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Manifestation of dark matter axions in spin effects in storage rings

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OUTLINE

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Pseudoscalar interactions caused by dark matter axions

CP-noninvariant interactions caused by dark matter axions are time-dependent. Like photons, moving axions form a wave which pseudoscalar field reads

$$a(\boldsymbol{r}, t) = a_0 \cos\left(E_a t - \boldsymbol{p}_a \cdot \boldsymbol{r} + \phi_a\right).$$

Here $E_a = \sqrt{m_a^2 + \boldsymbol{p}_a^2}$, $\boldsymbol{p}_a$, and m_a are the energy, momentum, and mass of axions . The Earth motion through our galactic define its velocity relative to dark matter, $V \sim 10^{-3}c$. Therefore, $|\boldsymbol{p}_a| \approx m_a V$ and axions and axion-like particles have momenta of the order of $|\nabla a| \sim 10^{-3}\dot{a}c$.

Even if the axion field is influenced by the electromagnetic interaction, this equation is used only for a determination of the frequency E_a and does not change in the weak-field approximation.

Axions manifest themselves in interactions with photons, gluons, and fermions (first of all, nucleons). The corresponding contributions to the Lagrangian density are defined by

$$\mathcal{L}_\gamma = -\frac{g_{a\gamma\gamma}}{4} a F_{\mu\nu}\widetilde{F}^{\mu\nu} = g_{a\gamma\gamma} a \boldsymbol{E}\cdot\boldsymbol{B},$$

$$\mathcal{L}_g = \frac{g_{QCD}^2 C_g}{32\pi^2 f_a} a G_{\mu\nu}\widetilde{G}^{\mu\nu}, \qquad \mathcal{L}_N = g_{aNN}\gamma^\mu\gamma^5\partial_\mu a,$$

where $g_{QCD}^2/(4\pi) \sim 1$ is the coupling constant for the color field, C_g, $g_{a\gamma\gamma}$ and g_{aNN} are model-dependent constants, f_a is the constant of interaction of axions with matter (axion decay constant), and the tilde denotes a dual tensor.

The oscillating contribution to the EDM (from gluons) is given by

$$\mathcal{L}_{aEDM} = -\frac{i}{2} g_d a \sigma^{\mu\nu} \gamma^5 F_{\mu\nu}$$

where the EDM is equal to $d_a = g_d a$ and g_d is proportional to g_{aNN}.

Another contribution to the Lagrangian introduced by Pospelov et al. is defined by the gradient interaction:

$$\mathcal{L}_N = g_{aNN} \gamma^\mu \gamma^5 \partial_\mu a$$

M. Pospelov, A. Ritz, and M. Voloshin, Bosonic super-WIMPs as keV-scale dark matter, Phys. Rev. D **78**, 115012 (2008).
V. A. Dzuba, V. V. Flambaum, and M. Pospelov, Atomic ionization by keV-scale pseudoscalar dark-matter particles, Phys. Rev. D **81**, 103520 (2010).
P. W. Graham and S. Rajendran, Phys. Rev. D **88**, 035023 (2013).
P. W. Graham et al., Phys. Rev. D **97**, 055006 (2018).
P. W. Graham et al., Phys. Rev. D **103**, 055010 (2021).

The Lagrangian $L = \overline{\psi}\mathcal{L}\psi$ describing electromagnetic interactions of a Dirac particle with allowance for a pseudoscalar axion field is defined by

$$\mathcal{L} = \gamma^{\mu}(i\hbar\partial_{\mu} - eA_{\mu}) - m + \frac{\mu'}{2}\sigma^{\mu\nu}F_{\mu\nu} - i\frac{d}{2}\sigma^{\mu\nu}\gamma^{5}F_{\mu\nu} + g_{aNN}\gamma^{\mu}\gamma^{5}\Lambda_{\mu},$$

$$\Lambda_{\mu} = \partial_{\mu}a, \qquad \gamma^{5} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

where μ' and d are the anomalous magnetic and electric dipole moments. In the last term, $a = a_0 \cos(m_a t - \boldsymbol{p}_a \cdot \boldsymbol{r})$ is the axion field.

The corresponding Hamiltonian in the Dirac representation reads

$$\mathcal{H} = \beta m + \boldsymbol{\alpha} \cdot (\boldsymbol{p} - e\boldsymbol{A}) + e\Phi + \mu'(i\boldsymbol{\gamma} \cdot \boldsymbol{E} - \boldsymbol{\Pi} \cdot \boldsymbol{B})$$
$$-d(\boldsymbol{\Pi} \cdot \boldsymbol{E} + i\gamma \cdot \boldsymbol{B}) - g_{aNN}(\gamma^5 \Lambda_0 + \boldsymbol{\Sigma} \cdot \boldsymbol{\Lambda}).$$

Relativistic Foldy-Wouthuysen transformation and the relativistic Hamiltonian in the Foldy-Wouthuysen representation

- For arbitrary-spin particles in external fields, the Foldy-Wouthuysen (FW) representation restores the Schrödinger form of relativistic quantum mechanics (QM) while the Dirac representation corrupts this form (Foldy, Wouthuysen, 1950; Silenko, 2003, 2008).

