Quantum field-theoretical description of neutrino oscillation in terms of distance-dependent propagators

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Introduction

The Standard Model is capable of describing a great number of various processes of elementary particle interaction with a high accuracy in the framework of the perturbative *S*-matrix formalism, and the results of the theoretical description are remarkably confirmed by experimental data in the vast majority of cases.

However, there are several phenomena that cannot be described in the framework of the standard perturbation theory.

In particular, these are the oscillation phenomena of neutral mesons and neutrinos occurring at macroscopic space and time intervals.

Neutrino oscillation is a well-known and experimentally confirmed phenomenon, which is usually understood as the transition from a neutrino flavor state to another neutrino flavor state depending on the distance traveled.

- S. Bilenky, “Introduction to the physics of massive and mixed neutrinos,”
Lecture Notes in Physics **817** (2010) 1.

However, the standard theoretical description of neutrino oscillations is <u>inconsistent</u> because the neutrino flavor states are ill-defined.

In accordance with the quantum superposition principle

$$|\psi\rangle = \alpha\,|\psi_1\rangle + \beta\,|\psi_2\rangle\,, \quad \alpha, \beta \in \mathbf{C},$$

if both $|\psi_1\rangle$ and $|\psi_2\rangle$ are states of the same quantum system, i.e. they evolve over time with the same Hamiltonian

$$|\psi_1(\mathrm{t})\rangle = \mathrm{e}^{-\mathrm{iHt}}\,|\psi_1\rangle\,, \quad |\psi_2(\mathrm{t})\rangle = \mathrm{e}^{-\mathrm{iHt}}\,|\psi_2\rangle\,.$$

Obviously, it is not the case for the neutrino mass eigenstates.

The problems of describing neutrino oscillations in terms of the neutrino flavor states were repeatedly discussed in the literature:

- C. Giunti, C.W. Kim, J.A. Lee and U.W. Lee, “On the treatment of neutrino oscillations without resort to weak eigenstates,” Phys. Rev. D **48** (1993) 4310,
- W. Grimus and P. Stockinger, “Real oscillations of virtual neutrinos,” Phys. Rev. D **54** (1996) 3414,
- A.E. Lobanov, “Particle quantum states with indefinite mass and neutrino oscillations,” Annals Phys. **403** (2019) 82.

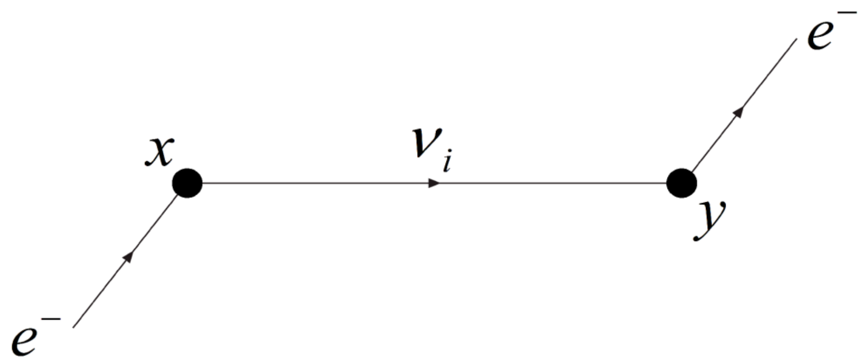
For the first time, a description of neutrino oscillations within the framework of QFT and S-matrix approach was put forward in the paper

- I.Yu. Kobzarev, B.V. Martemyanov, L.B. Okun and M.G. Shchepkin,

  "Sum rules for neutrino oscillations,"

  Sov. J. Nucl. Phys. 35 (1982) 708.

The authors considered a process, where a charged lepton produced a neutrino mass eigenstate at an infinitely heavy nucleus and then the neutrino produced a charged lepton at another infinitely heavy nucleus.



The incoming and outgoing leptons were described by plane waves, whereas the matrix elements of the charged weak hadron currents of the nuclei were taken to be proportional to the delta functions of their positions.

In calculating the matrix element of the process, there appeared the expression

$$\mathbf{S^c(p,\vec{n},L) = \int dz e^{ipz} S^c(z) \delta(\vec{z}-\vec{n}L)},$$

which we call the distance-dependent propagator of a virtual neutrino. The integral can be evaluated exactly, the result being given by

$$\mathbf{S^c(p,\vec{n},L) = \frac{e^{-i\vec{p}\vec{n}L}}{4\pi L}(\gamma^0 p^0 - \vec{\gamma}\vec{n}(p_{ms} + \frac{i}{L}) + m)\, e^{ip_{ms}L}},$$

$$\mathbf{p_{ms} = \sqrt{(p^0)^2 - m^2}}.$$

In this approach the produced neutrinos are off-shell and described by the standard Feynman propagators in the coordinate representation.

In the momentum representation, there appear distance-dependent propagators of virtual neutrinos.

Neutrino oscillation is now an ordinary interference process.

The incoming and outgoing leptons are described by plane waves, whereas the positions of nuclei are fixed by delta functions, which is a rather rough approximation resulting in violation of momentum conservation.

The idea was developed in the paper

- C. Giunti, C.W. Kim, J.A. Lee and U.W. Lee, “On the treatment of neutrino oscillations without resort to weak eigenstates,” Phys. Rev. D **48** (1993) 4310.

The localization of the incoming and outgoing particles or nuclei is described with the help of wave packets, which makes the calculations of amplitudes very complicated.

The reason is:

The standard S-matrix formalism is not appropriate for describing the processes passing at finite distances and lasting finite time intervals.

Modified perturbative formalism

We put forward a modification of the perturbative formalism, which allows one to describe the processes passing at finite distances during finite time intervals.

The approach is based on the Feynman diagram technique in the coordinate representation supplemented by modified rules of passing to the momentum representation. The latter reflect the geometry of neutrino oscillation experiments and lead to a modification of the Feynman propagators of the neutrino mass eigenstates in the momentum representation.

The idea behind the approach comes from the paper

- R.P. Feynman, “Space-Time Approach to Quantum Electrodynamics,” Phys. Rev. **76** (1949), 769.

It was developed in the papers

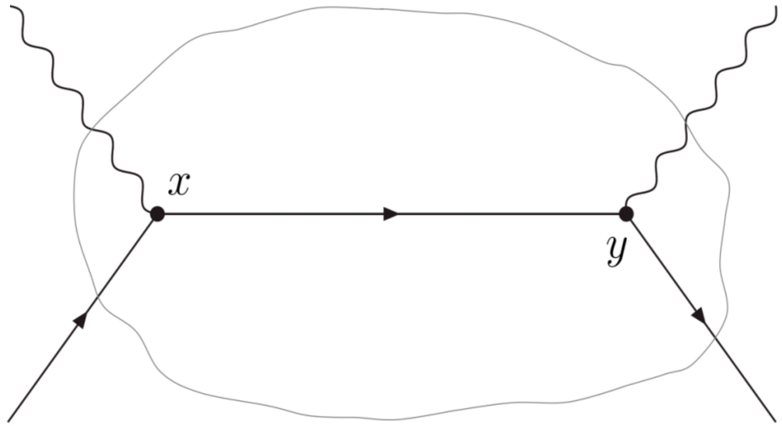
- V.O. Egorov and I.P. Volobuev, “Neutrino oscillation processes in a quantum field-theoretical approach,” Phys. Rev. D **97** (2018) no.9, 093002,
- V.O. Egorov and I.P. Volobuev, “Quantum field theory description of processes passing at finite space and time intervals,”
Theor. Math. Phys. **199** (2019) no.1, 562,
- V.O. Egorov and I.P. Volobuev, “Coherence length of neutrino oscillations in a quantum field-theoretical approach,” Phys. Rev. D **100** (2019) no.3, 033004.

