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Multiloop calculations in $\mathcal{N}=1$ SQED with N_f flavours regularized by higher derivatives

Higher order calculations in supersymmetric theories

The report is based on

I. E. Shirokov and K. V. Stepanyantz, JHEP 2204 (2022) 108.

Calculations of higher order quantum corrections in $N = 1$ supersymmetric theories are important for both theory and phenomenological applications.

L.Mihaila, Adv. High Energy Phys. **2013** (2013) 607807.

Most of these calculations were made in the $\overline{\text{DR}}$-scheme.

L.V.Avdeev, O.V.Tarasov, Phys.Lett. **B 112** (1982) 356; I.Jack, D.R.T.Jones, C.G.North, Phys.Lett. **B 386** (1996) 138; Nucl.Phys. **B 486** (1997) 479; R.V.Harlander, D.R.T.Jones, P.Kant, L.Mihaila, M.Steinhauser, JHEP **0612** (2006) 024.

This means that theory is regularized by dimensional reduction and divergences are removed by modified minimal subtraction.

W.A.Bardeen, A.J.Buras, D.W.Duke and T.Muta, Phys. Rev. D **B 18** (1978) 3998; W.Siegel, Phys.Lett. **B 84** (1979) 193.

Higher order calculations in supersymmetric theories

However, the dimensional reduction is not self-consistent.

W.Siegel, Phys.Lett. **B 94** (1980) 37.

Removing of the inconsistencies leads to the loss of explicit supersymmetry:

L.V.Avdeev, G.A.Chochia, A.A.Vladimirov, Phys.Lett. **B 105** (1981) 272.

As a consequence, supersymmetry can be broken by quantum corrections in higher loops.

L.V.Avdeev, Phys.Lett. **B 117** (1982) 317;
L.V.Avdeev, A.A.Vladimirov, Nucl.Phys. **B 219** (1983) 262.

Higher covariant derivative regularization

The higher covariant derivative regularization is a consistent regularization, which does not break supersymmetry.

A.A.Slavnov, Nucl.Phys., **B 31** (1971) 301; Theor.Math.Phys. **13** (1972) 1064.

In order to regularize a theory by higher derivatives it is necessary to add a term with higher degrees of covariant derivatives. Then divergences remain only in the one-loop approximation. These remaining divergences are regularized by inserting the Pauli–Villars determinants.

A.A.Slavnov, Theor.Math.Phys. **33** (1977) 977.

The higher covariant derivative regularization can be generalized to the $\mathcal{N}=1$ supersymmetric case

V.K.Krivoshchekov, Theor.Math.Phys. **36** (1978) 745;
P.West, Nucl.Phys. **B 268** (1986) 113.

In this talk we will discuss quantum corrections in SQED regularized by higher covariant derivatives.

NSVZ β-function for $\mathcal{N}=1$ SQED with N_f flavors

The simplest particular case of the $\mathcal{N}=1$ gauge theory is the $\mathcal{N}=1$ supersymmetric electrodynamics (SQED) with N_f flavors, which (in the massless case) is described by the action

$$S=\frac{1}{4e_0^2}\text{Re}\int d^4x\,d^2\theta\,W^aW_a+\sum_{f=1}^{N_f}\frac{1}{4}\int d^4x\,d^4\theta\,\Big(\phi_f^* e^{2V}\phi_f+\widetilde{\phi}_f^* e^{-2V}\widetilde{\phi}_f\Big),$$

where V is a real gauge superfield, ϕ_f and $\widetilde{\phi}_f$ with $f=1,\ldots,N_f$ are chiral matter superfields with opposite U(1) charges, and $W_a=\bar{D}^2D_aV/4$. In our notation the bare and renormalized coupling constants are denoted by e_0 and e, respectively.

