Critical Non-Abelian Vortex and Holography for Little String Theory

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1 Introduction

Seiberg and Witten 1994 : Confinement in the monopole vacuum of $\mathcal{N}=2$ supersymmetric QCD

Abelian confinement

In the search for a non-Abelian confinement

Non-Abelian vortex strings

were found in $\mathcal{N}=2$ U(N) QCD

Hanany, Tong 2003

Auzzi, Bolognesi, Evslin, Konishi, Yung 2003

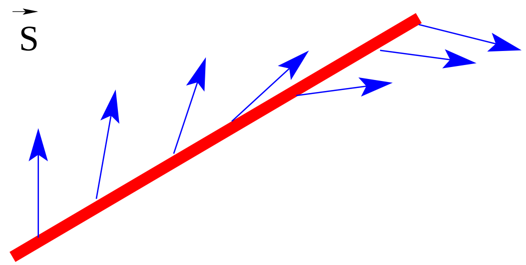
Shifman Yung 2004

Hanany Tong 2004

Non-Abelian string : Orientational zero modes

Rotation of color flux inside SU(N).

Non-Abelian vortex string is BPS and preserves $\mathcal{N}=(2,2)$ supersymmetry on its world sheet.



Shifman and Yung, 2015: Non-Abelian vortex in $\mathcal{N} = 2$ supersymmetric QCD can behave as a critical superstring

Idea:

Non-Abelian string has more moduli then Abrikosov-Nielsen-Olesen string.

It has translational + orientaional moduli

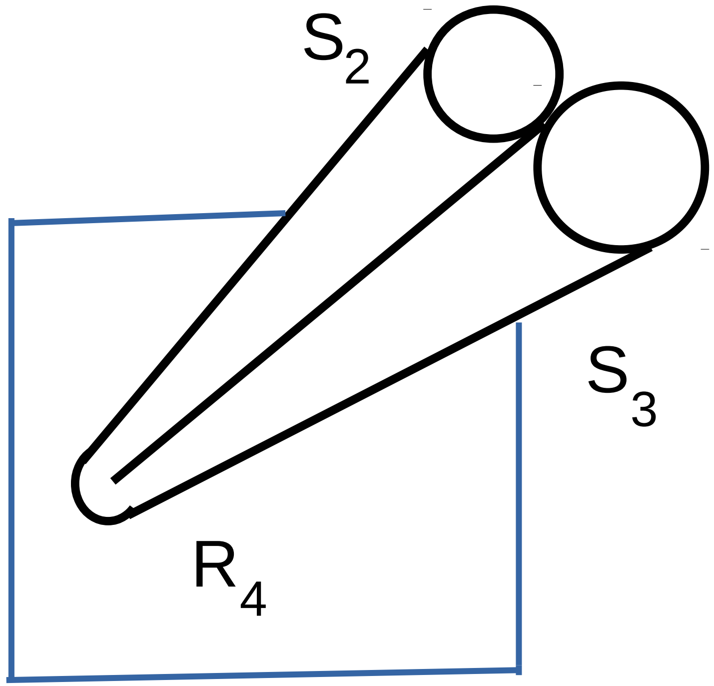
We can fulfill the criticality condition: In $\mathcal{N} = 2$ QCD with U$(N = 2)$ gauge group and $N_f = 4$ quark flavors.

- The solitonic non-Abelian vortex have six orientational and size moduli, which, together with four translational moduli, form a ten-dimensional space.
- For $N_f = 2N$ 2D world sheet theory on the string is conformal.

For $N = 2$ and $N_f = 4$ the target space of the 2D sigma model on the string world sheet is

$$R^4 \times Y_6,$$

where Y_6 is a non-compact Calabi-Yau manifold studied by Candelas, Witten and Vafa, namely conifold.



Compactification on Y_6

We studied states of closed type IIA string propagating on $R^4 \times Y_6$ and interpreted them as hadrons in 4D $\mathcal{N} = 2$ QCD.