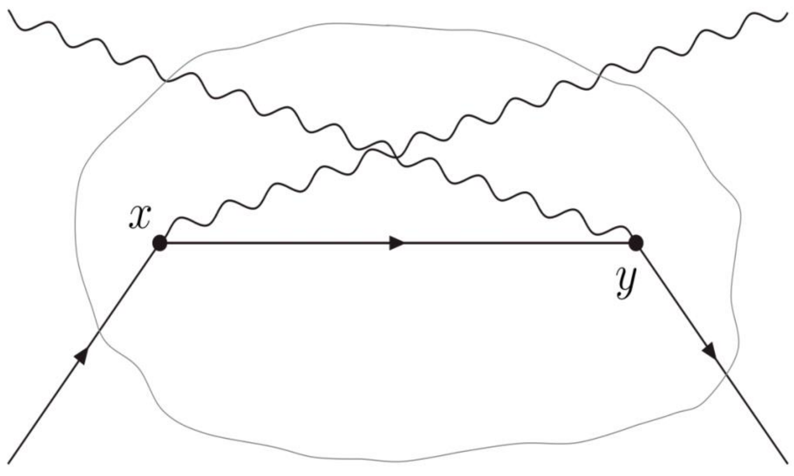
The modern perturbative S-matrix formalism and the diagram technique were formulated in the papers

- F.J. Dyson, “The S matrix in quantum electrodynamics,”
  Phys. Rev. **75** (1949) 1736,

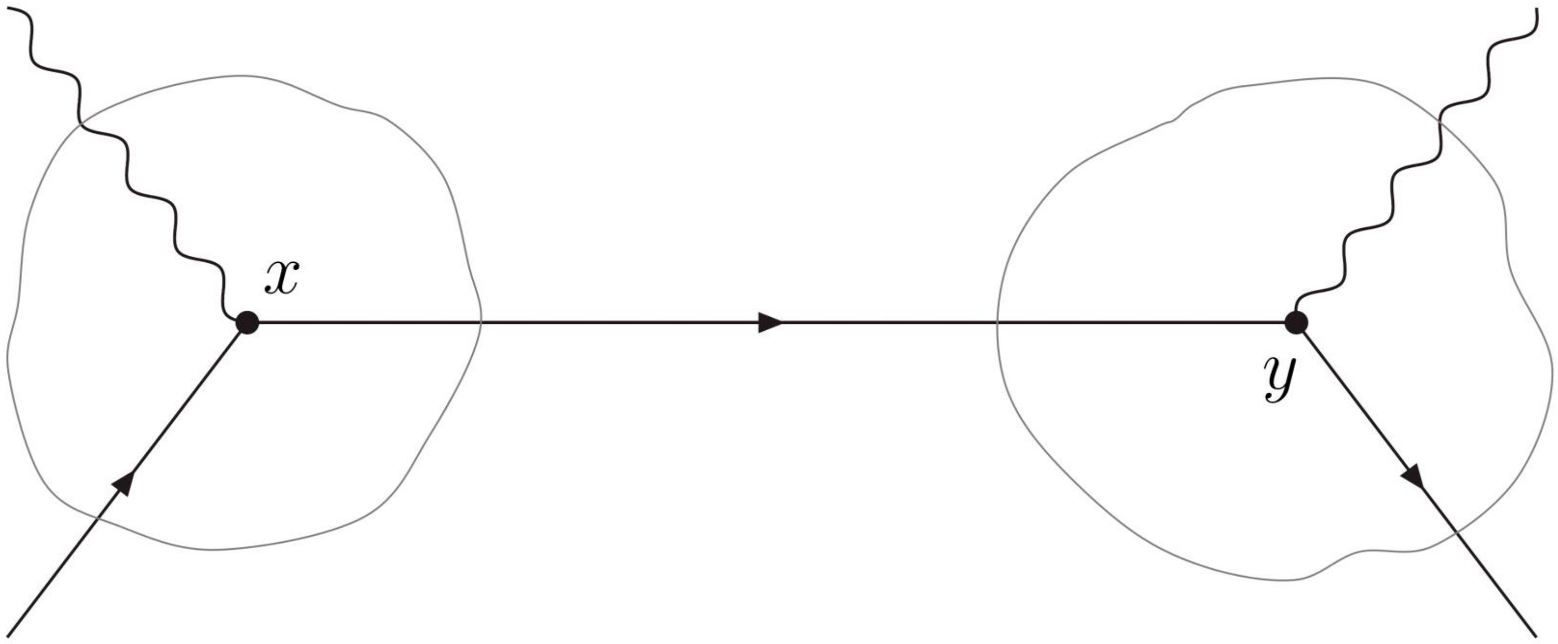
- R.P. Feynman, “Space-Time Approach to Quantum Electrodynamics,”
  Phys. Rev. **76** (1949), 769.

Standard scattering process



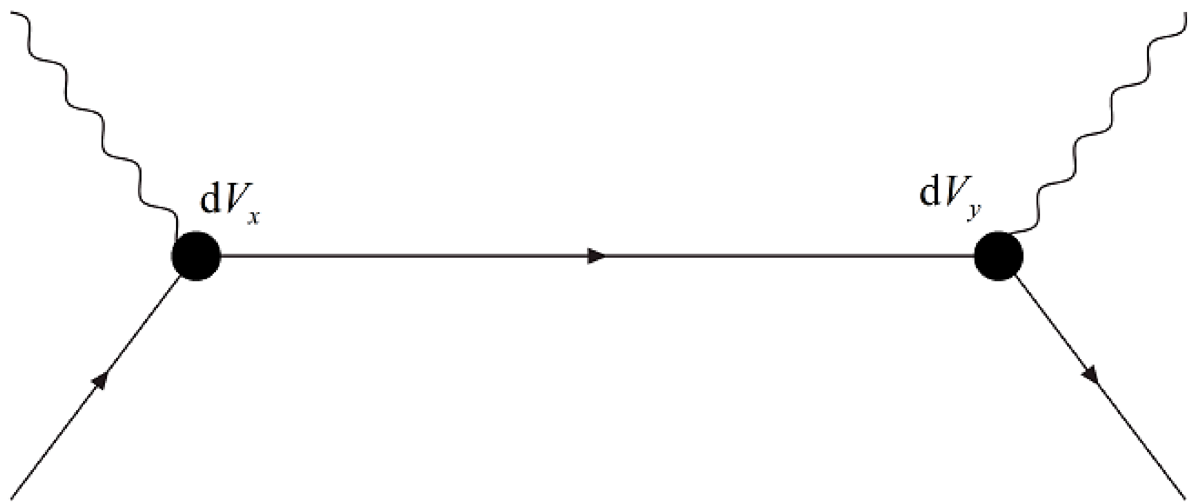


Scattering process in neutrino oscillation experiment setting



Distance-dependent propagators

To take into account the geometry of the experiment we have to integrate with respect to the coordinates x and y in such a way that the distance between the points x and y along the unit vector directed from the source to the detector is fixed and equal to L.



This can be achieved by introducing the delta function $\delta((\vec{\mathbf{y}}-\vec{\mathbf{x}})\vec{\mathbf{n}}-\mathbf{L})$ into the integrand. Formally, this is equivalent to replacing the standard Feynman fermion propagator $\mathbf{S^c(y-x)}$ by $\mathbf{S^c(y-x)}\delta((\vec{\mathbf{y}}-\vec{\mathbf{x}})\vec{\mathbf{n}}-\mathbf{L})$.

The Fourier transform of this product is called *the distance-dependent fermion propagator in the momentum representation*:

$$\mathbf{S^c(p,\vec{n},L)} = \int \mathbf{dz\, e^{ipz} S^c(z)}\delta(\vec{\mathbf{n}}\vec{\mathbf{z}}-\mathbf{L}).$$

The integral can be evaluated exactly:

$$\mathbf{S^c(p,\vec{n},L) = i\,\frac{\hat{p} + \vec{\gamma}\vec{n}\left(\vec{p}\vec{n} - \sqrt{(\vec{p}\vec{n})^2 + p^2 - m^2}\right) + m}{2\sqrt{(\vec{p}\vec{n})^{\,2} + p^2 - m^2}}\,e^{-i\left(\vec{p}\vec{n} - \sqrt{(\vec{p}\vec{n})^{\,2} + p^2 - m^2}\right)L}},$$

In what follows, we assume $\vec{p}\vec{n} \sim \left|\vec{p}\right|$.

In the paper

- W. Grimus and P. Stockinger, “Real oscillations of virtual neutrinos,” Phys. Rev. D **54** (1996) 3414

it was rigorously proved that the particles propagating over macroscopic distances are almost on the mass shell, i.e. for such particles

$$\left|p^2 - m^2\right| / (\vec{p}\vec{n})^2 << 1.$$

In this approximation we get the distance-dependent propagator in the simple form:

$$\mathbf{S}^c(\mathbf{p}, \vec{\mathbf{n}}, \mathbf{L}) = \mathrm{i}\,\frac{\hat{\mathbf{p}} + \mathbf{m}}{2\vec{\mathbf{p}}\vec{\mathbf{n}}}\,\mathrm{e}^{\mathrm{i}\,\frac{\mathbf{p}^2 - \mathbf{m}^2}{2\vec{\mathbf{p}}\vec{\mathbf{n}}}\mathbf{L}}.$$

Distant-dependent propagator of Okun et al.

$$\mathbf{S^c(z) \to S^c(z)\delta(\vec{z} - \vec{n}L)}.$$

For particles almost on the mass shell

$$\mathbf{S^c(p, \vec{n}, L) = \frac{e^{-i\vec{p}\vec{n}L}}{4\pi L}(\gamma^0 p^0 - \vec{\gamma}\vec{n}(p_{ms} + \frac{i}{L}) + m)\, e^{ip_{ms}L}}$$

$$\simeq \mathbf{\frac{\hat{p} + m}{4\pi L}}\, e^{i\,\mathbf{\frac{p^2 - m^2}{2\vec{p}\vec{n}} L}}.$$

Time-dependent propagator

$$\mathbf{S^c(z) \to S^c(z)\delta(z^0 - T)}, \quad \mathbf{S^c\,(p, T) = i\,\frac{\hat{p} + m}{2p^0}}\, e^{i\,\mathbf{\frac{p^2 - m^2}{2p^0} T}}.$$

