Fermionic contribution to anomalous dimension of twist-2 operators in $\mathcal{N}=4$ SYM

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Deep Inelastic Scattering

[Gross, Wilczek '73]

$$\mathcal{O}^{q}_{\mu_1,\ldots,\mu_j} = \bar{q}\ \gamma_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\, q + \text{symmetrisation} - \text{traces}$$

Twist = Canonical dimension - Lorentz spin j [Gross, Wilczek '73]

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Operators mix under renormalization → Matrix of anomalous dimensions

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$$\Gamma = \begin{pmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{pmatrix}$$

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$$\Gamma = \begin{pmatrix} \gamma_{qq} & \gamma_{qg} \\ \gamma_{gq} & \gamma_{gg} \end{pmatrix}$$

$$\gamma_{qq} = 2C_F\left[4S_1(j) - 3 - \frac{2}{j(j+1)}\right] \qquad \gamma_{qg} = -8T_R\frac{j^2+j+2}{j(j+1)(j+2)}$$

$$\gamma_{gq} = -4C_F\frac{j^2+j+2}{(j-1)j(j+1)} \qquad \gamma_{gg} = \left[8C_A\left(S_1(j) - \frac{1}{j(j-1)} - \frac{1}{(j+1)(j+2)} - \frac{11}{12}\right) + \frac{8}{3}T_R\right]$$

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$$S_1(j) = \sum_{k=1}^{j} \frac{1}{k} = \Psi(1) - \Psi(j+1)$$

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Dokshitzer relation

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Origin:

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Origin: $\mathcal{N} = 1$ Supersymmetric Yang-Mills theory

Wilson twist-2 operators:

[Gross, Wilczek '73]

$$\mathcal{O}^{g}_{\mu_1,\ldots,\mu_j} = G^{a}_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\ldots\mathcal{D}_{\mu_{j-1}}G^{a}_{\rho\mu_j}$$

$$\mathcal{O}^{\lambda}_{\mu_1,\ldots,\mu_j} = \bar{\lambda}^{a}_{i}\gamma_{\mu_1}\mathcal{D}_{\mu_2}\ldots\mathcal{D}_{\mu_j}\lambda^{a,\,i}$$

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Anomalous dimension matrix in leading order:

[Lipatov '00]

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Anomalous dimension matrix in leading order:

[Lipatov '00]

$$\Gamma = \begin{pmatrix} \gamma_{gg} & \gamma_{g\lambda} & \gamma_{g\phi} \\ \gamma_{\lambda g} & \gamma_{\lambda\lambda} & \gamma_{\lambda\phi} \\ \gamma_{\phi g} & \gamma_{\phi\lambda} & \gamma_{\phi\phi} \end{pmatrix}$$

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Anomalous dimension matrix in leading order:

[Lipatov '00]

$$\gamma^{(0)}_{gg} = -4S_1(j) + \frac{4}{j-1} - \frac{4}{j} + \frac{4}{j+1} - \frac{4}{j+2} \qquad \gamma^{(0)}_{\lambda g} = \frac{8}{j} - \frac{16}{j+1} + \frac{16}{j+2}$$

$$\gamma^{(0)}_{\phi g} = \frac{12}{j+1} - \frac{12}{j+2} \qquad \gamma^{(0)}_{g\lambda} = \frac{4}{j-1} - \frac{4}{j} + \frac{2}{j+1} \qquad \gamma^{(0)}_{\lambda\phi} = \frac{8}{j} \qquad \gamma^{(0)}_{\phi\lambda} = \frac{6}{j+1}$$

$$\gamma^{(0)}_{\lambda\lambda} = -4S_1(j) + \frac{4}{j} - \frac{4}{j+1} \qquad \gamma^{(0)}_{\phi\phi} = -4S_1(j) \qquad \gamma^{(0)}_{g\phi} = \frac{4}{j-1} - \frac{4}{j}$$

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[Gross, Wilczek '73]

$$\mathcal{O}^{g}_{\mu_1,\dots,\mu_j} = G^a_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\dots\mathcal{D}_{\mu_{j-1}}G^a_{\rho\mu_j} \qquad \tilde{\mathcal{O}}^{g}_{\mu_1,\dots,\mu_j} = G^a_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\dots\mathcal{D}_{\mu_{j-1}}\tilde{G}^a_{\rho\mu_j}$$

$$\mathcal{O}^{\lambda}_{\mu_1,\dots,\mu_j} = \bar{\lambda}^a_i\gamma_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\lambda^{a,\,i} \qquad \tilde{\mathcal{O}}^{\lambda}_{\mu_1,\dots,\mu_j} = \bar{\lambda}^a_i\gamma_5\gamma_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\lambda^{a,\,i}$$

$$\mathcal{O}^{\phi}_{\mu_1,\dots,\mu_j} = \bar{\phi}^a_r\mathcal{D}_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\phi^{a,\,r}$$

Anomalous dimension matrix in leading order:

[Lipatov '00]

$$\gamma^{(0)}_{gg} = -4S_1(j) + \frac{4}{j-1} - \frac{4}{j} + \frac{4}{j+1} - \frac{4}{j+2} \qquad \gamma^{(0)}_{\lambda g} = \frac{8}{j} - \frac{16}{j+1} + \frac{16}{j+2}$$

$$\gamma^{(0)}_{\phi g} = \frac{12}{j+1} - \frac{12}{j+2} \qquad \gamma^{(0)}_{g\lambda} = \frac{4}{j-1} - \frac{4}{j} + \frac{2}{j+1} \qquad \gamma^{(0)}_{\lambda\phi} = \frac{8}{j} \qquad \gamma^{(0)}_{\phi\lambda} = \frac{6}{j+1}$$

$$\gamma^{(0)}_{\lambda\lambda} = -4S_1(j) + \frac{4}{j} - \frac{4}{j+1} \qquad \gamma^{(0)}_{\phi\phi} = -4S_1(j) \qquad \gamma^{(0)}_{g\phi} = \frac{4}{j-1} - \frac{4}{j}$$

$$\widetilde{\Gamma} = \begin{pmatrix} \tilde{\gamma}_{gg} & \tilde{\gamma}_{g\lambda} \\ \tilde{\gamma}_{\lambda g} & \tilde{\gamma}_{\lambda\lambda} \end{pmatrix}$$

Wilson twist-2 operators:

[Gross, Wilczek '73]

$$\mathcal{O}^{g}_{\mu_1,\dots,\mu_j} = G^{a}_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\dots\mathcal{D}_{\mu_{j-1}}G^{a}_{\rho\mu_j} \qquad \tilde{\mathcal{O}}^{g}_{\mu_1,\dots,\mu_j} = G^{a}_{\rho\mu_1}\mathcal{D}_{\mu_2}\mathcal{D}_{\mu_3}\dots\mathcal{D}_{\mu_{j-1}}\tilde{G}^{a}_{\rho\mu_j}$$

$$\mathcal{O}^{\lambda}_{\mu_1,\dots,\mu_j} = \bar{\lambda}^{a}_{i}\gamma_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\lambda^{a,\,i} \qquad \tilde{\mathcal{O}}^{\lambda}_{\mu_1,\dots,\mu_j} = \bar{\lambda}^{a}_{i}\gamma_5\gamma_{\mu_1}\mathcal{D}_{\mu_2}\dots\mathcal{D}_{\mu_j}\lambda^{a,\,i}$$

