Quantum and classical local P-violation

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Main Topics

- Polarization as Anomalous transport: 4-velocity as gauge field, hydrodynamical helicity as topological charge
- Helicity as a classical source of Chiral Vortical and Magnetic Effects
- Vortex helicity flip as a (semi)classical model of anomaly
- Handedness as a quantum momentum0space counterpart of vortivity and helicity
- Conclusions

Local (C)P-violation

- May happen in medium (Kharzeev et al.)

- Related to axial anomaly

- Anomalous transport: Chiral Magnetic (problems with background: isobars), Vortical, Separation…Effects

Global polarization

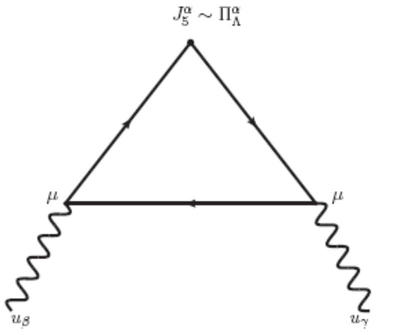
- Global polarization normal to REACTION plane
- Predictions (Z.-T.Liang et al.): large orbital angular momentum -> large polarization
- Search by STAR (Selyuzhenkov et al.'07) : polarization NOT found at % level!
- Maybe due to locality of LS coupling while large orbital angular momentum is distributed
- How to transform rotation to spin?

Anomalous mechanism – polarization –kind of anomalous transport similar to CM(V)E

- 4-Velocity is also a GAUGE FIELD (V.I. Zakharov et al)

$$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$

- Triangle anomaly leads to polarization of quarks and hyperons (Rogachevsky, Sorin, OT '10)
- Analogous to anomalous gluon contribution to nucleon spin (Efremov,OT'88)
- 4-velocity instead of gluon field!



O. Rogachevsky, A. Sorin, O. Teryaev
Chiral vortaic effect and neutron asymmetries in heavy-ion collisions
PHYSICAL REVIEW C 82, 054910 (2010)

One would expect that polarization is proportional to the anomalously induced axial current [7]

$$j_A^\mu \sim \mu^2 \left(1 - \frac{2\mu n}{3(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho} V_\nu \partial_\lambda V_\rho, \qquad (6)$$

where n and ϵ are the corresponding charge and energy densities and P is the pressure. Therefore, the μ dependence of polarization must be stronger than that of the CVE, leading to the effect's increasing rapidly with decreasing energy.

This option may be explored in the framework of the program of polarization studies at the NICA [17] performed at collision points as well as within the low-energy scan program at the RHIC.

Energy dependence

- Coupling -> chemical potential

$$Q_5^s = \frac{N_c}{2\pi^2} \int d^3x \, \mu_s^2(x) \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Field -> velocity; (Color) magnetic field strength -> vorticity;
- Topological current -> hydrodynamical helicity
- Rapid decrease with energy
- Large chemical potential: appropriate for NICA/FAIR energies

From (chiral) quarks to hadrons: quark-hadron duality via axial charge

- Induced axial charge

$$c_V = \frac{\mu_s^2 + \mu_A^2}{2\pi^2} + \frac{T^2}{6}, \quad Q_5^s = N_c \int d^3x \, c_V \gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

- Neglect axial chemical potential
- T-dependent term (Gravity anomaly: next talk of G. Prokhorov)
- Lattice simulations: suppressed due to collective effects

From axial charge to polarization (and from quarks to confined hadrons) – analog of Cooper-Frye

- Analogy of matrix elements and classical averages

$$< p_n|j^0(0)|p_n >= 2p_n^0 Q_n \qquad < Q >\equiv \frac{\sum_{n=1}^N Q_n}{N} = \frac{\int d^3x\, j^0_{class}(x)}{N}$$

- Axial current: charge -> polarization vector
- Lorentz boost: requires the sign change of helicity "below" and "above" the RP

$$\Pi^{\Lambda,lab} = (\Pi_0^{\Lambda,lab}, \Pi_x^{\Lambda,lab}, \Pi_y^{\Lambda,lab}, \Pi_z^{\Lambda,lab}) = \frac{\Pi_0^\Lambda}{m_\Lambda}(p_y, 0, p_0, 0)$$

$$< \Pi_0^\Lambda >= \frac{m_\Lambda \Pi_0^{\Lambda,lab}}{p_y} =< \frac{m_\Lambda}{N_\Lambda p_y} > Q_5^s \equiv < \frac{m_\Lambda}{N_\Lambda p_y} > \frac{N_c}{2\pi^2} \int d^3x\, \mu_s^2(x)\gamma^2 \epsilon^{ijk} v_i \partial_j v_k$$

Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions (Angular velocity ~ c/Compton wavelength)
- ~25 orders of magnitude faster than Earth's rotation
- Calculation in kinetic quark - gluon string model (DCM/QGSM) – Boltzmann type eqns + phenomenological string amplitudes): Baznat, Gudima, Sorin, OT:
- PRC'13 (helicity separation+P@NICA~1%), 16 (femto-vortex sheets, NICA), 17 (antihyperons, gravitational anomaly, STAR)

Rotation in HIC and related (classical) quantities

- Non-central collisions – orbital angular momentum
- L=Σ r x p
- Differential pseudovector characteristics – vorticity
- ω = curl v
- Pseudoscalar – helicity
- H ~ <(v curl v)>
- Maximal helicity – Beltrami chaotic flows v || curl v

Beltrami limit: is it close?

- Cauchy-Schwarz inequality:
- $r^2 = \langle (v \,\mathrm{curl}\, v) \rangle^2 / \langle (\mathrm{curl}\, v)^2 \rangle \langle v^2 \rangle \leq 1$

- Maximal
  average cosine ~ 0.1

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **88**, 061901(R) (2013)

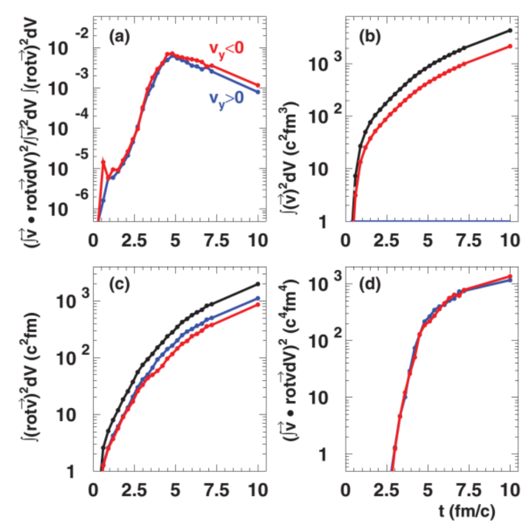
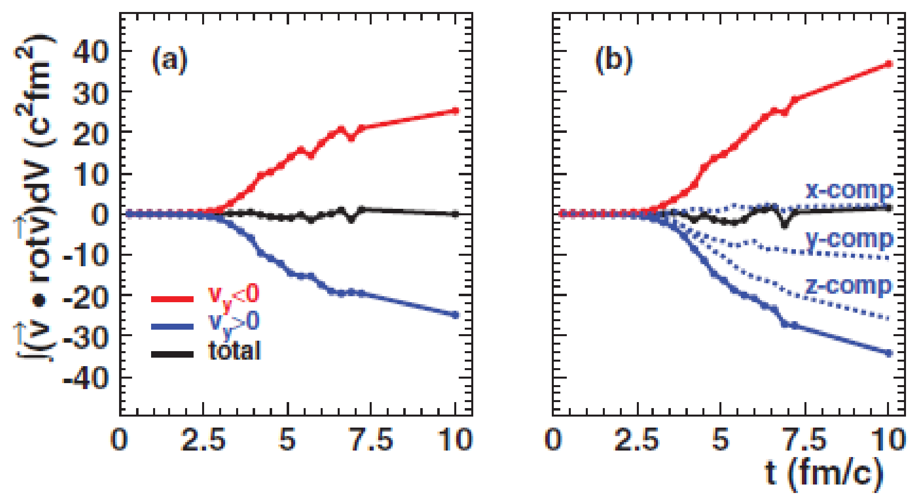


FIG. 4. (Color online) Time dependence of Cauchy-Schwarz bound for helicity in Au + Au at $s^{1/2} = 5$ GeV at impact parameter $b = 8$ fm(a); the integrated squares of velocity (b), vorticity (c), and helicity (d).

Helicity separation in QGSM **PRC88 (2013) 061901**

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane – required by boost!
- Confirmed in HSD (OT, Usubov, PR**C92 (2015) 014906**
- **zz: vorticity quadrupole structure**



Helicity conservation

- Induced current should be conserved: no divergence in effective theory

- Additional conservation laws at equilibrium (VI Zakharov et al.)

- No equilibrium at the stage of vorticity and helicity generation

Classical P-odd effects (VI Zakharov, OT, in preparation)

- $k\, v = \omega$
- $j = e\, n\, v = e\, n\, \omega / k$
- Classical CVE: NEW classical axial chemical potential $\mu_{5\,Cl} = en/k$ (c.f. Wiegmann and Abanov'22)
- HIC: Beltrami -> helicity separation
- $k \to H/\langle v^2 \rangle$, $\mu_5 \sim e\, \varepsilon / m\, H$

Classical CME

- QGP: opposite charges with same (?- model tests in progress) vorticity: opposite currents: no charge separation
- Magnetic field : $\omega \to \omega + e B/m$
- Contribution from opposite charges: Classical CME with $\mu_{5\,CL} \sim e^2 \varepsilon / m^2 H$

Classical CVE and CME in HIC

- Opposite H below and above reation plane: new source of charge separation
- Contribution of longitudinal components of v and ω to H: in plane sepration
- May contribute to the observed unexpected behaviour of charge separation in isobars (Ru/Zr) comparison at RHIC

Classical vortices and anomaly

- Anomaly comes from vortex as "particle" model (OT' On the semiclassical interpretation of axial anomaly, *Mod.Phys.Lett. A* 6 (1991) 2323-2325)

$$\bar{S} = \langle \bar{L} \rangle = \bar{L}_{||} = m\bar{R} \times \bar{V}_{\perp} = -\frac{e}{\pi c}\bar{H} \cdot G_{\perp} \qquad k = \frac{\bar{p}_{||}\bar{S}}{|P_{||}|} = S \cdot (\theta(P_{||}) - \theta(-P_{||}))$$

$$\dot{\bar{p}}_{||} = e\bar{E} \qquad \dot{k} = 2\bar{S}\dot{\bar{p}}_{||}\delta(p_{||}) = -2\frac{e^2}{\pi c}\bar{E}\bar{H}\delta(p_{||})G_{\perp}$$

$$\Delta p_{||} \ll \dot{k} \gg = -\frac{2e^2}{\pi c}\bar{E} \cdot \bar{H}G_{\perp} \quad \Delta p_{||} \sim \hbar/\Delta x_{||}.$$

$$\ll \dot{k} \gg \sim -\frac{\alpha}{\pi}\bar{E} \cdot \bar{H} \cdot V \qquad \alpha = \frac{e^2}{\hbar c}, V = \Delta X_{||}G_{\perp}.$$

- For heavy particles anomalous helicity breaking is opposite to explicit one (cancellation of physical and regulator fermions)

Uncertainty principle and vorticity

- Crucial role of uncertainty principle
- Suggestion (new): apply for vorticity and helicity
- Momentum correlations (handedness)
- Introduced by A.V. Efermov for jets initiated by polarized quarks and gluons
- Applied in HIC (OT, Usubov'15) (but relation with helicity not discussed)

Vorticity, helicity and handedness

- v -> <p/E> (different ways of average over elementary cell are possible)
- Curl v -> $1/\hbar \langle [p_i, p_k] \rangle$
- Ordering is necessary (size, species)
- <v curl v> -> $1/\hbar \langle\langle [p_i, p_k] p_j \rangle\rangle$
- Original Efremov's handedness
- The ordering should select the particle used to specify the cell (femtoscopy?)

