Renormalon-chain contributions to two-point correlators of nonlocal quark currents

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Outline

Introduction: What are QCD composite vertices and renormalon chains?

The correlator of two composite functions

$(x,\underline{0})$ moment of the correlator and mesonic distribution amplitudes

$(\underline{0},\underline{0})$ moment of the correlator

Summary

Nonlocal composite vertices in QCD

Meson distribution amplitude (DA)

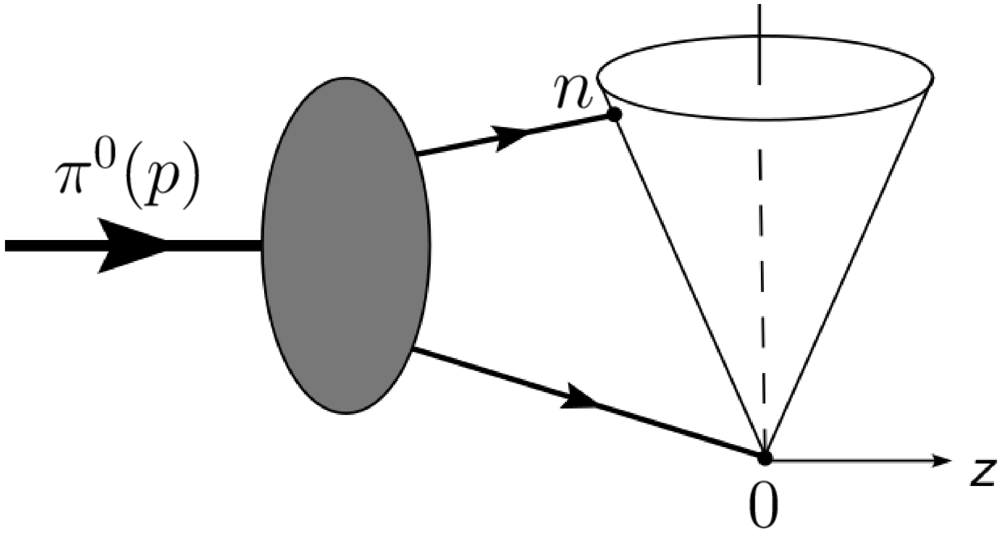
$$f_\pi \int_0^1 \mathrm{d}x e^{i(np)x} \boldsymbol{\Phi_\pi(x)} = \langle 0 | \bar{d}(n) \frac{\hat{\Gamma}}{(np)} [n, 0]\, u(0) | \pi(p) \rangle$$

Bilocal current (on the light cone)

Fourier transform

$$\hat{O}(x, 0)$$

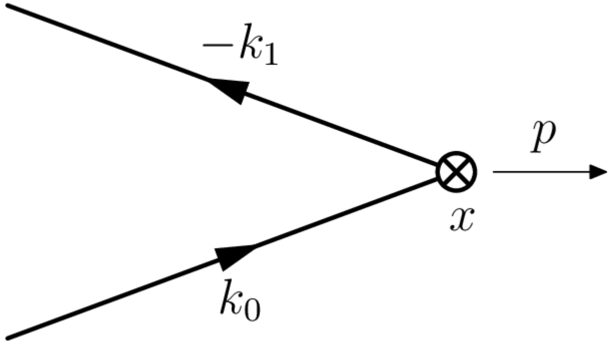
Composite operator



$$[n, 0] = \mathrm{P} \exp \left[i g t_a \int_0^n \mathrm{d}\mathbf{z}^\mu \, A_\mu^a(\mathbf{z}) \right]$$

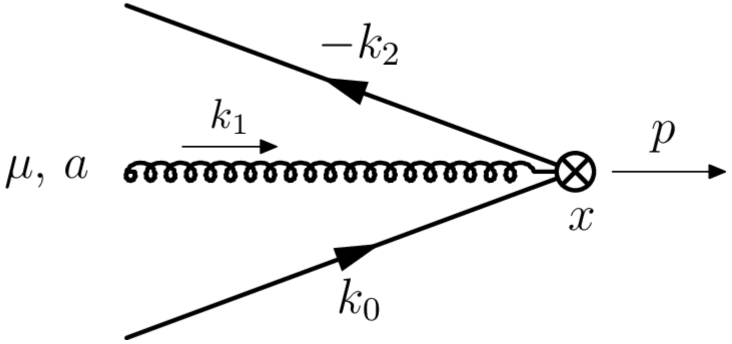
Feynman rules for composite vertices $\otimes$

$\hat{O}(x,0) =$



$= \Gamma\delta(x - \tilde{n}k_0)$ $\quad\quad \Gamma = \tilde{\not{n}}\gamma_5, \quad \sigma_{\mu\nu}\tilde{n}^\nu, \ldots$

$+$



$$= \frac{g}{2}\lambda_a\Gamma\tilde{n}_\mu\frac{1}{x_1}\left[\delta\left(x - x_0 - x_1\right) - \delta\left(x - x_0\right)\right]$$

$+$

…

Light-cone vector

$\tilde{n}^2 = 0, \qquad \tilde{n}p = 1, \qquad p = \sum k_i$

Correlators of composite vertices in QCD

$$= \Pi(x,y;p^2) = \int \mathrm{d}^D\mathbf{z}\, e^{ip\mathbf{z}} \langle 0| \mathrm{T}\left[\hat{O}(x;0)\hat{O}(y;\mathbf{z})\right]|0\rangle$$

The correlator of composite operators describes the perturbative content of DAs

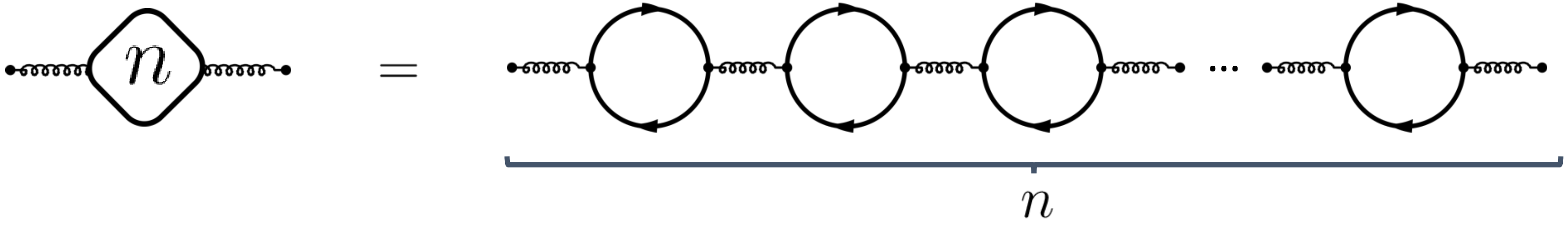
QCD SR

$$\Phi_{\text{meson}}(x) \sim \underset{-p^2\to M^2}{\textbf{Borel transform}} \left[\int_0^1 \mathrm{d}y\, \Pi\big(x,y;p^2\big)\right]$$

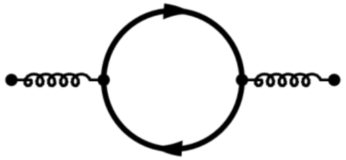
Feynman integrals in QCD after factorization:

$$f_1(x) \star \Pi(x,y;p^2) \star f_2(y) \qquad\qquad f_1(x)\star f_2(x) = \int_0^1 \mathrm{d}x f_1(x) f_2(x)$$

