Before inflation

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS,
Moscow - Chernogolovka, Russia

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Finiteness of inflationary stage

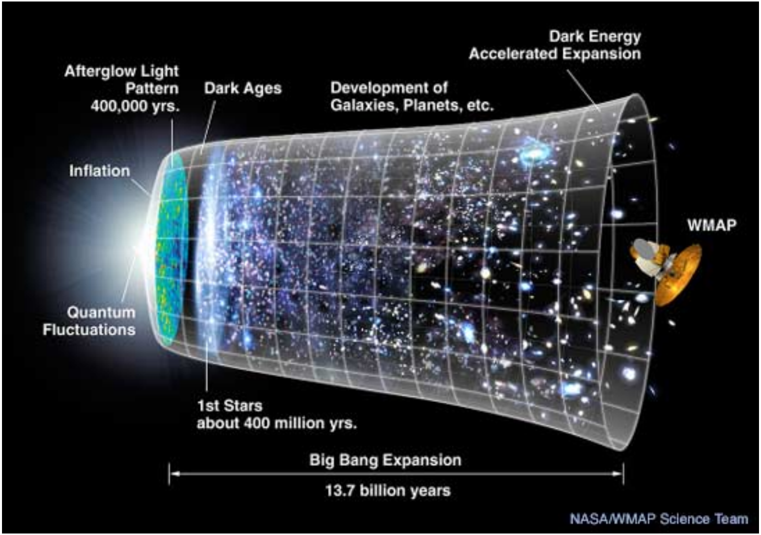
Pre-inflationary stage

Isotropic bounce in scalar-tensor gravity

Pre-inflationary anisotropic singularity

Bianchi-I type solutions in Horndeski gravity

Conclusions


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

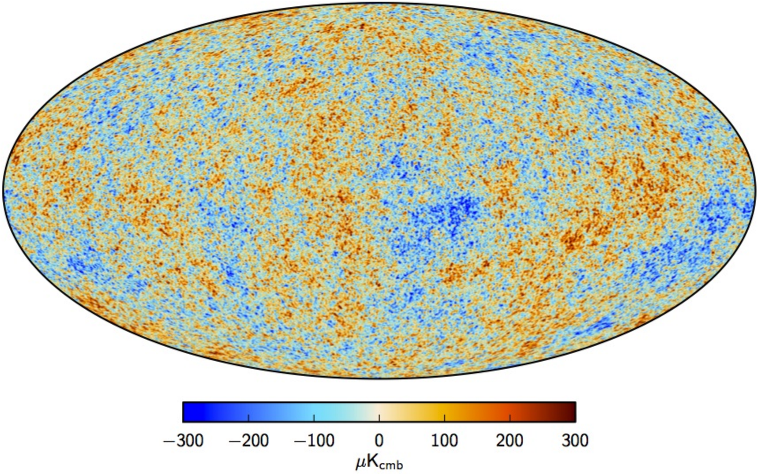
$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

> 60 $\quad\sim 55\quad$ 7.5 $\quad$ 0.5

CMB temperature anisotropy

Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation in the synchronous gauge with some additional conditions fixing it completely:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\mathcal{R}(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0,\ \ g^{(a)}_{,l}\, e_m^{l(a)} = 0,\ \ e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (primordial gravitational waves (GW)).
The most important quantities:

$$P_{\mathcal{R}}(k),\ \frac{d\ln P_{\mathcal{R}}(k)}{d\ln k} \equiv n_s(k) - 1,\ \ r(k) \equiv \frac{P_g}{P_{\mathcal{R}}}$$

Both $|n_s - 1|$ and $|r|$ are small during slow-roll inflation.

New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \;\; P_{\mathcal{R}}(k) = (2.10 \pm 0.03) \cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \;\mathrm{Mpc}^{-1}, \;\; n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s) N_H \sim 2$).

The most recent upper limits on r

1. BICEP/Keck Collaboration: P. A. R. Ade et al., Phys. Rev. Lett. 127, 151301 (2021); arXiv:2110.00483:

$r_{0.05} < 0.036$ at the 95% C.L.

2. M. Tristram et al., Phys. Rev. D 105, 083524 (2022); arXiv:2112.07961:

$r_{0.05} < 0.032$ at the 95% C.L.

For comparison, in the chaotic inflationary model $V(\varphi) \propto |\varphi|^n$, $r = \frac{4n}{N}$, $1 - n_s = \frac{n+2}{2N}$. The r upper bound gives $n < 0.5$ for $N_{0.05} = (55 - 60)$, but then $1 - n_s \leq 0.022$. Thus, this model is disfavoured by observational data.