The position and spin operators, as well as other operators, are counterparts of the corresponding classical variables only when they are defined in the FW representation but not in the Dirac one (Foldy, Wouthuysen, 1950; Zou, Zhang, Silenko, 2020).

The passage to the classical limit usually reduces to a replacement of the operators in quantum-mechanical Hamiltonians and equations of motion in the FW representatin with the corresponding classical quantities (Silenko, 2013).

The FW transformation belongs to the foundation of QM!

In the general case, it is convenient to present the Dirac Hamiltonian as follows:

$$\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \quad \beta\mathcal{M} = \mathcal{M}\beta, \quad \beta\mathcal{E} = \mathcal{E}\beta, \quad \beta\mathcal{O} = -\mathcal{O}\beta.$$

The even operators $\mathcal{M}$ and $\mathcal{E}$ and the odd operator $\mathcal{O}$ are diagonal and off-diagonal in two spinors, respectively.

FW transformation for relativistic arbitrary-spin particles in arbitrarily strong external fields – final results and the proof of validity:

A.J. Silenko, General method of the relativistic Foldy-Wouthuysen transformation and proof of validity of the Foldy-Wouthuysen Hamiltonian, Phys. Rev. A **91**, 022103 (2015).

If one holds only terms proportional to the zero and first powers of ℏ, the final FW Hamiltonian takes the form

$$\mathcal{H}_{FW} = \beta\epsilon + \mathcal{E} + \frac{1}{4}\left\{\frac{1}{2\epsilon^2 + \{\epsilon,\mathcal{M}\}}, (\beta[\mathcal{O},[\mathcal{O},\mathcal{M}]] - [\mathcal{O},[\mathcal{O},\mathcal{F}]])\right\}.$$

$$\epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}.$$

This is the only relativistic method which gives the *exact* form of leading terms in the FW Hamiltonian of the zero and first orders in $\hbar$ and does not need cumbersome derivations. The first-order terms in $\hbar$ describe spin effects.

In the considered case,

$$\mathcal{M} = m - \mu'\boldsymbol{\Sigma}\cdot\boldsymbol{B} - d\boldsymbol{\Sigma}\cdot\boldsymbol{E}, \qquad \mathcal{E} = e\Phi - g_{aNN}\boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda},$$

$$\mathcal{O} = \boldsymbol{\alpha}\cdot(\boldsymbol{p} - e\boldsymbol{A}) + i\mu'\boldsymbol{\gamma}\cdot\boldsymbol{E} - id\boldsymbol{\gamma}\cdot\boldsymbol{B} - g_{aNN}\gamma^5\Lambda_0.$$

The FW Hamiltonian has the form

$$\mathcal{H}_{FW} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_3,$$

$$\mathcal{H}_1 = \beta\epsilon' + e\Phi - \frac{1}{2}\left\{\left(\frac{\mu_0 m}{\epsilon'} + \mu'\right), \boldsymbol{\Pi}\cdot\boldsymbol{B}\right\}$$

$$+\frac{1}{4}\left\{\left(\frac{\mu_0 m}{\epsilon' + m} + \mu'\right)\frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma}\cdot[\boldsymbol{\pi}\times\boldsymbol{E}] - \boldsymbol{\Sigma}\cdot[\boldsymbol{E}\times\boldsymbol{\pi}] - \nabla\cdot\boldsymbol{E}\right)\right\}$$

$$+\frac{\mu'}{4}\left\{\frac{1}{\epsilon'(\epsilon' + m)}, \left[(\boldsymbol{B}\cdot\boldsymbol{\pi})(\boldsymbol{\Pi}\cdot\boldsymbol{\pi}) + (\boldsymbol{\Pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{B}) + 2\pi(\boldsymbol{\pi}\cdot\boldsymbol{j} + \boldsymbol{j}\cdot\boldsymbol{\pi})\right]\right\},$$

$$\mathcal{H}_2 = -d\boldsymbol{\Pi}\cdot\boldsymbol{E} + \frac{d}{4}\left\{\frac{1}{\epsilon'(\epsilon'+m)}, \left[(\boldsymbol{E}\cdot\boldsymbol{\pi})(\boldsymbol{\Pi}\cdot\boldsymbol{\pi}) + (\boldsymbol{\Pi}\cdot\boldsymbol{\pi})(\boldsymbol{\pi}\cdot\boldsymbol{E})\right]\right\}$$
$$-\frac{d}{4}\left\{\frac{1}{\epsilon'}, \left(\boldsymbol{\Sigma}\cdot[\boldsymbol{\pi}\times\boldsymbol{B}] - \boldsymbol{\Sigma}\cdot[\boldsymbol{B}\times\boldsymbol{\pi}]\right)\right\},$$