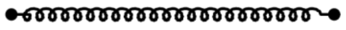
Renormalon-chain correlators



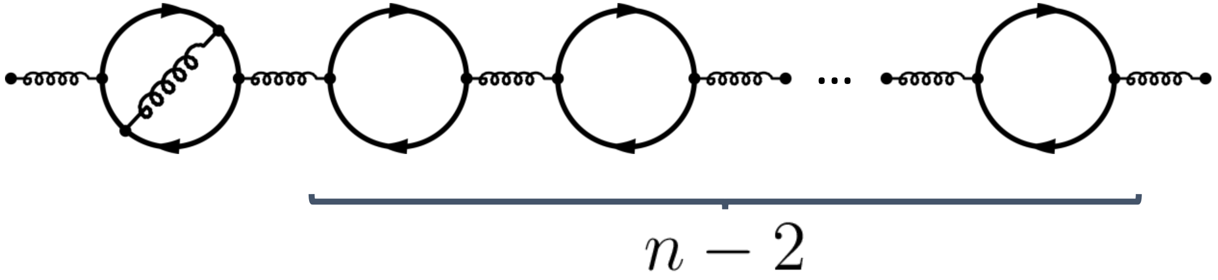
$\sim O(a_s^n \beta_0^n)$



$\sim \frac{4}{3} a_s T_F n_f \frac{h_1(\varepsilon)}{\varepsilon} \left(\frac{-p^2}{\mu^2} \right)^{D/2-2}$



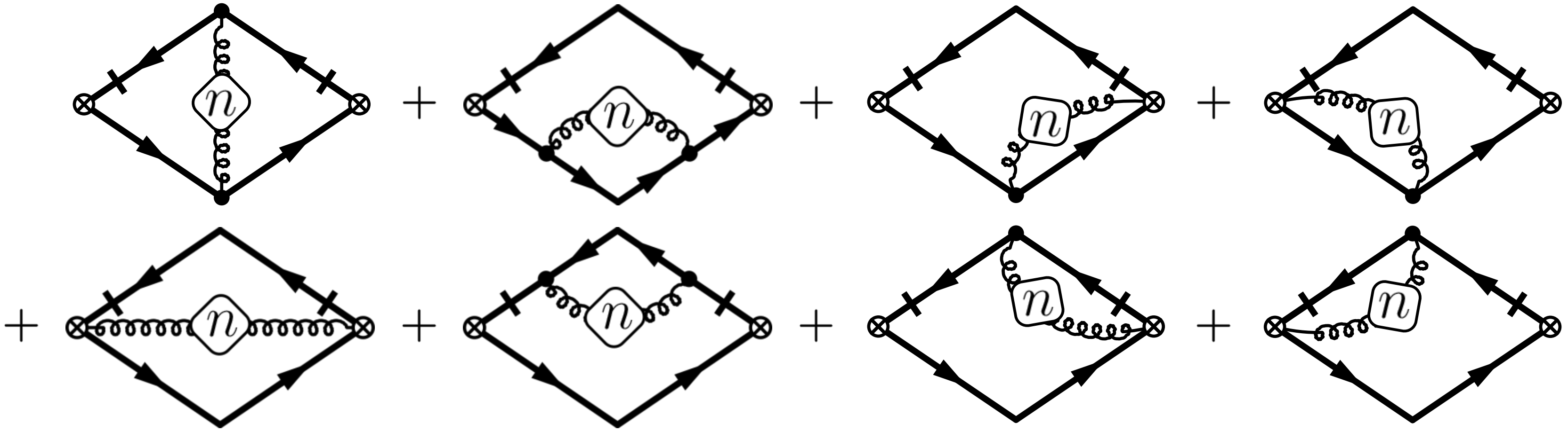
$\frac{4}{3} a_s T_F n_f \rightarrow -a_s \beta_0$



$\sim O(a_s^n C_F n_f^{n-1}) \sim O(a_s^n \beta_1 \beta_0^{n-2})$

The correlator $\Pi_n(x, y)$

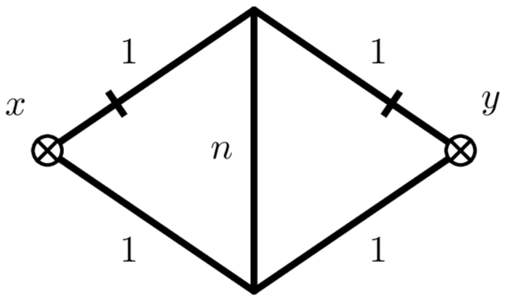
$$-i\frac{a_s}{\pi^2}N_cC_F\boldsymbol{A^n\Pi_n(x,y;L)} =$$



$$L = \ln\frac{-p^2}{\mu^2} \qquad a_s = \frac{\alpha_s}{4\pi} \qquad \boldsymbol{A = \frac{4}{3}a_sT_Fn_f} \text{ or } \boldsymbol{-a_s\beta_0}$$

Two-loop master integral

w **is a light-cone vector**



$$= \int \mathrm{d}^D k_1\, \mathrm{d}^D k_2\, \frac{\delta\left[x - (wk_1)/(wp)\right] \delta\left[y - (wk_2)/(wp)\right]}{k_1^2 k_2^2 (k_1 - p)^2 (k_2 - p)^2 \left[(k_1 - k_2)^2\right]^n} = \frac{(-1)^{1+n}\pi^D}{(-p^2)^{n+4-D}} \boldsymbol{G(n; x, y; D)},$$

$$G(n; \boldsymbol{x}, \boldsymbol{y}; D) = \frac{\Gamma(2 + \dot{n} - \lambda)\Gamma(\dot{n})}{\Gamma(n)\Gamma(2 + \dot{n})} \hat{\mathbf{S}} \left\{ \frac{H(x - y)}{|x - y|^{4-D-n}} \frac{z^{\lambda-1}}{\bar{z}^{\lambda-2}} \left[{}_3\boldsymbol{F_2}\left(\begin{matrix} 1,\ 1,\ \lambda \\ 1 - \dot{n},\ \dot{n} + 2 \end{matrix} \middle| \bar{z} \right) - \frac{\Gamma(2 + \dot{n})\Gamma(1 - \dot{n})\Gamma(n)\Gamma(1 + \dot{n})}{\bar{z}^{-\dot{n}}\Gamma(\lambda)\Gamma(2(\dot{n} + 1))} {}_2F_1\left(\begin{matrix} n,\ \dot{n} + 1 \\ 2(\dot{n} + 1) \end{matrix} \middle| \bar{z} \right) \right] \right\},$$

$$G(n; \boldsymbol{x}, \underline{\mathbf{0}}; D) = -\frac{\Gamma^2(-\dot{n})\Gamma(1 + \dot{n} - \lambda)\Gamma(\lambda)}{\Gamma(n)\Gamma(1 - \dot{n})\Gamma(\lambda - \dot{n})} (x\bar{x})^{\lambda-1} \left\{ \frac{\Gamma(n)\Gamma^2(\dot{n})\Gamma^3(1 - \dot{n})\Gamma(\lambda - \dot{n})}{\Gamma^2(\lambda)\Gamma(2\dot{n})\Gamma(1 - 2\dot{n})\Gamma(-\dot{n})} + \hat{\mathbf{S}} \left[x^{-\dot{n}}\, {}_3\boldsymbol{F_2}\left(\begin{matrix} 1,\ \lambda,\ -\dot{n} \\ 1 - \dot{n},\ \lambda - \dot{n} \end{matrix} \middle| x \right) \right] \right\},$$