Modelling of handedness (OT, Usubov,PRC’15)

- Small non-zero results for separate octants

TABLE I. Octant enumeration.

Octant	Momentum
0	$p_x > 0, p_y > 0, p_z > 0$
1	$p_x > 0, p_y > 0, p_z \leqslant 0$
2	$p_x > 0, p_y \leqslant 0, p_z > 0$
3	$p_x > 0, p_y \leqslant 0, p_z \leqslant 0$
4	$p_x \leqslant 0, p_y > 0, p_z > 0$
5	$p_x \leqslant 0, p_y > 0, p_z \leqslant 0$
6	$p_x \leqslant 0, p_y \leqslant 0, p_z > 0$
7	$p_x \leqslant 0, p_y \leqslant 0, p_z \leqslant 0$

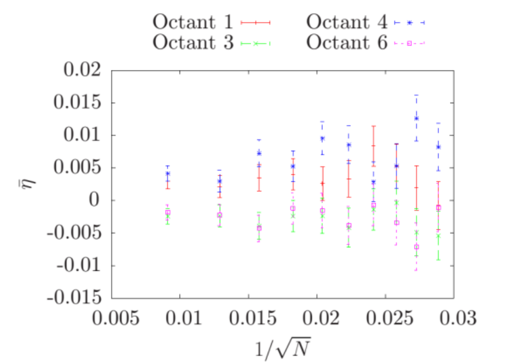
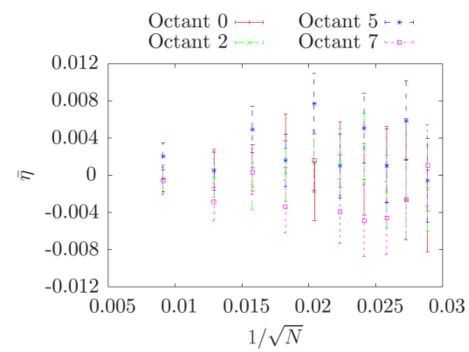


FIG. 8. (Color online) Dependence of $\bar{\eta}$ on $1/\sqrt{N}$, for impact parameter $b = 7$ fm. Octants 1, 3, 4, and 6.



- Counterpart of helicity separation and quadrupole structure?! It is difficult to say at the moment whether η possesses any special physical meaning;
- The two subjects of that paper may be related. Can the (P)HSD model based on Baym-Kadanoff equations contain this?

Conclusions/Outlook

- The anomalous transport have the “classical” counterpart due to helical flows
- The CME in Ru/Zr collisions get the extra contributions which may explain its unexpected behaviour
- The vortices supplemented by uncertainty principle lead to anomaly equation
- The P-odd momentum correlations may serve as a probe of vorticity
- This may be used as a hadronic-like probes for HIC and liquid-like description of hadrons.
- Cosmological helical flows?

Backup

Axial charge and properties of polarization

- Polarization is enhanced for particles with small transverse momenta – azimuthal dependence naturally emerges

- Antihyperons : same sign (C-even axial charge) and larger value (smaller N)

- More pronounced at lower energy. Baryon/antibaryon splitting due to magnetic field – increase (?!) with energy. Non-linear effects in H may be essential, cf vector mesons on the lattice: Luschevskaya, Solovjeva, OT: **JHEP 1709 (2017) 142**

Lambda vs Antilambda and role of vector mesons

- Difference at low energies too large – same axial charge carried by much smaller number
- Strange axial charge may be also carried by K* mesons
- Λ - accompanied by (+,anti 0) K* mesons with two sea quarks – small corrections
- Anti Λ – more numerous (-,0) K* mesons with single (sea) strange antiquark
- Dominance of one component of spin results also in tensor polarization (P-even source) –revealed in dilepton anisotropies (Bratkovskaya, Toneev,OT'95)

Chemical potential and flavour dependence

- Way via axial current/charge differs from “direct” TD
- TD-Universal, “flavor-blind” (only mass-dependent) polarization
- Axial current: polarization depends on baryon structure
- Most pronounced at low energies
- Comparison of hyperons polarization (c.f. hadronic collisions)

Other approach to baryons in confined phase: vortices in pionic superfluid (V.I. Zakharov,OT: 1705.01650;PRD96,09623)

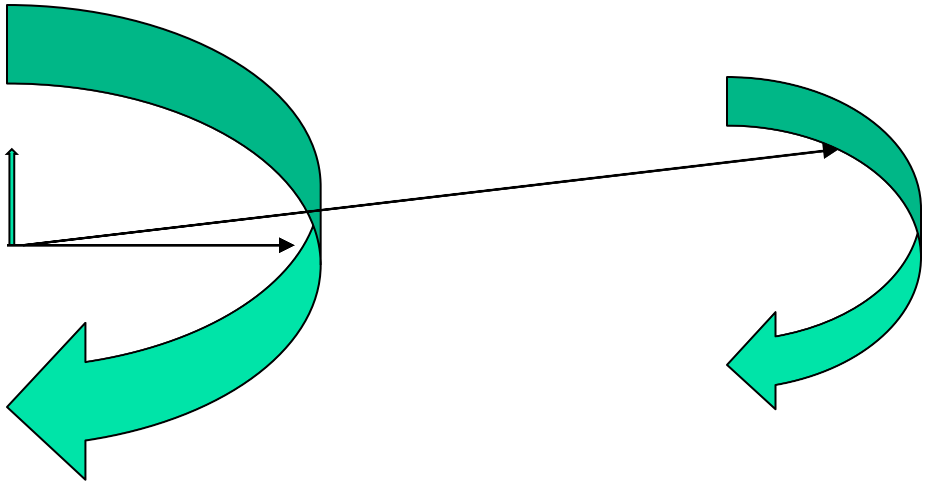
- Pions may carry the axial current due to quantized vortices in pionic superfluid (Kirilin,Sadofyev,Zakharov'12)

$$j_5^\mu = \frac{1}{4\pi^2 f_\pi^2}\epsilon^{\mu\nu\rho\sigma}(\partial_\nu\pi^0)(\partial_\rho\partial_\sigma\pi^0) \qquad \frac{\pi_0}{f_\pi} = \mu\cdot t + \varphi(x_i) \qquad \oint \partial_i\varphi dx_i = 2\pi n$$

$$\partial_i\varphi = \mu v_i$$

- Suggestion: core of the vortex- baryonic degrees of freedom- polarization

Core of quantized vortex

- Constant circulation – velocity increases when core is approached
- S 
- Helium ($v < v_{sound}$) bounded by intermolecular distances
- Pions ($v<c$) –> (baryon) spin in the center

Comparison of methods

- Wigner function – induced axial current (triangle diagram– V.I. Zakharov) – Prokhorov , OT,1707.02491

$$\alpha_\mu = \frac{1}{T} u^\nu \partial_\nu u_\mu = \frac{a_\mu}{T}, \quad w_\mu = \frac{1}{2T} \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha u^\beta = \frac{\omega_\mu}{T}$$

$$\langle : j^5_\mu : \rangle = \Big(\frac{1}{6} \big[T^2 + \frac{a^2 - \omega^2}{4\pi^2} \big] + \frac{\mu^2}{2\pi^2} \Big) \omega_\mu + \frac{1}{12\pi^2} (\omega \cdot a)\, a_\mu$$

$$\langle : j^5_\mu : \rangle = 2\pi \, \mathrm{Im} \Big[\big(\frac{1}{6} (T^2 + \varphi^2) + \frac{\mu^2}{2\pi^2} \big) \varphi_\mu \Big] \qquad \varphi_\mu = \frac{a_\mu}{2\pi} + \frac{i\omega_\mu}{2\pi}$$

- New terms of higher order in vorticity
- Topological universal acceleration-directed term

Role of mass effects (Prokhorov,OT, in preparation)

- Threshold effects in chemical potential and angular velocity

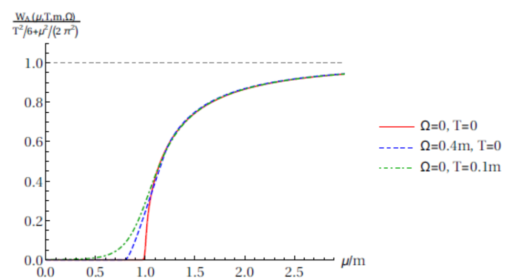


FIG. 1: Axial current coefficient $j_5 = W_A(\mu, T, m, \Omega)\Omega$ normalised to $T^2/6+\mu^2/(2\pi^2)$ as a function of chemical potential μ/m. Acceleration $a = 0$. There is a step at $\mu = m$ for $T = 0$ and $\Omega = 0$ (red solid line), which is smoothed for nonzero T (green dashed-dot line) and nonzero rotational velocity Ω (blue dashed line). For high chemical potential axial current asymptotically tends to its value $T^2/6 + \mu^2/(2\pi^2)$, corresponding to $a = \Omega = m = 0$.

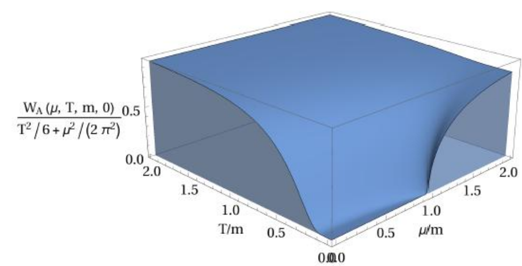


FIG. 3: Coefficient $W_A(\mu, T, m, 0)$ as a function of μ and T for zero rotational velocity $\Omega = 0$, normalised to zero mass value. For zero temperature $T = 0$ it vanishes below $\mu < m$.

