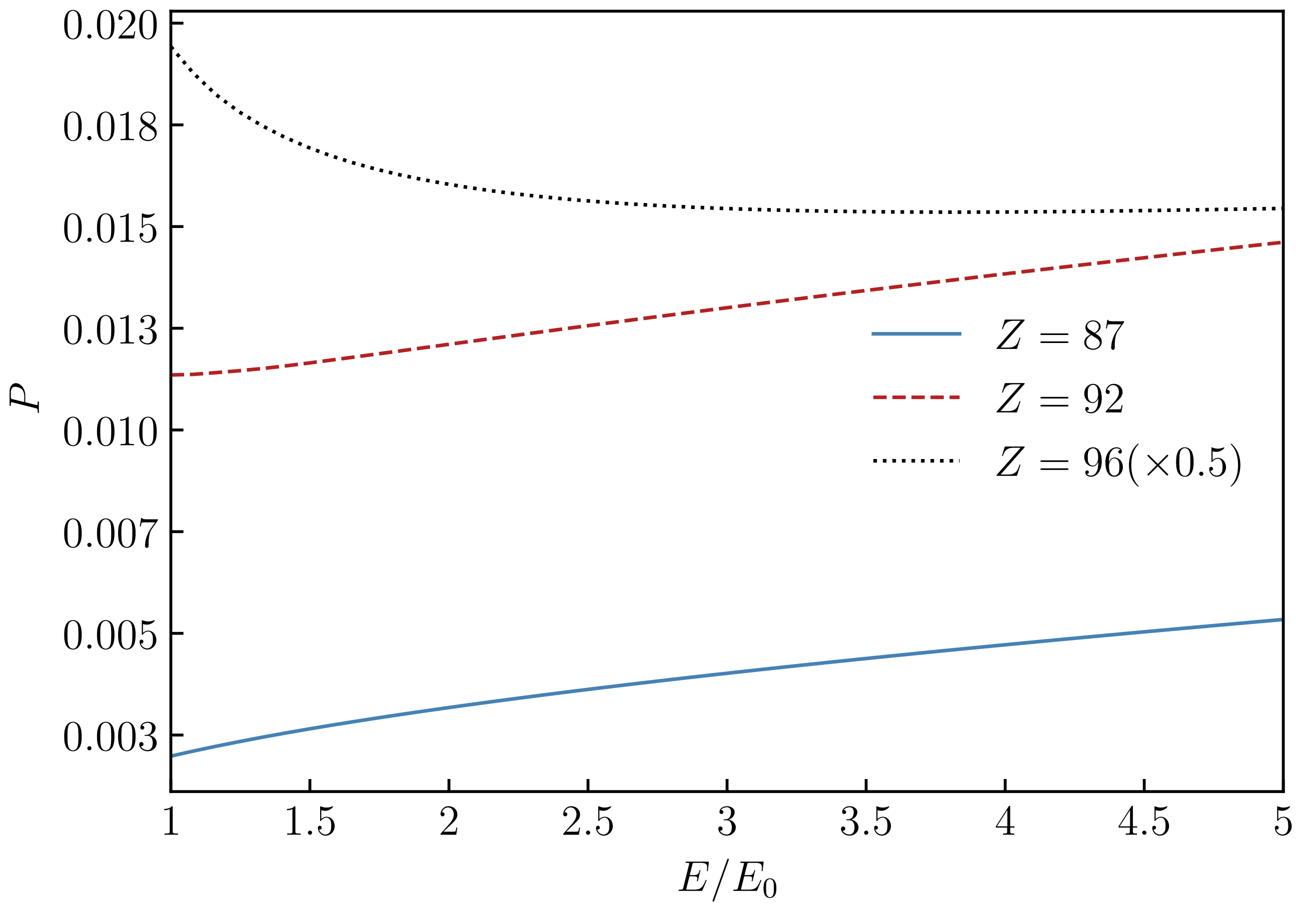
How to observe the vacuum decay

(I.A. Maltsev et al., PRL, 2019; R.V. Popov et al., PRD, 2020)