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$$\gamma^{(0)}_{\phi g} = \frac{12}{j+1} - \frac{12}{j+2} \qquad \gamma^{(0)}_{g\lambda} = \frac{4}{j-1} - \frac{4}{j} + \frac{2}{j+1} \qquad \gamma^{(0)}_{\lambda\phi} = \frac{8}{j} \qquad \gamma^{(0)}_{\phi\lambda} = \frac{6}{j+1}$$

$$\gamma^{(0)}_{\lambda\lambda} = -4S_1(j) + \frac{4}{j} - \frac{4}{j+1} \qquad \gamma^{(0)}_{\phi\phi} = -4S_1(j) \qquad \gamma^{(0)}_{g\phi} = \frac{4}{j-1} - \frac{4}{j}$$

$$\tilde{\gamma}^{(0)}_{gg} = -4S_1(j) - \frac{8}{j+1} + \frac{8}{j} \qquad \tilde{\gamma}^{(0)}_{\lambda g} = -\frac{8}{j} + \frac{16}{j+1}$$

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Anomalous dimension matrix in leading order:

[Lipatov '00]

$$\Gamma^{(0)} = \begin{pmatrix} \gamma_{gg}^{(0)} & \gamma_{g\lambda}^{(0)} & \gamma_{g\phi}^{(0)} \\ \gamma_{\lambda g}^{(0)} & \gamma_{\lambda\lambda}^{(0)} & \gamma_{\lambda\phi}^{(0)} \\ \gamma_{\phi g}^{(0)} & \gamma_{\phi\lambda}^{(0)} & \gamma_{\phi\phi}^{(0)} \end{pmatrix} \qquad \widetilde{\Gamma}^{(0)} = \begin{pmatrix} \tilde{\gamma}_{gg}^{(0)} & \tilde{\gamma}_{g\lambda}^{(0)} \\ \tilde{\gamma}_{\lambda g}^{(0)} & \tilde{\gamma}_{\lambda\lambda}^{(0)} \end{pmatrix}$$

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$$\Downarrow$$

$$\begin{pmatrix}S_1(j-2) & 0 & 0\\ 0 & S_1(j) & 0\\ 0 & 0 & S_1(j+2)\end{pmatrix}$$

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Eigenvalues of anomalous dimension matrix are expressed trough the same function $\gamma^{(0)}_{uni}(j+2) = S_1(j)$ with shifted argument

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[Lipatov '00]

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Origin:

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[Lipatov '00]

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Eigenvalues of anomalous dimension matrix are expressed trough the same function $\gamma^{(0)}_{uni}(j+2) = S_1(j)$ with shifted argument

Origin: All multiplicatively renormalizable operators in $\mathcal{N}=4$ SYM theory belong to the same supermultiplet

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Origin: All multiplicatively renormalizable operators in $\mathcal{N}=4$ SYM theory belong to the same supermultiplet

$$\mathcal{O}^{T}_{\mu_1,\dots,\mu_j} \quad = \quad \mathcal{O}^{g}_{\mu_1,\dots,\mu_j} + \mathcal{O}^{\lambda}_{\mu_1,\dots,\mu_j} + \mathcal{O}^{\phi}_{\mu_1,\dots,\mu_j}$$

[A.Belitsky, S.Derkachov, G.Korchemsky, A.Manashov '03]
[A.Bukhvostov, G.Frolov, L.Lipatov, E.Kuraev 1985]

Conformal operators: $P_n^{(a,b)}$ – Jacobi polynomials

$$\mathcal{O}_j = X_1 (i\partial_+)^n P_n^{(2j_1-1,2j_2-1)}\left(\overleftrightarrow{\mathcal{D}}^+/\partial^+\right) X_2$$

$$\partial \equiv \overrightarrow{\partial} + \overleftarrow{\partial}$$
$$\overleftrightarrow{\mathcal{D}} \equiv \overrightarrow{\mathcal{D}} - \overleftarrow{\mathcal{D}}$$

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Building supermultiplet:

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Building supermultiplet: Supersymmetry transformations for $\mathcal{N}=4$ SYM

[A.Belitsky, S.Derkachov, G.Korchemsky, A.Manashov '03]
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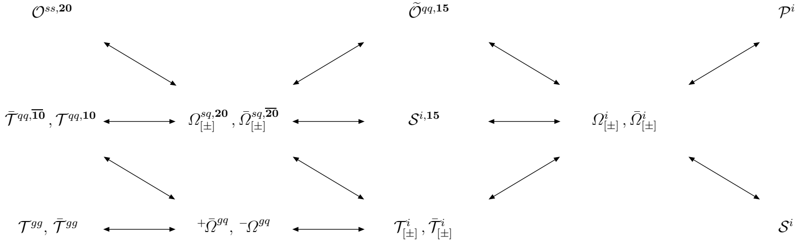
Building supermultiplet: Supersymmetry transformations for $\mathcal{N}=4$ SYM



$\mathcal{N}=4$ SYM theory: Two loops

$$\Gamma^{(1)} = \begin{pmatrix} \gamma^{(1)}_{gg} & \gamma^{(1)}_{g\lambda} & \gamma^{(1)}_{g\phi} \\ \gamma^{(1)}_{\lambda g} & \gamma^{(1)}_{\lambda\lambda} & \gamma^{(1)}_{\lambda\phi} \\ \gamma^{(1)}_{\phi g} & \gamma^{(1)}_{\phi\lambda} & \gamma^{(1)}_{\phi\phi} \end{pmatrix}$$

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$$\Downarrow$$

$$\begin{pmatrix} \gamma_{uni}^{(1)}(j-2) & \Gamma_{21} & \Gamma_{31} \\ 0 & \gamma_{uni}^{(1)}(j) & \Gamma_{32} \\ 0 & 0 & \gamma_{uni}^{(1)}(j+2) \end{pmatrix}$$

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$$\begin{pmatrix} \gamma^{(1)}_{uni}(j-2) & \Gamma_{21} & \Gamma_{31} \\ 0 & \gamma^{(1)}_{uni}(j) & \Gamma_{32} \\ 0 & 0 & \gamma^{(1)}_{uni}(j+2) \end{pmatrix} \qquad \begin{pmatrix} \gamma^{(1)}_{uni}(j-1) & \widetilde{\Gamma}_{21} \\ 0 & \gamma^{(1)}_{uni}(j+1) \end{pmatrix}$$

In the next-to-leading order (NLO) the matrix will triangle, but again the eigenvalues are expressed through the same function $\gamma^{(1)}_{uni}(j)$

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$$\gamma(j) \;\equiv\; \gamma_{uni}(j) = \hat{a}\gamma^{(0)}_{uni}(j) + \hat{a}^2\gamma^{(1)}_{uni}(j) + \hat{a}^3\gamma^{(2)}_{uni}(j) + \ldots,$$

[KLV '03]

$$\frac{1}{8}\,\gamma^{(1)}_{uni}(j+2) = (S_3(j) + S_{-3}(j)) - 2\,S_{-2,1}(j) + 2\,S_1(j)(S_2(j) + S_{-2}(j))$$

$$\Gamma^{(1)} = \begin{pmatrix} \gamma^{(1)}_{gg} & \gamma^{(1)}_{g\lambda} & \gamma^{(1)}_{g\phi} \\ \gamma^{(1)}_{\lambda g} & \gamma^{(1)}_{\lambda\lambda} & \gamma^{(1)}_{\lambda\phi} \\ \gamma^{(1)}_{\phi g} & \gamma^{(1)}_{\phi\lambda} & \gamma^{(1)}_{\phi\phi} \end{pmatrix} \qquad \widetilde{\Gamma}^{(1)} = \begin{pmatrix} \tilde{\gamma}^{(1)}_{gg} & \tilde{\gamma}^{(1)}_{g\lambda} \\ \tilde{\gamma}^{(1)}_{\lambda g} & \tilde{\gamma}^{(1)}_{\lambda\lambda} \end{pmatrix}$$