Neutrino oscillations in vacuum

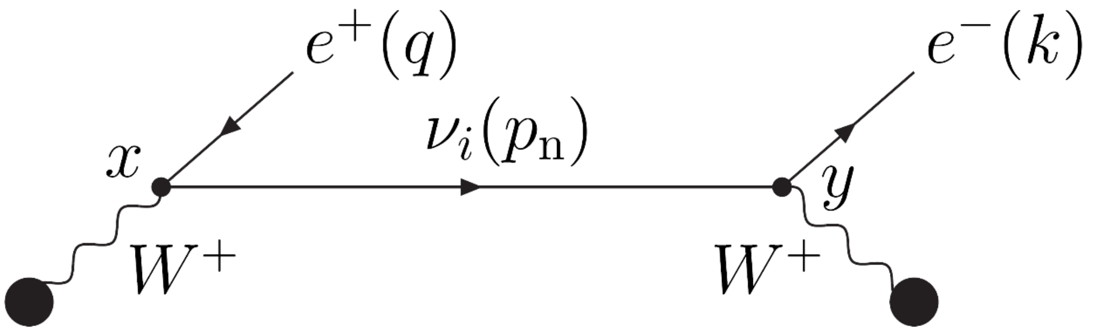
We work in the minimal extension of the SM by the right neutrino singlets. The interaction Lagrangian of the leptons looks like

$$L_{\rm int}^{\rm lep} = -\frac{g}{2\sqrt{2}}\left(\sum_{i,k=1}^{3} \bar{l}_i\,\gamma^\mu\,(1-\gamma^5)\,U_{ik}\,\nu_k\,W_\mu^- + \text{h.c.}\right) +$$

$$+\frac{g\sin^2\theta_{\rm w}}{\cos\theta_{\rm w}}\sum_{i=1}^{3}\bar{l}_i\,\gamma^\mu\,l_i\,Z_\mu - \frac{g}{4\cos\theta_{\rm w}}\sum_{i=1}^{3}\bar{l}_i\,\gamma^\mu\,(1-\gamma^5)\,l_i\,Z_\mu +$$

$$+\frac{g}{4\cos\theta_{\rm w}}\sum_{k=1}^{3}\bar{\nu}_k\,\gamma^\mu\,(1-\gamma^5)\,\nu_k\,Z_\mu\,.$$

First, we consider the processes, where neutrinos are produced and detected in the charged-current interaction with nuclei.



The process amplitude in the momentum representation for $\vec{n}\left(\vec{y}-\vec{x}\right)=L$ in the approximation of zero neutrino masses (everywhere, except in the exponential)

$$M=-\mathrm{i}\,\frac{G_{\mathrm{F}}^2}{4\vec{p}_{\mathrm{n}}\vec{n}}\left(\sum_{i=1}^{3}\left|U_{1i}\right|^2 \mathrm{e}^{-\mathrm{i}\frac{m_i^2-p_{\mathrm{n}}^2}{2\vec{p}_{\mathrm{n}}\vec{n}}L}\right)J_{\rho}^{(2)}\left(\vec{P}^{(2)},\vec{P}^{(2')}\right)\times$$

$$\times\,\overline{u}\left(\vec{k}\right)\gamma^{\rho}\left(1-\gamma^5\right)\hat{p}_{\mathrm{n}}\,\gamma^{\mu}\left(1-\gamma^5\right)\upsilon\left(\vec{q}\right)J_{\mu}^{(1)}\left(\vec{P}^{(1)},\vec{P}^{(1')}\right).$$

The amplitude is the sum of three terms, interference of which gives rise to neutrino oscillations.

The squared amplitude summed over the particle polarizations factorizes in the approximation of massless neutrinos:

$$\left\langle |M|^2 \right\rangle = \underbrace{\left\langle |M_{\mathrm{P}}|^2 \right\rangle}_{\text{Production process}} \underbrace{\left\langle |M_{\mathrm{D}}|^2 \right\rangle}_{\text{Detection process}} \frac{1}{4(\vec{p}_{\mathrm{n}}\vec{n})^2} \left[1 - 4 \sum_{\substack{i,k=1 \\ i>k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2\left(\frac{\Delta m_{ik}^2}{4\vec{p}_{\mathrm{n}}\vec{n}} L \right) \right].$$

$$\Delta m_{ik}^2 = m_i^2 - m_k^2$$

To find the corresponding differential probability we need to multiply the squared amplitude by the delta function of energy-momentum conservation $(2\pi)^4 \delta\left(P^{(1)} + P^{(2)} - P^{(1')} - q - P^{(2')} - k\right)$ and by $2\pi\,\delta\left(P^{(1)} - P^{(1')} - q - p\right)$, where $p^2 = 0$ and $\vec{p}$ is directed from the source to the detector, and to integrate with respect to the momenta of the outgoing particles and nuclei.

As a result we get:

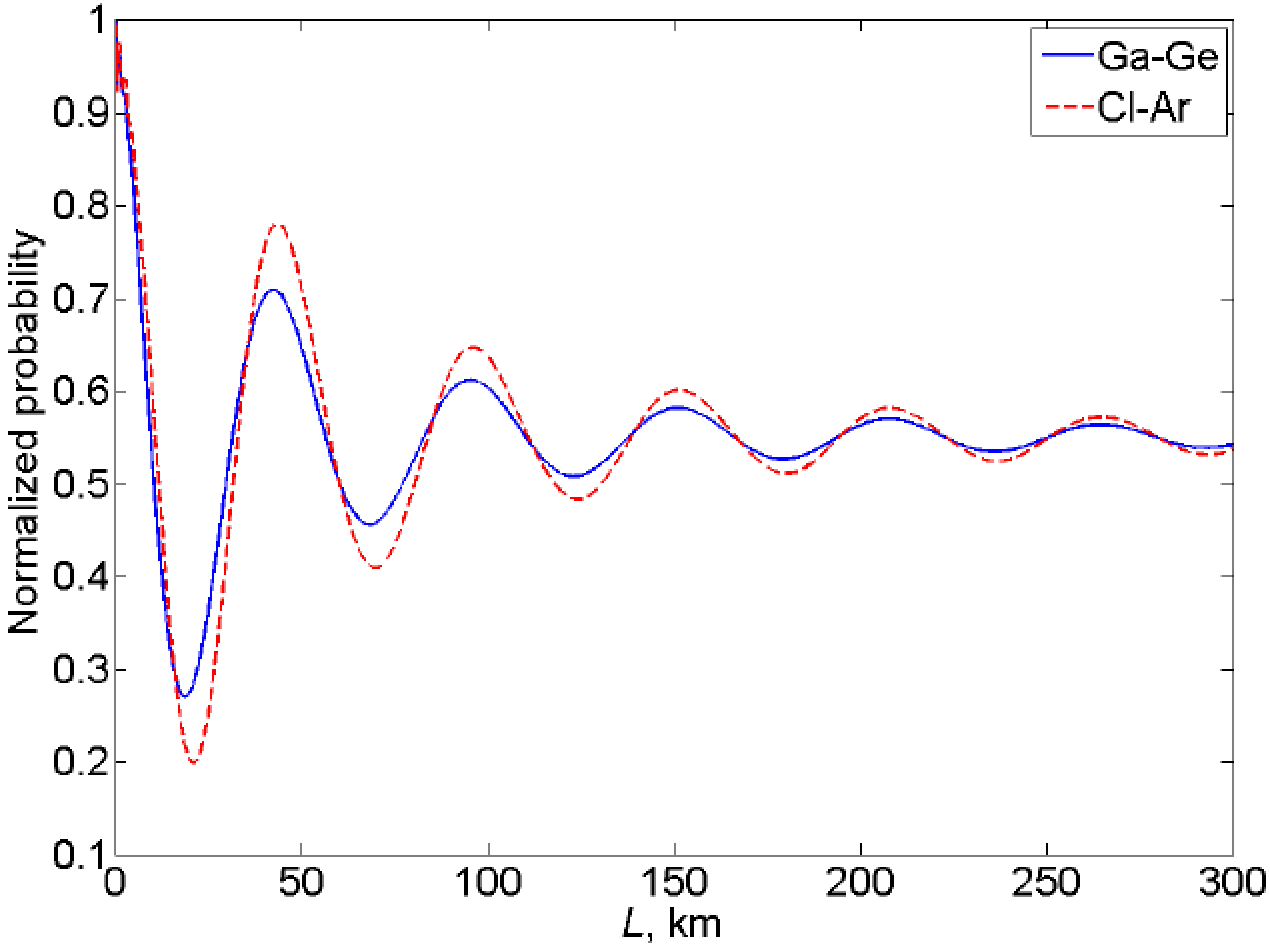
$$\frac{\mathrm{d}^3 W}{\mathrm{d}^3 p} = \frac{\mathrm{d}^3 W_\mathrm{P}}{\mathrm{d}^3 p} W_\mathrm{D} \underbrace{\left[1 - 4\sum_{\substack{i,k=1 \\ i>k}}^{3} |U_{1i}|^2 |U_{1k}|^2 \sin^2\left(\frac{\Delta m_{ik}^2}{4|\vec{p}|} L\right)\right]}_{P_{ee}(|\vec{p}|, L) \text{ is the standard oscillating factor}}. \quad \left(\vec{p}\vec{n} = |\vec{p}| \right)$$