$\mathcal{N} = 1$ SQED with N_f flavors, regularized by higher derivatives

In order to regularize the theory by higher derivatives, it is necessary to add the higher derivative term to the action:

$$S_{\text{reg}} = \frac{1}{4e_0^2}\text{Re}\int d^4x\, d^2\theta\, W^a R(\partial^2/\Lambda^2) W_a + \sum_{f=1}^{N_f} \frac{1}{4}\int d^4x\, d^4\theta\, \Big(\phi_f^* e^{2V}\phi_f + \widetilde{\phi}_f^* e^{-2V}\widetilde{\phi}_f\Big),$$

where $R(\partial^2/\Lambda^2)$ is a regulator, e.g. $R = 1 + \partial^{2n}/\Lambda^{2n}$.
Another similar regulator function appears in the gauge fixing term

$$S_{\text{gf}} = -\frac{1}{32\xi_0 e_0^2}\int d^4x\, d^4\theta\, D^2V K(\partial^2/\Lambda^2)\bar{D}^2 V,$$

where ξ_0 is the bare gauge parameter. The minimal (Feynman) gauge corresponds to $\xi_0 = 1$ and $R(x) = K(x)$. However, we will make calculations for an arbitrary ξ_0 and $K(x) \neq R(x)$.

Pauli–Villars determinants

Adding the higher derivative term allows to remove all divergences beyond the one-loop approximation. To remove one-loop divergences, we insert in the generating functional the Pauli–Villars determinants:

$$Z[\text{sources}] = \int DV \left(\prod_{\alpha=1}^{N_f} D\phi_\alpha D\widetilde{\phi}_\alpha\right) \text{Det}(PV, M)^{N_f} \exp\Big(iS_{\text{reg}} + iS_{\text{gf}} + iS_{\text{ист}}\Big)$$

$$\text{Det}(PV, M)^{-1} = \int D\Phi\, D\widetilde{\Phi}\, \exp(iS_\Phi).$$

Here the action for the massive Pauli–Villars superfields is given be the expression

$$S_\Phi = \frac{1}{4}\int d^4x\, d^4\theta \left(\Phi^* e^{2V}\Phi + \widetilde{\Phi}^* e^{-2V}\widetilde{\Phi}\right) + \left(\frac{M}{2}\int d^4x\, d^2\theta\, \widetilde{\Phi}\,\Phi + \text{c.c.}\right),$$

and it is important that the ratio of the Pauli–Villars mass M to the regularization parameter Λ should not depend on the coupling constant.

Renormalization

The considered theory is renormalizable.

A. A. Slavnov, Nucl. Phys. **B 97** (1975) 155.

Therefore the ultraviolet divergences can be absorbed into the renormalization of the coupling constant, of the gauge parameter, and of the chiral matter superfields ϕ_α and $\widetilde{\phi}_\alpha$. All chiral superfields have the same renormalization constant Z, such that $\phi_{\alpha,R} = \sqrt{Z}\phi_\alpha$, $\widetilde{\phi}_{\alpha,R} = \sqrt{Z}\widetilde{\phi}_\alpha$ for all values of $\alpha = 1,\ldots,N_f$.

The renormalization group functions

It is convenient to encode the ultraviolet divergences in RGFs. It is neccesary to distinguish between RGFs defined in terms of the bare coupling constant,

$$\beta(\alpha_0) = \frac{d\alpha_0}{d\ln\Lambda}\Big|_{\alpha=\text{const}}; \qquad \gamma(\alpha_0) = -\frac{d\ln Z}{d\ln\Lambda}\Big|_{\alpha=\text{const}},$$

and the ones standardly defined in terms of the renormalized coupling constant by the equations

$$\widetilde{\beta}(\alpha) = \frac{d\alpha}{d\ln\mu}\Big|_{\alpha_0=\text{const}}; \qquad \widetilde{\gamma}(\alpha) = \frac{d\ln Z}{d\ln\mu}\Big|_{\alpha_0=\text{const}},$$

where μ is a renormalization point.

A.L.Kataev and K.V.Stepanyantz, Nucl. Phys. B **875** (2013) 459.