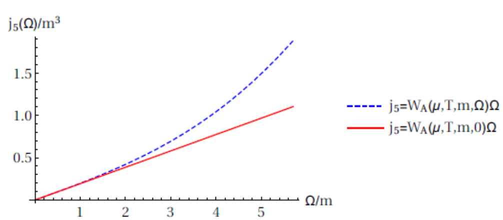


FIG. 2: Typical behaviour of axial current as a function of rotational velocity Ω (blue dashed line) in comparison with its value in linear approximation over Ω (red line). Chemical potential $\mu = m$, temperature $T = m$. One can see, that rotational velocity dependence in the coefficient W_A increases axial current value. Acceleration $a = 0$.

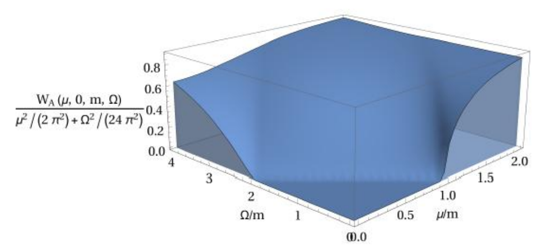


FIG. 4: Coefficient $W_A(\mu, 0, m, \Omega)$ as a function of μ and Ω for zero temperature $T = 0$, normalised to zero mass limit value. There is an area with vanishing $W_A(\mu, 0, m, \Omega) = 0$ for low μ and Ω, border is $\Omega = 2(m - \mu)$. In particular $W_A(m, 0, m, 0) = W_A(0, 0, m, 2m) = 0$. For high rotational velocity and chemical potential W_A tends to zero mass limit value.

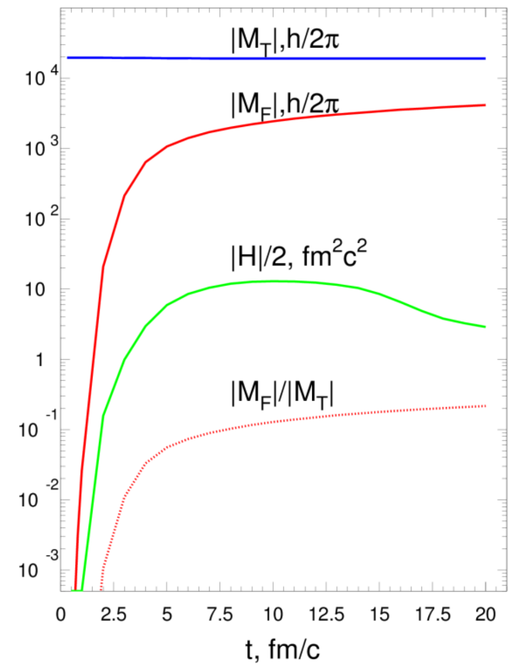
“Hidden anomaly”

- Chemical potential (follows already from M. Buzzegoli, E. Grossi and F. Becattini, JHEP 1710 (2017) 091) and angular velocity – “phase structure”

- Anomalous current recovered in chiral limit and integration over all momenta

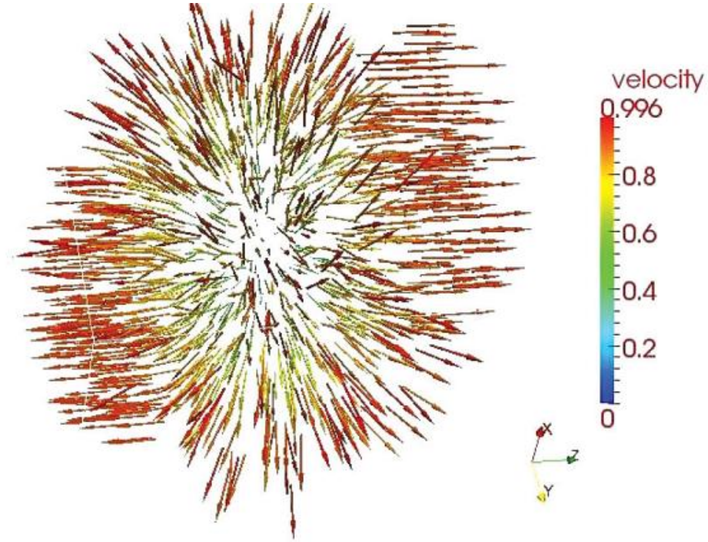
Angular momentum conservation and helicity

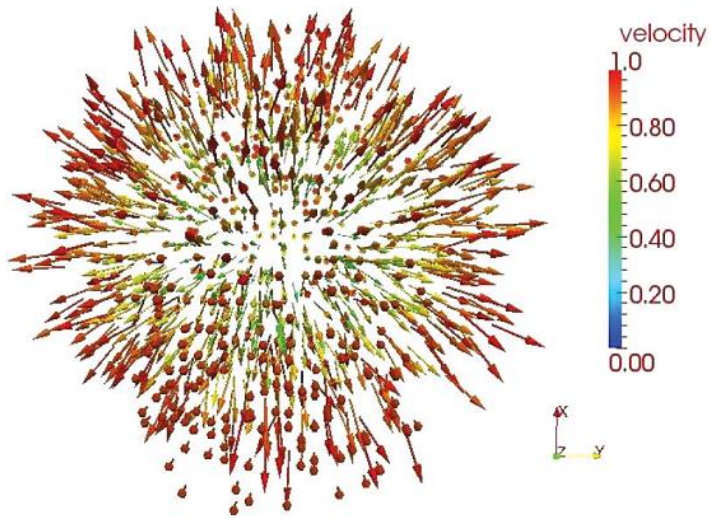
- Helicity vs orbital angular momentum (OAM) of fireball
- (~10% of total)
- Conservation of OAM with a good accuracy!



Distribution of velocity (“Little Bang”)

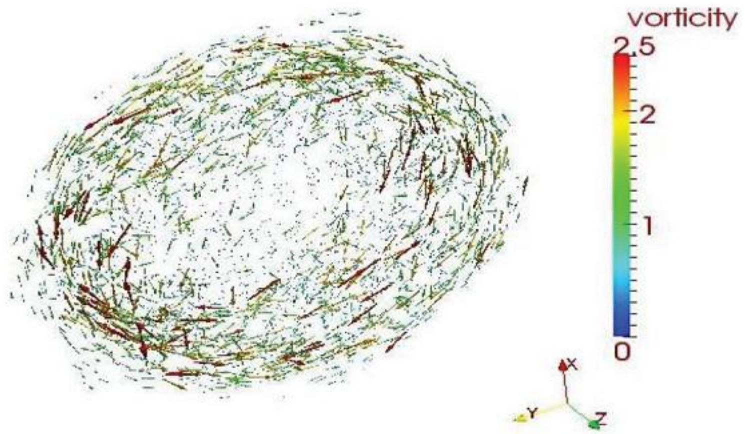
- 3D/2D projection
- z-beams direction
- x-impact paramater

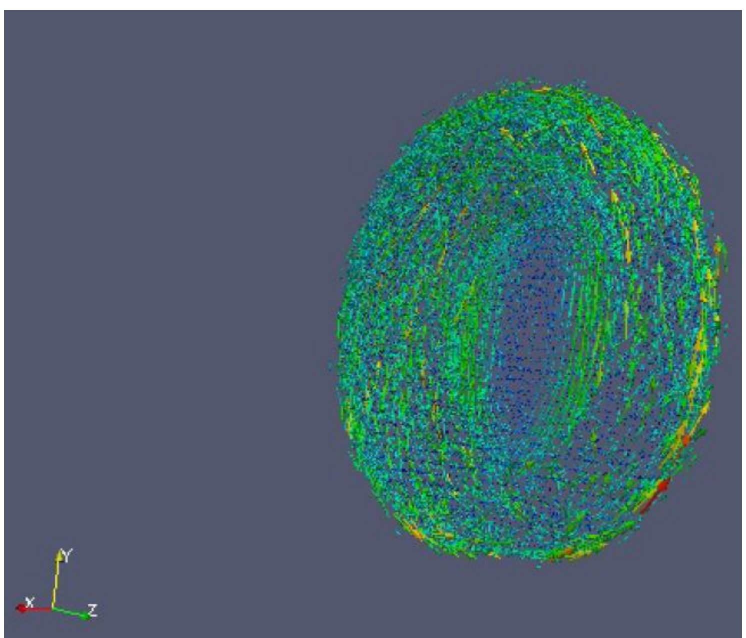


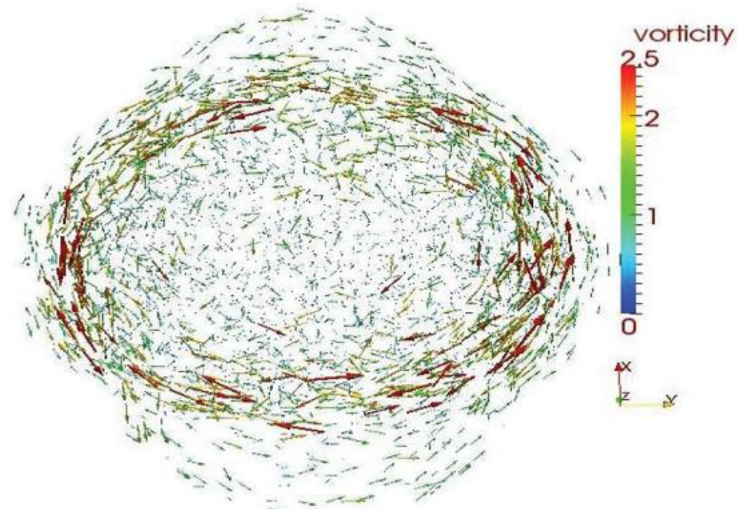


Distribution of vorticity (“Little galaxies”)