Probability of $\nu(\vec{p})$ production

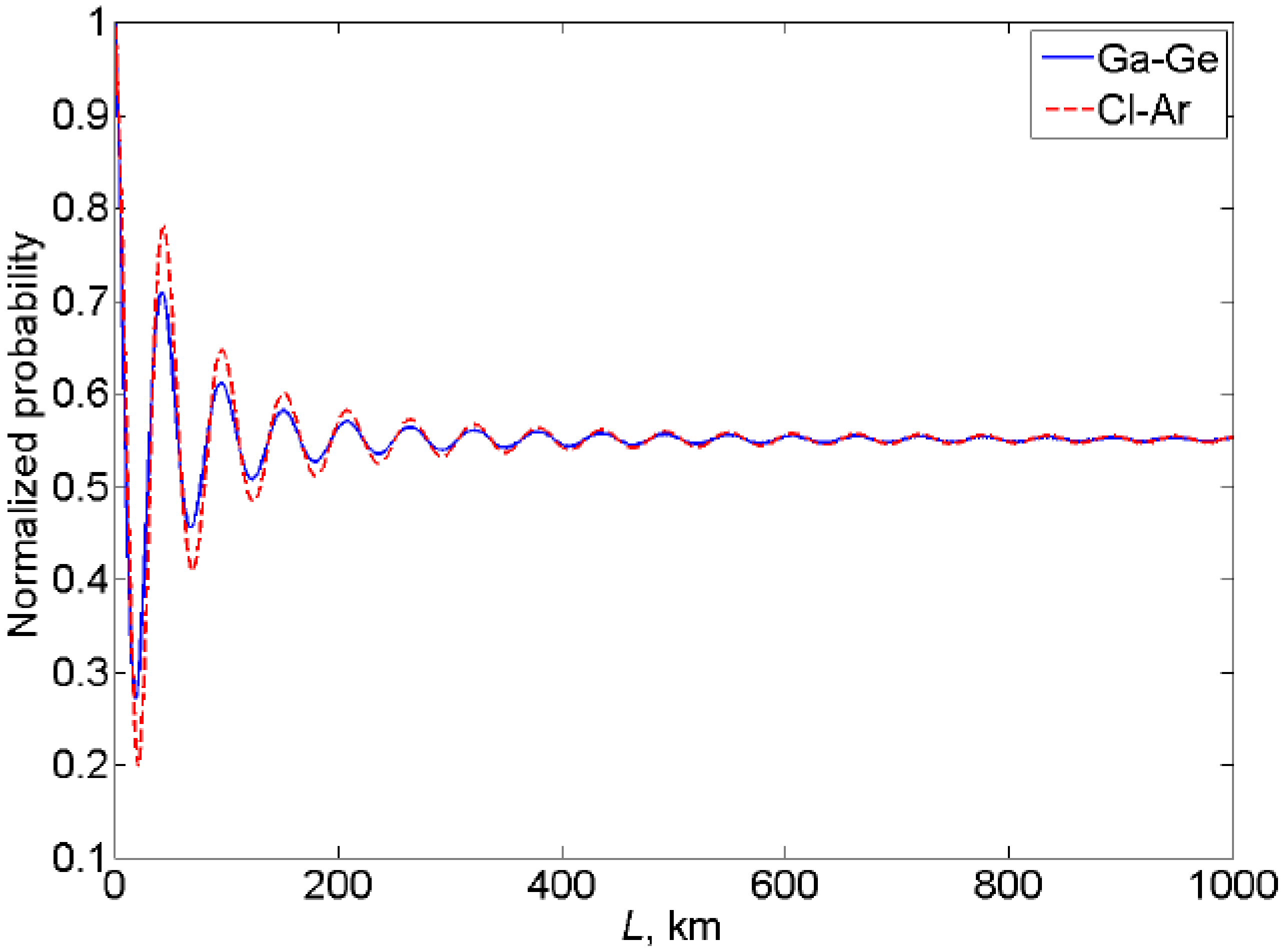
Finally, since the experimental setting determines the direction of the neutrino momentum, but not its magnitude, to find the probability of detecting an electron we have to integrate this expression with respect to $|\vec{p}|$:

$$\frac{\mathrm{d}W}{\mathrm{d}\Omega} = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{\mathrm{d}^3 W}{\mathrm{d}^3 p} |\vec{p}|^2 \mathrm{d}|\vec{p}| = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{\mathrm{d}^3 W_\mathrm{P}}{\mathrm{d}^3 p} W_\mathrm{D}\, P_{ee}(|\vec{p}|, L)\, |\vec{p}|^2 \mathrm{d}|\vec{p}| \,.$$

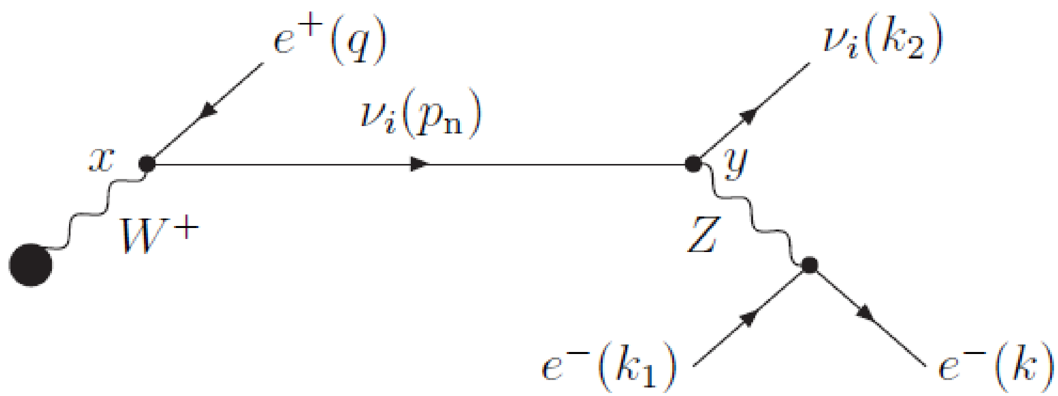
Normalized probabilities of the neutrino oscillation processes with the neutrino production in the ^{15}O decay and the registration by Cl-Ar and Ga-Ge detectors.

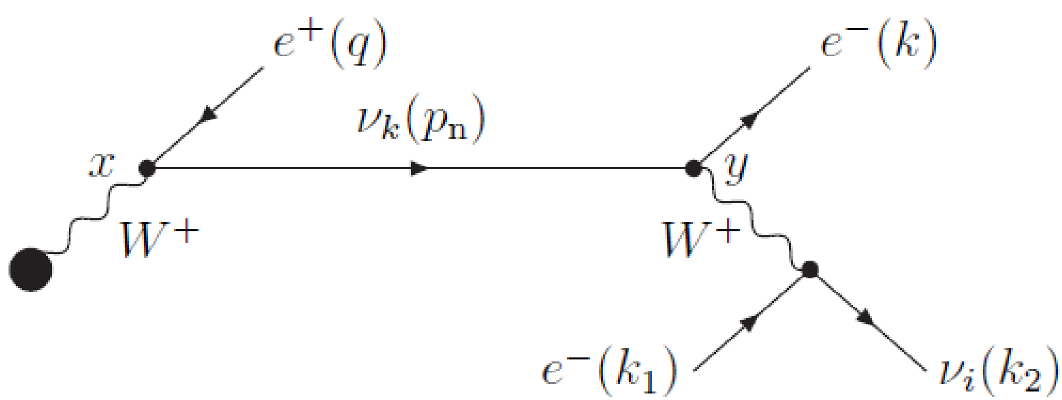


Normalized probabilities of the neutrino oscillation processes with the neutrino production in the ^{15}O decay and the registration by Cl-Ar and Ga-Ge detectors.