Conifold == non-compact CY.

Looking for states with normalizable wave function over Y_6

String states localized near the conifold singularity. They are 4D SQCD states

2 Non-Abelian vortex and Little String Theory

Ghoshal, Vafa, 1995; Giveon Kutasov 1999

Critical string on a conifold is equivalent to non-critical $c = 1$ string

$$\mathcal{R}^4 \times \mathcal{R}_\phi \times S^1,$$

$\mathcal{R}_\phi$ is a real line associated with the Liouville field ϕ and the theory has a linear in ϕ dilaton, such that string coupling is given by

$$g_s = e^{-\frac{Q}{2}\phi}.$$

Aharony, Berkooz, Kutasov, Seiberg, 1998

Non-gravitational string theories – ”Little String Theories”

$$T_{--} = -\frac{1}{2}\left[(\partial_z \phi)^2 + Q\,\partial_z^2 \phi + (\partial_z Y)^2\right]$$

$$Y \sim Y + 2\pi Q \qquad Q = \sqrt{2}, \qquad c^{SUSY}_{\phi+Y} = 3 + 3Q^2 = 9$$

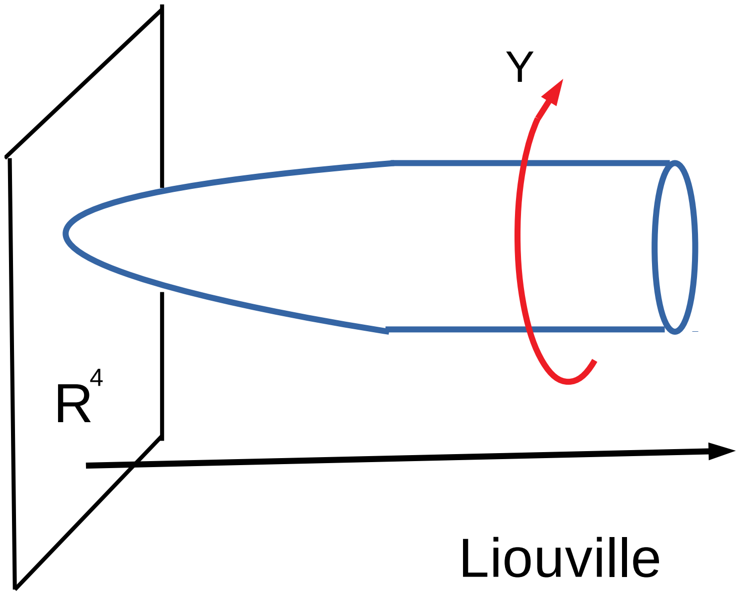
Liouville interaction

$$\delta L = b \int d^2\theta\, e^{-\frac{\phi+iY}{Q}}$$

Mirror description: $SL(2,R)/U(1)$ WZNW model at level $k=1$.

Bosonic part is 2D Witten's black hole with target space forming semi-infinite cigar.

Lioville field ϕ – motion along the cigar.



The spectrum of primary operators was computed exactly.

Dixon, Peskin, Lykken, 1989; Mukhi, Vafa, 1993; Evans, Gaberdiel, Perry, 1998

$$V_{j,m} \approx \exp\left(\sqrt{2}j\phi + i\sqrt{2}mY\right), \qquad \phi \to \infty$$

$$V_{j;m} = g_s \Psi_{j;m}(\phi, Y) = e^{-\frac{\phi}{\sqrt{2}}}\, \Psi_{j;m}(\phi, Y), \qquad \Psi_{j;m} \sim e^{\sqrt{2}(j+\frac{1}{2})\phi + i\sqrt{2}mY}$$