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Nested harmonic sums (level $\ell = |a| + |b| + |c| + \ldots$):

$$S_a(j) \;=\; \sum_{k=1}^{j} \frac{(\mathrm{sign}(a))^k}{k^a}, \qquad S_{a,b,c,\cdots}(j) \;=\; \sum_{k=1}^{j} \frac{(\mathrm{sign}(a))^k}{k^a}\, S_{b,c,\cdots}(k)$$

Two-loop result in $\mathcal{N}=4$ SYM theory:

[KLV '03]

$$\frac{1}{8}\,\gamma_{uni}^{(1)}(j+2) = -S_3(j) - S_{-3}(j) + 2\,S_{1,-2}(j) + 2\,S_{2,1}(j) + 2\,S_{1,2}(j)$$

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Two-loop result in QCD:

$$\mathbf{N}_{\pm}\,S_{\vec{m}} \;=\; S_{\vec{m}}(N\pm 1)$$

$$\begin{aligned}\gamma_{\rm ns}^{(1)+}(N) \;&= 4C_AC_F\left(2\mathbf{N}_+S_3-\frac{17}{24}-2S_{-3}-\frac{28}{3}S_1+(\mathbf{N}_-+\mathbf{N}_+)\left[\frac{151}{18}S_1+2S_{1,-2}-\frac{11}{6}S_2\right]\right)\\ &+4C_Fn_f\left(\frac{1}{12}+\frac{4}{3}S_1-(\mathbf{N}_-+\mathbf{N}_+)\left[\frac{11}{9}S_1-\frac{1}{3}S_2\right]\right)+4{C_F}^2\left(4S_{-3}+2S_1+2S_2-\frac{3}{8}\right.\\ &\left.+\mathbf{N}_-\left[S_2+2S_3\right]-(\mathbf{N}_-+\mathbf{N}_+)\left[S_1+4S_{1,-2}+2S_{1,2}+2S_{2,1}+S_3\right]\right)\end{aligned}$$

Two loops: QCD vs. $\mathcal{N}=4$ SYM theory

Two-loop result in $\mathcal{N}=4$ SYM theory:

[KLV '03]

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$$C_A = N_c, \quad C_F = N_c, \quad n_f = 2N_c\times 0$$

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[A.Kotikov, L.Lipatov arXiv:hep-ph/0112346]

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$$C_A = N_c, \quad C_F = N_c, \quad n_f = 2N_c \times 0$$

Main result of two-loop calculations:

confirmation of maximal transcendentality principle [KL '02]

$$S_{a,b,c,\cdots}(j) \;=\; \sum_{k=1}^{j} \frac{(\mathrm{sign}(a))^k}{k^a}\, S_{b,c,\cdots}(k)$$

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Transcendentality: sum of the absolute values of indices $|a|+|b|+|c|+\ldots$

$\mathcal{N}=4$ SYM theory: Maximal transcendentality

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Eigenvalues of the anomalous dimension matrix for twist-2 operators in $\mathcal{N}=4$ SYM theory are expressed only through harmonic sums with maximal transcendentality

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Eigenvalues of the anomalous dimension matrix for twist-2 operators in $\mathcal{N}=4$ SYM theory are expressed only through harmonic sums with maximal transcendentality

In ℓ order of perturbative theory is equal to $2\ell-1$

$\mathcal{N} = 4$ SYM theory: Maximal transcendentality

Main result of two-loop calculations:

confirmation of maximal transcendentality principle [KL '02]

$$S_{a,b,c,\cdots}(j) \;=\; \sum_{k=1}^{j} \frac{(\mathrm{sign}(a))^k}{k^a}\, S_{b,c,\cdots}(k)$$

Transcendentality: sum of the absolute values of indices $|a| + |b| + |c| + \ldots$

Eigenvalues of the anomalous dimension matrix for twist-2 operators in $\mathcal{N} = 4$ SYM theory are expressed only through harmonic sums with maximal transcendentality

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$\mathcal{N} = 4$ SYM theory: Maximal transcendentality

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One loop: S_1
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Using the maximal transcendentality principle we can obtain the universal anomalous dimension in $\mathcal{N}=4$ SYM theory without any calculations from the results obtained in QCD

Three-loop anomalous dimension in QCD: 10 years

[Moch, Vermaseren, Vogt '04]