The processes, where neutrinos are produced and detected in both charged and neutral-current interactions with electrons:





Calculating the probability of the process with the help of the above formulated rules, we arrive at the result

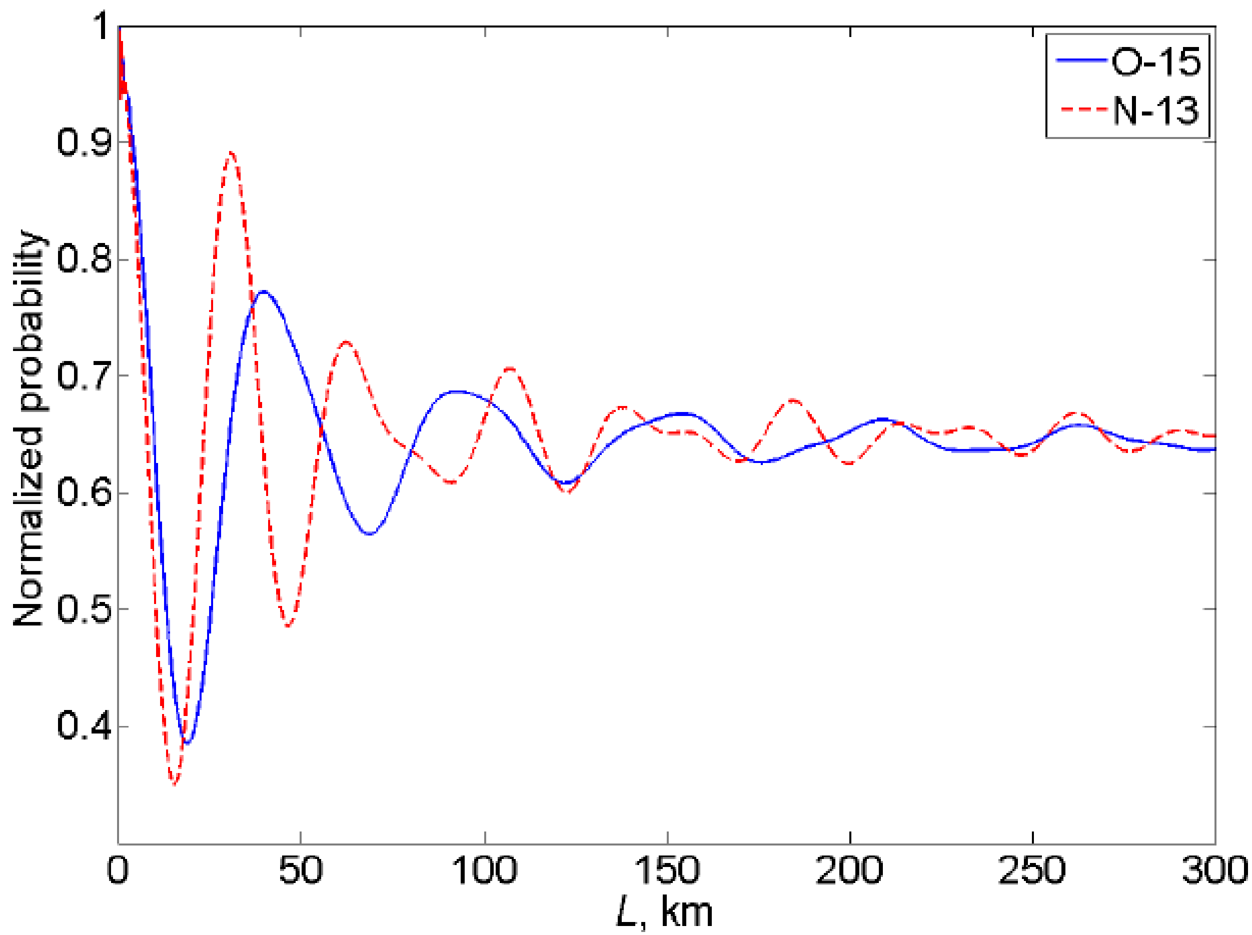
$$\frac{dW}{d\Omega} = \int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{d^3W}{d^3p} \left|\vec{p}\right|^2 d\left|\vec{p}\right| = \int\limits_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \frac{d^3W_1}{d^3p} W_2 \left|\vec{p}\right|^2 d\left|\vec{p}\right|,$$

where

$$W_2 = \frac{G_{\mathrm{F}}^2 m}{2\pi} \frac{2\left|\vec{p}\right|^2}{2\left|\vec{p}\right| + m} \left[1 - 2\sin^2\theta_{\mathrm{W}} \left(1 + \frac{2\left|\vec{p}\right|}{2\left|\vec{p}\right| + m}\right) + 4\sin^4\theta_{\mathrm{W}} \left(1 + \frac{1}{3}\left(\frac{2\left|\vec{p}\right|}{2\left|\vec{p}\right| + m}\right)^2\right) + \right.$$
$$\left. + 4\sin^2\theta_{\mathrm{W}} \left(1 + \frac{2\left|\vec{p}\right|}{2\left|\vec{p}\right| + m}\right) P_{ee}\left(\left|\vec{p}\right|, L\right)\right]$$

is the probability of neutrino scattering in the detector.

Normalized probability of the neutrino oscillation process with the neutrino production in the ^{15}O and ^{13}N decays and the registration by a water-based Cherenkov detector.



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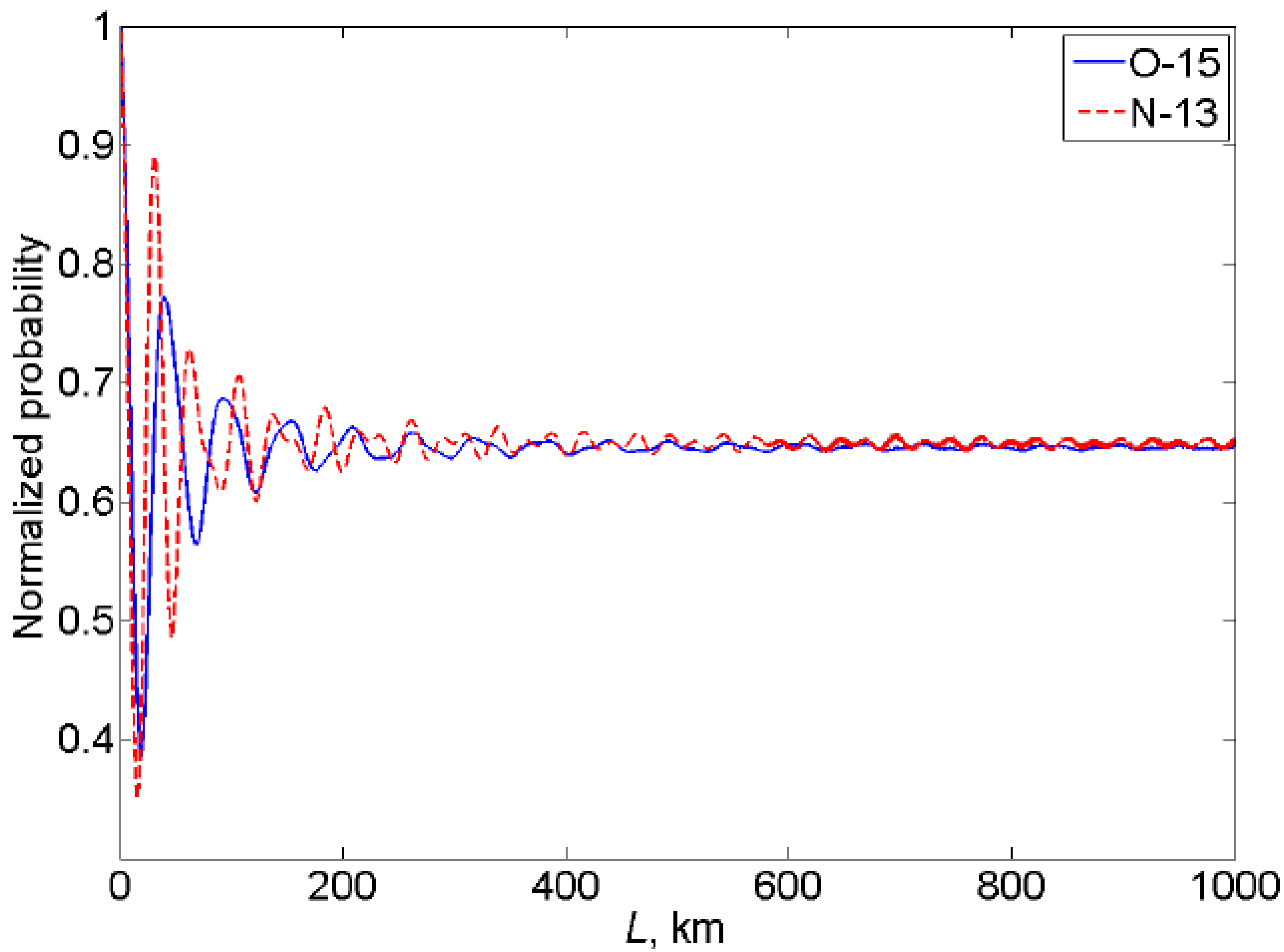


Neutrino oscillations in constant homogeneous magnetic field

Neutrino oscillations in magnetic field were first considered within the framework of QFT in the paper

- M.B. Voloshin, M.I. Vysotskiĭ and L.B. Okun, "Neutrino electrodynamics and possible consequences for solar neutrinos," Sov. Phys. JETP **64** (1986) no.3, 446.

The equation of motion of a neutrino in the field:

$$\left(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m_i - \frac{1}{2}\mu_0 m_i F_{\mu\nu}\sigma^{\mu\nu}\right)\nu_i\left(x\right) = 0$$

(the transition moments are neglected).