$$G(n; \underline{\mathbf{0}}, \underline{\mathbf{0}}; D) = 2 \frac{\Gamma^2(-\dot{n})\Gamma(1 - \lambda + \dot{n})\Gamma^2(\lambda)}{\Gamma(n)\Gamma(1 - \dot{n})\Gamma(2\lambda - \dot{n})} \left[\frac{\Gamma(n)\Gamma(3\lambda - n)}{\Gamma(\lambda)\Gamma(2\lambda)} \pi\dot{n} \cot(\pi\dot{n}) - {}_3\boldsymbol{F_2}\left(\begin{matrix} 1,\ \lambda,\ -\dot{n} \\ 1 - \dot{n},\ 2\lambda - \dot{n} \end{matrix} \middle| 1 \right) \right],$$

where $\lambda = \frac{D}{2} - 1$, $\omega/2 = D - 4 - n$, $\dot{n} = n - \lambda$, $\hat{\mathbf{S}} f(x) = f(x) + f(\bar{x})$, $z = \frac{\bar{x}y}{x\bar{y}}$, and $\bar{x} = 1 - x$.

Mellin transform $f(\underline{a}) = \int_0^1 \mathrm{d}x\, x^a f(x)$

Mikhailov, Volchanskiy, JHEP (2019) 202

The correlator $\Pi_n(x,y)$

$$\frac{a_s}{\pi^2}N_c C_F A^n \Pi'_n(x,y;L;\varepsilon) = i\,\hat{\mathbf{R}}'\left[\langle VV\rangle_n(x,y)\right], \qquad A = \frac{4}{3}a_s T_F n_f \text{ or } -a_s\beta_0$$

$$\Pi'_n(x,y;L;\varepsilon) = \frac{\mathrm{B}(\bar\varepsilon,\varepsilon)e^{-\varepsilon L}}{2(n+1)}\hat{\mathbf{P}}\left[(y\bar y)^{1-\varepsilon}\left(1-\hat{\mathbf{K}}\right)\frac{V(x,y;\varepsilon)_{+(x)}}{\varepsilon^{n+1}h_1(\varepsilon)}\right] + \frac{(1+\varepsilon)\,\hat{\mathbf{S}}\left[H(x-y)I_z(\bar\varepsilon,\varepsilon)\right]}{2(n+1)(n+2)\varepsilon^{n+2}h_2(\varepsilon)} - \frac{(y\bar x + x\bar y - 1)\,\tilde L^{n+2}(|y-x|)}{(n+1)(n+2)}$$
$$-\frac{1}{(n+1)(n+2)}\left\{\frac{V(x,y;\varepsilon)(y\bar y)^{1-\varepsilon}|y-x|^{\varepsilon}}{\varepsilon^{n+3}\mathrm{B}(\bar\varepsilon,\varepsilon)h_2(\varepsilon)} - y\bar y V(x,y;0)\tilde L^{n+2}(|y-x|) + \left[2(y\bar x + x\bar y)-1\right]\frac{\bar\varepsilon\,\hat{\mathbf{S}}\left[H(x-y)I_z(\bar\varepsilon,\varepsilon)\right]}{2\varepsilon^{n+2}h_2(\varepsilon)}\right\}_+$$
$$+\delta(x-y)x\bar x\sum_{k=0}^{n+2}(-)^{n-k}\frac{n!}{k!}\left(1-2^{k-n-3}\right)\hat{\mathbf{S}}\left[(x-\bar x)\tilde L^k(x)\right] - (xy+\bar x\bar y)F^{(n)}(x,y) - (x\bar x + y\bar y)\,G^{(n)}(x,y) + O(\varepsilon)$$

$$V(x,y;\varepsilon) = 2\,\hat{\mathbf{S}}\left[H(y-x)\left(\frac{x}{y}\right)^{1-\varepsilon}\left(1-\varepsilon+\frac{1}{y-x}\right)\right],$$

generalized ERBL evolution kernel

$$\tilde L(x) = L + \ln(x) - \frac{5}{3}, \qquad L = \ln\frac{-p^2}{\mu^2},$$

$$h_m(\varepsilon) = \frac{(1-\varepsilon)^m\Gamma(1+\varepsilon)\Gamma^3(1-\varepsilon)}{(1-2\varepsilon/3)^m(1-2\varepsilon)^m\Gamma(1-2\varepsilon)}$$

one-loop fermion bubble

$$F(x,y;\delta) = (1-\hat{\mathbf{K}})e^{5/3-L}G(x,y;1+\delta),$$

$$G(x,y;\delta) = \hat{\mathbf{S}}\left\{\frac{\pi H(x-y)e^{\delta(L-5/3)}}{2(x\bar y)^{1-\delta}\sin(\pi\delta)}\left[{}_2f_1\left(\begin{matrix}\delta,\ \bar\delta\\ 2\bar\delta\end{matrix}\middle|\,\bar z\right) - \bar z^{\delta}\,{}_3f_2\left(\begin{matrix}1,\ 1,\ 1\\ 1+\delta,\ 1+\bar\delta\end{matrix}\middle|\,\bar z\right)\right]\right\},$$

two-loop master integral in 4 dimensions

$$f(x,y)_{+(x)} = f(x,y) - \delta(x-y)\int_0^1 dt\, f(t,y),$$

plus distribution

$$\hat{\mathbf{S}}\,f(x,y) = f(x,y) + f(\bar x,\bar y), \qquad \hat{\mathbf{P}}\,f(x,y) = f(x,y) + f(y,x), \qquad \hat{\mathbf{K}}\sum_{n=-\infty}^{\infty}a_n\varepsilon^n = \sum_{n=1}^{\infty}\frac{a_{-n}}{\varepsilon^n}$$

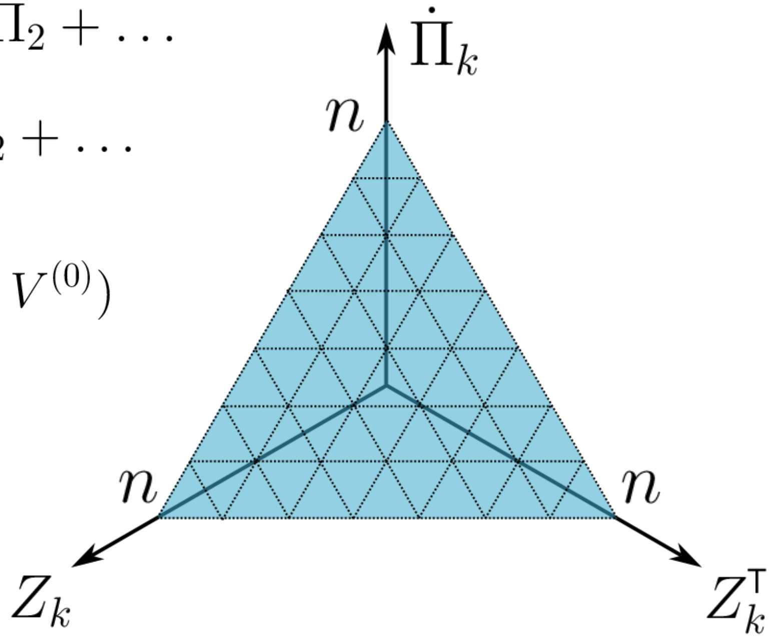
Counterterm structure for $\Pi_n(x,y)$

$$\dot{\Pi}(x,y;L) \overset{\text{def}}{=} \frac{\mathrm{d}}{\mathrm{d}L}\Pi(x,y;L) \qquad \dot{\Pi} = \dot{\Pi}_0 + a_s\dot{\Pi}_1 + a_s^2\dot{\Pi}_2 + \dots$$