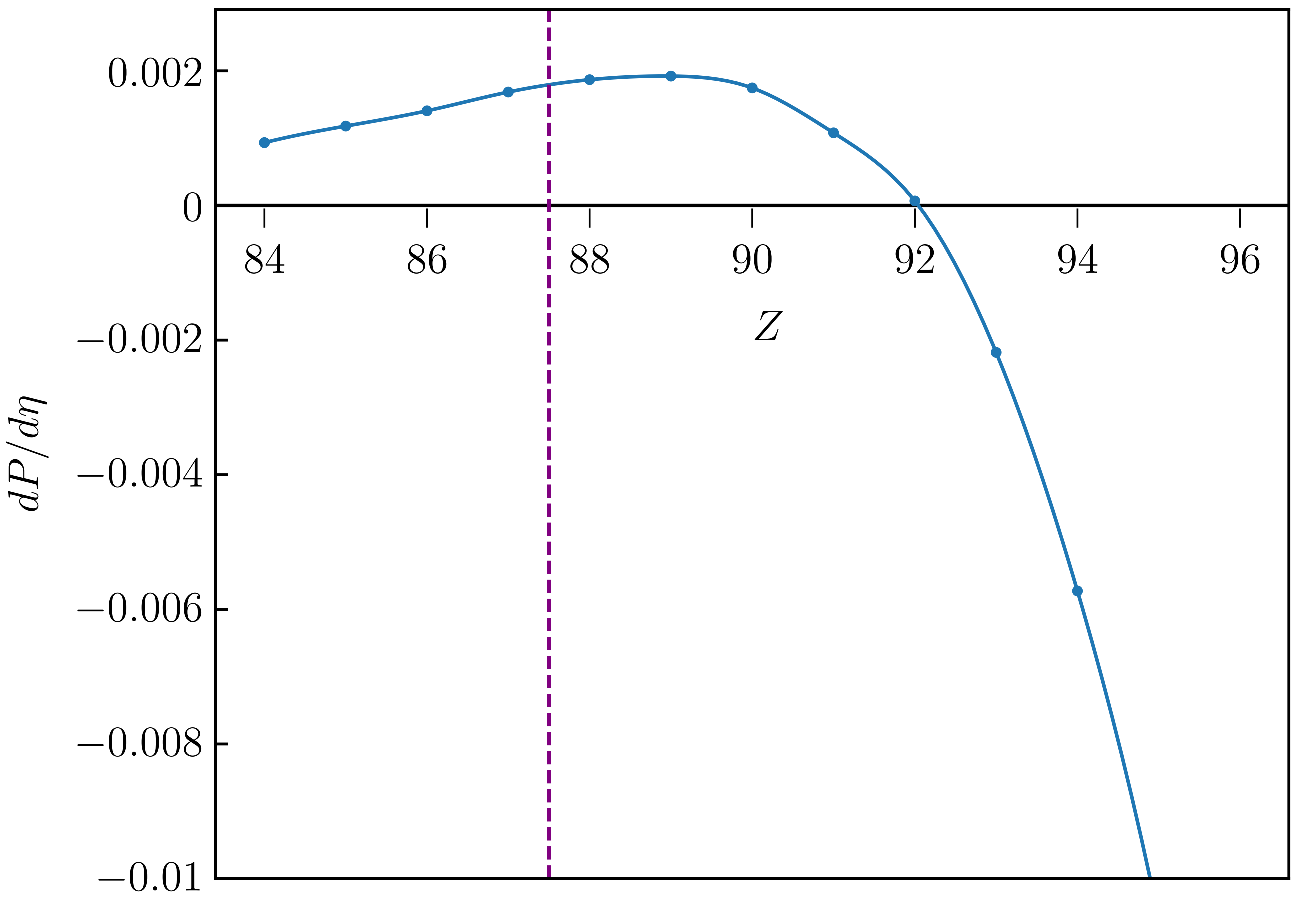
We consider only the trajectories for which the minimal internuclear distance is the same: $R_{\rm min} = 17.5$ fm. We introduce $\eta = E/E_0 \geq 1$.