Green's function in the momentum representation looks like:

$$S_i^{\mathrm{c}}(p) = \mathrm{i}\left\{\left(p^2 - m_i^2\right)\left(p^2 - m_i^2 + \mathrm{i}\varepsilon\right) - \mu_0^2 m_i^2\left[\left(p^2 + m_i^2\right)F^{\mu\nu}F_{\mu\nu} - 4F^{\mu\nu}p_{\nu}F_{\mu\sigma}p^{\sigma}\right] + \frac{1}{4}\mu_0^4 m_i^4\left[\left(F^{\mu\nu}F_{\mu\nu}\right)^2 + \left(F^{\mu\nu}\tilde{F}_{\mu\nu}\right)^2\right]\right\}^{-1} \times$$
$$\times\left\{\left(p^2 - m_i^2\right)\left(\hat{p} + m_i\right) - \frac{1}{2}\mu_0^2 m_i^2 F^{\mu\nu}F_{\mu\nu}\left(\hat{p} - m_i\right) - 2\mu_0^2 m_i^2 F_{\mu\nu}F^{\nu\sigma}p_{\sigma}\gamma^{\mu} + 2\mu_0 m_i^2\tilde{F}_{\mu\nu}p^{\nu}\gamma^{\mu}\gamma^5 + \right.$$
$$\left. + \mathrm{i}\mu_0 m_i\left[\frac{1}{2}\left(p^2 + m_i^2\right)F_{\mu\nu} - \frac{1}{4}\mu_0^2 m_i^2 F^{\rho\sigma}\left(F_{\rho\sigma}F_{\mu\nu} + \tilde{F}_{\rho\sigma}\tilde{F}_{\mu\nu}\right) - 2F_{\mu\rho}p^{\rho}p_{\nu}\right]\gamma^{\mu\nu} - \frac{\mathrm{i}}{2}\mu_0^2 m_i^3 F_{\mu\nu}\tilde{F}^{\mu\nu}\gamma^5\right\}.$$

For the field under consideration, $F_{\mu\nu}=\varepsilon_{\mu\nu k0}H^k$, the dispersion relation looks like

$$\left(p^0\right)^2 = \vec{p}^{\,2} + m_i^2 + \mu_0^2 m_i^2 \vec{H}^2 \pm 2\mu_0 m_i \sqrt{\vec{p}^{\,2}\vec{H}_\perp^2 + m_i^2 \vec{H}^2}\,, \quad (\perp \text{ with respect to } \vec{p}\,)$$

which coincides with the one for the neutron in such field, found in the paper

- I.M. Ternov, V.G. Bagrov and A.M. Khapaev, "Electromagnetic radiation from a neutron in an external magnetic field," Sov. Phys. JETP **21** (1965) no.3, 613.

In what follows, we neglect the terms of order 2 and larger in μ_0. Substituting $S_i^c(p)$ in the definition of distance-dependent propagator, we get:

$$S_i^c(p,\vec{L}) = \mathrm{i}\,\frac{\hat{p}\left(1-\mathrm{i}\vec{j}\vec{\gamma}\right)}{4p^0}\,\mathrm{e}^{\mathrm{i}\left(\frac{p^2-m_i^2}{2|\vec{p}|}+\mu_0 m_i H_\perp\right)L} + \mathrm{i}\,\frac{\hat{p}\left(1+\mathrm{i}\vec{j}\vec{\gamma}\right)}{4p^0}\,\mathrm{e}^{\mathrm{i}\left(\frac{p^2-m_i^2}{2|\vec{p}|}-\mu_0 m_i H_\perp\right)L},$$

where

$$\vec{j} \equiv \frac{\left[\vec{n}\times\vec{h}\right]}{\sqrt{1-\left(\vec{n}\vec{h}\right)^2}}, \quad \vec{h} \equiv \frac{\vec{H}}{\left|\vec{H}\right|}, \quad \frac{\vec{p}}{\left|\vec{p}\right|} = \vec{n}, \quad \vec{j}^{\,2} = 1, \quad p^0 = \left|\vec{p}\right|.$$

In the magnetic field, each neutrino mass eigenstate splits into two states, corresponding to two possible spin orientations and energies.

This propagator can also be used, if the field changes adiabatically along the neutrino path, i.e.,

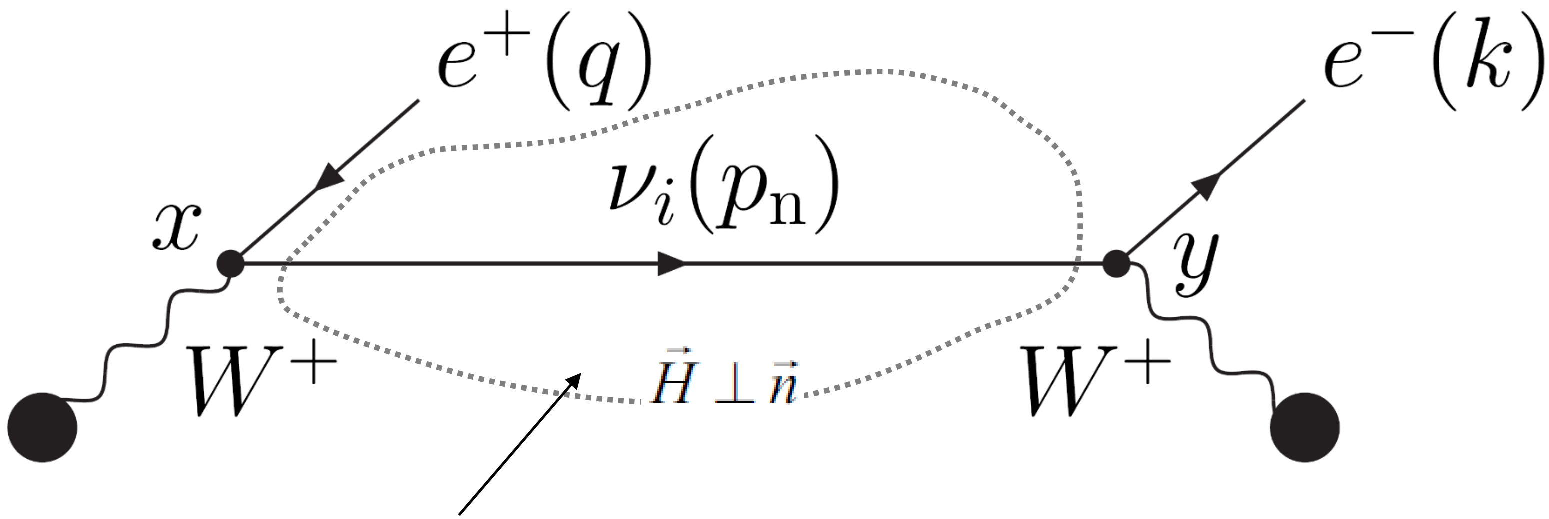
$$\mu_0 m_{max} \vec{n} \vec{\nabla} |\vec{H}| \ll \frac{|\vec{p}|}{d}$$

d denoting the characteristic size of the field region.

In this case, we should use the average value of the field, substituting

$$H \rightarrow \bar{H} = \frac{1}{L} \int_0^L H(l)\, dl \ .$$

Now we consider neutrino oscillation processes in the presence of the field:



Region of the field. The processes of production and detection occur out of it

The amplitude in the momentum representation for $\vec{H} \perp \vec{n}$:

$$M = -\mathrm{i}\frac{G_{\mathrm{F}}^{2}}{8p_{\mathrm{n}}^{0}} J_{\rho}^{(2)}\left(\vec{P}^{(2)}, \vec{P}^{(2')}\right) \overline{u}\left(\vec{k}\right) \gamma^{\rho}\left(1-\gamma^{5}\right) \hat{p}_{\mathrm{n}} \sum_{i=1}^{3} \left|U_{1i}\right|^{2} \left[\left(1-\mathrm{i}\vec{j}\vec{\gamma}\right) \mathrm{e}^{\mathrm{i}\left(\frac{p_{\mathrm{n}}^{2}-m_{i}^{2}}{2|\vec{p}_{\mathrm{n}}|}+\mu_{0} m_{i} \overline{H}\right)L} + \left(1+\mathrm{i}\vec{j}\vec{\gamma}\right) \mathrm{e}^{\mathrm{i}\left(\frac{p_{\mathrm{n}}^{2}-m_{i}^{2}}{2|\vec{p}_{\mathrm{n}}|}-\mu_{0} m_{i} \overline{H}\right)L}\right] \times$$

$$\times \gamma^{\mu}\left(1-\gamma^{5}\right) v(\vec{q}) \, J_{\mu}^{(1)}\left(\vec{P}^{(1)}, \vec{P}^{(1')}\right).$$