Renormalization constant of $Z(x,y)$ composite operator: $Z = \mathbb{1} + a_s Z_1 + a_s^2 Z_2 + \dots$

$$\mathbb{1} = \delta(x-y) \qquad Z_1 = -\frac{1}{\varepsilon}V^{(0)} \qquad Z_2 = -\frac{1}{2\varepsilon}V^{(1)} + \frac{1}{2\varepsilon^2}V^{(0)} \otimes (\beta_0\mathbb{1} + V^{(0)})$$

$$\hat{\mathbf{R}}\,\dot{\Pi}_n = Z \otimes \dot{\Pi}_n(Z_a a_s) \otimes Z^{\mathsf{T}} \quad \Longrightarrow \quad \boxed{\hat{\mathbf{R}}\,\dot{\Pi}_n = \frac{1}{2}\sum_{\sum r=n}\left\{Z_{r_1}, \dot{\Pi}_{r_2} \otimes Z_{r_3}\right\}}$$



Renormalized n-loop correlator is expressed through a m-loop charge-renormalized correlators ($m \leqslant n$) and ERBL evolution kernels of no more than $n-1$ loops.

$$V^{(n)}(x,y) = \frac{\mathrm{d}^n}{\mathrm{d}a^n}V(x,y;a)\Big|_{a=0} \qquad (F_1 \otimes F_2)(x,y) = \int_0^1 \mathrm{d}t\, F_1(x,t)F_2(t,y) \qquad \{F_1, F_2\}(x,y) = (F_1 \otimes F_2)(x,y) + (F_2 \otimes F_1)(x,y)$$

$(x,\underline{0})$ moment of the correlator

Exponential generating function:

$$\sum_{n\geqslant 0}\frac{A^n}{n!}\dot{\Pi}_n(x,\underline{0};L)$$

$$=\hat{\mathbf{S}}\Bigg\{\frac{e^{A(L-5/3)}x^A}{A^2(1+A)(2+A)}\Bigg[-\left[A+x\bar{x}(4+A^2)\right]+2x\bar{x}\frac{(\pi A)^2\cot(\pi A)}{x^A\sin(\pi A)}+x(2\bar{x}+A)\frac{\pi A}{\sin(\pi A)}$$

$$+\frac{2x^2\bar{x}A}{(1+A)^2}{}_3\boldsymbol{F}_2\left(\begin{matrix}1,\,1,\,1-A\\2-A,\,2-A\end{matrix}\middle|\,x\right)-x(2\bar{x}+A)\mathbf{B}_{\boldsymbol{x}}(1-A,A)$$

$$+xA\left[(1+A)(2-x)-2x\bar{x}\right]+x\bar{x}(x-\bar{x})\left(1-\frac{A}{2}x^{-A}\ln\frac{\bar{x}}{x}\right)\Bigg]\Bigg\}$$

$$-\frac{1}{2}\sum_{n>0}\frac{A^n}{[(n+1)!]^2}\left[\left(\frac{d}{da}\right)^{n+1}\int_0^1 dy\,\frac{y\bar{y}V(x,y;a)_{+(x)}}{h_1(a)}\right]_{a=0}$$

$(x,\underline{0})$ moment of the correlator

$$\dot{\Pi}_n(x,\underline{0};L) = \frac{\mathrm{d}}{\mathrm{d}L}\Pi_n(x,\underline{0};L) = (-1)^{n+1}n!\sum_{k=0}^{n}\frac{(-L)^k}{k!}\Pi_n^{k+1}(x,\underline{0})$$

$$\boldsymbol{\Pi_n^{n+1}(x,\underline{0})} = \frac{1}{2}\,\hat{\mathrm{S}}\,(x\ln x) + \delta_{0,n}\left[-\frac{1}{2}\,\hat{\mathrm{S}}\,(x\ln x) + \frac{1}{2}x\bar{x}\left(\frac{\pi^2}{3} - 5 - \boldsymbol{\ln^2\frac{x}{\bar{x}}}\right)\right]$$

$$\boldsymbol{\Pi_n^{n}(x,\underline{0})} = \hat{\mathrm{S}}\Bigg\{x\bar{x}\left(-3\boldsymbol{\mathrm{Li}_3\,x} + \ln x\,\mathrm{Li}_2\,x + \frac{\pi^2}{6}\ln x\right) - \frac{x}{2}\left(\mathrm{Li}_2\,x - \frac{\pi^2}{6} - \frac{1}{2}\ln^2 x + \frac{19}{6}\ln x\right) + \delta_{1,n}\frac{1}{24}x\ln x(7+3\ln x)$$
$$+\,\delta_{1,n}\frac{1}{2}x\bar{x}\left[\boldsymbol{\mathrm{Li}_3\,x} - \ln x\,\mathrm{Li}_2\,x + \frac{1}{6}\ln^3 x - \frac{1}{2}\ln x\ln^2\bar{x} - \frac{5}{6}\ln^2 x - \frac{5}{3}\ln x\ln\bar{x} - \frac{5\pi^2}{36} + \frac{7}{12}\right]\Bigg\}$$

$$\boldsymbol{\Pi_n^{n-1}(x,\underline{0})} \sim \hat{\mathrm{S}}\,\boldsymbol{\mathrm{Li}_4\,x} \qquad\qquad \boldsymbol{\Pi_n^{n-2}(x,\underline{0})} \sim \hat{\mathrm{S}}\,\boldsymbol{\mathrm{H}_{3,2}(x)}$$

$(x,\underline{0})$ moment of the correlator

$$\dot{\Pi}_n(x,\underline{0};L) = \frac{\mathrm{d}}{\mathrm{d}L}\Pi_n(x,\underline{0};L) = (-1)^{n+1} n! \sum_{k=0}^{n} \frac{(-L)^k}{k!}\Pi_n^{k+1}(x,\underline{0})$$

$$\Pi_n^{k+1}(x,\underline{0}) \sim \hat{\mathrm{S}}\,\mathrm{H}_\mu(x), \qquad \mu = m_1,\ldots m_r : \sum_{i=1}^{r} m_i = n-k+2$$

Kalmykov, Kniehl, NPB 809 (2009) 365

Harmonic polylogarithms without trailing zeroes or negative indices:

$$\mathrm{H}_{\mathbf{k}}(z) = \mathrm{Li}_{\mathbf{k}}(z) = \sum_{\sigma} z^{m_1} \prod_{i=1}^{n} \frac{1}{m_i^{k_i}}, \qquad |z| < 1,$$