Normalizable states have $j \leq -\frac{1}{2}$

We have

- Discrete series

$$j = -\frac{1}{2}, -1, -\frac{3}{2}, ..., \qquad m = \pm\{j, j-1, j-2, ...\}$$

- Prinipal continues representations

  $j = -\frac{1}{2} + is$, $m =$ integer or half-integer

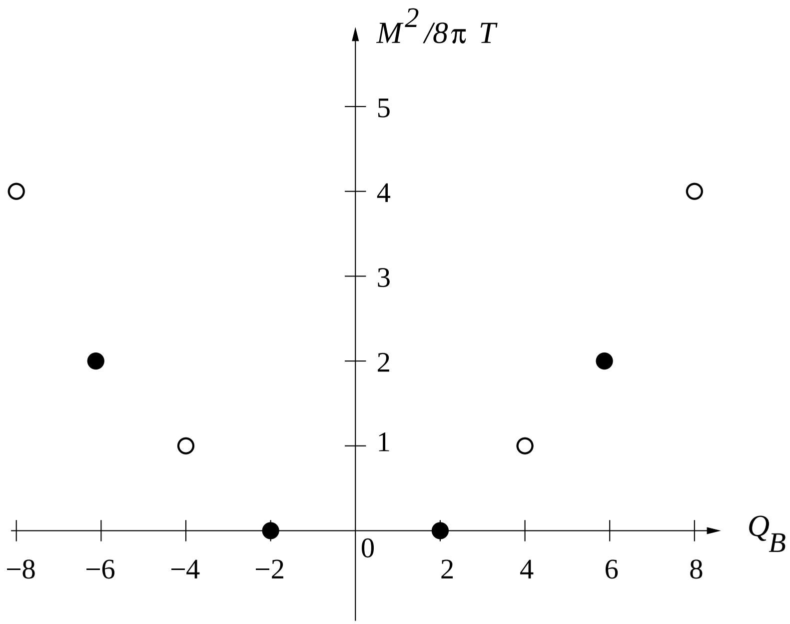
Another requirement: No negative norm states

$$-\frac{k+2}{2} < j < 0$$

For our value k = 1 we are left with

$$j = -\frac{1}{2}, \qquad m = \pm \left\{ \frac{1}{2}, \frac{3}{2}, \ldots \right\}$$

$$j = -1, \qquad m = \pm \{ 1, 2, \ldots \}$$



Global group of the 4D QCD:

$$SU(2) \times SU(2) \times U(1)_B$$

U(1) - "baryonic" symmetry.

$$Q_B = 4m$$

3 Solitonic gauge-string duality vs AdS/CFT

Of course, there are conceptual differences.

- The origin of the 10D space.
- The scales of the string tension are dramatically different.

Let us 'forget' it for a minute and take a pragmatic point of view

- AdS/CFT correspondence is based on the presence of $N_b \to \infty$ parallel D-branes, while we have no branes.
- AdS/CFT correspondence assumes holography. Off-shell correlation functions on the field theory side correspond to string theory correlation functions on the "boundary", infinitely far away from the branes.

  In our solitonic string-gauge duality all non-trivial 'real' physics localized near the tip of the cigar.

  We consider states with $j \leq -1/2$

The first distinction suggests that we can think of our solitonic string-gauge duality as of $N_b = 0$ limit of AdS/CFT correspondence. The simplest example is the Klebanov-Witten's construction of N_b D3-branes filling the R^4 space near the conifold singularity in type IIB superstring. If $N_b \to 0$ warp-factors disappear and we get our conifold geometry.

The second distinction seems crucial. Our goal is to clarify this issue.

