Solution of the kinetic equation taking into account resonant the Compton process in a magnetized medium

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Введение

- The process of photon propagation in a magnetized equilibrium e^+e^- plasma taking into account the resonance in the Compton scattering reaction is considered.
  $B \lesssim B_e$, $B_e = m^2/e$. The natural system of units is used:
  $c = \hbar = k_b = 1$.
  in CGS $B_e = m^2c^3/e\hbar \simeq 4.41 \cdot 10^{13}$ Гс
- Formulation of the problem. Find a solution of the kinetic equation for the distribution function of photons in a magnetized cold plasma for the process of Compton scattering, taking into account the resonance on a virtual electron.
- Previously, a similar problem was set in the work:
  - Mushtukov A.A. et al. Compton scattering S-matrix and cross section in strong magnetic field Phys. Rev. D. 2016. Vol. 93.
  - Mushtukov A.A. et al. Statistical features of multiple Compton scattering in a strong magnetic field at arxiv 2204.12271v1 2022

- An equilibrium plasma is considered at a temperature $T \ll m$ and field $B \sim 10^{12}$ directed along the z axis. The photon distribution function is non-equilibrium.
- Stationary case $\frac{\partial f_\omega^{(\lambda)}}{\partial t} = 0$
- Then the kinetic equation given by:

$$(\vec{n}, \vec{\bigtriangledown}_r f_\omega^{(\lambda)}) = \sum_{\lambda'=1}^{2} \int dW_{\lambda \to \lambda'} \times$$

$$\times \{ f_{E'}(1 - f_E) f_{\omega'}^{(\lambda')}(1 + f_\omega^{(\lambda)}) - f_E(1 - f_{E'}) f_\omega^{(\lambda)}(1 + f_{\omega'}^{\lambda'}) \,.\}$$

$\lambda\lambda' = 1, 2$ – polarization states of photons.
$f_\omega, f_{\omega'}$ – distribution functions of the final and initial photons.
$f_E, f_{E'}$ – distribution equilibrium functions of the final and initial electrons.
$dW_{\lambda \to \lambda'}$ – differential photon absorption rate (M.V.Chistyakov, D.A.Rumyantsev, EPJ 2016).

At $T \ll m$ and $B \gtrsim 10^{12}$ G, the electrons will occupy the ground Landau level. Therefore, we can put the Landau level of the initial and final electrons $\ell = \ell' = 0$, and for $n = 1$, by studying the behavior of the photon distribution function near the resonance.

$$x \frac{df_\omega^{(\lambda)}(z, x)}{dz} = \sum_{\lambda'=1}^{2} \int_{-1}^{1} dx' \varphi_\omega^{\lambda\lambda'}(x, x') \left[f_\omega^{(\lambda')}(z, x') - f_\omega^{(\lambda)}(z, x) \right]$$

$x = cos(\theta)$, $x' = cos(\theta')$ – the angles between the momentum of the initial and final photons and the magnetic field, respectively.

$$\varphi_\omega^{\lambda\lambda'}(x,x') = \frac{n_e}{32\pi m\omega} \sum_{s''=\pm 1} \int_0^{2\pi} \frac{d\eta}{2\pi} \times$$
$$\times \frac{\left|\mathcal{M}_{e_1\to e_0\gamma^{(\lambda')}}\right|^2 \left|\mathcal{M}_{e_0\gamma^{(\lambda')}\to e_1^{(s'')}}\right|^2}{[\omega^2(1-x^2)+2\omega m-2eB]^2+(\Gamma_1^{s''}P_0/2)^2} \times$$
$$\times \frac{(m+\omega-\omega x x' - \sqrt{m+\omega-\omega x x')^2 - x'^2 2eB}}{\sqrt{(m+\omega-\omega x x')^2 - x'^2 2eB}}$$

Where $\Gamma_1^{s''}$ – total electron absorption width.

$$E_1''\Gamma_1^{\pm} \simeq \frac{e^2(eB)^2}{\pi M_1} \frac{1}{M_1 \pm m} \int_0^{\zeta} dx e^{-x} \frac{1-\zeta\cdot x}{\sqrt{x^2-\zeta\cdot x+1}}$$

Here $M_n = \sqrt{m^2 + 2\cdot eBn}$ and $\zeta = \frac{M_1^2+m^2}{eB}$

The equation can be conveniently rewritten in the following way

$$\frac{df_{\omega}^{(\lambda)}(x)}{dz} + \chi_{\omega}^{(\lambda)}(x) f_{\omega}^{(1)}(x) =$$
$$= \frac{1}{x} \int_{-1}^{1} dx' \cdot \left\{ \varphi_{\omega}^{\lambda 1}(x, x') f_{\omega}^{(1)}(z, x') + \varphi_{\omega}^{\lambda 2} f_{\omega}^{(2)}(z, x') \right\} ,$$

where

$$\chi_{\omega}^{(\lambda)} \equiv \frac{1}{x} \int_{-1}^{1} dx' \left\{ \varphi_{\omega}^{\lambda 1}(x') + \varphi_{\omega}^{\lambda 2}(x') \right\}$$