- Layer (on core - corona borderline) patterns

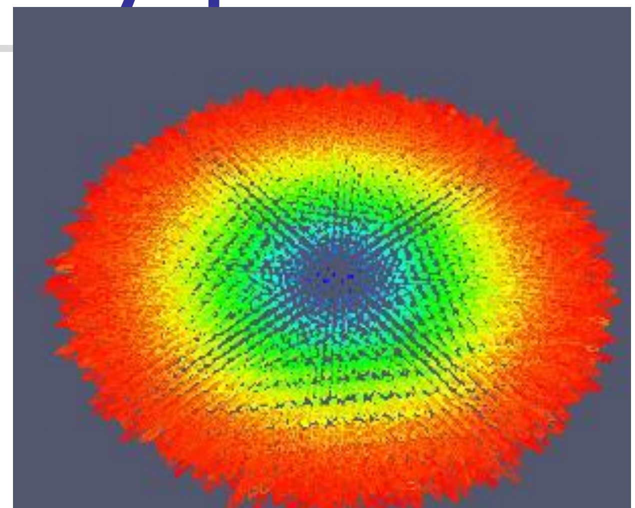




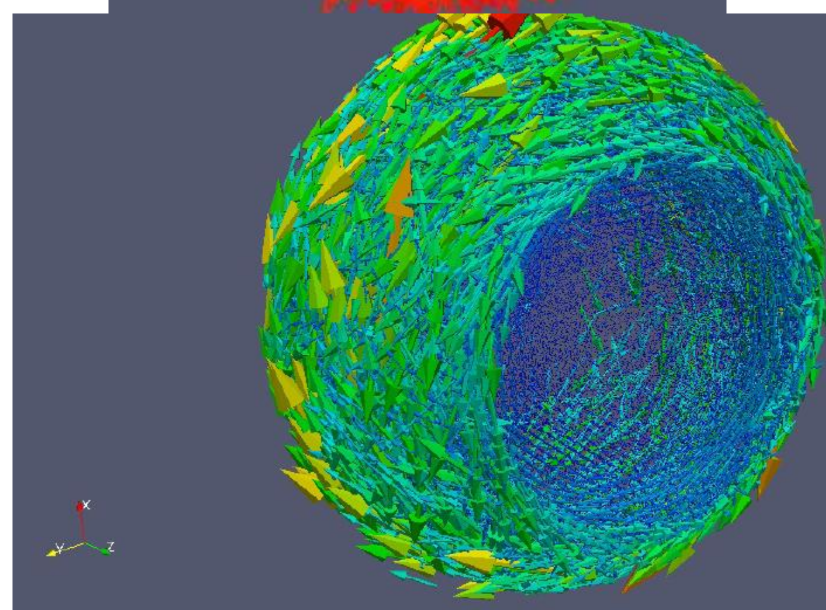


Velocity and vorticity patterns

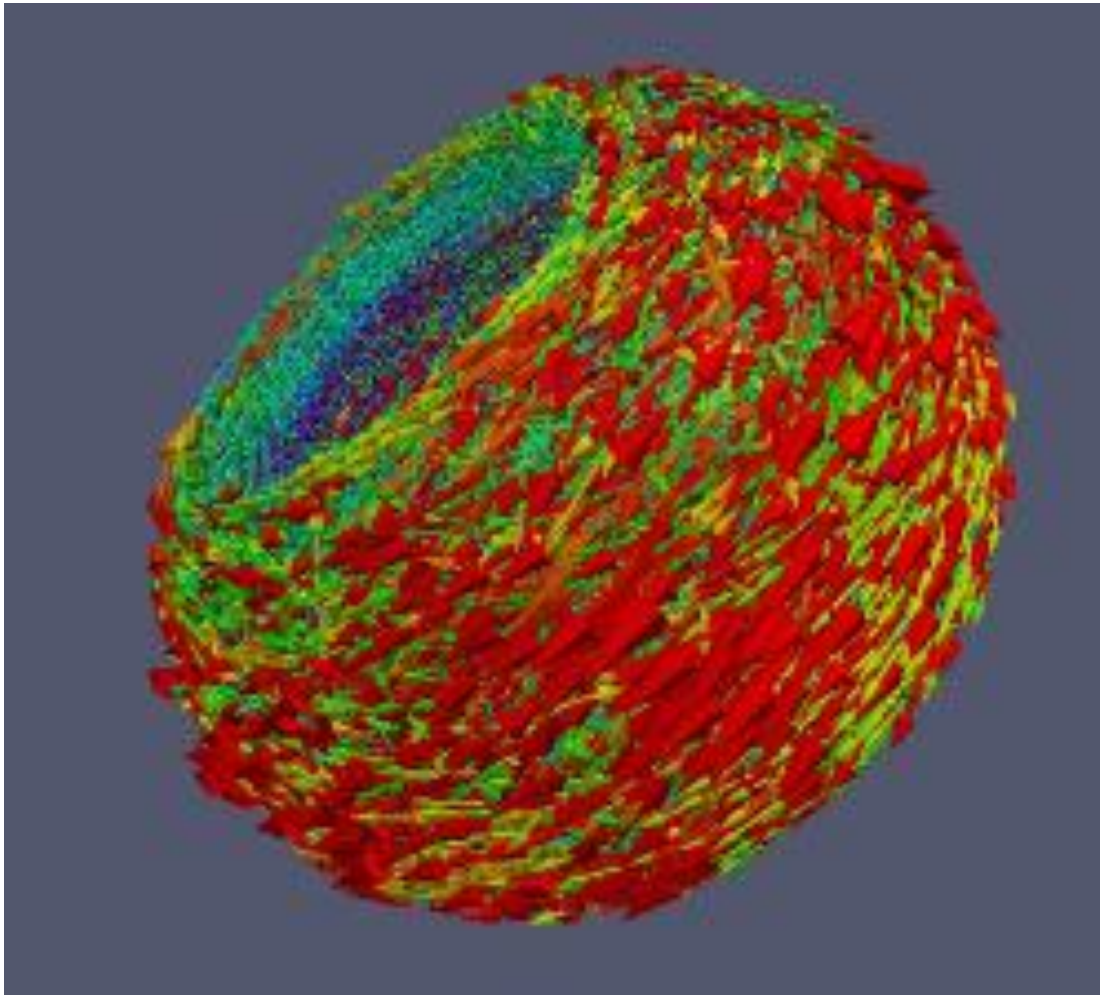
- Velocity



- Vorticity pattern – vortex sheets

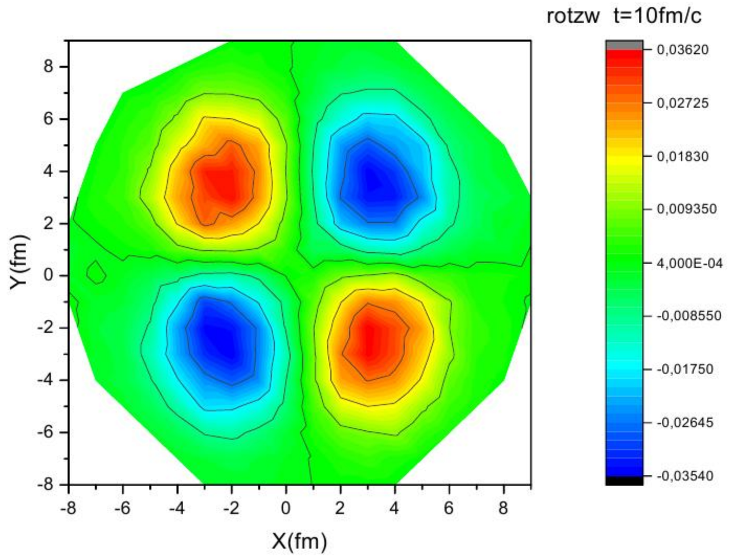


Vortex sheet (cf talk Q. Wang)



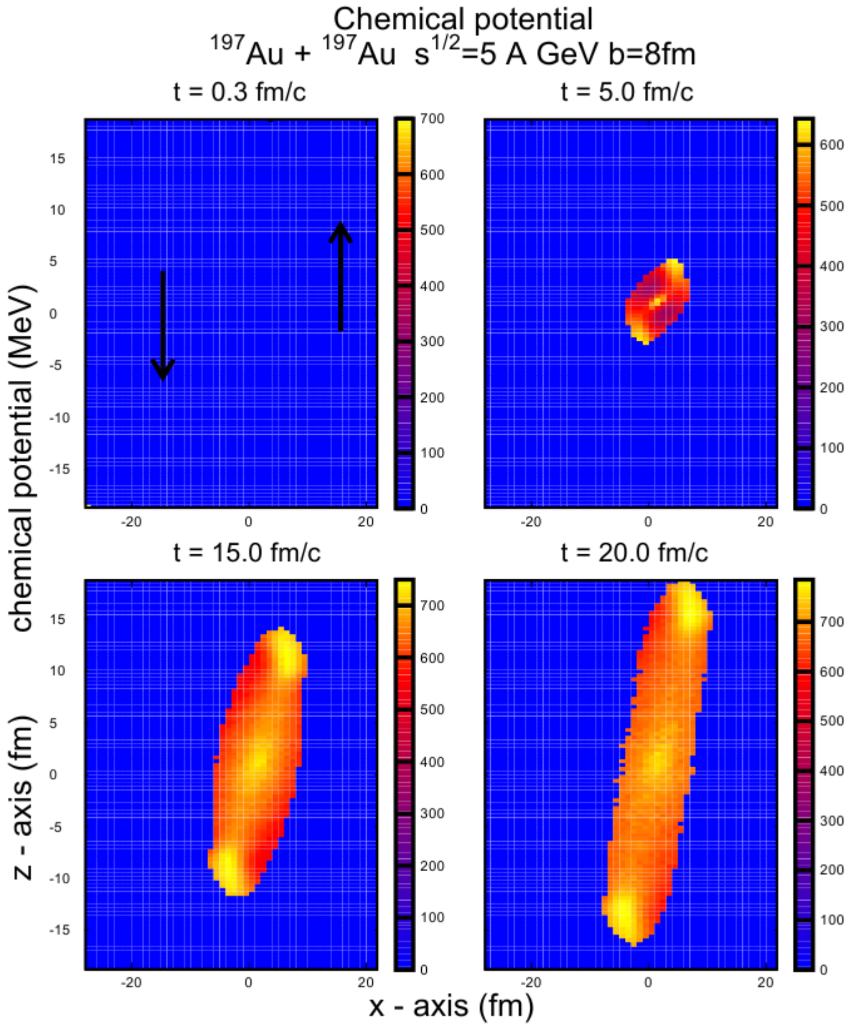
Structure of vorticity

- y-component: constant vorticity, velocity changes sign
- **z-component: quadrupole structure of vorticity**