NSVZ β-function in $\mathcal{N}=1$ supersymmetric theories

In $\mathcal{N}=1$ supersymmetric theories the β-function is related to the anomalous dimension of the matter superfields by the equation

$$\beta(\alpha,\lambda) = -\frac{\alpha^2\Big(3C_2 - T(R) + C(R)_i{}^j\gamma_j{}^i(\alpha,\lambda)/r\Big)}{2\pi(1-C_2\alpha/2\pi)}, \qquad \text{where}$$

$$\text{tr}\,(T^A T^B) \equiv T(R)\,\delta^{AB}; \qquad (T^A)_i{}^k (T^A)_k{}^j \equiv C(R)_i{}^j;$$

$$f^{ACD} f^{BCD} \equiv C_2 \delta^{AB}; \qquad r \equiv \delta_{AA}.$$

V.Novikov, M.A.Shifman, A.Vainshtein, V.I.Zakharov, Nucl.Phys. **B 229** (1983) 381; Phys.Lett. **B 166** (1985) 329; M.A.Shifman, A.I.Vainshtein, Nucl.Phys. **B 277** (1986) 456; D.R.T.Jones, Phys.Lett. **B 123** (1983) 45.

The NSVZ β-function was obtained from different arguments: instantons, anomalies etc.

The NSVZ relation with the HD regularization

With the higher covariant derivative regularization loop integrals giving a β-function defined in terms of the bare coupling constant are integrals of total derivatives

A.Soloshenko, K.V.Stepanyantz, hep-th/0304083.

and even integrals of double total derivatives

A.V.Smilga, A.I.Vainshtein, Nucl.Phys. **B 704** (2005) 445.

This allows to calculate one of the loop integrals analytically and to obtain the NSVZ relation for the RG functions defined in terms of the bare coupling constant. In the Abelian case this has been done in all loops

K.V.Stepanyantz, Nucl.Phys. **B 852** (2011) 71; JHEP **1408** (2014) 096.

In the non-Abelian case proof is more complicated, it has also been done in all loops

K.V.Stepanyantz, Eur. Phys. J. **C 80** (2020) no.10, 911; JHEP **2001** (2020) 192.

NSVZ β-function in $\mathcal{N}=1$ SQED with N_f flavours

RGFs defined in terms of the bare coupling constant are independent of a renormalization prescription for a fixed regularization, but depend on a regularization. In considered theory and in the case of using the higher derivative regularization described above they satisfy the NSVZ equation

$$\frac{\beta(\alpha_0)}{\alpha_0^2} = \frac{N_f}{\pi}\Big(1-\gamma(\alpha_0)\Big)$$

M.A.Shifman, A.I.Vainshtein, V.I.Zakharov, JETP Lett. **42** (1985) 224; Phys.Lett. **B 166** (1986) 334.

in all loops for an arbitrary renormalization prescription.
In contrast, RGFs defined in terms of the renormalized coupling constant depend on both a regularization and a subtraction scheme. For them NSVZ realtion is valid only for some certain renormalization prescriptions called "the NSVZ schemes". Some of them are given by the HD+MSL renormalization prescription, when the theory is regularized by higher derivatives and divergences are removed by minimal subtractions of logarithms.

A.L.Kataev and K.V.Stepanyantz, Nucl. Phys. B **875** (2013) 459.

Anomalous dimension defined in terms of the bare coupling constant

Calculating superdiagrams with two external lines of the matter superfields one can obtain the function G related to the corresponding part of the effective action by the equation

$$\Gamma^{(2)}_\phi = \frac{1}{4}\int \frac{d^4p}{(2\pi)^4}\, d^4\theta \sum_{\alpha=1}^{N_f}\Big(\phi^*_\alpha(p,\theta)\phi_\alpha(-p,\theta)+\widetilde{\phi}^*_\alpha(p,\theta)\widetilde{\phi}_\alpha(-p,\theta)\Big)G(\alpha_0,\Lambda/p)$$

If the function G is known, then the anomalous dimension defined in terms of the bare coupling constant can be obtained with the help of the equation

$$\gamma(\alpha_0) = \frac{d\ln G}{d\ln\Lambda}\Big|_{\alpha=\text{const};\, p\to 0}$$

where the condition $p\to 0$ removes terms proportional to powers of p/Λ.