The target prediction for r in the 3 simplest (one-parametric) inflationary models having $n_s - 1 = -\frac{2}{N}$ (the $R + R^2$, Higgs and combined Higgs-R^2 models) is

$$r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.004$$

Kinematic origin of scalar perturbations

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different in different points of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then

$$\mathcal{R}(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B 117, 175 (1982) in the case of one-field inflation.

Visualizing small differences in the number of e-folds

Duration of inflation in terms of e-folds was finite for all points inside our past light cone. For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\mathcal{R}(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1, P_{\mathcal{R}} = P_0$,

$$\ell(\ell+1)\left\langle(\Delta T/T_\gamma)^2_{lm}\right\rangle = \frac{2\pi}{25}P_0$$

For $\frac{\Delta T}{T} \sim 10^{-5}$, $\delta N \sim 5\times 10^{-5}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, like in the minimal (one-parametric) inflationary models, $\delta t \sim 5 t_{Pl}$!

Planck time intervals are seen by the naked eye!

Pre-inflationary stage

Different possibilities were considered historically.

1. Creation of inflation "from nothing" (Grishchuk and Zeldovich, 1981).
One possibility among infinite number of others.
2. Our Universe was not an individual entity before inflationary stage, it was a part of some "Superuniverse" ("Multiverse" in modern terminology) (AS, Quantum Gravity, 1981).
3. More generally, any process may be responsible for the formation of inflationary stage in our Universe, that was called "creation from anything" in AS and Ya. B. Zeldovich, Sov. Sci. Rev. 1988.
Possible relation to de Sitter entropy.

More conservative classical dynamical scenarios

4. De Sitter "Genesis": beginning from the exact contracting full de Sitter space-time at $t \to -\infty$ (AS, PLB 91, 99 (1980)). Requires adding an additional term

$$R_i^l R_l^k - \frac{2}{3} R R_i^k - \frac{1}{2} \delta_i^k R_{lm} R^{lm} + \frac{1}{4} \delta_i^k R^2$$

to the rhs of the gravitational field equations. Not generic. May not be the "ultimate" solution: a quantum system may not spend an infinite time in an unstable state.

5. Isotropic bounce due to a positive spatial curvature (AS, Sov. Astron. Lett. 4, 82 (1978)).

Generic, but probability of a bounce is small for a large initial size of a universe $W \sim 1/Ma_0$. It is difficult to reach inflation from a low curvature state.

6. Generic anisotropic and inhomogeneous singularity with curvature much exceeding that during inflation.

Isotropic bounce with zero spatial curvature in scalar-tensor gravity

D. Polarski, A. A. Starobinsky, Y. Verbin. Bouncing cosmological isotropic solutions in scalar-tensor gravity. J. Cosm. Astropart. Phys. 2022, 052 (2022); arXiv:2111.07319. In contrast to GR, scalar-tensor gravity admits breaking of weak and null energy conditions, so isotropic bounce is possible even in the absence of spatial curvature.

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - U(\Phi) - \frac{\xi}{2}R\Phi^2\right)$$

Bouncing solutions have been found for polynomial $U(\phi)$ negative in some range but bounded from below.

However, in all solutions either the Hubble function $H(t)$ becomes divergent at some finite moment of time before the bounce, or the effective gravitational constant $G_{eff} = G/(1 - \xi\kappa^2\Phi^2)$ becomes negative around the bounce.

As was shown in AS, Sov. Astron. Lett. 7, 36 (1981), in such solutions, arbitrarily small anisotropic perturbations diverge in the point there $G_{eff}^{-1} = 0$ and this results in the formation of generic anisotropic and inhomogeneous singularity at this moment preventing the transition to the region where G_{eff} is negative.

Bianchi-I type models in $f(R)$ gravity

Analytical and numerical investigation for $f(R) = R + R^2$ gravity in the Bianchi-I type model in D. Muller, A. Ricciardone, A. A. Starobinsky and A. V. Toporensky, Eur. Phys. J. C 78, 311 (2018). Two main types of singularities in $f(R)$ gravity with the same generic structure at $t \to 0$:

$$ds^2 = dt^2 - \sum_{i=1}^{3} |t|^{2p_i} \, a_l^{(i)} a_m^{(i)} \, dx^l dx^m, \; 0 < s \leq 3/2, \; u = s(2-s)$$

where $p_i < 1, \; s = \sum_i p_i, \; u = \sum_i p_i^2$ and $a_l^{(i)}, \; p_i$ are functions of $\mathbf{r}$. Here $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$.