Goncharov, Math. Res. Letters 4 (1997) 617

$$\mathbf{k} = k_1,\ldots k_n, \qquad \sigma = \{\forall m_i \in \mathbb{N}, i = 1,\ldots n : m_1 > m_2 > \cdots > m_n > 0\}$$

$$\mathrm{H}_{\mathbf{k}}(z) = \int_0^z dt\, \underbrace{\omega_0(t) \circ \cdots \circ \omega_0}_{k_1-1} \circ\, \omega_1 \circ \underbrace{\omega_0 \circ \cdots \circ \omega_0}_{k_2-1} \circ\, \omega_1 \circ \cdots \circ \underbrace{\omega_0 \circ \cdots \circ \omega_0}_{k_n-1} \circ\, \omega_1$$

Remiddi, Vermaseren, IJMPA 15 (2000) 725

$$\omega_0(t) = \frac{1}{t}, \qquad \omega_1(t) = \frac{1}{1-t}, \qquad \omega_{k_1}(t_1) \circ \omega_{k_2} = \omega_{k_1}(t_1) \int_0^{t_1} dt_2\, \omega_{k_2}(t_2)$$

Borel transformation

QCD SR

$$\Phi_{\text{meson}}(x) \sim \underset{-p^2\to M^2}{\textbf{Borel transform}} \left[\Pi\left(x, \underline{0}; L = \ln\frac{-p^2}{\mu^2}\right)\right]$$

$$\hat{\mathsf{B}}\left[f(t)\right](\mu) = \lim_{\substack{t=n\mu \\ n\to\infty}} \frac{(-t)^n}{\Gamma(n)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} f(t),$$

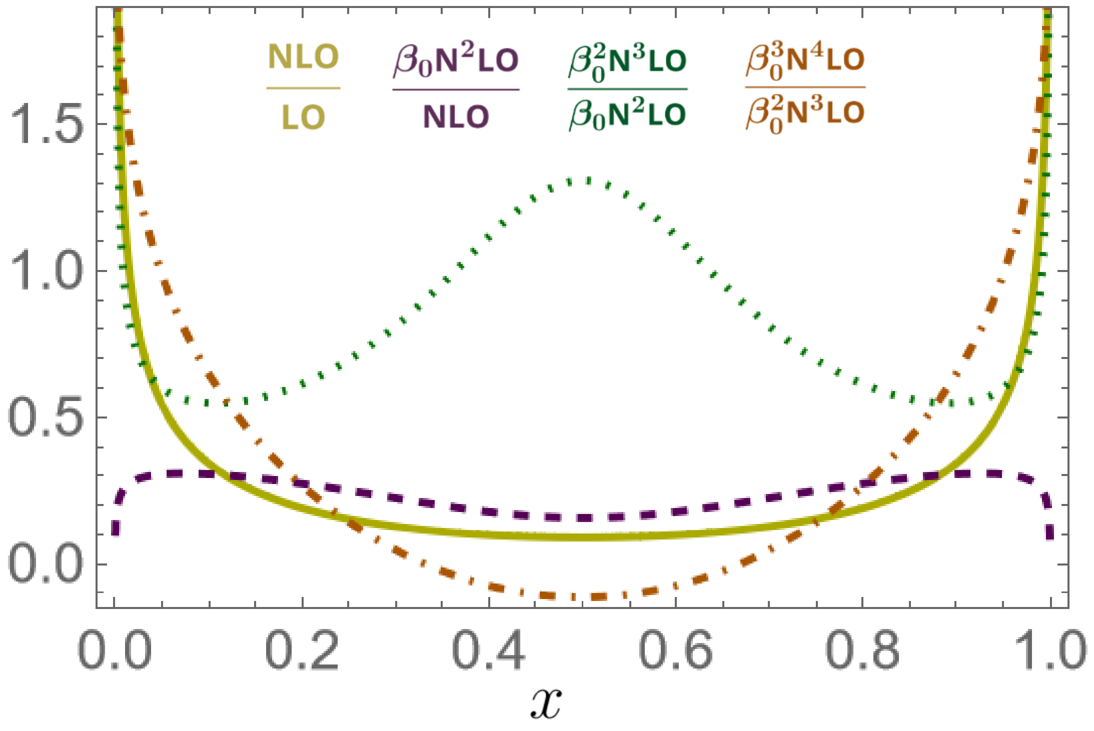
$$\hat{\mathsf{B}}\left[t^{-a}\right](\mu) = \frac{\mu^{-a}}{\Gamma(a)}, \quad a > 0, \qquad \hat{\mathsf{B}}\left[e^{-at}\right](\mu) = \delta(1-\mu a), \quad a > 0.$$

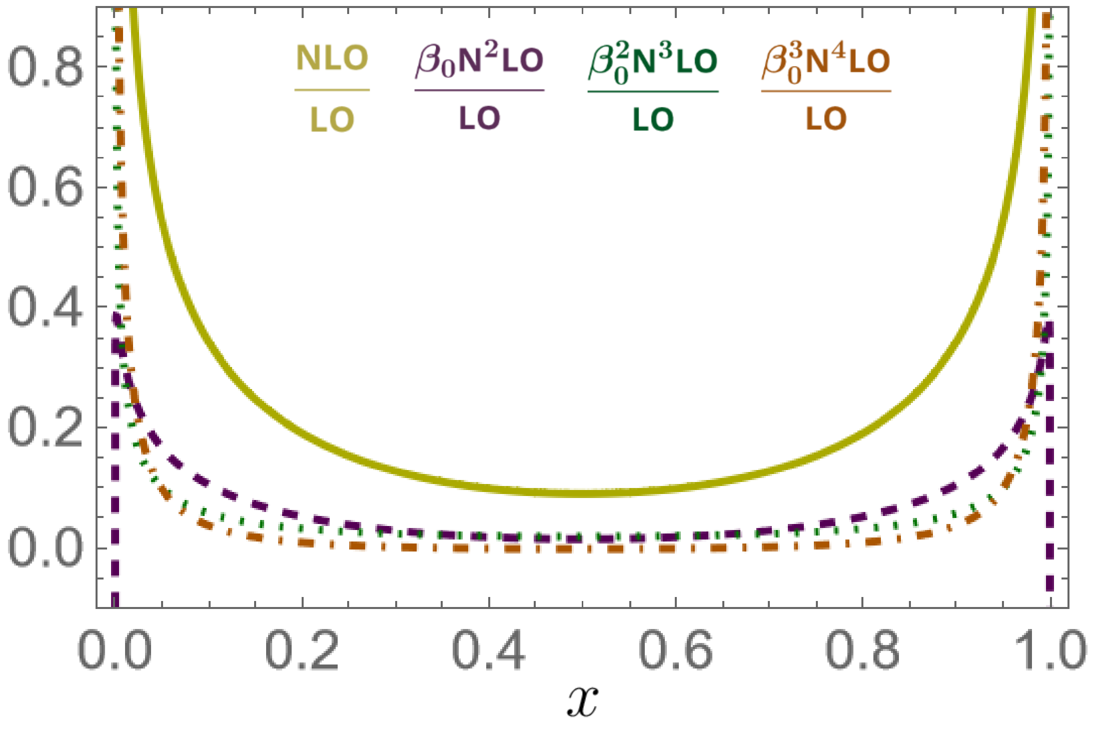
$$\hat{\mathsf{B}}\left[\ln^m(t)\right](\mu) = m(-1)^m \left(\frac{d}{da}\right)^{m-1} \frac{e^{-al}}{\Gamma(1+a)}\Bigg|_{a=0} = -m! \sum_{s=0}^{m-1} \frac{1}{s!} \left[\ln\left(\mu e^{\gamma_{\mathsf{E}}}\right)\right]^s \sum_{\forall\Pi} \prod_{i=1}^{N} \frac{(-\zeta_{p_i})^{r_i}}{p_i^{r_i} r_i!}$$

Here, $\Pi = (p_1^{r_1}, p_2^{r_2}, \ldots, p_N^{r_N})$ is a partition of $n \in \mathbb{N}$,

i.e. $n = \sum_{i=1}^{N} p_i r_i$: $1 < p_1 < p_2 < \cdots < p_N$ with p_i, $r_i \in \mathbb{N}$.