Computer program for supergarph calculations

Explicit calculations in the framework of $N = 1$ superspace are rather complicated, so special C++ program was created to deal with them. At the present moment the programm can deal with $N = 1$ SQED with N_f flavours and can calculate two-point Green function of matter fields. The program makes such steps of calculation:

1. Generation of all supergraphs in desired order of perturbation theory using given vertexes and propagators.
2. Evaluation of so-called "D-algebra", using standart proceedure of removing supersymmetric covariant derivatives and taking integral over superspace.
3. Removing objects with spinor indices by taking γ-matrix traces etc.
4. Reducing of remaining impulse intregrals, by collecting terms using some integral transformations.

At present moment impulse integrals must be taken by hand or by using other software.

Example, one-loop calculation: Input

```
Type:
F#_1^1{-1}*F_1^1{1}
Option:
SQED
Nf^0
Loops:
1
Order:
2
Propagators:
V_1^1{1}*V_2^1{1}
-1/4*i*e^0*[K4{1}-K5{1}]*I{1^-4}*D#_1{^1}(D_1(D#_1{_1}(d_{1}{12})))
2*i*e^0*I{1^-2}*K4{1}*d_{1}{12}
F#_1^1{1}*F_2^1{1}
-1/4*i*e^0*I{1^-2}*D_1(D#_2(d_{1}{12}))
Vertexes:
1/2*i*e*F#_1^1{6}*V_1^1{-3}*F_1_1{-2}
1/2*i*e^2*F#_1^1{30}*V_1^1{-3}*V_1^1{-5}*F_1_1{-2}
```

Example, one-loop and two-loop calculation: Output

```
Результат:
-1/2*e^2*F#_1^1{-1}*F_1_2{1}*I{2^-4}*K5^-1{2}
e^4*F#_1^1{-1}*F_1_2{1}*I{3^-2}*I{2^-4}*I{6^-2}*K5^-2{2}
-2*e^4*F#_1^1{-1}*F_1_2{1}*K3{3}*I{2^-6}*K3{6}*K5^-2{2}
-2*e^4*F#_1^1{-1}*F_1_2{1}*K2{6}*I{2^-6}*K2{3}*I{3^2}*K5^-2{2}
e^4*F#_1^1{-1}*F_1_2{1}*K2{3}*I{2^-4}*K2{6}*K5^-2{2}
2*e^4*F#_1^1{-1}*F_1_2{1}*K2{3}*I{2^-6}*K5^-2{2}
e^4*F#_1^1{-1}*F_1_2{1}*I{2^-4}*I{3^-2}*I{6^-2}*K5^-1{2}*K5^-1{3}
```

$$\gamma(\alpha_0) = \frac{d\ln G}{d\ln\Lambda}\bigg|_{q=0} = -\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{2e_0^2}{K^4R_K^2}\Bigg\{R_K$$

$$-2e_0^2N_f\int\frac{d^4L}{(2\pi)^4}\left(\frac{1}{L^2(L+K)^2}-\frac{1}{(L^2+M^2)((L+K)^2+M^2)}\right)\Bigg\}$$

$$+\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{e_0^4}{R_KR_L}\left(\frac{4}{K^2L^4(K+L)^2}-\frac{2}{K^4L^4}\right)+O(e_0^6).$$

Result coincides with

S.S.Aleshin, *et al.* Nucl. Phys. B **956** (2020), 115020;
A.L.Kataev and K.V.Stepanyantz, Theor.Math.Phys. **181** (2014) 1531.

Timing information

Operating system: Windows 10 x64
Processor: AMD Ryzen 5 1600 Six-Core Processor 3.20 GHz
RAM: 8 Гб
Compiler: GNU GCC Compiler
Compilation options: -march=native, -O3

term $\sim N_f^0$	1 loop	2 loops	3 loops
$\xi_0 = 1$	0.052 s	0.14 s	2.6 s
$\xi_0 \neq 1$	0.067 s	0.57 s	2 m 27 s

term $\sim N_f^1$	2 loops	3 loops
$\xi_0 = 1,\, m = 0$	0.16 s	6.6 s
$\xi_0 \neq 1,\, m = 0$	0.52 s	13 m 49 s
$\xi_0 = 1,\, m \neq 0$	0.41 s	41.5 s
$\xi_0 \neq 1,\, m \neq 0$	1.23 s	3 h 54 m

term $\sim N_f^2$	3 loops
$\xi_0 = 1,\, m = 0$	4.2 s
$\xi_0 \neq 1,\, m = 0$	6.6 s
$\xi_0 = 1,\, m \neq 0$	35 s
$\xi_0 \neq 1,\, m \neq 0$	2 m 58 s