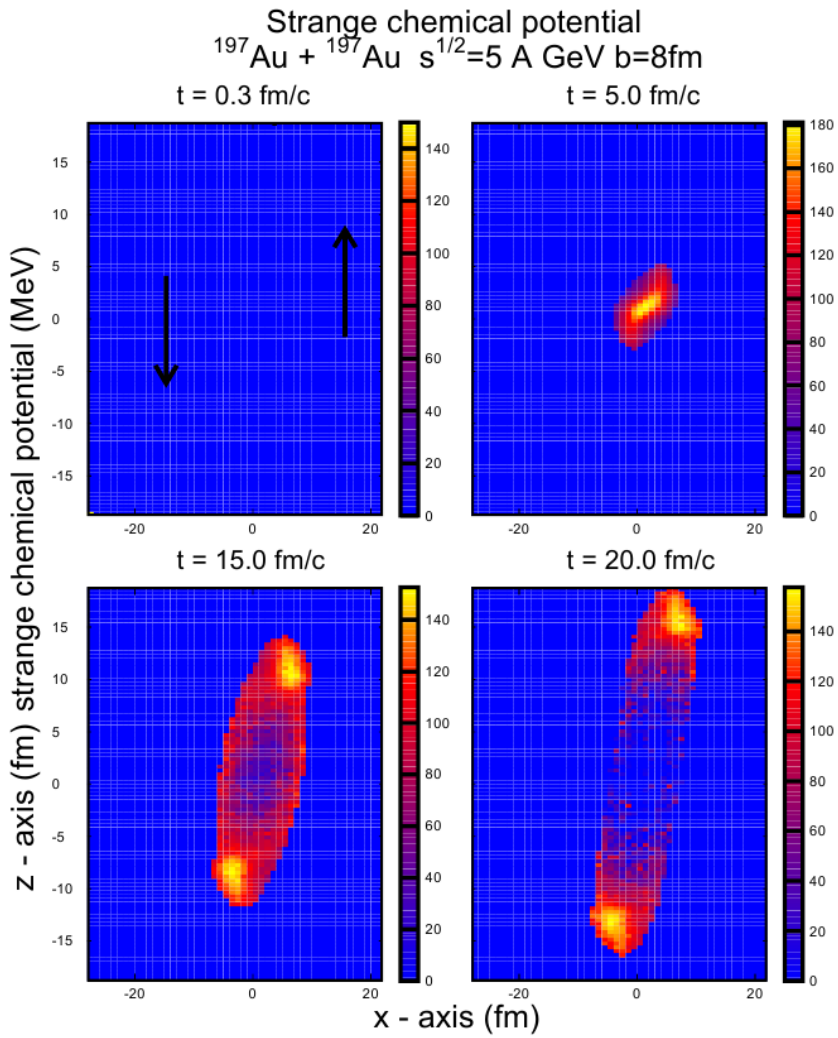
Chemical potential : Kinetics -> TD

- TD and chemical equilibrium
- Conservation laws
- Chemical potential from equilibrium distribution functions
- 2d section: y=0

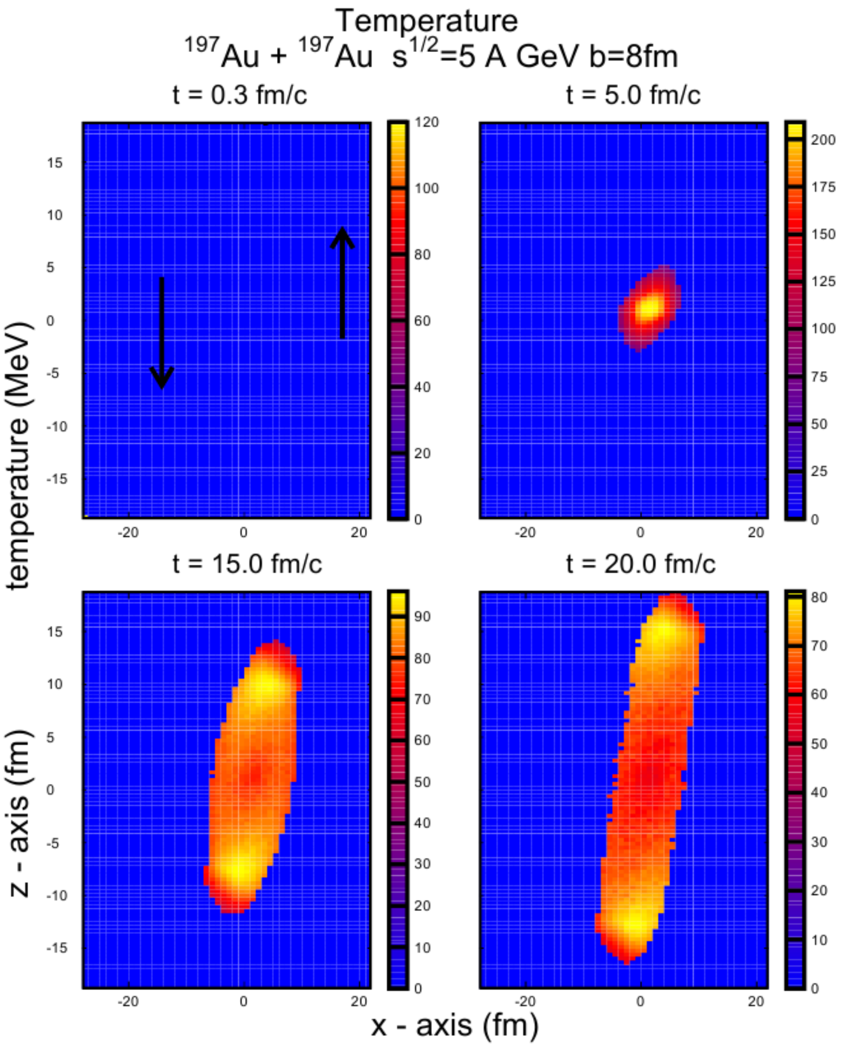


Strange chemical potential (polarization of Lambda is carried mostly by strange quark!)

- Non-uniform in space and time



Temperature

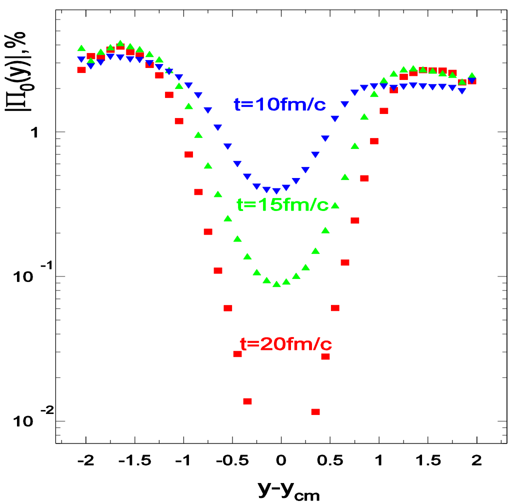


QGSM numerics for polarization

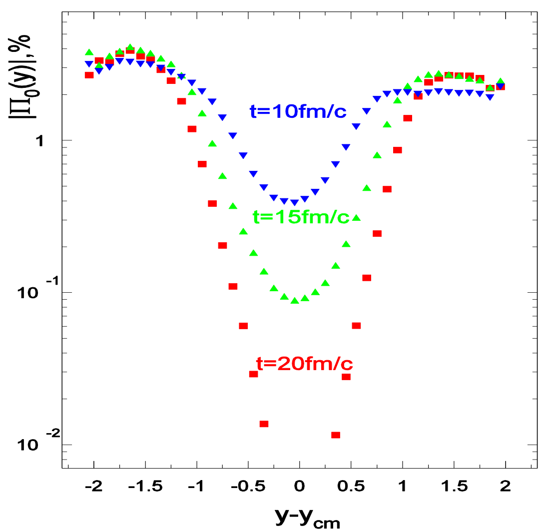
- Helicity ~ 0th component of polarization in lab. frame + effect of boost to Lambda rest frame

$$\Pi_0(y)=1/(4\pi^2)\int \gamma^2(x)\mu_s^2(x)|v\cdot rot(v)|n_\Lambda(y,x)w_1d^3x / \int n_\Lambda(y,x)w_2d^3x$$

$w_1=1$, $w2= 1$

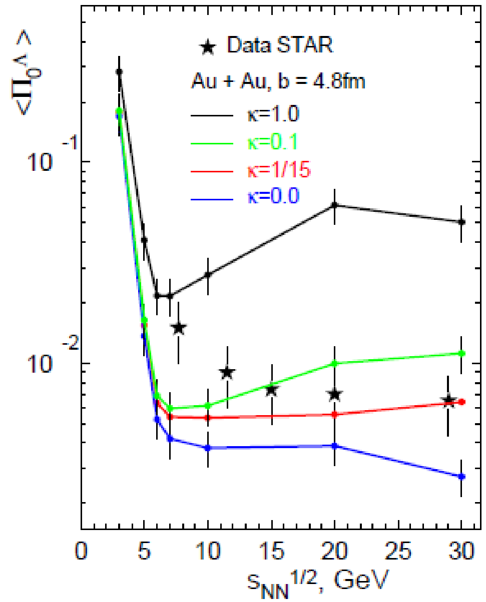


$w_1=1$, $w_2=p_y/m$



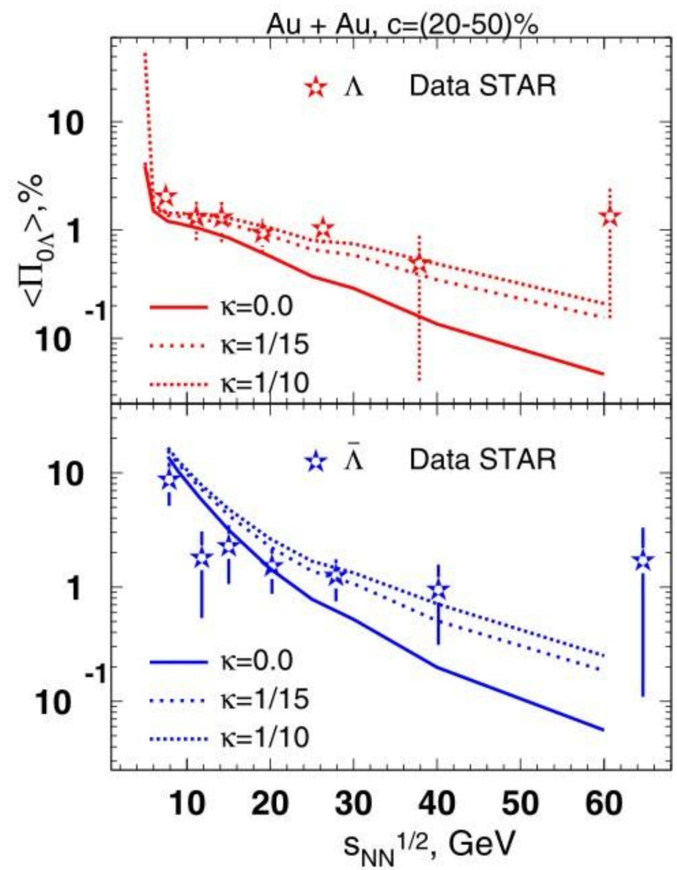
The role of (gravitational anomaly related) T^2 term