$(x, \underline{0})$ moment of the correlator





$$a_s = \frac{\alpha_s(\mu^2 = 1\ \text{GeV}^2)}{4\pi} = \frac{0.494}{4\pi} \qquad n_f = 3, \qquad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f = 9$$

Agaev et al, PRD 83 (2011) 054020

$(\underline{-1}, \underline{0})$ moment of the correlator

$\left.\dfrac{\hat{\mathrm{B}}\langle V(\underline{-1}) V(\underline{0})\rangle_{\beta_0^{n-1}\mathrm{N}^n\mathrm{LO}}}{\hat{\mathrm{B}}\langle V(\underline{-1}) V(\underline{0})\rangle_{\beta_0^{n-2}\mathrm{N}^{n-1}\mathrm{LO}}}\right\|_{L_{\mathrm{B}}=0}$	
$\dfrac{\mathrm{NLO}}{\mathrm{LO}}$	26%
$\dfrac{\beta_0 \mathrm{N}^2\mathrm{LO}}{\mathrm{LO}}$	7%
$\dfrac{\beta_0^2 \mathrm{N}^3\mathrm{LO}}{\mathrm{LO}}$	5%
$\dfrac{\beta_0^3 \mathrm{N}^4\mathrm{LO}}{\mathrm{LO}}$	5%
$\dfrac{\beta_0^4 \mathrm{N}^5\mathrm{LO}}{\mathrm{LO}}$	9%

(0,0) moment of the correlator

$$\dot{\Pi}_n(\underline{0},\underline{0};L) = (-1)^{n+1} n! \frac{1}{3} \sum_{k=0}^{n} \frac{(-L)^k}{k!} \sum_{m=0}^{n-k} \frac{\left(\frac{5}{3}\right)^r}{r!} \tilde{\boldsymbol{\Psi}}_{\boldsymbol{n-k-r}},$$

Broadhurst, Z. Phys. C 58 (1993) 339
Broadhurst, Kataev, PLB315 (1993) 179

where $\tilde{\boldsymbol{\Psi}}_{\boldsymbol{n}} = (n+1)\left(n + \frac{n+6}{2^{n+3}}\right) - 8 \sum_{\ell=1}^{\left[\frac{n+1}{2}\right]} \ell(1-2^{-2\ell})(1-2^{2\ell-n-2})\zeta_{2\ell+1}$

(0,0) moment of the correlator

$$\dot{\Pi}_n(\underline{0},\underline{0};L) = (-1)^{n+1} n! \frac{1}{3} \sum_{k=0}^{n} \frac{(-L)^k}{k!} \sum_{m=0}^{n-k} \frac{\left(\frac{5}{3}\right)^r}{r!} \tilde{\boldsymbol{\Psi}}_{\boldsymbol{n-k-r}},$$

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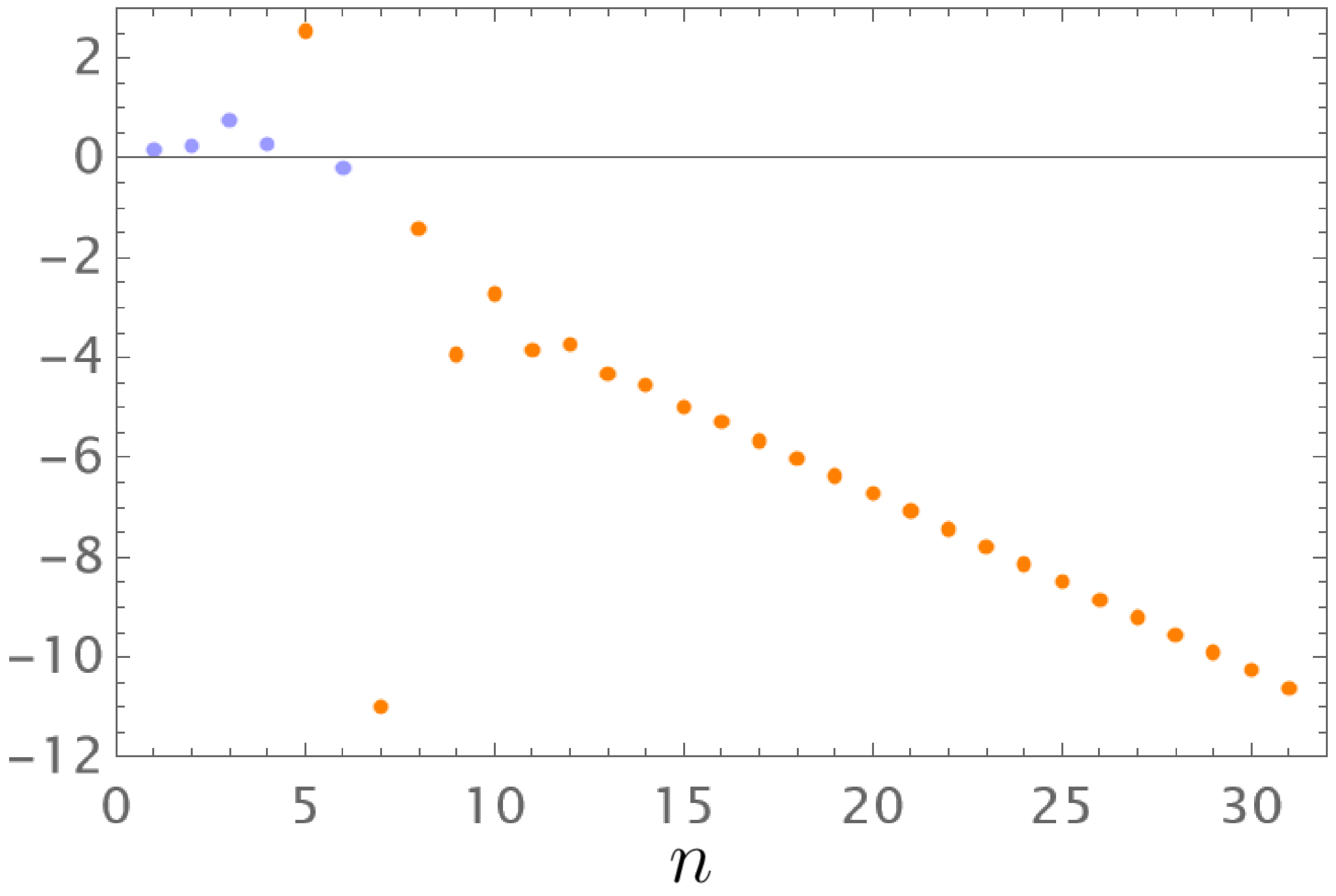
Exponential generating function:

$$\sum_{n \geqslant 0} \frac{A^n}{n!} \dot{\Pi}_n(\underline{0},\underline{0};L)$$

$$= \frac{2}{3} \frac{e^{A(L-5/3)}}{(1+A)(2+A)} \left[\frac{1}{2}\psi_1\left(\frac{1+A}{2}\right) - \frac{1}{2}\psi_1\left(\frac{A}{2}\right) + \frac{\pi^2 \cot(\pi A)}{\sin(\pi A)} + \frac{1}{A^2} - \frac{3+2A}{(1+A)^2(2+A)^2} \right]$$