Type A. $1 \leq s \leq 3/2, \; |f'(R)| \propto |t|^{1-s} \to +\infty \, .$
Type B. $0 < s < 1, \; R \to R_0 < 0, \; f'(R_0) = 0 \, .$
In addition, there can exist an isotropic Big Rip type singularity with $s = -3(n-1)(2n-1)/(n-2), \; u = s^2/3$ in the future evolution if $f(R) \propto R^n, \; n > 2$ for $R \to \infty$.

Bianchi-I type models with inflation in R^2 gravity

For $f(R) = R^2$, even an exact solution can be found.

$$ds^2 = \tanh^{2\alpha}\left(\frac{3H_0 t}{2}\right)\left(dt^2 - \sum_{i=1}^{3} a_i^2(t)dx_i^2\right)$$

$$a_i(t) = \sinh^{1/3}(3H_0 t)\tanh^{\beta_i}\left(\frac{3H_0 t}{2}\right), \ \sum_i \beta_i = 0, \ \sum_i \beta_i^2 < \frac{2}{3}$$

$$\alpha^2 = \frac{\frac{2}{3} - \sum_i \beta_i^2}{6}, \quad \alpha > 0$$

Next step: relate arbitrary functions of spatial coordinates in the generic solution near a curvature singularity to those in the quasi-de Sitter solution. Spatial gradients may become important for some period before the beginning of inflation.

The same structure of generic singularity for a non-minimally coupled scalar field (scalar-tensor gravity)

$$S = \int \left(f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) \right) \sqrt{-g}\, d^4x + S_m$$

$$f(\phi) = \frac{1}{2\kappa^2} - \xi\phi^2$$

Type A. $\xi < 0, |\phi| \to \infty$.
Type B. $\xi > 0, |\phi| \to 1/\sqrt{2\xi}\kappa$.
The asymptotic regimes and a number of exact solutions in the Bianchi type I model are presented in A. Yu. Kamenshchik, E. O. Pozdeeva, A. A. Starobinsky, A. Tronconi, G. Venturi and S. Yu. Vernov, Phys. Rev. D 97, 023536 (2018) with some of them borrowed from A. A. Starobinsky, MS Degree thesis, Moscow State University, 1972, unpublished.
Anisotropy grows towards singularity generically like in GR.

Horndeski gravity

Generalization of scalar-tensor gravity in the scalar sector without introducing new (scalar) degrees of freedom.

$$\mathcal{S} = \int (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5) \sqrt{-g}\, d^4x \,,$$

$$\mathcal{L}_2 = G_2(\phi, X), \quad \mathcal{L}_3 = -G_3(\phi, X)\, \Box\phi \,,$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4X}(\phi, X) \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \,,$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu}\, \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X}(\phi, X) \times$$

$$\times \left[(\Box\phi)^3 - 3\Box\phi (\nabla_\mu \nabla_\nu \phi)^2 + 2\, (\nabla_\mu \nabla_\nu \phi)^3 \right] .$$

Here $X = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$, $G_{AX} \equiv \partial G_A / \partial X$. R and $G_{\mu\nu}$ are the Ricci scalar and the Einstein tensor, $R = -G^\mu_\mu$.

Specific types of Horndeski gravity

1. $G_2 = X - V(\phi),\ G_3 = G_5 = 0,\ G_4 = const$ - GR + a self-interacting scalar field minimally coupled to gravity.
2. $G_2 = X - V(\phi),\ G_3 = G_5 = 0,\ G_4 = G_4(\phi)$ - scalar-tensor gravity.
3. $G_2 = G_2(\phi, X),\ G_3 = G_5 = 0,\ G_4 = const$ - K-essence theory.
4. $G_3(\phi, X) \neq 0$ - Kinetic Gravity Braiding (KGB) theory.

The theory with $G_4 = G_4(\phi),\ G_5 = 0$ is the most general Horndeski model in which the sound speed of tensor perturbations is exactly equal to the speed of light in the presence of the scalar field ϕ. However, we will not restrict ourselves to this constraint.

Bianchi-I type solutions of Horndeski gravity

R. Galeev, R. Muharlyamov, A. A. Starobinsky, S. V. Sushkov and M. S. Volkov, Phys. Rev. D 103, 104015 (2021); arXiv:2102.10981.