- Different values of coefficient probed



- LQCD suppression by collective effects supported

Λ vs Anti Λ



Au + Au, c=(20-50)%
Λ Data STAR
$\bar{\Lambda}$ Data STAR
κ=0.0
κ=1/15
κ=1/10
κ=0.0
κ=1/15
κ=1/10
<Π0Λ>,%
10
1
10^-1
10
1
10^-1
10
20
30
40
50
60
sNN^1/2, GeV

Conclusions/Outlook

- Polarization may provide the new probe of anomaly in quark-gluon matter (to be studied at NICA!?)
- Quark-hadron duality via axial charge/pionic superfluid

- Predictions
- Energy dependence: confirmed, reproduced
- Same sign and larger magnitude of antihyperon polarization: splitting decreases with energy
- Flavor dependence of size and sign of polarization as a probe of anomaly
- Induced extra current (with new topological term) from Wigner functions – "hidden anomaly" in averaged TD (Firenze) polarization at m ->0

- Femto-vortex sheets

Gravitational Formfactors

$$\langle p'|T^{\mu\nu}_{q,g}|p\rangle = \bar{u}(p')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}p^{\nu)} + B_{q,g}(\Delta^2)P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M\Big]u(p)$$

- Conservation laws - zero Anomalous Gravitomagnetic Moment : $\mu_G = J$ (g=2)

$$P_{q,g} = A_{q,g}(0) \qquad A_q(0) + A_g(0) = 1$$

$$J_{q,g} = \frac{1}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right] \qquad A_q(0) + B_q(0) + A_g(0) + B_g(0) = 1$$

- May be extracted from high-energy experiments/NPQCD calculations
- Describe the partition of angular momentum between quarks and gluons
- Describe interaction with both classical and TeV gravity

Generalized Parton Diistributions (related to matrix elements of non local operators) – models for both EM and Gravitational Formfactors (Selyugin,OT '09)

- Smaller mass square radius (attraction vs repulsion!?)

$$\rho(b) = \sum_q e_q \int dx q(x,b) \qquad = \int d^2 q F_1(Q^2 = q^2) e^{i\vec{q}\,\vec{b}}$$

$$= \int_0^\infty \frac{q dq}{2\pi} J_0(qb) \frac{G_E(q^2) + \tau G_M(q^2)}{1+\tau}$$

$$\rho_0^{\rm Gr}(b) = \frac{1}{2\pi} \int_\infty^0 dq q J_0(qb) A(q^2)$$

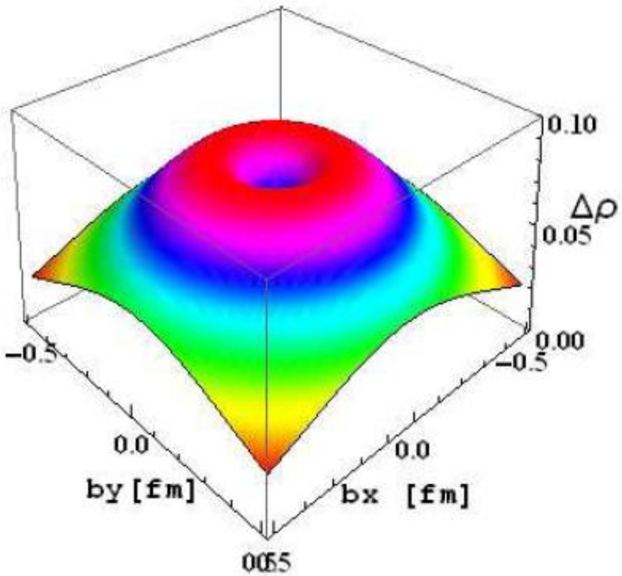


FIG. 17: Difference in the forms of charge density F_1^P and "matter" density (A)

Electromagnetism vs Gravity

- Interaction – field vs metric deviation

$$M = \langle P'|J_q^\mu|P\rangle A_\mu(q)$$

$$M = \frac{1}{2}\sum_{q,G}\langle P'|T_{q,G}^{\mu\nu}|P\rangle h_{\mu\nu}(q)$$

- Static limit

$$\langle P|J_q^\mu|P\rangle = 2e_q P^\mu$$

$$\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

$$h_{00} = 2\phi(x)$$

$$M_0 = \langle P|J_q^\mu|P\rangle A_\mu = 2e_q M\phi(q)$$

$$M_0 = \frac{1}{2}\sum_{q,G}\langle P|T_i^{\mu\nu}|P\rangle h_{\mu\nu} = 2M\cdot M\phi(q)$$

- Mass as charge – equivalence principle

Gravitomagnetism

- Gravitomagnetic field (weak, except in gravity waves) – action on spin from $M = \frac{1}{2}\sum_{q,G} \langle P'|T^{\mu\nu}_{q,G}|P\rangle h_{\mu\nu}(q)$

$$\vec{H}_J = \frac{1}{2} rot\vec{g};\ \vec{g}_i \equiv g_{0i}$$

spin dragging twice smaller than EM

- Lorentz force – similar to EM case: factor ½ cancelled with 2 from $h_{00} = 2\phi(x)$ Larmor frequency same as EM

$$\omega_J = \frac{\mu_G}{J} H_J = \frac{H_L}{2} = \omega_L\ \vec{H}_L = rot\vec{g}$$

- Orbital and Spin momenta dragging – the same - Equivalence principle

One more gravitational formfactor

- Quadrupole

$$\langle P+q/2|T^{\mu\nu}|P-q/2\rangle = C(q^2)(g^{\mu\nu}q^2 - q^\mu q^\nu) + \ldots$$

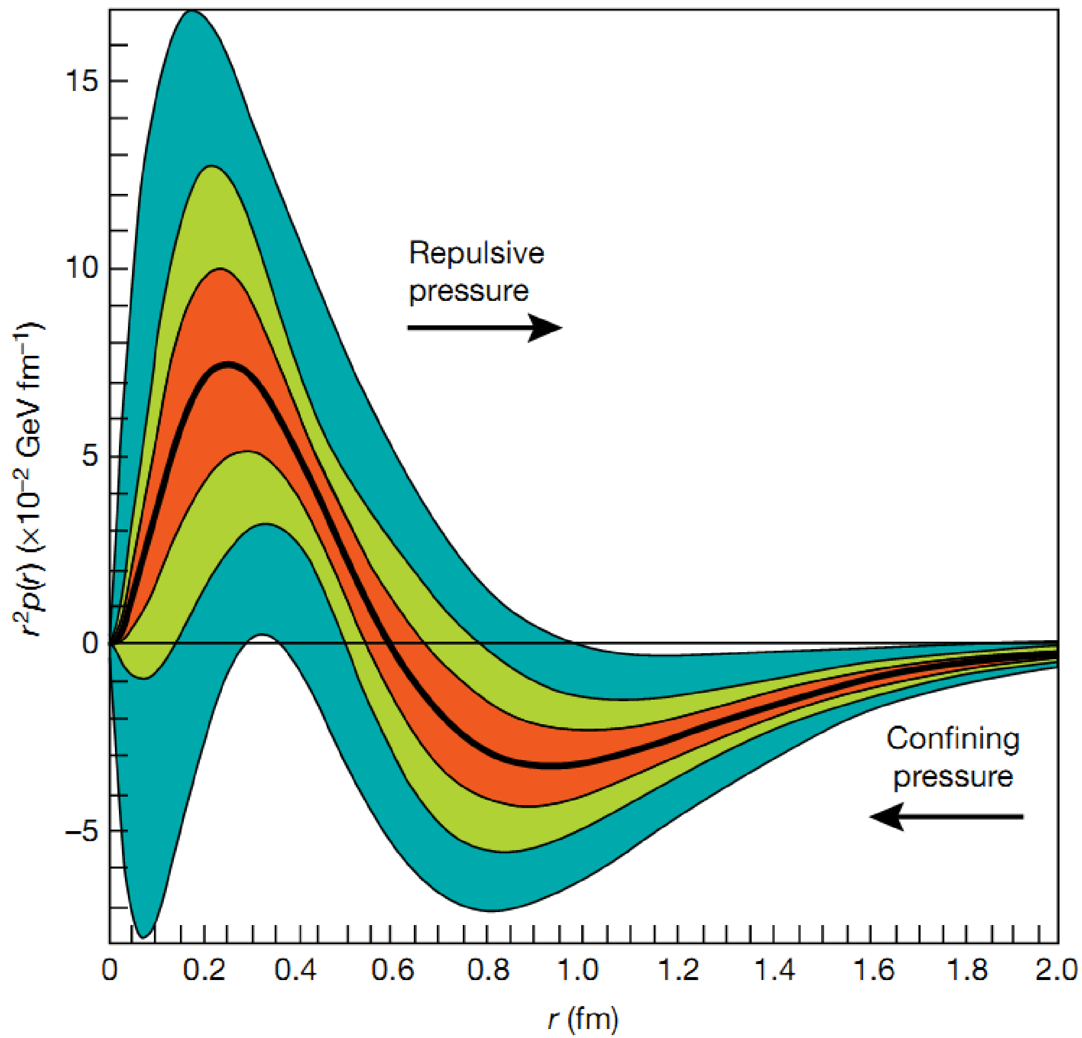
- Cf vacuum matrix element – cosmological constant $\langle 0|T^{\mu\nu}|0\rangle = \Lambda g^{\mu\nu}$
- Kinematical factor – moment of pressure C~<p r^4> (<p r^2> =0)
  M.Polyakov'03
- Stable equilibrium C>0: $\Lambda = C(q^2)q^2$
- Inflation ~ annihilation (q^2>0)

LETTER

NATURE | www.nature.com/nature
https://doi.org/10.1038/s41586-018-0060-z

The pressure distribution inside the proton

V. D. Burkert[1]*, L. Elouadrhiri[1] & F. X. Girod[1]



Stability

- All the known cases (hadrons, Q-balls)
  Schweitzer e.a.
  – stable objects

- Photon (but no rest frame!): C ~ln2
  Gabdrakhmanov, OT '12

Equivalence principle

- Newtonian – “Falling elevator” – well known and checked (also for elementary particles)
- Post-Newtonian – gravity action on SPIN – known since 1962 (Kobzarev and Okun’); rederived from conservarion laws - Kobzarev and Zakharov
- Anomalous gravitomagnetic (and electric-CP-odd) moment iz ZERO or
- Classical and QUANTUM rotators behave in the SAME way
- - not checked on purpose but in fact checked in atomic spins experiments at % level (Silenko,OT’07)

Dirac Eq and Foldy - Wouthausen transformation

- Metric of the type

$$ds^2 = V^2c^2dt^2 - \delta_{\hat{a}\hat{b}}W^{\hat{a}}{}_cW^{\hat{b}}{}_d(dx^c - K^c c dt)(dx^d - K^d c dt).$$