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$$
\begin{aligned}
\gamma_{\rm ns}^{(2)+}(N) \;=\; & 16 C_A C_F n_f \Big(\frac{3}{2}\zeta_3 - \frac{5}{4} + \frac{10}{9}S_{-3} - \frac{10}{9}S_3 + \frac{4}{3}S_{1,-2} - \frac{2}{3}S_{-4} + 2S_{1,1} - \frac{25}{9}S_2 + \frac{257}{27}S_1 - \frac{2}{3}S_{-3,1} - \mathbf{N}_+\Big[S_{2,1} - \frac{2}{3}S_{3,1} - \frac{2}{3}S_4\Big] - (\mathbf{N}_+ - 1)\Big[\frac{23}{18}S_3 - S_2\Big] - (\mathbf{N}_- + \mathbf{N}_+)\Big[S_{1,1} \\
& + \frac{1237}{216}S_1 + \frac{11}{18}S_3 - \frac{317}{108}S_2 + \frac{16}{9}S_{1,-2} - \frac{2}{3}S_{1,-2,1} - \frac{1}{3}S_{1,-3} - \frac{1}{2}S_{1,3} - \frac{1}{2}S_{2,1} - \frac{1}{3}S_{2,-2} + S_1\zeta_3 + \frac{1}{2}S_{3,1}\Big]\Big) + 16 C_F {C_A}^2 \Big(\frac{1657}{576} - \frac{15}{4}\zeta_3 + 2S_{-5} + \frac{31}{6}S_{-4} - 4S_{-4,1} - \frac{67}{9}S_{-3} + 2S_{-3,-2} \\
& + \frac{11}{3}S_{-3,1} + \frac{3}{2}S_{-2} - 6S_{-2}\zeta_3 - 2S_{-2,-3} + 3S_{-2,-2} - 4S_{-2,-2,1} + 8S_{-2,1,-2} - \frac{1883}{54}S_1 - 10S_{1,-3} - \frac{16}{3}S_{1,-2} + 12S_{1,-2,1} + 4S_{1,3} - 4S_{2,-2} - \frac{5}{2}S_4 + \frac{1}{2}S_5 + \frac{176}{9}S_2 + \frac{13}{3}S_3 \\
& + (\mathbf{N}_- + \mathbf{N}_+ - 2)\Big[3S_1\zeta_3 + 11S_{1,1} - 4S_{1,1,-2}\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[\frac{9737}{432}S_1 - 3S_{1,-4} + \frac{19}{6}S_{1,-3} + 8S_{1,-3,1} + \frac{91}{9}S_{1,-2} - 6S_{1,-2,-2} - \frac{29}{3}S_{1,-2,1} + 8S_{1,1,-3} - 16S_{1,1,-2,1} - 4S_{1,1,3} - \frac{19}{4}S_{1,3} \\
& + 4S_{1,3,1} + 3S_{1,4} + 8S_{2,-2,1} + 2S_{2,3} - S_{3,-2} + \frac{11}{12}S_{3,1} - S_{4,1} - 4S_{2,-3} + \frac{1}{6}S_{2,-2} - \frac{1967}{216}S_2 + \frac{121}{72}S_3\Big] - (\mathbf{N}_- - \mathbf{N}_+)\Big[3S_2\zeta_3 + 7S_{2,1} - 3S_{2,1,-2} + 2S_{2,-2,1} - \frac{1}{4}S_{2,3} - \frac{3}{2}S_{3,-2} - \frac{29}{6}S_{3,1} \\
& + \frac{11}{4}S_{4,1} + \frac{1}{2}S_{2,-3} - S_{2,-2}\Big] + \mathbf{N}_+\Big[\frac{28}{9}S_3 - \frac{2376}{216}S_2 - \frac{8}{3}S_4 - \frac{5}{2}S_5\Big]\Big) + 16 C_F n_f^2 \Big(\frac{17}{144} - \frac{13}{27}S_1 + \frac{2}{9}S_2 + (\mathbf{N}_- + \mathbf{N}_+)\Big[\frac{2}{9}S_1 - \frac{11}{54}S_2 + \frac{1}{18}S_3\Big]\Big) + 16 {C_F}^2 C_A \Big(\frac{45}{4}\zeta_3 - \frac{151}{64} - 10S_{-5} \\
& - \frac{89}{6}S_{-4} + 20S_{-4,1} + \frac{134}{9}S_{-3} - 2S_{-3,-2} - \frac{31}{3}S_{-3,1} + 2S_{-3,2} - \frac{9}{2}S_{-2} + 18S_{-2}\zeta_3 + 10S_{-2,-3} - 6S_{-2,-2} + 8S_{-2,-2,1} - 28S_{-2,1,-2} + 46S_{1,-3} + \frac{26}{3}S_{1,-2} - 48S_{1,-2,1} + \frac{28}{3}S_{1,2} - \frac{185}{6}S_3 \\
& - 8S_{1,3} + 2S_{3,-2} - 4S_5 - (\mathbf{N}_- + \mathbf{N}_+ - 2)\Big[9S_1\zeta_3 - \frac{133}{36}S_1 + \frac{209}{6}S_{1,1} - 14S_{1,1,-2} - \frac{242}{18}S_2 + 9S_{2,-2} + \frac{33}{4}S_4 - 3S_{3,1} + \frac{14}{3}S_{2,1}\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[17S_{1,-4} - \frac{107}{6}S_{1,-3} - 32S_{1,-3,1} \\
& - \frac{173}{9}S_{1,-2} + 16S_{1,-2,-2} + \frac{103}{3}S_{1,-2,1} - 2S_{1,-2,2} - 36S_{1,1,-3} + 56S_{1,1,-2,1} + 8S_{1,1,3} - \frac{109}{9}S_{1,2} - 4S_{1,2,-2} + \frac{43}{3}S_{1,3} - 8S_{1,3,1} - 11S_{1,4} + \frac{11}{3}S_{2,2} + 21S_{2,-3} - 30S_{2,-2,1} - 4S_{2,1,-2} \\
& - 5S_{2,3} - S_{4,1} + \frac{31}{6}S_{2,-2} - \frac{67}{9}S_{2,1}\Big] + (\mathbf{N}_- - \mathbf{N}_+)\Big[9S_2\zeta_3 + 2S_{2,-3} + 4S_{2,-2,1} - 12S_{2,1,-2} - 2S_{2,3} + 13S_{4,1} + \frac{1}{2}S_{2,-2} + \frac{11}{2}S_4 - \frac{33}{2}S_{3,1} + \frac{59}{9}S_3 + \frac{127}{6}S_{2,1} - \frac{1153}{72}S_2\Big] + \mathbf{N}_+\Big[8S_{3,-2} \\
& + \frac{4}{3}S_{3,1} - 2S_{3,2} + 14S_5 + \frac{23}{6}S_4 + \frac{73}{3}S_3 + \frac{151}{24}S_2\Big]\Big) + 16 {C_F}^2 n_f \Big(\frac{23}{16} - \frac{3}{2}\zeta_3 + \frac{4}{3}S_{-3,1} - \frac{59}{36}S_2 + \frac{4}{3}S_{-4} - \frac{20}{9}S_{-3} + \frac{20}{9}S_1 - \frac{8}{3}S_{1,-2} - \frac{8}{3}S_{1,1} - \frac{4}{3}S_{1,2} + \mathbf{N}_+\Big[\frac{25}{9}S_3 - \frac{4}{3}S_{3,1} - \frac{1}{3}S_4\Big] \\
& - (\mathbf{N}_+ - 1)\Big[\frac{67}{36}S_2 - \frac{4}{3}S_{2,1} + \frac{4}{3}S_3\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[S_1\zeta_3 - \frac{325}{144}S_1 - \frac{2}{3}S_{1,-3} + \frac{32}{9}S_{1,-2} - \frac{4}{3}S_{1,-2,1} + \frac{4}{3}S_{1,1} + \frac{16}{9}S_{1,2} - \frac{4}{3}S_{1,3} + \frac{11}{18}S_2 - \frac{2}{3}S_{2,-2} + \frac{10}{9}S_{2,1} + \frac{1}{2}S_4 - \frac{2}{3}S_{2,2} - \frac{8}{9}S_3\Big]\Big) \\
& + 16 {C_F}^3 \Big(12S_{-5} - \frac{29}{32} - \frac{15}{2}\zeta_3 + 9S_{-4} - 24S_{-4,1} - 4S_{-3,-2} + 6S_{-3,1} - 4S_{-3,2} + 3S_{-2} + 25S_3 - 12S_{-2}\zeta_3 - 12S_{-2,-3} + 24S_{-2,1,-2} - 52S_{1,-3} + 4S_{1,-2} + 48S_{1,-2,1} - 4S_{3,-2} + \frac{67}{2}S_2 - 17S_4 \\
& + (\mathbf{N}_- + \mathbf{N}_+ - 2)\Big[6S_1\zeta_3 - \frac{31}{8}S_1 + 35S_{1,1} - 12S_{1,1,-2} + S_{1,2} + 10S_{2,-2} + S_{2,1} + 2S_{2,2} - 2S_{3,1} - 3S_5\Big] + (\mathbf{N}_- + \mathbf{N}_+)\Big[23S_{1,-3} - 22S_{1,-4} + 32S_{1,-3,1} - 2S_{1,-2} - 8S_{1,-2,-2} - 30S_{1,-2,1} - 6S_{1,3} \\
& + 4S_{1,-2,2} + 40S_{1,1,-3} - 48S_{1,1,-2,1} + 8S_{1,2,-2} + 4S_{1,2,2} + 8S_{1,3,1} + 4S_{1,4} + 28S_{2,-2,1} + 4S_{2,1,2} + 4S_{2,2,1} + 4S_{3,1,1} - 4S_{3,2} + 8S_{2,1,-2} - 26S_{2,-3} - 2S_{2,3} - 4S_{3,-2} - 3S_{2,-2} - 3S_{2,2} + \frac{3}{2}S_4\Big] \\
& + (\mathbf{N}_- - \mathbf{N}_+)\Big[12S_{2,1,-2} - 6S_2\zeta_3 - 2S_{2,-3} + 3S_{2,3} + 2S_{3,-2} - \frac{81}{4}S_{2,1} + 14S_{3,1} - 5S_{2,-2} - \frac{1}{2}S_{2,2} + \frac{15}{8}S_2 + \frac{1}{2}S_3 - 13S_{4,1} + 4S_5\Big] + \mathbf{N}_+\Big[14S_4 - \frac{265}{8}S_2 - \frac{87}{4}S_3 - 4S_{4,1} - 4S_5\Big]\Big)
\end{aligned}
$$