The Bianchi-I type metric:

$$ds^2 = -dt^2 + \mathrm{a}_1^2\, dx_1^2 + \mathrm{a}_2^2\, dx_2^2 + \mathrm{a}_3^2\, dx_3^2\,.$$

Parametrization of the three scale factors:

$$\mathrm{a}_1 = \mathrm{a}\, e^{\beta_+ + \sqrt{3}\beta_-},\ \mathrm{a}_2 = \mathrm{a}\, e^{\beta_+ - \sqrt{3}\beta_-},\ \mathrm{a}_3 = \mathrm{a}\, e^{-2\beta_+}\,.$$

Let us denote

$$\mathcal{G} = 2G_4 - 2G_{4X}\dot{\phi}^2 + G_{5\phi}\dot{\phi}^2\,,\ \sigma^2 = \dot{\beta}_+^2 + \dot{\beta}_-^2\,,\quad H = \frac{\dot{\mathrm{a}}}{\mathrm{a}}\,,$$

$$H_1 = H + \dot{\beta}_+ + \sqrt{3}\dot{\beta}_-\,,\ H_2 = H + \dot{\beta}_+ - \sqrt{3}\dot{\beta}_-\,,\ H_3 = H - 2\dot{\beta}_+\,.$$

4 equations for 4 variables

The field equation:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}(\mathrm{a}^3\mathcal{J}) = \mathcal{P}\,,$$

$$\mathcal{J} = \dot\phi\Big[G_{2X} - 2G_{3\phi} + 3H\dot\phi(G_{3X} - 2G_{4X\phi}) + + G_0^0\,(-2G_{4X} -$$

$$-2\dot\phi^2 G_{4XX} + 2G_{5\phi} + G_{5X\phi}\dot\phi^2) + H_1H_2H_3(3G_{5X}\dot\phi + G_{5XX}\dot\phi^3)\Big]\,,$$

$$\mathcal{P} = G_{2\phi} - \dot\phi^2(G_{3\phi\phi} + G_{3X\phi}\ddot\phi) + RG_{4\phi} + 2G_{4X\phi}\dot\phi(3\ddot\phi H - \dot\phi G_0^0) +$$

$$+G_0^0 G_{5\phi\phi}\dot\phi^2 + G_{5X\phi}\dot\phi^3 H_1H_2H_3\,.$$

The first order G_0^0 equation:

$$3\left(H^2-\sigma^2\right)\left(\mathcal{G}-2G_{4X}\dot{\phi}^2-2G_{4XX}\dot{\phi}^4+2G_{5\phi}\dot{\phi}^2+G_{5X\phi}\dot{\phi}^4\right)=$$

$$=-G_2+\dot{\phi}^2G_{2X}+3G_{3X}H\dot{\phi}^3-G_{3\phi}\dot{\phi}^2-6G_{4\phi}H\dot{\phi}-6G_{4X\phi}H\dot{\phi}^3+$$

$$+\dot{\phi}^3(5G_{5X}+G_{5XX}\dot{\phi}^2)(H-2\dot{\beta}_+)\left[(H+\dot{\beta}_+)^2-3\dot{\beta}_-^2\right].$$

Two first order equations for anisotropy factors:

$$\mathcal{G}\dot{\beta}_++G_{5X}\dot{\phi}^3\left(\dot{\beta}_-^2-\dot{\beta}_+^2-H\dot{\beta}_+\right)=\frac{C_+}{\mathrm{a}^3},$$

$$\mathcal{G}\dot{\beta}_-+G_{5X}\dot{\phi}^3\left(2\dot{\beta}_+\dot{\beta}_--H\dot{\beta}_-\right)=\frac{C_-}{\mathrm{a}^3}.$$

Behaviour of anisotropy towards curvature singularity

Anisotropy grows towards singularity like in GR in models where $G_4 = G_4(\phi)$, $G_5 = 0$. However, if $G_4 = G_4(X)$, or $G_5 = G_5(X)$, it grows first but then decrease. This effect was first found earlier in A. A. Starobinsky, S. V. Sushkov and M. S. Volkov, Phys. Rev. D 101, 064039 (2020); arXiv:1912.12320 for the specific case $G_5 = const$. Such models show gradient instabilities at early times (c_s^2, or c_t^2, or both become negative). Thus, their initial phase, although not anisotropic, cannot be isotropic either. It should therefore be inhomogeneous. At the same time, it is possible that a systematic analysis of theories with more general $G_4(\phi, X)$ and/or $G_5(\phi, X)$ may reveal models free of instabilities.

Conclusions

- In scalar-tensor gravity with $U(\phi)$ negative in some range but bounded from below, isotropic bouncing solutions with a finite Hubble function $H(t)$ are possible even in the absence of spatial curvature, but they require G_{eff} becoming negative around the bounce and, thus, are unstable with respect to the formation of generic and inhomogeneous curvature singularity at the moment when $G_{eff}^{-1} = 0$. Thus, they are unphysical.
- In Horndeski gravity, anisotropy behaviour near singularity may be completely different from that in GR for models in which the GW velocity in presence of the scalar field is different from the light one, though this difference may be arbitrarily small at present. Moreover, even the structure of the curvature singularity itself may be different.