where $\mathcal{H}_1$ defines the CP-conserving part of the total Hamiltonian $\mathcal{H}_{FW}$, $\mu_0 = e\hbar/(2m)$ is the Dirac magnetic moment, $\epsilon' = \sqrt{m^2+\boldsymbol{\pi}^2}$, and $\boldsymbol{j} = \frac{1}{4\pi}\left(c\nabla\times\boldsymbol{B} - \frac{\partial\boldsymbol{E}}{\partial t}\right)$ is the density of external electric current.

The new terms describing the interaction with the axion fields are given by

$$\mathcal{H}_3 = \frac{g_{aNN}}{2}\left\{\frac{\boldsymbol{\Pi}\cdot\boldsymbol{p}}{\epsilon'}, \Lambda_0\right\}$$
$$-\frac{g_{aNN}}{2}\left[\left\{\frac{m}{\epsilon'}, \boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda}\right\} + \frac{(\boldsymbol{\Sigma}\cdot\boldsymbol{p})}{\epsilon'(\epsilon'+m)}(\boldsymbol{p}\cdot\boldsymbol{\Lambda}) + (\boldsymbol{\Lambda}\cdot\boldsymbol{p})\frac{(\boldsymbol{\Sigma}\cdot\boldsymbol{p})}{\epsilon'(\epsilon'+m)}\right].$$

Corrections to the spin motion. A comparison with similar corrections caused by the axion-induced EDM

In the semiclassical approximation, the angular velocity of the spin rotation has the form

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}_{TBMT} + \boldsymbol{\Omega}_{EDM} + \boldsymbol{\Omega}_{axion},$$

$$\boldsymbol{\Omega}_{TBMT} = -\frac{e}{2m}\left\{\left(g-2+\frac{2}{\gamma}\right)\boldsymbol{B} - \frac{(g-2)\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{B}) - \left(g-2+\frac{2}{\gamma+1}\right)(\boldsymbol{\beta}\times\boldsymbol{E})\right\},$$

$$\boldsymbol{\Omega}_{EDM} = -\frac{e\eta}{2m}\left[\boldsymbol{E} - \frac{\gamma}{\gamma+1}\boldsymbol{\beta}(\boldsymbol{\beta}\cdot\boldsymbol{E}) + \boldsymbol{\beta}\times\boldsymbol{H}\right],$$

$$\boldsymbol{\Omega}_{axion} = 2g_{aNN}\left(\Lambda_0\boldsymbol{\beta} - \frac{\boldsymbol{\Lambda}}{\gamma} - \frac{\gamma}{\gamma+1}(\boldsymbol{\beta}\cdot\boldsymbol{\Lambda})\boldsymbol{\beta}\right),$$

where $\boldsymbol{\Omega}_{TBMT}$ is determined by the Thomas-Bargmann-Michel-Telegdi equation and the factors $g = 4(\mu_0+\mu')m/e$ and $\eta = 4dm/e$ are introduced.

The equation describes the spin motion relative to the Cartesian coordinate axes. In accelerators and storage rings, the spin dynamics is determined relative to the radial, longitudinal and vertical axes. The angular velocity of the spin motion relative to the latter axes is given by

$$\boldsymbol{\Omega}' = \boldsymbol{\Omega} - \boldsymbol{\omega}_c,$$

where $\boldsymbol{\omega}_c$ is the vector which is parallel to the vertical axis and defines

the angular velocity of the cyclotron motion.

$\mathcal{H}_3 = -g_{aNN}\boldsymbol{\Sigma}\cdot\boldsymbol{\Lambda}$ in the nonrelativistic limit. However, a particle

motion can be relativistic.

The newly added first term in Ω_{axion} is three orders of magnitude larger than the second term. This fact significantly increases an importance of a search for a possible manifestation of the axion field in storage ring experiments.

Summary

- The Dirac Hamiltonian with an allowance for the field of dark matter axions has been written down and its relativistic Foldy-Wouthuysen thansformation has been carried out. The obtained equations of spin motion are exact.
- The derived equation of spin motion contains a new term. In comparison with the previous results obtained in the static limit, this term extraordinarily (three orders of magnitude) enhances the real value of the axion-induced signal. This fact significantly increases an importance of a search for a possible manifestation of the axion field in storage ring experiments.
- The newly obtained contribution results in the spin rotation about the radial axis.

Thank you for your attention