- Tetrads in Schwinger gauge

$$e_i^{\hat{0}} = V\delta_i^0, \qquad e_i^{\hat{a}} = W^{\hat{a}}{}_b(\delta_i^b - cK^b\delta_i^0),$$

$$e_{\hat{0}}^i = \frac{1}{V}(\delta^i{}_0 + \delta^i{}_a cK^a), \qquad e_{\hat{a}}^i = \delta^i{}_b W^b{}_{\hat{a}}, \qquad a = 1, 2, 3,$$

- Dirac eq $(i\hbar\gamma^\alpha D_\alpha - mc)\Psi = 0, \qquad \alpha = 0, 1, 2, 3.$

$$D_\alpha = e_\alpha^i D_i, \qquad D_i = \partial_i + \frac{iq}{\hbar}A_i + \frac{i}{4}\sigma^{\alpha\beta}\Gamma_{i\alpha\beta}.$$

Dirac hamiltonian

- Connection

$$\Gamma_{i\hat{a}\hat{0}} = \frac{c^2}{V} W^b{}_{\hat{a}} \partial_b V e_i{}^{\hat{0}} - \frac{c}{V} \mathcal{Q}_{(\hat{a}\hat{b})} e_i{}^{\hat{b}},$$

$$\Gamma_{i\hat{a}\hat{b}} = \frac{c}{V} \mathcal{Q}_{[\hat{a}\hat{b}]} e_i{}^{\hat{0}} + (\mathcal{C}_{\hat{a}\hat{b}\hat{c}} + \mathcal{C}_{\hat{a}\hat{c}\hat{b}} + \mathcal{C}_{\hat{c}\hat{b}\hat{a}}) e_i{}^{\hat{c}}.$$

$$\mathcal{Q}_{\hat{a}\hat{b}} = g_{\hat{a}\hat{c}} W^d{}_{\hat{b}} \left(\frac{1}{c} \dot{W}^{\hat{c}}{}_d + K^e \partial_e W^{\hat{c}}{}_d + W^{\hat{c}}{}_e \partial_d K^e \right),$$

$$\mathcal{C}_{\hat{a}\hat{b}}{}^{\hat{c}} = W^d{}_{\hat{a}} W^e{}_{\hat{b}} \partial_{[d} W^{\hat{c}}{}_{e]}, \qquad \mathcal{C}_{\hat{a}\hat{b}\hat{c}} = g_{\hat{c}\hat{d}} \mathcal{C}_{\hat{a}\hat{b}}{}^{\hat{d}}.$$

- Hermitian Hamiltonian

$$i\hbar \frac{\partial \psi}{\partial t} = \mathcal{H} \psi. \qquad \psi = (\sqrt{-g} e_{\hat{0}}^0)^{\frac{1}{2}} \Psi.$$

$$\mathcal{H} = \beta m c^2 V + q\Phi + \frac{c}{2} (\pi_b \mathcal{F}^b{}_a \alpha^a + \alpha^a \mathcal{F}^b{}_a \pi_b)$$
$$+ \frac{c}{2} (\boldsymbol{K} \cdot \boldsymbol{\pi} + \boldsymbol{\pi} \cdot \boldsymbol{K}) + \frac{\hbar c}{4} (\boldsymbol{\Xi} \cdot \boldsymbol{\Sigma} - \Upsilon \gamma_5).$$

$$\Upsilon = V \epsilon^{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{a}\hat{b}\hat{c}} = -V \epsilon^{\hat{a}\hat{b}\hat{c}} \mathcal{C}_{\hat{a}\hat{b}\hat{c}},$$

$$\Xi_{\hat{a}} = \frac{V}{c} \epsilon_{\hat{a}\hat{b}\hat{c}} \Gamma_{\hat{0}}{}^{\hat{b}\hat{c}} = \epsilon_{\hat{a}\hat{b}\hat{c}} \mathcal{Q}^{\hat{b}\hat{c}}.$$

Foldy-Wouthuysen transformation

- Even and odd parts

$$\mathcal{H} = \beta\mathcal{M} + \mathcal{E} + \mathcal{O}, \qquad \beta\mathcal{M} = \mathcal{M}\beta,$$
$$\beta\mathcal{E} = \mathcal{E}\beta, \qquad \beta\mathcal{O} = -\mathcal{O}\beta.$$

- FW transformation (Silenko '08)

$$U = \frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}\beta, \qquad \psi_{\mathrm{FW}} = U\psi, \qquad \mathcal{H}_{\mathrm{FW}} = U\mathcal{H}U^{-1} - i\hbar U\partial_t U^{-1}.$$

$$U^{-1} = \beta\frac{\beta\epsilon + \beta\mathcal{M} - \mathcal{O}}{\sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}}. \qquad \epsilon = \sqrt{\mathcal{M}^2 + \mathcal{O}^2}$$

$$\begin{aligned}\mathcal{H}' = \beta\epsilon + \mathcal{E} + \frac{1}{2T}&([T,[T,(\beta\epsilon + Z)]] \\ &+ \beta[\mathcal{O},[\mathcal{O},\mathcal{M}]] - [\mathcal{O},[\mathcal{O},Z]] \\ &- [(\epsilon + \mathcal{M}),[(\epsilon + \mathcal{M}),Z]] - [(\epsilon + \mathcal{M}),[\mathcal{M},\mathcal{O}]] \\ &- \beta\{\mathcal{O},[(\epsilon + \mathcal{M}),Z]\} + \beta\{(\epsilon + \mathcal{M}),[\mathcal{O},Z]\})\frac{1}{T},\end{aligned}$$

$$T = \sqrt{(\beta\epsilon + \beta\mathcal{M} - \mathcal{O})^2}$$

$$Z = \mathcal{E} - i\hbar\frac{\partial}{\partial t}$$

$$\mathcal{H}' = \beta\epsilon + \mathcal{E}' + \mathcal{O}', \qquad \beta\mathcal{E}' = \mathcal{E}'\beta, \qquad \beta\mathcal{O}' = -\mathcal{O}'\beta,$$

$$\mathcal{H}_{\mathrm{FW}} = \beta\epsilon + \mathcal{E}' + \frac{1}{4}\beta\left\{\mathcal{O}'^2, \frac{1}{\epsilon}\right\}.$$

FW for arbitrary gravitational field

- Result

$$\mathcal{H}_{\mathrm{FW}} = \mathcal{H}_{\mathrm{FW}}^{(1)} + \mathcal{H}_{\mathrm{FW}}^{(2)}.$$

$$\epsilon' = \sqrt{m^2c^4V^2 + \frac{c^2}{4}\delta^{ac}\{p_b, \mathcal{F}^b{}_a\}\{p_d, \mathcal{F}^d{}_c\}},$$

$$\mathcal{T} = 2\epsilon'^2 + \{\epsilon', mc^2V\}.$$

$$\mathcal{M} = mc^2V,$$

$$\mathcal{E} = q\Phi + \frac{c}{2}(\boldsymbol{K}\cdot\boldsymbol{\pi} + \boldsymbol{\pi}\cdot\boldsymbol{K}) + \frac{\hbar c}{4}\boldsymbol{\Xi}\cdot\boldsymbol{\Sigma},$$

$$\mathcal{O} = \frac{c}{2}(\pi_b\mathcal{F}^b{}_a\alpha^a + \alpha^a\mathcal{F}^b{}_a\pi_b) - \frac{\hbar c}{4}\Upsilon\gamma_5.$$

$$\mathcal{H}_{\mathrm{FW}}^{(1)} = \beta\epsilon' + \frac{\hbar c^2}{16}\left\{\frac{1}{\epsilon'}, (2\epsilon^{cae}\Pi_e\{p_b, \mathcal{F}^d{}_c\partial_d\mathcal{F}^b{}_a\} + \Pi^a\{p_b, \mathcal{F}^b{}_a\Upsilon\})\right\} + \frac{\hbar mc^4}{4}\epsilon^{cae}\Pi_e\left\{\frac{1}{\mathcal{T}}, \{p_d, \mathcal{F}^d{}_c\mathcal{F}^b{}_a\partial_b V\}\right\},$$

$$\mathcal{H}_{\mathrm{FW}}^{(2)} = \frac{c}{2}(K^a p_a + p_a K^a) + \frac{\hbar c}{4}\Sigma_a\Xi^a + \frac{\hbar c^2}{16}\left\{\frac{1}{\mathcal{T}}, \left\{\Sigma_a\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \left[\epsilon^{abc}\left(\frac{1}{c}\dot{\mathcal{F}}^f{}_c \;\ldots - \mathcal{F}^d{}_c\partial_d K^f + K^d\partial_d\mathcal{F}^f{}_c\right) - \frac{1}{2}\mathcal{F}^f{}_d(\delta^{db}\Xi^a - \delta^{da}\Xi^b)\right]\right\}\right\}\right\},$$

Operator EOM

- Polarization operator $\mathbf{\Pi} = \beta\mathbf{\Sigma}$

$$\frac{d\mathbf{\Pi}}{dt} = \frac{i}{\hbar}[\mathcal{H}_{\rm FW}, \mathbf{\Pi}] = \mathbf{\Omega}_{(1)} \times \mathbf{\Sigma} + \mathbf{\Omega}_{(2)} \times \mathbf{\Pi}.$$

- Angular velocities

$$\Omega^a_{(1)} = \frac{mc^4}{2}\left\{\frac{1}{\mathcal{T}}, \{p_e, \epsilon^{abc}\mathcal{F}^e{}_b\mathcal{F}^d{}_c\partial_d V\}\right\} + \frac{c^2}{8}\left\{\frac{1}{\epsilon'}, \{p_e, (2\epsilon^{abc}\mathcal{F}^d{}_b\partial_d\mathcal{F}^e{}_c + \delta^{ab}\mathcal{F}^e{}_b\Upsilon)\}\right\},$$

$$\Omega^a_{(2)} = \frac{\hbar c^2}{8}\left\{\frac{1}{\mathcal{T}}, \left\{\{p_e, \mathcal{F}^e{}_b\}, \left\{p_f, \left[\epsilon^{abc}\left(\frac{1}{c}\dot{\mathcal{F}}^f{}_c - \mathcal{F}^d{}_c\partial_d K^f + K^d\partial_d\mathcal{F}^f{}_c\right) - \frac{1}{2}\mathcal{F}^f{}_d(\delta^{db}\Xi^a - \delta^{da}\Xi^b)\right]\right\}\right\}\right\} + \frac{c}{2}\Xi^a.$$