Three-loop anomalous dimension

$$\gamma(\alpha_0) = \frac{d\ln G}{d\ln\Lambda}\Big|_{P=0} = -\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{2e_0^2}{K^4R_K} + \frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{e_0^4}{R_KR_L}\Big(\frac{4}{K^2L^4(K+L)^2}$$
$$-\frac{2}{K^4L^4}\Big) + N_f\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{4e_0^4}{R_K^2K^4}\Big(\frac{1}{L^2(L+K)^2} - \frac{1}{(L^2+M^2)((L+K)^2+M^2)}\Big)$$
$$+\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{d^4Q}{(2\pi)^4}\frac{8e_0^6}{R_KR_LR_Q}\Big[-\frac{1}{3K^4L^4Q^4} + \frac{1}{K^4L^2Q^4(Q+L)^2} + \frac{1}{K^2L^4(K+L)^2}$$
$$\times\frac{1}{(Q+K+L)^2}\Big(\frac{1}{Q^2} - \frac{2}{(Q+L)^2}\Big)\Big] + N_f\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{d^4Q}{(2\pi)^4}\frac{16e_0^6\,K_\mu L^\mu}{R_K^2R_LK^4L^4(K+L)^2}$$
$$\times\Big(\frac{1}{Q^2(Q+K)^2} - \frac{1}{(Q^2+M^2)((Q+K)^2+M^2)}\Big) + N_f\frac{d}{d\ln\Lambda}\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{d^4Q}{(2\pi)^4}\frac{8e_0^6}{R_K^2R_LK^4}$$
$$\times\frac{1}{L^2}\Big(\frac{2(Q+K+L)^2-K^2-L^2}{Q^2(Q+K)^2(Q+L)^2(Q+K+L)^2} - \frac{2(Q+K+L)^2-K^2-L^2}{(Q^2+M^2)((Q+K)^2+M^2)((Q+L)^2+M^2)}$$
$$\times\frac{1}{((Q+K+L)^2+M^2)} + \frac{4M^2}{(Q^2+M^2)^2((Q+K)^2+M^2)((Q+L)^2+M^2)}\Big) - (N_f)^2\frac{d}{d\ln\Lambda}$$
$$\times\int\frac{d^4K}{(2\pi)^4}\frac{d^4L}{(2\pi)^4}\frac{d^4Q}{(2\pi)^4}\frac{8e_0^6}{R_K^3K^4}\Big(\frac{1}{Q^2(Q+K)^2} - \frac{1}{(Q^2+M^2)((Q+K)^2+M^2)}\Big)\Big(\frac{1}{L^2(L+K)^2}$$
$$-\frac{1}{(L^2+M^2)((L+K)^2+M^2)}\Big) + O(e_0^8),$$

Chebishev polinomials method

The integrals are calculated using the Chebyshev polynomials method.

J. L. Rosner, Annals Phys. **44** (1967), 11.

The Chebyshev polynomials are defined as:

$$C_n(\cos\theta) \equiv \frac{\sin((n+1)\theta)}{\sin\theta}$$

and (for $t < 1$) satisfy the important equation

$$\frac{1}{1-2tz+t^2} = \sum_{n=0}^{\infty} t^n C_n(z).$$

Consequently, the function $(K-L)^{-2} = (K^2 - 2KL\cos\theta + L^2)^{-1}$, where θ is the angle between the Euclidian four-vectors K_μ and L_μ, can be presented in the form

$$\frac{1}{(K-L)^2} = \begin{cases} \dfrac{1}{K^2}\displaystyle\sum_{n=0}^{\infty}\Big(\frac{L}{K}\Big)^n C_n(\cos\theta), & \text{если} \quad K > L; \\ \dfrac{1}{L^2}\displaystyle\sum_{n=0}^{\infty}\Big(\frac{K}{L}\Big)^n C_n(\cos\theta), & \text{если} \quad L > K. \end{cases}$$