Three-loop anomalous dimension in QCD: 10 years

[Moch, Vermaseren, Vogt '04]

$$
\begin{aligned}
\gamma_{\rm ns}^{(2)+}(N) = {} & 16C_AC_Fn_f\Big(\frac{3}{2}\zeta_3-\frac{5}{4}+\frac{10}{9}S_{-3}-\frac{10}{9}S_3+\frac{4}{3}S_{1,-2}-\frac{2}{3}S_{-4}+2S_{1,1}-\frac{25}{9}S_2+\frac{257}{27}S_1-\frac{2}{3}S_{-3,1}-\mathbf{N}_+\Big[S_{2,1}-\frac{2}{3}S_{3,1}-\frac{2}{3}S_4\Big]-(\mathbf{N}_+-1)\Big[\frac{23}{18}S_3-S_2\Big]-(\mathbf{N}_-+\mathbf{N}_+)\Big[S_{1,1} \\
& +\frac{1237}{216}S_1+\frac{11}{18}S_3-\frac{317}{108}S_2+\frac{16}{9}S_{1,-2}-\frac{2}{3}S_{1,-2,1}-\frac{1}{3}S_{1,-3}-\frac{1}{2}S_{1,3}-\frac{1}{2}S_{2,1}-\frac{1}{3}S_{2,-2}+S_1\zeta_3+\frac{1}{2}S_{3,1}\Big]\Big)+16C_FC_A{}^2\Big(\frac{1657}{576}-\frac{15}{4}\zeta_3+2S_{-5}+\frac{31}{6}S_{-4}-4S_{-4,1}-\frac{67}{9}S_{-3}+2S_{-3,-2} \\
& +\frac{11}{3}S_{-3,1}+\frac{3}{2}S_{-2}-6S_{-2}\zeta_3-2S_{-2,-3}+3S_{-2,-2}-4S_{-2,-2,1}+8S_{-2,1,-2}-\frac{1883}{54}S_1-10S_{1,-3}-\frac{16}{3}S_{1,-2}+12S_{1,-2,1}+4S_{1,3}-4S_{2,-2}-\frac{5}{2}S_4+\frac{1}{2}S_5+\frac{176}{9}S_2+\frac{13}{3}S_3 \\
& +(\mathbf{N}_-+\mathbf{N}_+-2)\Big[3S_1\zeta_3+11S_{1,1}-4S_{1,1,-2}\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[\frac{9737}{432}S_1-3S_{1,-4}+\frac{19}{6}S_{1,-3}+8S_{1,-3,1}+\frac{91}{9}S_{1,-2}-6S_{1,-2,-2}-\frac{29}{3}S_{1,-2,1}+8S_{1,1,-3}-16S_{1,1,-2,1}-4S_{1,1,3}-\frac{19}{4}S_{1,3} \\
& +4S_{1,3,1}+3S_{1,4}+8S_{2,-2,1}+2S_{2,3}-S_{3,-2}+\frac{11}{12}S_{3,1}-S_{4,1}-4S_{2,-3}+\frac{1}{6}S_{2,-2}-\frac{1967}{216}S_2+\frac{121}{72}S_3\Big]-(\mathbf{N}_--\mathbf{N}_+)\Big[3S_2\zeta_3+7S_{2,1}-3S_{2,1,-2}+2S_{2,-2,1}-\frac{1}{4}S_{2,3}-\frac{3}{2}S_{3,-2}-\frac{29}{6}S_{3,1} \\
& +\frac{11}{4}S_{4,1}+\frac{1}{2}S_{2,-3}-S_{2,-2}\Big]+\mathbf{N}_+\Big[\frac{28}{9}S_3-\frac{2376}{216}S_2-\frac{8}{3}S_4-\frac{5}{2}S_5\Big]\Big)+16C_Fn_f^2\Big(\frac{17}{144}-\frac{13}{27}S_1+\frac{2}{9}S_2+(\mathbf{N}_-+\mathbf{N}_+)\Big[\frac{2}{9}S_1-\frac{11}{54}S_2+\frac{1}{18}S_3\Big]\Big)+16C_F{}^2C_A\Big(\frac{45}{4}\zeta_3-\frac{151}{64}-10S_{-5} \\
& -\frac{89}{6}S_{-4}+20S_{-4,1}+\frac{134}{9}S_{-3}-2S_{-3,-2}-\frac{31}{3}S_{-3,1}+2S_{-3,2}-\frac{9}{2}S_{-2}+18S_{-2}\zeta_3+10S_{-2,-3}-6S_{-2,-2}+8S_{-2,-2,1}-28S_{-2,1,-2}+46S_{1,-3}+\frac{26}{3}S_{1,-2}-48S_{1,-2,1}+\frac{28}{3}S_{1,2}-\frac{185}{6}S_3 \\
& -8S_{1,3}+2S_{3,-2}-4S_5-(\mathbf{N}_-+\mathbf{N}_+-2)\Big[9S_1\zeta_3-\frac{133}{36}S_1+\frac{209}{6}S_{1,1}-14S_{1,1,-2}-\frac{242}{18}S_2+9S_{2,-2}+\frac{33}{4}S_4-3S_{3,1}+\frac{14}{3}S_{2,1}\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[17S_{1,-4}-\frac{107}{6}S_{1,-3}-32S_{1,-3,1} \\
& -\frac{173}{9}S_{1,-2}+16S_{1,-2,-2}+\frac{103}{3}S_{1,-2,1}-2S_{1,-2,2}-36S_{1,1,-3}+56S_{1,1,-2,1}+8S_{1,1,3}-\frac{109}{9}S_{1,2}-4S_{1,2,-2}+\frac{43}{3}S_{1,3}-8S_{1,3,1}-11S_{1,4}+\frac{11}{3}S_{2,2}+21S_{2,-3}-30S_{2,-2,1}-4S_{2,1,-2} \\
& -5S_{2,3}-S_{4,1}+\frac{31}{6}S_{2,-2}-\frac{67}{9}S_{2,1}\Big]+(\mathbf{N}_--\mathbf{N}_+)\Big[9S_2\zeta_3+2S_{2,-3}+4S_{2,-2,1}-12S_{2,1,-2}-2S_{2,3}+13S_{4,1}+\frac{1}{2}S_{2,-2}+\frac{11}{2}S_4-\frac{33}{2}S_{3,1}+\frac{59}{9}S_3+\frac{127}{6}S_{2,1}-\frac{1153}{72}S_2\Big]+\mathbf{N}_+\Big[8S_{3,-2} \\
& +\frac{4}{3}S_{3,1}-2S_{3,2}+14S_5+\frac{23}{6}S_4+\frac{73}{3}S_3+\frac{151}{24}S_2\Big]\Big)+16C_F{}^2n_f\Big(\frac{23}{16}-\frac{3}{2}\zeta_3+\frac{4}{3}S_{-3,1}-\frac{59}{36}S_2+\frac{4}{3}S_{-4}-\frac{20}{9}S_{-3}+\frac{20}{9}S_1-\frac{8}{3}S_{1,-2}-\frac{8}{3}S_{1,1}-\frac{4}{3}S_{1,2}+\mathbf{N}_+\Big[\frac{25}{9}S_3-\frac{4}{3}S_{3,1}-\frac{1}{3}S_4\Big] \\
& -(\mathbf{N}_+-1)\Big[\frac{67}{36}S_2-\frac{4}{3}S_{2,1}+\frac{4}{3}S_3\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[S_1\zeta_3-\frac{325}{144}S_1-\frac{2}{3}S_{1,-3}+\frac{32}{9}S_{1,-2}-\frac{4}{3}S_{1,-2,1}+\frac{4}{3}S_{1,1}+\frac{16}{9}S_{1,2}-\frac{4}{3}S_{1,3}+\frac{11}{18}S_2-\frac{2}{3}S_{2,-2}+\frac{10}{9}S_{2,1}+\frac{1}{2}S_4-\frac{2}{3}S_{2,2}-\frac{8}{9}S_3\Big]\Big) \\
& +16C_F{}^3\Big(12S_{-5}-\frac{29}{32}-\frac{15}{2}\zeta_3+9S_{-4}-24S_{-4,1}-4S_{-3,-2}+6S_{-3,1}-4S_{-3,2}+3S_{-2}+25S_3-12S_{-2}\zeta_3-12S_{-2,-3}+24S_{-2,1,-2}-52S_{1,-3}+4S_{1,-2}+48S_{1,-2,1}-4S_{3,-2}+\frac{67}{2}S_2-17S_4 \\
& +(\mathbf{N}_-+\mathbf{N}_+-2)\Big[6S_1\zeta_3-\frac{31}{8}S_1+35S_{1,1}-12S_{1,1,-2}+S_{1,2}+10S_{2,-2}+S_{2,1}+2S_{2,2}-2S_{3,1}-3S_5\Big]+(\mathbf{N}_-+\mathbf{N}_+)\Big[23S_{1,-3}-22S_{1,-4}+32S_{1,-3,1}-2S_{1,-2}-8S_{1,-2,-2}-30S_{1,-2,1}-6S_{1,3} \\
& +4S_{1,-2,2}+40S_{1,1,-3}-48S_{1,1,-2,1}+8S_{1,2,-2}+4S_{1,2,2}+8S_{1,3,1}+4S_{1,4}+28S_{2,-2,1}+4S_{2,1,2}+4S_{2,2,1}+4S_{3,1,1}-4S_{3,2}+8S_{2,1,-2}-26S_{2,-3}-2S_{2,3}-4S_{3,-2}-3S_{2,-2}-3S_{2,2}+\frac{3}{2}S_4\Big] \\
& +(\mathbf{N}_--\mathbf{N}_+)\Big[12S_{2,1,-2}-6S_2\zeta_3-2S_{2,-3}+3S_{2,3}+2S_{3,-2}-\frac{81}{4}S_{2,1}+14S_{3,1}-5S_{2,-2}-\frac{1}{2}S_{2,2}+\frac{15}{8}S_2+\frac{1}{2}S_3-13S_{4,1}+4S_5\Big]+\mathbf{N}_+\Big[14S_4-\frac{265}{8}S_2-\frac{87}{4}S_3-4S_{4,1}-4S_5\Big]\Big)
\end{aligned}
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Three-loop anomalous dimension in QCD: 10 years