Semi-classical limit

- Average spin

$$\frac{d\mathbf{s}}{dt} = \mathbf{\Omega} \times \mathbf{s} = (\mathbf{\Omega}_{(1)} + \mathbf{\Omega}_{(2)}) \times \mathbf{s},$$

$$\Omega^a_{(1)} = \frac{c^2}{\epsilon'} \mathcal{F}^d{}_c p_d \Big(\frac{1}{2} \Upsilon \delta^{ac} - \epsilon^{aef} V C_{ef}{}^c + \frac{\epsilon'}{\epsilon' + mc^2 V} \epsilon^{abc} W^e{}_{\hat{b}} \partial_e V \Big),$$

$$\Omega^a_{(2)} = \frac{c}{2} \Xi^a - \frac{c^3}{\epsilon'(\epsilon' + mc^2 V)} \epsilon^{abc} Q_{(bd)} \delta^{dn} \mathcal{F}^k{}_n p_k \mathcal{F}^l{}_c p_l,$$

Application to anisotropic universe (Kamenshchik,OT) – no suppression ~ G M/Rc^2

- Bianchi-1 Universe

$$ds^2 = dt^2 - a^2(t)(dx^1)^2 - b^2(t)(dx^2)^2 - c^2(t)(dx^3)^2.$$

- Particular case $W_1^{\hat{1}} = a(t),\ W_2^{\hat{2}} = b(t),\ W_3^{\hat{3}} = c(t).$

$$W_{\hat{1}}^{1} = \frac{1}{a(t)},\ W_{\hat{2}}^{2} = \frac{1}{b(t)},\ W_{\hat{3}}^{3} = \frac{1}{c(t)}.$$

- No anholonomity $\Upsilon = 0$

$$\Omega^{\hat{1}}_{(2)} = \frac{\gamma}{\gamma+1} v_{\hat{2}} v_{\hat{3}} \left(\frac{\dot{b}}{b} - \frac{\dot{c}}{c}\right).$$

$$Q_{\hat{1}\hat{1}} = -\frac{\dot{a}}{a},\ Q_{\hat{2}\hat{2}} = -\frac{\dot{b}}{b},\ Q_{\hat{3}\hat{3}} = -\frac{\dot{c}}{c}.$$

Kasner solution

- t-dependence

$$a(t) = a_0 t^{p_1},\ b(t) = b_0 t^{p_2},\ c(t) = c_0 t^{p_3},$$

$$p_1 + p_2 + p_3 = 1,\ \ p_1^2 + p_2^2 + p_3^2 = 1.$$

- Euler-type expressions

$$\Omega^{\bar{1}}_{(2)} = \frac{\gamma}{\gamma + 1} v_{\bar{2}} v_{\bar{3}} \left(\frac{p_2 - p_3}{t} \right)$$

Heckmann-Schucking solution

- Dust admixture

$$a(t) = a_0 t^{p_1} (t_0 + t)^{\frac{2}{3} - p_1}, \; b(t) = b_0 t^{p_2} (t_0 + t)^{\frac{2}{3} - p_2},$$
$$c(t) = c_0 t^{p_3} (t_0 + t)^{\frac{2}{3} - p_3}.$$

- Modification:

$$\Omega^1_{(2)} = \frac{\gamma}{\gamma + 1} v_2 v_3 \frac{(p_2 - p_3) t_0}{t(t_0 + t)}$$

$$= \frac{\gamma}{\gamma + 1} v_2 v_3 \frac{(p_2 - p_3) t_0}{t^2} \left(1 + o\left(\frac{t_0}{t}\right)\right)$$

Biancki-IX Universe

- Metric $W^{\hat{b}}{}_{a} = \begin{pmatrix} -a\sin x^3 & a\sin x^1\cos x^3 & 0 \\ b\cos x^3 & b\sin x^1\sin x^3 & 0 \\ 0 & c\cos x^1 & c \end{pmatrix}$ $W^{c}{}_{\hat{b}} = \begin{pmatrix} -\frac{1}{a}\sin x^3 & \frac{1}{b}\cos x^3 & 0 \\ \frac{1}{a}\frac{\cos x^3}{\sin x^1} & \frac{1}{b}\frac{\sin x^3}{\sin x^1} & 0 \\ -\frac{1}{a}\frac{\cos x^1\cos x^3}{\sin x^1} & -\frac{1}{b}\frac{\sin x^3\cos x^1}{\sin x^1} & \frac{1}{c} \end{pmatrix}$
- Anholonomity coefficients
- $C^{\hat{3}}_{\hat{1}\hat{2}} = \frac{c}{ab}$ + cyclic permutations
- -> non-zero $\Upsilon = 2\left(\frac{c}{ab} + \frac{b}{ac} + \frac{a}{bc}\right)$

$$\Omega^{\hat{1}}_{(1)} = v^{\hat{1}}\left(\frac{c}{ab} + \frac{b}{ac} - \frac{a}{bc}\right)$$

Approach to singularity

- Chaotic oscillations – sequence of Kasner regimes $p_1 = -\frac{u}{1+u+u^2},\ p_2 = \frac{1+u}{1+u+u^2},\ p_3 = \frac{u(1+u)}{1+u+u^2}$
- If Lifshitz-Khalatnıkov parameter u >1 – "epochs" $p_1' = p_2(u-1),\ p_2' = p_1(u-1),\ p_3' = p_3(u-1)$

$$p_1' = p_1\left(\frac{1}{u}\right),\ p_2' = p_3\left(\frac{1}{u}\right),\ p_3' = p_2\left(\frac{1}{u}\right)$$

- If u< 1 – "eras"
- Change of eras – chaotic mapping of [0,1]interrval

$$Tx = \left\{\frac{1}{x}\right\},\quad x_{s+1} = \left\{\frac{1}{x_s}\right\}$$

Angular velocities

- New epoch: u -> -u
- New era – changed sign

$$\Omega^{\hat{1}}_{(2)} = \frac{\gamma}{(\gamma+1)t} v_{\hat{2}} v_{\hat{3}} \cdot \frac{1-u^2}{1+u+u^2},$$
$$\Omega^{\hat{2}}_{(2)} = \frac{\gamma}{(\gamma+1)t} v_{\hat{1}} v_{\hat{3}} \cdot \frac{2u+u^2}{1+u+u^2},$$
$$\Omega^{\hat{3}}_{(2)} = -\frac{\gamma}{(\gamma+1)t} v_{\hat{1}} v_{\hat{2}} \cdot \frac{1+2u}{1+u+u^2}.$$

- Odd velocity

$$\Omega^{\hat{1}}_{(1)} \sim -v^{\hat{1}}(t)^{\left(-1-\frac{2u}{1+u+u^2}\right)},$$
$$\Omega^{\hat{b}}_{(1)} \sim v^{\hat{b}}(t)^{\left(-1-\frac{2u}{1+u+u^2}\right)},\ b = 2, 3.$$

- New epoch
- New era - preserved

$$\Omega^{\hat{2}}_{(1)} \sim -v^{\hat{2}}(t)^{\left(-1-\frac{2u-2}{1-u+u^2}\right)},$$
$$\Omega^{\hat{a}}_{(1)} \sim v^{\hat{a}}(t)^{\left(-1-\frac{2u-2}{1-u+u^2}\right)},\ a = 1, 3.$$

Possible applications

- Anisotropy (c.f. crystals) ~ magnetic field
- Spin precession + equivalence principle = helicity flip (~AMM effect)
- Dirac neutrino – transformed to sterile component in early (bounced) Universe
- Angular velocity ~ 1/t -> amount of decoupled ~ 1
- Possible new candidate for dark matter?!
- Other fields AFTER inflation?

Properties of SSA

The same for the case of initial or final state polarization.

Various possibilities to measure the effects: change sign of $\vec{n}$ or $\vec{P}$: left-right or up-down asymmetry.

Qualitative features of the asymmetry

Transverse momentum required (to have $\vec{n}$)

Transverse polarization (to maximize $(\vec{P}\vec{n})$)

Interference of amplitudes

IMAGINARY phase between amplitudes - absent in Born approximation

Phases and T-oddness

Clearly seen in relativistic approach:

$\rho = \frac{1}{2}(\hat{p} + m)(1 + \hat{s}\gamma_5)$

Than: $d\sigma \sim Tr[\gamma_5 ....] \sim im\varepsilon_{sp_1p_2p_3}...$

Imaginary parts (loop amplitudes) are required to produce real observable.

$\varepsilon_{abcd} \equiv \varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$ each index appears once: $P-$ (compensate S) and $T-$ odd.

However: no real $T-$violation: interchange $|i> \leftrightarrow |f>$ is the nontrivial operation in the case of nonzero phases of $<f|S|i>^* = <i|S|f>$.

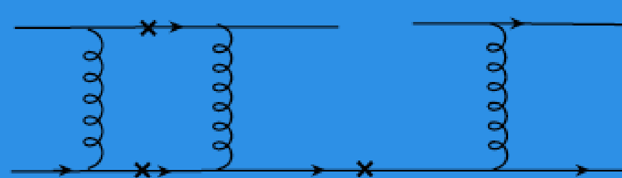
SSA - either T-violation or the phases.

DIS - no phases ($Q^2 < 0$)- real T-violation.

Perturbative PHASES IN QCD

QCD factorization: where to borrow imaginary parts?

Simplest way: from short distances - loops in partonic subprocess. Quarks elastic scattering (like $q - e$ scattering in DIS):

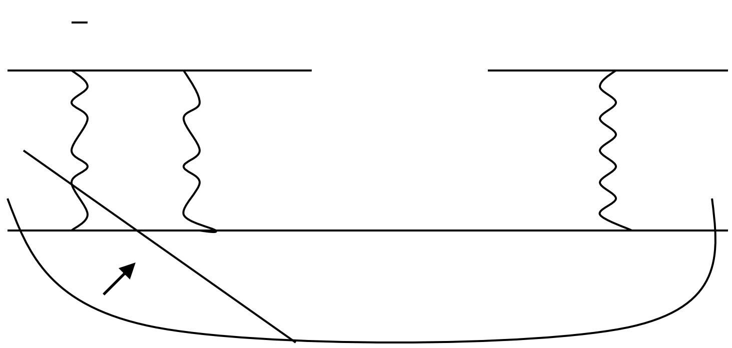


$$A \sim \frac{\alpha_S m p_T}{p_T^2 + m^2}$$

Large SSA "...contradict QCD or its applicability"

Short+ large overlap– twist 3

- Quarks – only from hadrons
- Various options for factorization – shift of SH separation



- New option for SSA: Instead of 1-loop twist 2 – Born twist 3 (quark-gluon correlator): Efremov, OT (85, Fermionic poles); Qiu, Sterman (91, GLUONIC poles)
- Further shift to large distances – T-odd fragmentation functions (Collins, dihadron, handedness)

Polarization at NICA/MPD (A. Kechechyan)

- QGSM Simulations and recovery accounting for MPD acceptance effects