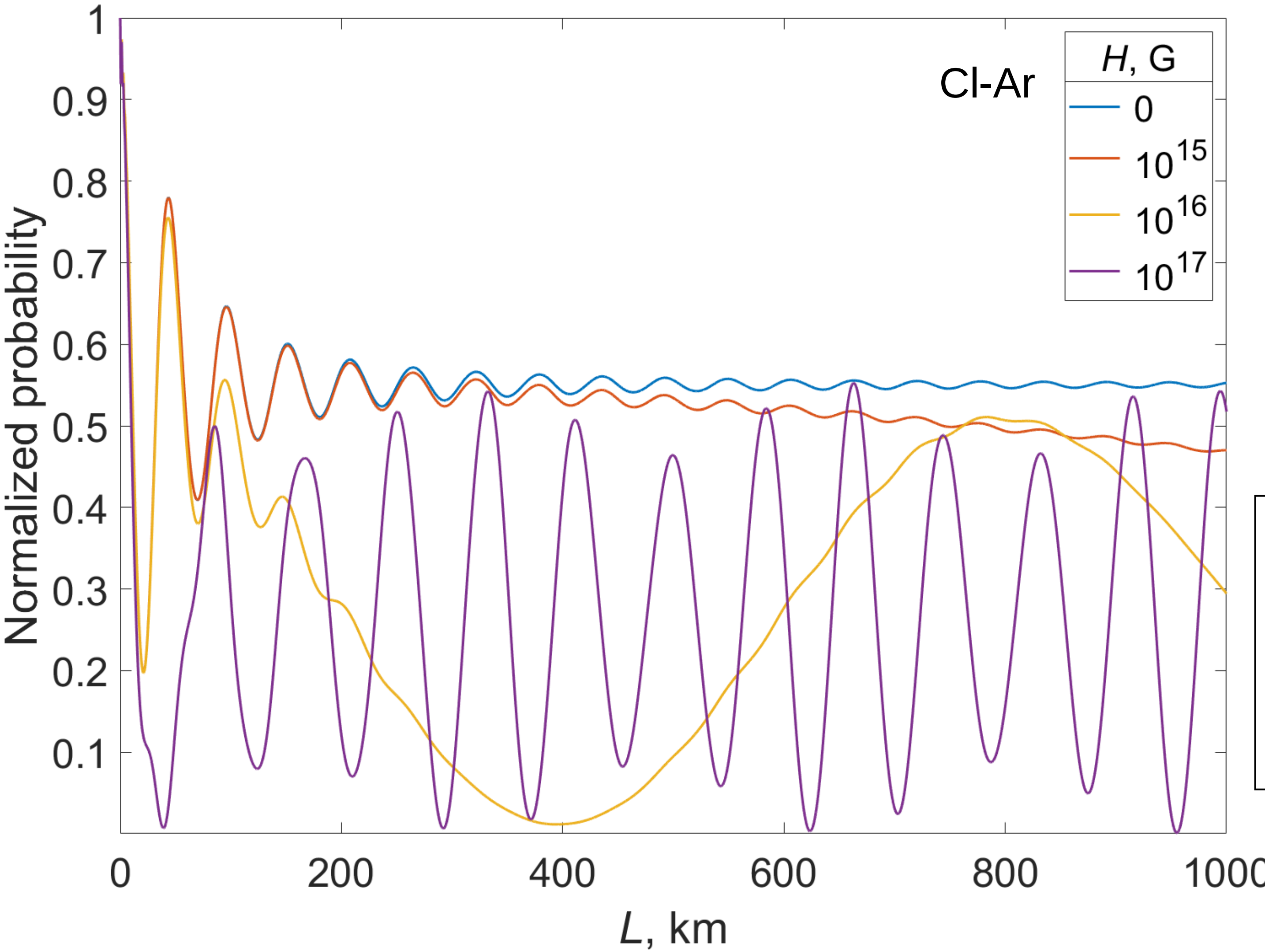
The squared amplitude factorizes again. Performing the outlined procedure, we arrive at the probability of the process:

$$\frac{\mathrm{d}W}{\mathrm{d}\Omega} = \int_{|\vec{p}|_{\min}}^{|\vec{p}|_{\max}} \mathrm{d}\left|\vec{p}\right| \left|\vec{p}\right|^{2} \frac{\mathrm{d}^{3} W_{\mathrm{P}}}{\mathrm{d}^{3} p} W_{\mathrm{D}} P_{ee}\left(\left|\vec{p}\right|, L, \overline{H}\right), \qquad \Delta m_{ik} = m_i - m_k \qquad \Sigma m_{ik} = m_i + m_k$$

$$P_{ee}\left(\left|\vec{p}\right|, L, \overline{H}\right) = \sum_{i=1}^{3} \left|U_{1i}\right|^{4} \cos^{2}\left(\mu_{0} m_{i} \overline{H} L\right) + \sum_{\substack{i,k=1 \\ i<k}}^{3} \left|U_{1i}\right|^{2} \left|U_{1k}\right|^{2} \cos\left(\frac{\Delta m_{ik}^{2}}{2\left|\vec{p}\right|} L\right) \left[\cos\left(\mu_{0} \Delta m_{ik} \overline{H} L\right) + \cos\left(\mu_{0} \Sigma m_{ik} \overline{H} L\right)\right].$$

In the case of two neutrino flavors and a constant field the last formula coincides with the one in the paper *Popov A., Studenikin A., Eur. Phys. J. C.* ***79*** *(2019), 144.*

Normalized probabilities of the neutrino oscillation processes in magnetic field with the neutrino production in the ^{15}O decay and the registration by Cl-Ar detector.

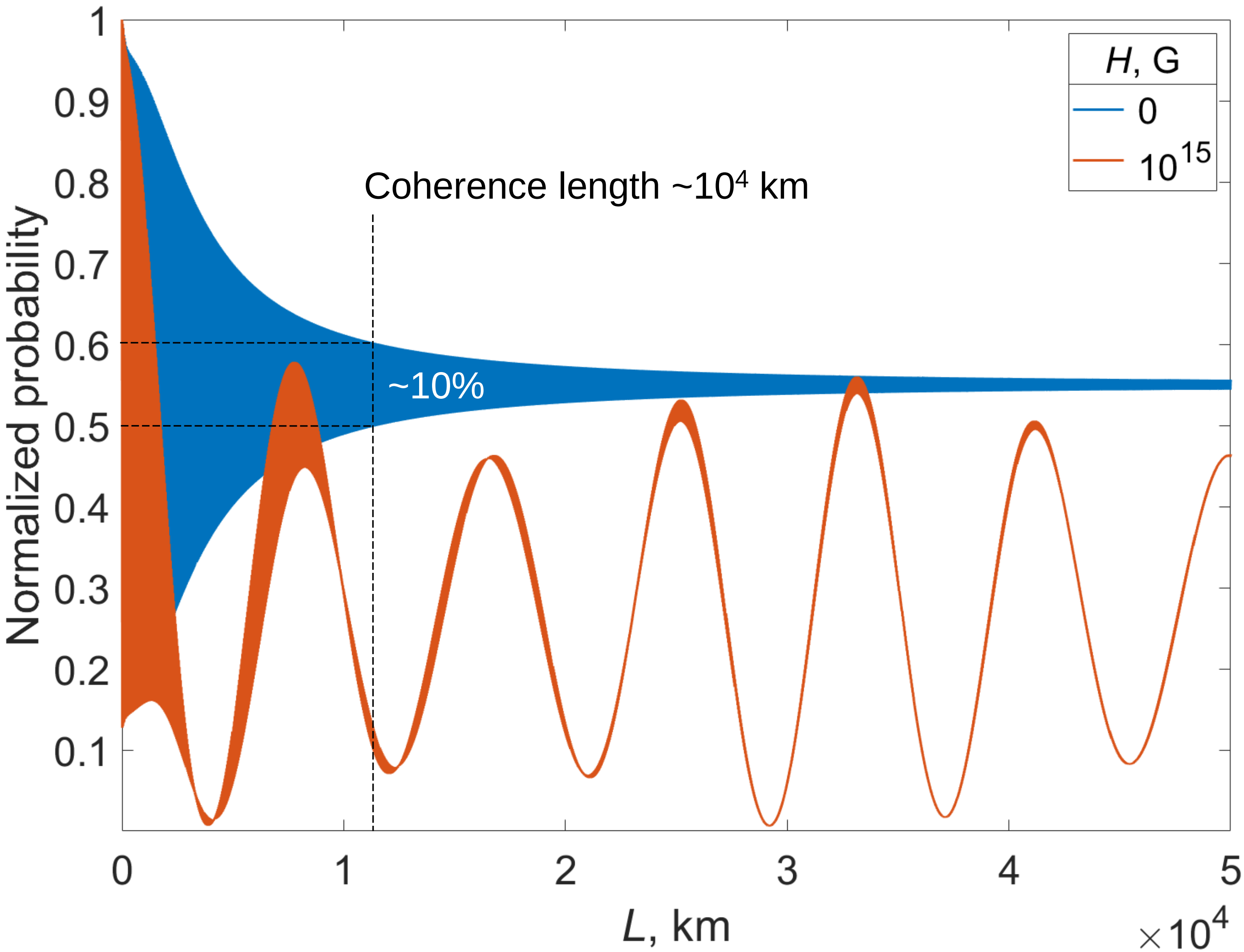


Inside an atom:
$\sim 10^7$ G $\sim$
$\sim 10^6$ eV^2

Magnetars:
up to
$\sim 10^{17}$ G $\sim$
$\sim 10^{16}$ eV^2

Similar curves in the monochromatic case have been found in *A.V. Chukhnova, A.E. Lobanov, Phys. Rev. D* ***101*** *(2020) 1, 013003*

The same for the reaction $^{7}\mathrm{Be}+e^{-}\rightarrow {}^{7}\mathrm{Li}+\nu_i$.