How to observe the vacuum decay



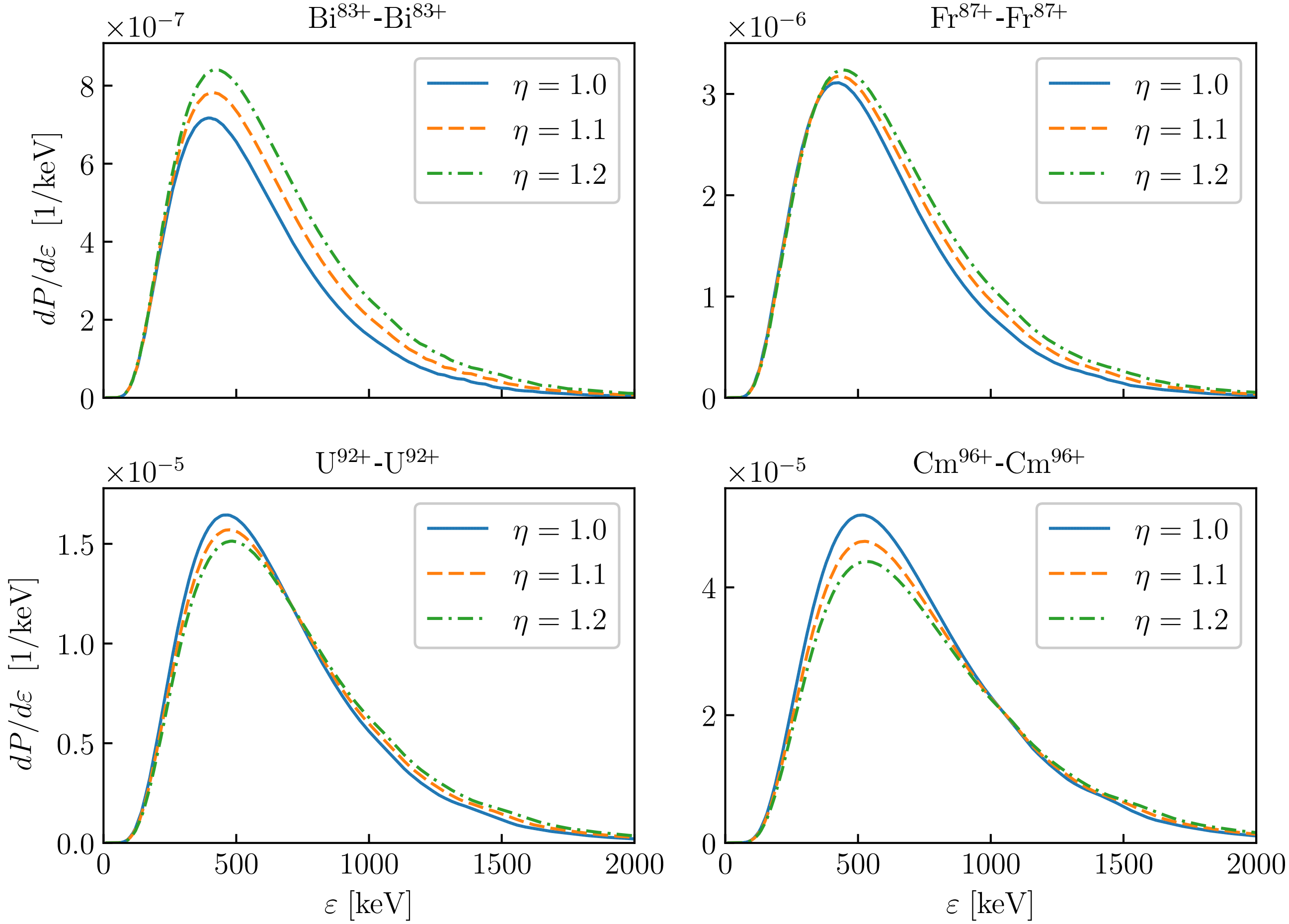
Total pair-production probability for symmetric ($Z = Z_1 = Z_2$) collisions as a function of the collision energy at $R_{\rm min} = 17.5$ fm.

How to observe the vacuum decay



The derivative of the pair-production probability with respect to the energy $dP/d\eta$, where $\eta = E/E_0$, at the point $\eta = 1$ as a function of the nuclear charge number $Z = Z_1 = Z_2$ at $R_{\rm min} = 17.5$ fm.

How to observe the vacuum decay



Positron spectra in symmetric ($Z = Z_1 = Z_2$) collisions for different collision energy $\eta = E/E_0$ at $R_{\min} = 17.5$ fm.

Conclusion

The experimental study of the proposed scenarios would either prove the vacuum decay in the supercritical Coulomb field or lead to discovery of a new physical phenomenon, which can not be described within the presently used QED formalism.

The same scenarios can be applied to observe the vacuum decay in collisions of bare nuclei with neutral atoms.

For details:

I.A. Maltsev, V.M. Shabaev, R.V. Popov, Y.S. Kozhedub, G. Plunien, X. Ma, Th. Stöhlker, and D.A. Tumakov, Phys. Rev. Lett. 123, 113401 (2019).

R.V. Popov, V.M. Shabaev, D.A. Telnov, I.I. Tupitsyn, I.A. Maltsev, Y.S. Kozhedub, A.I. Bondarev, N.V. Kozin, X. Ma, G. Plunien, T. Stöhlker, D.A. Tumakov, and V.A. Zaytsev, Phys. Rev. D 102, 076005 (2020).