The formal solution of the equation can be represented as follows ($C_{\omega}^{(\lambda)}$ determined by the boundary conditions)

$$f_{\omega}^{(\lambda)}(z, x) = C_{\omega}^{(\lambda)} e^{-\chi_{\omega}^{(\lambda)}(x) \cdot z} + \frac{1}{x} \int_{0}^{z} dz' \int_{-1}^{1} dx' e^{-\chi_{\omega}^{(\lambda)}(x) \cdot (z - z')} \times$$
$$\times \left\{ \varphi_{\omega}^{\lambda 1}(x, x') f_{\omega}^{(1)}(z', x') + \varphi_{\omega}^{\lambda 2} f_{\omega}^{(2)}(z' x') \right\}$$

An obtained equation with z is the Volterra equation, which can be solved using the Laplace transform.

Laplace transform from distribution functions

$$\overline{\mathcal{U}}^{(\lambda)}(s,x) \risingdotseq f^{(\lambda)}(z,x)$$

The resulting equation for $\mathcal{U}^{(\lambda)}(s,x)$

$$\overline{\mathcal{U}}^{(\lambda)}(s,x) = C_{\omega}^{(\lambda)}\frac{1}{\chi(x)+s} + \frac{1}{x}\int_{-1}^{1} dx' \frac{1}{\chi(x)+s} \times$$
$$\times \left\{ \varphi^{\lambda 1}(x,x')_{\omega}\overline{\mathcal{U}}^{(1)}(s,x') + \varphi_{\omega}^{\lambda 2}(x,x')\overline{\mathcal{U}}^{(2)}(s,x) \right\}$$

Using the expansion in Legendre polynomials

$$f_\omega^{(\lambda)}(z,x) = \sum_{\ell=0}^{\infty} A_\ell^{(\lambda)}(z,\omega)\mathcal{P}_\ell(x)\,,$$

$$\overline{\mathcal{U}}^{(\lambda)}(s,x) = \sum_{\ell=0}^{\infty} \overline{A}_\ell^{(\lambda)}(s,\omega)\mathcal{P}_\ell(x)\ ,$$

We obtain the system of algebraic equations:

$$\overline{A}_{\ell'}^{(\lambda)}(z,\omega) = f_0 \int_{-1}^{1} dx \frac{\mathcal{P}_{\ell'}(x)}{\chi_\omega^{(\lambda)}(x)+s} + \sum_{\ell=0}^{\infty}\int_{-1}^{1} dx \int_{-1}^{1} dx' \frac{\mathcal{P}_{\ell'}(x)\mathcal{P}_\ell(x')}{x\chi_\omega^{(\lambda)}(x)+sx} \times$$
$$\times \left\{\overline{A}_\ell^{(1)}(s,\omega)\varphi_\omega^{\lambda 1}(x,x') + \overline{A}_\ell^{(2)}(s,\omega)\varphi_\omega^{\lambda 2}(x,x')\right\}\ ,$$

где $f_0 = \frac{1}{e^{\omega/T}-1}$

Finally, the distribution functions of photons for two possible polarization states $\lambda = 1, 2$ can be represented as follows:

$$f_{\omega}^{(\lambda)}(z, x) = \frac{1}{2\pi i} \sum_{\ell=0}^{\infty} P_{\ell}(x) \int_{\sigma-i\infty}^{\sigma+i\infty} ds \cdot e^{sz} \overline{A}_{\ell}^{(\lambda)}(s, x)$$

Коэффициент A при $\lambda = 1$ $\ell = 0$

$$\overline{A}_0^{(1)} = -\overline{f(1/3)}\left(32\left(\pi - i\ln\left(\frac{3\overline{s}+8}{3\overline{s}-8}\right)\right)\left(9\overline{s}\ln\left(\frac{3\overline{s}+8}{3\overline{s}-8}\right) - 112\right)\right) :$$

$$: \left(-48i\overline{s}\left(9\overline{s}^2 - 64\right) + i\left(81\overline{s}^4 - 4096\right)\ln\left(\frac{3\overline{s}+8}{3\overline{s}-8}\right) + 4096\pi\right)$$

$$\overline{f(\gamma)} = f_0/\frac{\rho\cdot\omega^2}{\Delta_-};$$

$$\overline{s} = s/\frac{\rho\cdot\omega^2}{\Delta_-}$$

$\beta = eB/m^2 \simeq 0.023$, $\rho \simeq 8\pi\alpha^2 n_e$

Выводы

- The solution of the kinetic equation for finding the distribution function of photons of two possible polarizations in an equilibrium e^+e^- plasma in a relatively strong magnetic field in the cold plasma approximat.ion and taking into account resonance on a virtual electron is considered.
- The original equation is reduced to the Volterra equation in z and the Fredholm equation in terms of the angular distribution.
- Using the Laplace transform and the expansion of the distribution function in Legendre polynomials, the problem is reduced to a system of algebraic equations, the coefficients of which can be easily calculated numerically.
- The quadrature solution for the distribution function of two possible photon polarizations is obtained.