[Moch, Vermaseren, Vogt '04]

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$$\begin{aligned}\frac{1}{32}\,\gamma^{(2)}_{uni}(j) \quad = \quad & 2\,S_{-3}\,S_2 - S_5 - 2\,S_{-2}\,S_3 - 3\,S_{-5} + 24\,S_{-2,1,1,1} \\ & +6(S_{-4,1} + S_{-3,2} + S_{-2,3}) - 12(S_{-3,1,1} + S_{-2,1,2} + S_{-2,2,1}) \\ & -(S_2 + 2\,S_1^2)(3\,S_{-3} + S_3 - 2\,S_{-2,1}) - S_1(8\,S_{-4} + S_{-2}^2 \\ & +4\,S_2\,S_{-2} + 2\,S_2^2 + 3\,S_4 - 12\,S_{-3,1} - 10\,S_{-2,2} + 16\,S_{-2,1,1})\end{aligned}$$

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&-(S_2+2\,S_1^2)(3\,S_{-3}+S_3-2\,S_{-2,1}) - S_1(8\,S_{-4}+S_{-2}^2 \\
&+4\,S_2\,S_{-2}+2\,S_2^2+3\,S_4-12\,S_{-3,1}-10\,S_{-2,2}+16\,S_{-2,1,1})
\end{aligned}
$$

In particular case $j=2$

$$
\gamma_{uni}(2) = 12\,g^2 - 48\,g^4 + 336\,g^6 + \cdots\,, \qquad g^2 = \frac{g_{YM}^2\,N}{(4\pi^2)^2}
$$

$\mathcal{N} = 4$ SYM theory: Three loops

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& -(S_2 + 2\,S_1^2)(3\,S_{-3} + S_3 - 2\,S_{-2,1}) - S_1(8\,S_{-4} + S_{-2}^2 \\
& +4\,S_2\,S_{-2} + 2\,S_2^2 + 3\,S_4 - 12\,S_{-3,1} - 10\,S_{-2,2} + 16\,S_{-2,1,1})
\end{aligned}
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$$

Confirmation of the result for the anomalous dimension of Konishi in $\mathcal{N} = 4$ SYM theory from integrability (ABA)

Anomalous dimension of twist-2 operators $\mathrm{Tr} Z \mathcal{D}_{\mu_1} \mathcal{D}_{\mu_2} \ldots \mathcal{D}_{\mu_j} Z$

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  [Minahan, Zarembo, Beisert, Staudacher, V.Kazakov, Frolov, Tseytlin, Arutyunov '02-'05]
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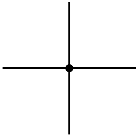
$\mathcal{N}=4$ SYM:

[Fiamberti, Santambrogio, Sieg, Zanon '07]



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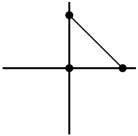

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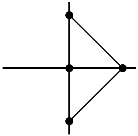

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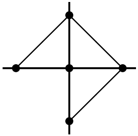

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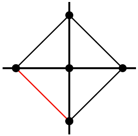

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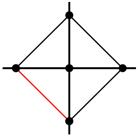

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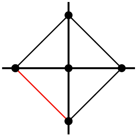

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Superstring theory:

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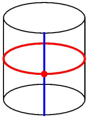

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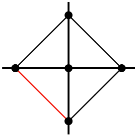

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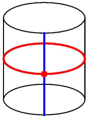

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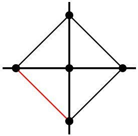

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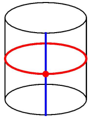

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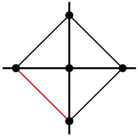

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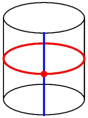

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The result in $\mathcal{N}$=4 SYM theory is the most complicated part of the corresponding QCD result with $C_A=N_c$ and $C_F=N_c$

Eigenvalue of BFKL kernel

[Balitsky, Fadin, Kuraev, Lipatov '75-'79]

$$\frac{\omega}{-4\,g^2} = \Psi\left(-\frac{\gamma^{\mathrm{BFKL}}}{2}\right) + \Psi\left(1 + \frac{\gamma^{\mathrm{BFKL}}}{2}\right) - 2\,\Psi\,(1)$$

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Double-logarithmic equation

[Kirschner, Lipatov '83]

$$\gamma_{\mathrm{NS}}(\omega + \gamma_{\mathrm{NS}}) = -2C_F a_s$$

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Leading N_f contribution

[Gracey '94]

$$\gamma_{\mathrm{NS}}(N) = a_s^2 C_F T_F N_f\left[\frac{2}{3}S_2 - \frac{10}{9}S_1 + \frac{3}{4} + \frac{11N^2+5N-3}{N^2(N+1)^2}\right]$$

$$+\, a_s^3 C_F T_F^2 N_f^2\left[\frac{2}{9}S_3 - \frac{10}{27}S_2 - \frac{2}{27}S_1 + \frac{17}{12} - \frac{12N^4+2N^3-12N^2-2N+3}{N^3(N+1)^3}\right] + \ldots$$