(0,0) moment of the correlator

$$\frac{(-\beta_0 a_s)^n \Pi_n(\underline{0},\underline{0};L)}{(-\beta_0 a_s)^{n-1}\Pi_{n-1}(\underline{0},\underline{0};L)}$$



Agaev et al, PRD 83 (2011) 054020

$$a_s = \frac{\alpha_s(\mu^2 = 1\ \text{GeV}^2)}{4\pi} = \frac{0.494}{4\pi}$$

$$n_f = 3, \qquad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f = 9$$

($\underline{0}$,$\underline{0}$) moment of the correlator

	$\left.\dfrac{\hat{\mathrm{B}}\langle V(\underline{0})V(\underline{0})\rangle_{\beta_0^{n-1}\mathrm{N}^n\mathrm{LO}}}{\hat{\mathrm{B}}\langle V(\underline{0})V(\underline{0})\rangle_{\beta_0^{n-2}\mathrm{N}^{n-1}\mathrm{LO}}}\right\|_{L_\mathrm{B}=0}$
$\dfrac{\mathrm{NLO}}{\mathrm{LO}}$	16%
$\dfrac{\beta_0\mathrm{N}^2\mathrm{LO}}{\mathrm{LO}}$	3.8%
$\dfrac{\beta_0^2\mathrm{N}^3\mathrm{LO}}{\mathrm{LO}}$	2.8%
$\dfrac{\beta_0^3\mathrm{N}^4\mathrm{LO}}{\mathrm{LO}}$	0.82%
$\dfrac{\beta_0^4\mathrm{N}^5\mathrm{LO}}{\mathrm{LO}}$	2.1%

	$\left.\dfrac{\hat{\mathrm{B}}\langle V(\underline{0})V(\underline{0})\rangle_{\mathrm{N}^n\mathrm{LO}}}{\hat{\mathrm{B}}\langle V(\underline{0})V(\underline{0})\rangle_{\mathrm{N}^{n-1}\mathrm{LO}}}\right\|_{L_\mathrm{B}=0}$	
$\dfrac{\mathrm{NLO}}{\mathrm{LO}}$	16%	
$\dfrac{\mathrm{N}^2\mathrm{LO}}{\mathrm{LO}}$	4.1% $\mathrm{N}^2\mathrm{LO} = \beta_0 b[1] + b[0]$	**Large β_0 approx. works!**
$\dfrac{\mathrm{N}^3\mathrm{LO}}{\mathrm{LO}}$	−0.76% $\mathrm{N}^3\mathrm{LO} = \beta_0^2 b[2] + \beta_0 b[1] + \beta_1 b[0,1] + b[0]$	**Large β_0 approx. is not applicable**
$\dfrac{\mathrm{N}^4\mathrm{LO}}{\mathrm{LO}}$	−0.093% $\mathrm{N}^4\mathrm{LO} = \beta_0^3 b[3] + \beta_0^2 b[2] + \beta_0 b[1] + \beta_2 b[0,0,1] + \beta_1 b[0,1] \beta_0\beta_1 b[1,1] + b[0]$	**Large β_0 approx. is not applicable**
$\dfrac{\mathrm{N}^5\mathrm{LO}}{\mathrm{LO}}$	?	

Summary

We have evaluated correlators of two vector composite quark currents of order β_0^nN^{n+1}LO in QCD, $n \geqslant 0$. The lower Mellin moments of the correlator has been calculated. The double-zeroth moment as well as some other fixed-order special cases agree with previous calculations in the literature.

Exponential generating functions for the correlator has been constructed.

The correlator at any fixed order $a_s^{n+1}\beta_0^n$ can be expressed in terms of harmonic polylogarithms of weight $n+2$.

We have estimated quantitative significance of the lower-order fermion-bubble chain contributions to the perturbative part of QCD sum rules for the lighter-meson distribution amplitudes (pion and longitudinal rho).

The correlator $\Pi_n(x, y)$

$$\dot{\Pi}_n(x,y;L) = \frac{\mathrm{d}}{\mathrm{d}L}\Pi_n(x,y;L) = (-1)^{n+1} n! \sum_{k=0}^{n} \frac{(-L)^k}{k!} \boldsymbol{\Pi_n^{k+1}(x,y)}$$

$$\boldsymbol{\Pi_n^k(x,y)} = (1 - y\bar{x} - x\bar{y}) E^{[n+2-k]}(|y-x|) + \frac{(-1)^n \delta_{1,k}}{(n+1)!} \hat{\mathbf{P}} \sum_{r=0}^{n+1} G^{(1)}_{n+1-r}(1) \left[y\bar{y} V^{[r]}(x,y) \right]_{+(x)}$$
$$+ \left[y\bar{y} V(x,y;0) E^{[n+2-k]}(|y-x|) \right]_{+} + \delta(x-y) x\bar{x}\, \hat{\mathbf{S}} \left[(x-\bar{x}) H_{n+2-k}(x) \right]$$
$$- \sum_{r=0}^{n+1-k} \sum_{s=0}^{n+1-k-r} F_3^{n-k-s,[r]}(x,y) E^{[s]}(|y-x|) - \sum_{r=0}^{n+1-k} F_2^{[r]}(x,y) E^{[n+1-k-r]}(x\bar{y})$$

$$F_3^n(x,y;\delta) = \hat{\mathbf{S}} \frac{H(x-y)}{2x\bar{y}} \left[(x\bar{x} + y\bar{y})(-1)^n + (xy + \bar{x}\bar{y})(x-y) \right] {}_3F_2\left(\begin{matrix} 1,\, 1,\, 1 \\ 1+\delta,\, 2-\delta \end{matrix} \middle| \bar{z} \right)$$

$$F_2(x,y;\delta) = \frac{\pi\delta}{\sin(\pi\delta)} \frac{\Gamma^2(1+\delta)}{\Gamma(1+2\delta)} \hat{\mathbf{S}} \left\{ H(x-y) \left[\frac{1+y-x}{\delta^2} \frac{d}{d\bar{z}} + xy + \bar{x}\bar{y} \right] {}_2F_1\left(\begin{matrix} \delta,\, \delta \\ 1+2\delta \end{matrix} \middle| \bar{z} \right) \right\}$$

$$E(x;a) = \exp\left[a \left(\frac{5}{3} - \ln x \right) \right] \qquad E^{[r]}(x) = \frac{1}{r!} \left(\frac{d}{da} \right)^r E(x;a) \Big|_{a=0} = \frac{1}{r!} \left(\frac{5}{3} - \ln x \right)^r$$