The closest example of AdS/CFT holography:

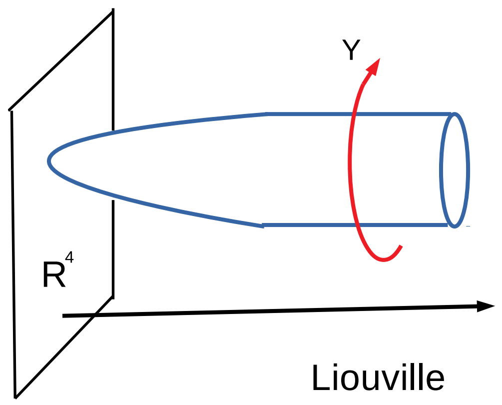
Aharony, Berkooz, Kutasov, Seiberg, 1998 ; Giveon, Kutasov, 1999;

Aharony, Giveon, Kutasov, 2004

6D LST on the world volume of k NS 5-branes, $k \to \infty$ $\Longleftarrow\!\!\Longrightarrow$ String theory on $R^6 \times SL(2,R)_k/U(1) \times SU(2)_k/U(1)$

Holography:

Off-shell correlation functions in QFT correspond to correlation functions of non-normalizable vertex operators on the cigar.



Aharony, Giveon, Kutasov, 2004

$$V^{\mathrm{non-norm}} \sim \frac{1}{p_\mu^2 + M^2} V^{\mathrm{norm}}$$

LSZ poles

We test this holography for our 4D SQCD.

4 Correlation functions

Vertex operator for arbitrary j

$$V_{j,m_L,m_R} = e^{iQ(m_L Y_L + m_R Y_R)} \left\{ e^{Qj\phi} + R(j, m_L, m_R; k) e^{-Q(j+1)\phi} + \cdots \right\},$$

where $Q^2 = 2/k$, $k \to 1$ and reflection coefficient

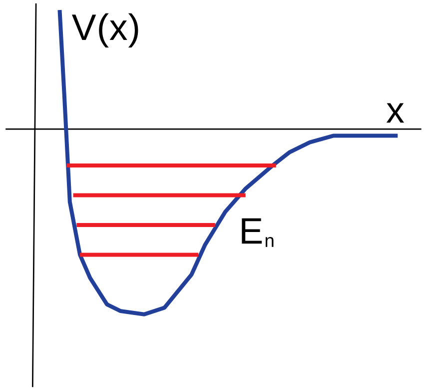
$$R(j, m_L, m_R; k) = \left[\frac{1}{\pi} \frac{\Gamma\left(1 + \frac{1}{k}\right)}{\Gamma\left(1 - \frac{1}{k}\right)} \right]^{2j+1} \frac{\Gamma\left(1 - \frac{2j+1}{k}\right) \Gamma(m_L + j + 1) \Gamma(m_R + j + 1) \Gamma(-2j - 1)}{\Gamma\left(1 + \frac{2j+1}{k}\right) \Gamma(m_L - j) \Gamma(m_R - j) \Gamma(2j + 1)}$$

At $j < -1/2$ $\quad e^{Qj\phi} \to$ falling wave function at $\phi \to \infty$

With $\tilde{j} = -(1 + j)$ $\quad e^{Q\tilde{j}\phi} \to$ rising wave function at $\phi \to \infty$

$$\Delta_{j,m} = \frac{1}{k} \left\{ -j(j+1) + m^2 \right\}$$

Example in quantum mechanics



$$\left\{-\partial_x^2 + V(x)\right\}\Psi = E\Psi$$

At $x \to \infty$ $\quad \Psi = Ae^{-\alpha_n x}, \quad E_n = -\alpha_n^2, \quad \alpha_n > 0$

If we allow rising exponents

$$\Psi = Ae^{-\alpha x} + Be^{\alpha x}$$

then there is no quantization of energy and

$$B \sim A\left(E - E_n\right)$$

If $j \to j_0$, where j_0 is from the discrete spectrum $j_0 = -1, -3/2, \ldots$

$$V_{j,m_L,m_R} = e^{iQ(m_L Y_L + m_R Y_R)} \left\{ e^{Qj\phi} + R(j, m_L, m_R; k) e^{-Q(j+1)\phi} + \cdots \right\},$$

$R(j, m_L, m_R; k) \to 0$ and

$$V_{j,m_L,m_R} \to e^{iQ(m_L Y_L + m_R Y_R)} \, e^{Qj\phi}$$

become normalizable.