Leading N_f contribution in QCD

Calculation of critical indices in $1/N_f$ expansion [Vasiliev, Pismak, Khonkonen '81]

$$\beta(g) = (D-4)g + \left(\frac{2}{3}T_F N_f - \frac{11}{6}C_A\right)g^2 + \cdots$$

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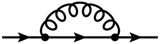
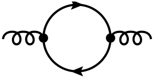
Skeleton Dyson equations with dressed propagators

$0 \;=\; \psi^{-1} \;+$  $\qquad \psi = \dfrac{A\,\not{k}}{(k^2)^{\mu-\alpha}}$

$0 \;=\; \mathcal{A}_{\mu\nu}^{-1} \;+$  $\qquad \mathcal{A}_{\mu\nu} = \dfrac{B\,g_{\mu\nu}}{(k^2)^{\mu-\beta}}$

Calculation of critical indices in $1/N_f$ expansion [Vasiliev, Pismak, Khonkonen '81]

$$\beta(g) = (D-4)g + \left(\frac{2}{3}T_F N_f - \frac{11}{6}C_A\right)g^2 + \cdots$$

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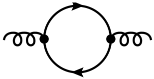
Using one-loop integral from Tkachov '81

$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$ 

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Leading N_f contribution in QCD

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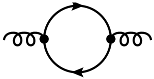
$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$ 

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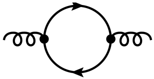
$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$ 

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$$\eta_1 = \frac{(\mu-2)(2\mu-1)\Gamma[2\mu]}{4\,\Gamma[\mu]^2\Gamma[2-\mu]\Gamma[1+\mu]}\,\frac{C_F}{T_F}$$

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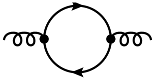

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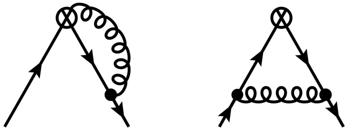
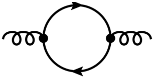
$$\gamma_q = -a_s^2 C_F N_f + \frac{5}{9}a_s^3 C_F N_f^2 + \frac{35}{81}a_s^4 C_F N_f^3 + \left(\frac{83}{243} - \frac{16}{27}\right)a_s^5 C_F N_f^4 + \ldots$$



[Gracey '93]

$$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$$  $$\psi = \frac{A\,\not{k}}{(k^2)^{\mu-\alpha}} \qquad \mathcal{A}_{\mu\nu} = \frac{B\,g_{\mu\nu}}{(k^2)^{\mu-\beta}}$$

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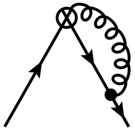
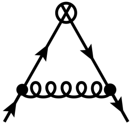
Leading N_f contribution to anomalous dimension in QCD

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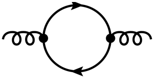
$0 = \mathcal{A}^{-1}_{\mu\nu} +$ 

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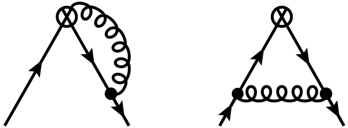

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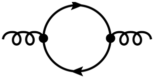
$$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$$  $$\psi = \frac{A\,\not{k}}{(k^2)^{\mu-\alpha}} \qquad \mathcal{A}_{\mu\nu} = \frac{B\,g_{\mu\nu}}{(k^2)^{\mu-\beta}}$$

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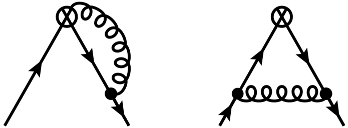
$$\times\left[\frac{\mu(1-\mu)}{(\mu+N-2)(\mu+N-1)} + \frac{2\mu}{\mu-1}\sum_{j=1}^{N}\frac{1}{\mu+j-2} - \frac{2\mu}{(\mu-1)^2+1}\right]$$



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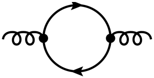
$$0 \quad = \quad \mathcal{A}^{-1}_{\mu\nu} \; + $$  $$\psi = \frac{A\, \not{k}}{(k^2)^{\mu-\alpha}} \qquad \mathcal{A}_{\mu\nu} = \frac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$$

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$$\gamma_{\rm NS}(N) \quad = \quad a_s^2 C_F T_F N_f \left[\frac{2}{3}S_2 - \frac{10}{9}S_1 + \frac{3}{4} + \frac{11N^2+5N-3}{N^2(N+1)^2}\right] +$$

$$+\; a_s^3 C_F T_F^2 N_f^2 \left[\frac{2}{9}S_3 - \frac{10}{27}S_2 - \frac{2}{27}S_1 + \frac{17}{12} - \frac{12N^4+2N^3-12N^2-2N+3}{N^3(N+1)^3}\right] + \ldots$$

[A.Belitsky, S.Derkachov, G.Korchemsky, A.Manashov '03]
[A.Bukhvostov, G.Frolov, L.Lipatov, E.Kuraev 1985]

Conformal operators: $P_n^{(a,b)}$ – Jacobi polynomials

$$\mathcal{O}_j = X_1 (i\partial_+)^n P_n^{(2j_1-1,2j_2-1)}\left(\overleftrightarrow{\mathcal{D}}^+/\partial^+\right) X_2$$

$$\partial \equiv \overrightarrow{\partial} + \overleftarrow{\partial}$$
$$\overleftrightarrow{\mathcal{D}} \equiv \overrightarrow{\mathcal{D}} - \overleftarrow{\mathcal{D}}$$

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Building supermultiplet:

Twist-2 in $\mathcal{N}=4$ SYM: Supermultiplet

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Building supermultiplet: Supersymmetry transformations for $\mathcal{N}=4$ SYM

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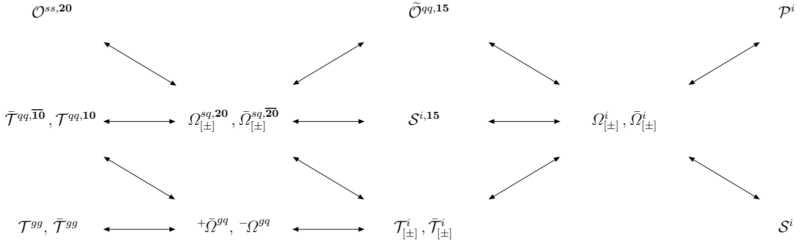
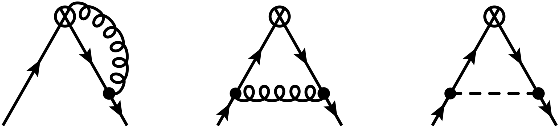
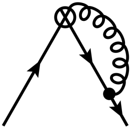
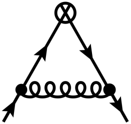


Fermionic contribution to anomalous dimension in $\mathcal{N}$=4 SYM

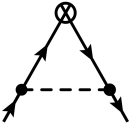




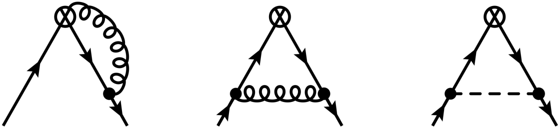
$$8S_1 - 8$$



$$-\frac{4}{N(N+1)}$$



$$2 \times \frac{2}{N(N+1)}$$



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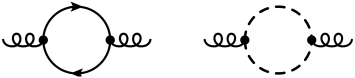

$\mathcal{A}_{\mu\nu} = \frac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$

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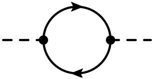

$\psi = \frac{A\, \not{k}}{(k^2)^{\mu-\alpha}} \qquad \phi = \frac{C}{(k^2)^{\mu-\gamma}}$