The correlator $\Pi_n(x, y)$

$$\sum_{n\geqslant 0} \frac{A^n}{n!}\dot{\Pi}_n(x,y;L)$$

$$= (x\bar{x} + y\bar{y})\, \frac{e^{A(L-5/3)}}{1-A}\, \hat{\mathbf{S}}\left[\frac{H(x-y)}{2(x\bar{y})^{1-A}}\bar{z}^A {}_3F_2\left(\begin{matrix} 1,\,1,\,1 \\ 1+A,\,2-A \end{matrix}\middle|\, \bar{z}\right)\right]$$

$$+ (xy + \bar{x}\bar{y})\, \frac{e^{A(L-5/3)}}{1+A}\, \hat{\mathbf{S}}\left[\frac{H(x-y)}{2(x\bar{y})^{-A}}\bar{z}^{1+A} {}_3F_2\left(\begin{matrix} 1,\,1,\,1 \\ 1-A,\,2+A \end{matrix}\middle|\, \bar{z}\right)\right]$$

$$- Ae^{A(L-5/3)}\frac{\pi A}{\sin(\pi A)}\, \hat{\mathbf{S}}\left[\frac{H(x-y)}{2(x\bar{y})^{-A}}(1+y-x)\frac{\Gamma^2(1-A)}{\Gamma(2-2A)}{}_2F_1\left(\begin{matrix} 1-A,\,1-A \\ 2-2A \end{matrix}\middle|\, \bar{z}\right)\right]$$

$$+ (xy + \bar{x}\bar{y})\, \frac{\pi}{\sin(\pi A)}\, \hat{\mathbf{S}}\left\{H(x-y)\left[\frac{e^{A(L-5/3)}}{(x\bar{y})^{-A}}\frac{\Gamma^2(1-A)}{\Gamma(1-2A)}{}_2F_1\left(\begin{matrix} -A,\,-A \\ 1-2A \end{matrix}\middle|\, \bar{z}\right) - 1\right]\right\}$$

$$+ \frac{1}{A}\left[e^{A(L-5/3)}|x-y|^A - 1\right](1 - y\bar{x} - x\bar{y}) + \frac{1}{A}\left\{\left[e^{A(L-5/3)}|x-y|^A - 1\right] W(x,y;0)\right\}_+$$

$$+ \delta(x-y)x\bar{x}\frac{1}{A}\,\hat{\mathbf{S}}\left[(x-\bar{x})\left(\frac{e^{A(L-5/3)}x^A}{A(1+A)(2+A)} - \frac{1}{2}\ln x\right)\right] - \frac{1}{2}\sum_{n\geqslant 0}\frac{A^n}{[(n+1)!]^2}\hat{\mathbf{P}}\left[\left(\frac{d}{da}\right)^{n+1}\frac{y\bar{y}V(x,y;a)_{+(x)}}{h_1(a)}\right]_{a=0}$$

$(x, \underline{0})$ moment of the correlator

$$\dot{\Pi}_n(x,\underline{0};L) = \frac{\mathrm{d}}{\mathrm{d}L}\Pi_n(x,\underline{0};L) = (-1)^{n+1}n!\sum_{k=0}^{n}\frac{(-L)^k}{k!}\Pi_n^{k+1}(x,\underline{0})$$

$$\Pi_n^k(x,\underline{0}) = \hat{\mathrm{S}}\Bigg\{\bar{x}\Bigg[\sum_{r=0}^{n-k}(-1)^r\mathbf{Li}_{r+2}\left(-\frac{\bar{x}}{x}\right)H_{n-k-r}(x) + P_3^{n+2-k}(x) + \frac{(-1)^n\delta_{1,k}}{2(n+1)!}\left(K_n - \sum_{s=1}^{n+1}\frac{(-1)^s\ln^s\bar{x}}{s!}L_{n-s}\right)\Bigg]$$
$$+ 2x\bar{x}\Bigg[P_1^{n+3-k}(x) - \sum_{r=0}^{n+1-k}(-1)^r\mathbf{Li}_{r+2}\left(-\frac{\bar{x}}{x}\right)H_{n+1-k-r}(x) - \sum_{r=0}^{n+1-k}\frac{x}{r!}\left(\frac{d}{da}\right)^r {}_3F_2\left(\begin{matrix}1,\,1,\,1-a\\2-a,\,2-a\end{matrix}\Bigg|\,x\right)P_2^{n+1-k-r}(x)$$
$$+ \frac{(-1)^n\delta_{1,k}}{2(n+1)!}\left(G_{n+2}^{(1)}(x) + G_{n+1}^{(1)}(x)\ln\bar{x} - \sum_{r=0}^{n}\mathbf{Li}_{r+2}(x)G_{n-r}^{(1)}(x)\right)\Bigg]\Bigg\}$$

$$P_1^n(x) = 2\sum_{s=1}^{[n/2]}\zeta_{2s}\left(1-\frac{1}{2^{2s-1}}\right)(2s-1)H_{n-2s}(1) - P_3^n(x) + H_n(x) - H_n(1)$$

$$P_2^n(x) = H_n(x) + \frac{1}{2}\sum_{s=0}^{n-1}\frac{1}{s!}\left(\frac{5}{3}-\ln x\right)^s(n+1-s)(n-s)$$

$$P_3^n(x) = 2\sum_{s=1}^{[n/2]}\zeta_{2s}\left(1-\frac{1}{2^{2s-1}}\right)H_{n-2s}(\bar{x}) + H_n(\bar{x}) - H_n(x) - \ln(x)H_{n-1}(x)$$

$$H_n(x) = \sum_{s=0}^{n}\frac{1}{s!}\left(\frac{5}{3}-\ln x\right)^s\left(1-\frac{1}{2^{n+1-s}}\right)$$

$$G_n^{(a)}(x) = \frac{1}{n!}\left[\left(\frac{d}{d\delta}\right)^n\frac{x^{-a}}{h_a(\delta)}\right]_{\delta=0}$$

$$F_n^{(a)}(x) = \sum_{s=0}^{n}\frac{x^{s-n}}{s!}\left[\left(\frac{d}{d\delta}\right)^s\frac{1}{h_a(\delta)}\right]_{\delta=0}$$

$$K_n = 4F_{n+1}^{(1)}(1) - F_{n+1}^{(1)}(2) + 2F_n^{(1)}(-1) + F_n^{(1)}(2) - 4G_{n+2}^{(1)}(1) - 3L_n$$

$$L_n = F_{n+1}^{(1)}(-2) + 2F_n^{(1)}(-1) - F_n^{(1)}(-2)$$

(<u>-1</u>,<u>0</u>) moment of the correlator

	$\left.\dfrac{\hat{\mathrm{B}}\langle V(\underline{-1})V(\underline{0})\rangle_{\beta_0^{n-1}\mathrm{N}^n\mathrm{LO}}}{\hat{\mathrm{B}}\langle V(\underline{-1})V(\underline{0})\rangle_{\beta_0^{n-2}\mathrm{N}^{n-1}\mathrm{LO}}}\right\|_{L_{\mathrm{B}}=0}$
$\dfrac{\mathrm{NLO}}{\mathrm{LO}}$	26%
$\dfrac{\beta_0\mathrm{N}^2\mathrm{LO}}{\mathrm{LO}}$	7%
$\dfrac{\beta_0^2\mathrm{N}^3\mathrm{LO}}{\mathrm{LO}}$	5%
$\dfrac{\beta_0^3\mathrm{N}^4\mathrm{LO}}{\mathrm{LO}}$	5%
$\dfrac{\beta_0^4\mathrm{N}^5\mathrm{LO}}{\mathrm{LO}}$	9%