Vise verse for $\tilde{j} = -(1+j) \to 0, 1/2, 1, \ldots$

$$R(\tilde{j}, m_L, m_R; k) \sim \frac{1}{j - j_0}$$

develops a pole and

$$V^{\mathrm{non-norm}}_{\tilde{j},m_L,m_R} \to e^{iQ(m_L Y_L + m_R Y_R)} \frac{e^{Qj\phi}}{j - j_0} \sim \frac{1}{p_\mu^2 + M^2} V^{\mathrm{norm}}_{j,m_L,m_R}$$

reduces to the normalizable $e^{Qj\phi}$ with $j < -1/2$

We checked this in our theory for 2pt correlation function for $\tilde{j} = -(1+j) \to 0$,

$$\left\langle V_{\tilde{j};m,-m} V_{\tilde{j}';-m,m} \right\rangle = -\frac{1}{1+j} \frac{m^2}{2\pi}$$

and 3pt correlation function

$$\left\langle V_{\tilde{j};m_1,-m_1} V_{j_2=-1/2;m_2,-m_2} V_{j_3=-1/2;m_3,-m_3} \right\rangle = -\frac{1}{2\pi} \frac{1}{1+j} \delta_{m_1+m_2+m_3,0}$$

Poles precisely corresponds to a $j = -1$ discrete series.

Holography works!

For $j = -1/2$ two exponents coincide and there is no pole.

$$\left\langle V_{-1/2;m,-m} V_{-1/2;-m,m} \right\rangle = 1$$

This confirms the interpretation of $j = -1/2$ states as physical states (logarithmically) localized near the tip of the cigar.

No holography in these channels

5 Conclusions

- We calculated 3pt correlation functions of normalizable operators which determine interactions of physical states in 4D SQCD.
- Holography works for most channels in our theory because non-normalizable vertex operators reduce to normalizable ones near special values of j associated with discrete spectrum.
- For operators of $j = -1/2$ series on the borderline between normalizable and non-normalizable operators holography does not work. These vertex operators correspond to physical states in 4D SQCD.

4D Massless states = Deformations of 10D metric preserving Ricci flatness

Deformations preserving Ricci-flat metric on a Calabi-Yau manifold are Kahler form deformations and deformations of the complex structure.

Conifold == non-compact CY.

Looking for states with normalizable wave function over Y_6

String states localized near the conifold singularity. They are 4D SQCD states

The only 4D (logarithmically) normalizable state is associated with deformations of the conifold complex structure

Complex structure modulus b = massless BPS baryon in 4D SQCD

10D "tachyon"

$$V^S_{j,m}(p_\mu) = e^{-\varphi}\, e^{ip_\mu x^\mu}\, V_{j,m}\,, \qquad j = -\frac{1}{2}, \qquad m = \pm\left\{\frac{1}{2}, \frac{3}{2}, \dots\right\}$$

$$\frac{p_\mu p^\mu}{8\pi T} + m^2 - j(j+1) = \frac{1}{2}$$

$$\frac{(M^S)^2}{8\pi T} = -\frac{p_\mu p^\mu}{8\pi T} = m^2 - \frac{1}{2} - j(j+1) = m^2 - \frac{1}{4} = 0, 2, 6, \dots$$

Massless state at $m = \pm\frac{1}{2}$ – b-state

Spin-2 states

$$V^G_{j,m}(p_\mu) = \xi_{\mu\nu}\psi^\mu_L\psi^\nu_R\, e^{-\varphi}\, e^{ip_\mu x^\mu}\, V_{j,m}, \qquad j = -1, \qquad m = \pm\{1, 2, \dots\}$$

$$\frac{(M^G)^2}{8\pi T} = m^2 = 1, 4, 9, \dots$$

No massless graviton