Asymptotic formulas for the normalized probabilities.

Detection through charged current interaction:

$$W_{\mathrm{asym}} = \sum_{i=1}^{3} |U_{1i}|^4 - \sum_{i=1}^{3} |U_{1i}|^4 \sin^2 \delta_i \,, \quad \delta_i = \mu_0 m_i \int_D H\,(l)\, \mathrm{d}l \,,$$

where $\sum_{i=1}^{3} |U_{1i}|^4 \simeq 0.550$ is the asymptotic value in vacuum.

Detection through neutral current interaction:

$$W_{\mathrm{asym}} = \sum_{i=1}^{3} |U_{1i}|^4 - \sum_{i=1}^{3} |U_{1i}|^4 \sin^2 \delta_i \, + \\ + C_{\mathrm{nc}} \left(1 - \sum_{i=1}^{3} |U_{1i}|^2 \sin^2 \delta_i - \sum_{i=1}^{3} |U_{1i}|^4 + \sum_{i=1}^{3} |U_{1i}|^4 \sin^2 \delta_i \right),$$

where

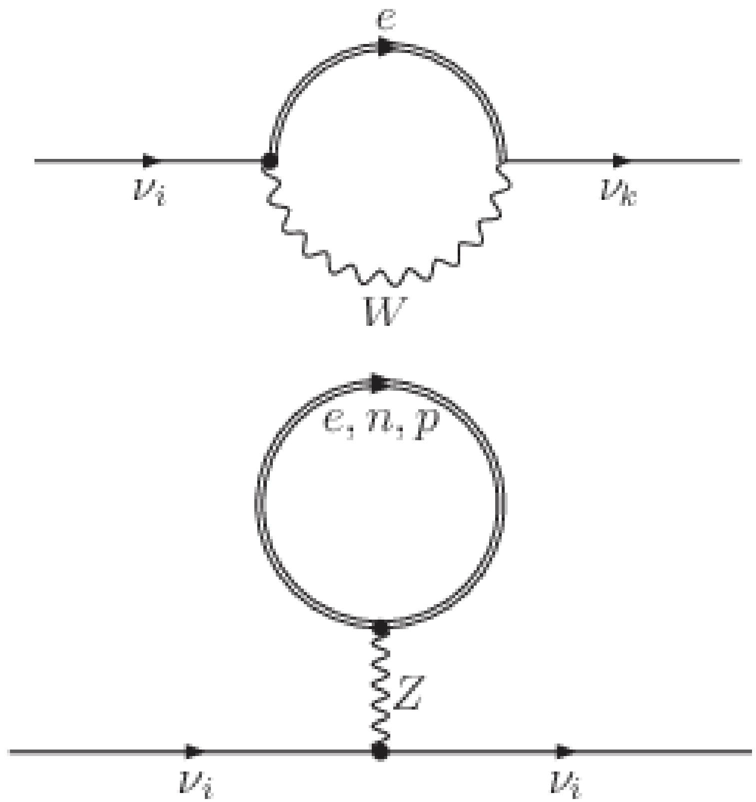
$$C_{\rm nc} = \int\limits_{|\vec{p}|_{\rm min}}^{|\vec{p}|_{\rm max}} \frac{\mathrm{d}^3 W_{\rm P}}{\mathrm{d}^3 p}\, W_{\nu_\mu e}\, |\vec{p}\,|^2\, \mathrm{d}|\vec{p}\,| \left(\int\limits_{|\vec{p}|_{\rm min}}^{|\vec{p}|_{\rm max}} \frac{\mathrm{d}^3 W_{\rm P}}{\mathrm{d}^3 p}\, W_{\nu_e e}\, |\vec{p}\,|^2\, \mathrm{d}|\vec{p}\,| \right)^{-1}$$

and for neutrino energies $420\,\mathrm{keV} \le |\vec{p}\,|_{\rm min} < |\vec{p}\,|_{\rm max} \le 14\,\mathrm{MeV}$

takes values in the interval $0.177 < C_{\rm nc} < 0.321.$

Neutrino oscillations in matter

- L. Wolfenstein, “Neutrino oscillations in matter,” Phys. Rev. D, **17**:9 (1978), 2369 – 2374.
- S.P. Mikheev and A. Y. Smirnov, Sov. J. Nucl. Phys., “Resonance Amplification of Oscillations in Matter and Spectroscopy of Solar Neutrinos,” 42, 913–917, 1985.
- S.P. Mikheev and A. Y. Smirnov, “Resonant amplification of ν oscillations in matter and solar-neutrino spectroscopy,” Nuovo Cim., C9, 17–26, 1986.



- A.E. Lobanov, “Neutrino oscillations in dense matter,” Izvestiya Vysshikh Uchebnykh Zavedenii, Fizika, **59** (2016), 141-144.

Green's function in a medium

$$\mathbf{G_{ik}(x,y)},\ \ \mathbf{i,k=1,2,3},$$

satisfies the equation

$$\left[\left(\mathbf{i}\gamma^\mu\partial_\mu - \mathbf{m_i} - \frac{\mathbf{1}}{\mathbf{2}}\hat{\mathbf{f}}_{\mathrm{N}}(\mathbf{1}-\gamma^{\mathbf{5}})\right)\delta_{\mathbf{ik}} - \mathbf{u_{1i}}\bar{\mathbf{u}}_{\mathbf{1k}}\frac{\mathbf{1}}{\mathbf{2}}\hat{\mathbf{f}}_{\mathrm{C}}(\mathbf{1}-\gamma^{\mathbf{5}})\right]\mathbf{G_{kl}(x,y)} = -\delta_{\mathbf{il}}\delta(\mathbf{x}-\mathbf{y}),$$

where the effective 4-potentials are defined by

$$\mathbf{f}^\mu_{\mathbf{N}} = \sqrt{\mathbf{2}}\mathbf{G_F}\sum_{\mathbf{c=e,p,n}}\left(\mathbf{j}^{\mu(\mathbf{c})}\left(\mathbf{T}_{\mathbf{3}}^{(\mathbf{c})} - \mathbf{2Q}^{(\mathbf{c})}\sin^{\mathbf{2}}\theta_{\mathbf{W}}\right) - \lambda^{\mu(\mathbf{c})}\mathbf{T}_{\mathbf{3}}^{(\mathbf{c})}\right),$$

$$\mathbf{f}^\mu_{\mathbf{C}} = \sqrt{\mathbf{2}}\mathbf{G_F}\left(\mathbf{j}^{\mu(\mathbf{e})} - \lambda^{\mu(\mathbf{e})}\right).$$

If the potentials $\mathbf{f}_{\mathbf{N}}^{\mu}$ and $\mathbf{f}_{\mathbf{C}}^{\mu}$ are constant, the equation for Green's function in the momentum representation takes the form

$$\left[\left(\hat{\mathbf{p}} - \mathbf{m_i} - \frac{1}{2}\hat{\mathbf{f}}_{\mathbf{N}}(1-\gamma^5)\right)\delta_{\mathbf{ik}} - \mathbf{u_{1i}}\bar{\mathbf{u}}_{\mathbf{1k}}\frac{1}{2}\hat{\mathbf{f}}_{\mathbf{C}}(1-\gamma^5)\right]\mathbf{G_{kl}}(\mathbf{p}) = -\delta_{\mathbf{il}}.$$

In the case of two flavors, this Green's function leads to the oscillation length defined by

$$\mathbf{\Delta m_M^2} = \sqrt{(\mathbf{\Delta m^2} - \mathbf{2pf_C})^2 + \mathbf{8\Delta m^2 pf_C}\sin^2\theta},$$

which coincides with the same expression in the standard approach

$$\mathbf{\Delta m_M^2} = \sqrt{(\mathbf{\Delta m^2}\cos\mathbf{2}\theta - \mathbf{2pf_C})^2 + (\mathbf{\Delta m^2}\sin\mathbf{2}\theta)^2}.$$

Conclusion

- It is shown that the Standard Model is capable of describing neutrino oscillations in the framework of a modified perturbative quantum field-theoretical formalism based on the use of distance-dependent propagators.

- The approach correctly reproduces the results of the standard description of neutrino oscillations in vacuum and magnetic field. Unlike the standard approach, it allows one to calculate corrections to the amplitudes of neutrino oscillation processes at low neutrino energies.

- The approach makes use of only the neutrino mass eigenstates, while the flavor states are not needed and should be amputated by Occam’s razor.

- The advantages of the approach are its extreme similarity to the standard Feynman diagram technique, the physical clearness and technical simplicity due to the use of plane waves only.

Thank you!