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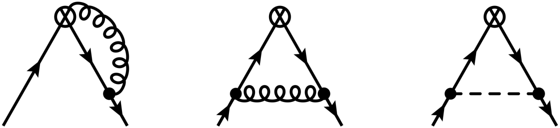
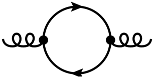
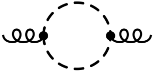
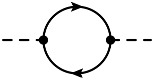
Fermionic contribution to anomalous dimension in $\mathcal{N}$=4 SYM



$8S_1 - 8$ $\qquad -\dfrac{4}{N(N+1)}$ $\qquad 2\times\dfrac{2}{N(N+1)}$

$0 = \mathcal{A}^{-1}_{\mu\nu} +$   $\qquad \mathcal{A}_{\mu\nu} = \dfrac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$

$0 = \phi^{-1} +$  $\qquad \psi = \dfrac{A\, \not{k}}{(k^2)^{\mu-\alpha}} \qquad \phi = \dfrac{C}{(k^2)^{\mu-\gamma}}$

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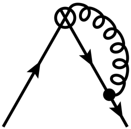
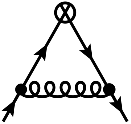
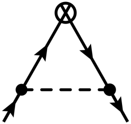
$$\beta_{\text{SUSY}} = \left(\frac{11}{3} - \frac{2}{3}N_f - \frac{1}{6}N_s\right)$$



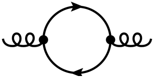
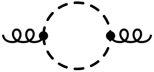
$8S_1 - 8$

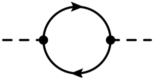


$-\dfrac{4}{N(N+1)}$



$2 \times \dfrac{2}{N(N+1)}$

$0 = \mathcal{A}^{-1}_{\mu\nu} +$   $\mathcal{A}_{\mu\nu} = \dfrac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$

$0 = \phi^{-1} +$  $\psi = \dfrac{A\, \not{k}}{(k^2)^{\mu-\alpha}} \qquad \phi = \dfrac{C}{(k^2)^{\mu-\gamma}}$

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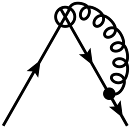
$$\beta_{\text{SUSY}} = \left(\frac{11}{3} - \frac{2}{3}N_f - \frac{1}{6}N_s\right) \quad N_s = 2N_f - 2$$



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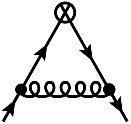

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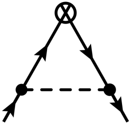
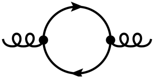
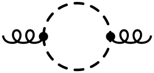
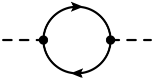


$2 \times \dfrac{2}{N(N+1)}$

$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$   $\qquad \mathcal{A}_{\mu\nu} = \dfrac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$

$0 \;=\; \phi^{-1} \;+$  $\qquad \psi = \dfrac{A\, \not{k}}{(k^2)^{\mu-\alpha}} \qquad \phi = \dfrac{C}{(k^2)^{\mu-\gamma}}$

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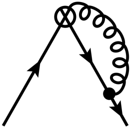
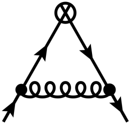
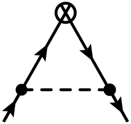
5 diagrams: $\quad 2S_1 - S_2$



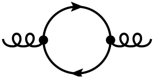
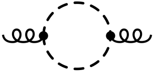
$8S_1 - 8$



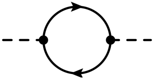
$-\dfrac{4}{N(N+1)}$



$2 \times \dfrac{2}{N(N+1)}$

$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$  

$$\mathcal{A}_{\mu\nu} = \frac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$$

$0 \;=\; \phi^{-1} \;+$ 

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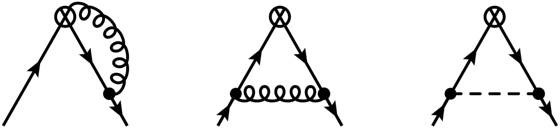
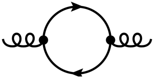
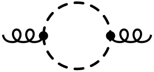
$$\frac{2}{3}S_2 - \frac{10}{9}S_1 + \frac{3}{4} + \frac{11N^2 + 5N - 3}{N^2(N+1)^2}$$



$$8S_1 - 8 \qquad -\frac{4}{N(N+1)} \qquad 2\times\frac{2}{N(N+1)}$$

$$0 \;=\; \mathcal{A}^{-1}_{\mu\nu} \;+$$





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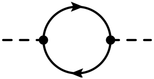

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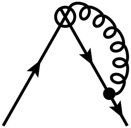
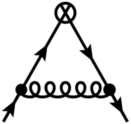
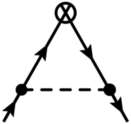
Fermionic contribution to anomalous dimension in $\mathcal{N}$=4 SYM



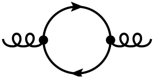
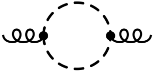
$8S_1 - 8$

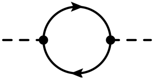


$-\frac{4}{N(N+1)}$



$2 \times \frac{2}{N(N+1)}$

$0 = \mathcal{A}^{-1}_{\mu\nu} +$   $\qquad \mathcal{A}_{\mu\nu} = \frac{B\, g_{\mu\nu}}{(k^2)^{\mu-\beta}}$

$0 = \phi^{-1} +$  $\qquad \psi = \frac{A\, \not{k}}{(k^2)^{\mu-\alpha}} \qquad \phi = \frac{C}{(k^2)^{\mu-\gamma}}$

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$$2S_1 - S_2,\quad 2S_2 - S_3,\quad 2S_3 - S_4 - S_1\zeta_3, \ldots$$

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Solution of the multiploop Baxter equation

Relation with integrability in $\mathcal{N}$=4 SYM

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Solution of the multiploop Baxter equation

$$\Big(u+\frac{i}{2}\Big)Q(u+i)+\Big(u-\frac{i}{2}\Big)Q(u-i)=\Big(u^2-\frac{1}{2}-M(M+1)\Big)Q(u)$$

$$\eta_1 = \frac{1}{N_f} \frac{\Gamma[2\mu-1]}{8\,\Gamma[\mu]^2\Gamma[1-\mu]\Gamma[\mu-1]} \sum_{j=1}^{N} \frac{1}{\mu+j-2}$$

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Solution of the multiploop Baxter equation [Beccaria, Belitsky, Kotikov, Zieme '09]

$$\Big(u + \frac{i}{2}\Big) Q(u+i) + \Big(u - \frac{i}{2}\Big) Q(u-i) = \Big(u^2 - \frac{1}{2} - M(M+1)\Big) Q(u)$$

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$$Q_\delta^{(0)}(u) = {}_3F_2\left(\begin{array}{c} -M,\ M+1+2\delta,\ \frac{1}{2}+iu \\ 1,\ 1 \end{array} \middle| 1 \right)$$

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$$Q_{N_f}(u)=\frac{\Gamma[2\mu-1]}{8\,\Gamma[\mu]^2\Gamma[1-\mu]\Gamma[\mu-1]}Q_{2-\mu}^{(0)}(u),\qquad \epsilon=2-\mu$$

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$$Q_1(u)=\frac{1}{4}\left(2\gamma_0\partial_{\delta_1}-\partial_{\delta_2}^2-\partial_{\delta_3}^2\right)\,{}_3F_2\left(\left.\begin{matrix}-M,\ M+1+2\,\delta_1,\ \frac{1}{2}+iu+\delta_2\\ 1+\delta_1+\delta_2+\delta_3,\ 1+\delta_2-\delta_3\end{matrix}\right|1\right)\Bigg|_{\delta_i=0}$$

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https://rscf.ru/en/project/22-22-00803