Then the angular parts can be calculated with the help of the useful identities

$$\int \frac{d\Omega_Q}{2\pi^2} C_m\Big(\frac{K_\mu Q^\mu}{KQ}\Big) C_n\Big(\frac{Q_\nu L^\nu}{QL}\Big) = \frac{1}{n+1}\delta_{mn} C_n\Big(\frac{K_\mu L^\mu}{KL}\Big);$$

$$\int \frac{d\Omega}{2\pi^2} C_m(\cos\theta) C_n(\cos\theta) = \delta_{mn},$$

where $d\Omega$ is the element of a solid angle on a sphere S^3 in the momentum space.

Anomalous dimension and β-function defined in terms of the bare coupling constant

$$\gamma(\alpha_0) = -\frac{\alpha_0}{\pi} + \frac{\alpha_0^2}{2\pi^2} + \frac{\alpha_0^2 N_f}{\pi^2}\Big(\ln a + 1 + \frac{A_1}{2}\Big) - \frac{\alpha_0^3}{2\pi^3} - \frac{\alpha_0^3 N_f}{\pi^3}\Big(\ln a + \frac{3}{4} + C\Big)$$
$$-\frac{\alpha_0^3 (N_f)^2}{\pi^3}\Big((\ln a + 1)^2 - \frac{A_2}{4} + D_1 \ln a + D_2\Big) + O(\alpha_0^4),$$

where A_1, A_2, C, D_1, and D_2 are numerical parameters depending on the regulator function $R(x)$. To find the β-function defined in terms of the bare coupling constant, we substitute the expression into the NSVZ equation. For RGFs defined in terms of the bare coupling constant it is valid in all loops for an arbitrary renormalization prescription supplementing the higher derivative regularization. Therefore, the four-loop β-function takes the form

$$\beta(\alpha_0) = \frac{\alpha_0^2 N_f}{\pi} + \frac{\alpha_0^3 N_f}{\pi^2} - \frac{\alpha_0^4 N_f}{2\pi^3} - \frac{\alpha_0^4 (N_f)^2}{\pi^3}\Big(\ln a + 1 + \frac{A_1}{2}\Big) + \frac{\alpha_0^5 N_f}{2\pi^4} + \frac{\alpha_0^5 (N_f)^2}{\pi^4}$$
$$\times\Big(\ln a + \frac{3}{4} + C\Big) + \frac{\alpha_0^5 (N_f)^3}{\pi^4}\Big((\ln a + 1)^2 - \frac{A_2}{4} + D_1 \ln a + D_2\Big) + O(\alpha_0^6).$$

Anomalous dimension and β-function defined in terms of the renormalized coupling constant

$$\widetilde{\gamma}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{2\pi^2} + \frac{\alpha^2 N_f}{\pi^2}\Big(\ln a + 1 + \frac{A_1}{2} + g_{1,0} - b_{1,0}\Big) - \frac{\alpha^3}{2\pi^3} + \frac{\alpha^3 N_f}{\pi^3}\Big(- \ln a - \frac{3}{4} - C - b_{2,0} + b_{1,0} - g_{2,0} + g_{1,0}\Big) + \frac{\alpha^3 (N_f)^2}{\pi^3}\Big\{ - \Big(\ln a + 1 - b_{1,0}\Big)^2 + \frac{A_2}{4} - D_1 \ln a - D_2 + b_{1,0} A_1 - g_{2,1}\Big\} + O(\alpha^4).$$

$$\frac{\widetilde{\beta}(\alpha)}{\alpha^2} = -\frac{d}{d\ln\mu}\Big(\frac{1}{\alpha}\Big)\Big|_{\alpha_0=\text{const}} = \frac{N_f}{\pi} + \frac{\alpha N_f}{\pi^2} - \frac{\alpha^2 N_f}{2\pi^3} - \frac{\alpha^2 (N_f)^2}{\pi^3}\Big(\ln a + 1 + \frac{A_1}{2} + b_{2,0} - b_{1,0}\Big) + \frac{\alpha^3 N_f}{2\pi^4} + \frac{\alpha^3 (N_f)^2}{\pi^4}\Big(\ln a + \frac{3}{4} + C + b_{3,0} - b_{1,0}\Big) + \frac{\alpha^3 (N_f)^3}{\pi^4}\Big\{\Big(\ln a + 1 - b_{1,0}\Big)^2 - b_{1,0} A_1 + b_{3,1} - \frac{A_2}{4} + D_1 \ln a + D_2\Big\} + O(\alpha^4).$$

It is known that the terms which do not contain N_f in anomalous dimension and terms that are proportional to N_f^1 in β-function are scheme independent. One can choose so-called "minimal scheme" where the finite constants $b_{1,0}, b_{2,0}, b_{3,0}, g_{1,0}$, $g_{2,0}, g_{3,0}$ can be chosen so as to set all scheme dependent terms to 0.
It can be done in all loops.

A. L. Kataev and K. V. Stepanyantz, Phys. Lett. B 730 (2014) 184;
I. E. Shirokov and K. V. Stepanyantz, JHEP 2204 (2022) 108.

Anomalous dimension and β-function in the "minimal scheme"

Result in the "minimal scheme":

$$\widetilde{\gamma}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{2\pi^2} - \frac{\alpha^3}{2\pi^3} + O(\alpha^4).$$

$$\widetilde{\beta}(\alpha) = \frac{\alpha^2 N_f}{\pi} + \frac{\alpha^3 N_f}{\pi^2} - \frac{\alpha^4 N_f}{2\pi^3} + \frac{\alpha^5 N_f}{2\pi^4} + O(\alpha^6).$$

Scheme-dependent terms coincide with the result obtained in $\overline{\text{DR}}$-scheme.

$$\widetilde{\gamma}_{\overline{\text{DR}}}(\alpha) = -\frac{\alpha}{\pi} + \frac{\alpha^2}{2\pi^2} + \frac{\alpha^2 N_f}{2\pi^2} - \frac{\alpha^3}{2\pi^3} + \frac{\alpha^3 N_f}{\pi^3}\Big(1 - \frac{3}{2}\zeta(3)\Big) + \frac{\alpha^3 (N_f)^2}{4\pi^3} + O(\alpha^4),$$

$$\frac{\widetilde{\beta}_{\overline{\text{DR}}}(\alpha)}{\alpha^2} = \frac{N_f}{\pi} + \frac{\alpha N_f}{\pi^2} - \frac{\alpha^2 N_f}{2\pi^3} - \frac{3\alpha^2 (N_f)^2}{4\pi^3} + \frac{\alpha^3 N_f}{2\pi^4} + \frac{\alpha^3 (N_f)^2}{\pi^4}\Big(-\frac{5}{6} + \frac{3}{2}\zeta(3)\Big)$$
$$+ \frac{\alpha^3 (N_f)^3}{12\pi^4} + O(\alpha^4)$$

I. Jack, D. R. T. Jones and C. G. North, Nucl. Phys. B 473 (1996), 308-322

Conclusion

- A new computer program, that can deal with supergraphs was created in the framework of $\mathcal{N} = 1$ SQED with N_f flavours.
- Two-point Green-function was calculated with help of this program in up to three loops.
- In two loops result coincides with one obtained earlier by hand.
- All calculations were made in arbitrary ξ-gauge. Gauge invariance of the result is useful correctness check.
- Using Chebishev polinomials techinique integrals were taken.
- Four-loop β-function was obtained using NSVZ-relation.
- All RGFs were also rewritten in terms of the renormalized coupling constant. Scheme independent terms coincide with ones obtained in $\overline{\text{DR}}$-scheme.
- Result was rewritten in the "minimal scheme". Existance of such scheme was proved in all loops in (S)QED.

Thank you for the attention!