Large neutrino mixing from the evolution of the renormalization group in the super-weak theory

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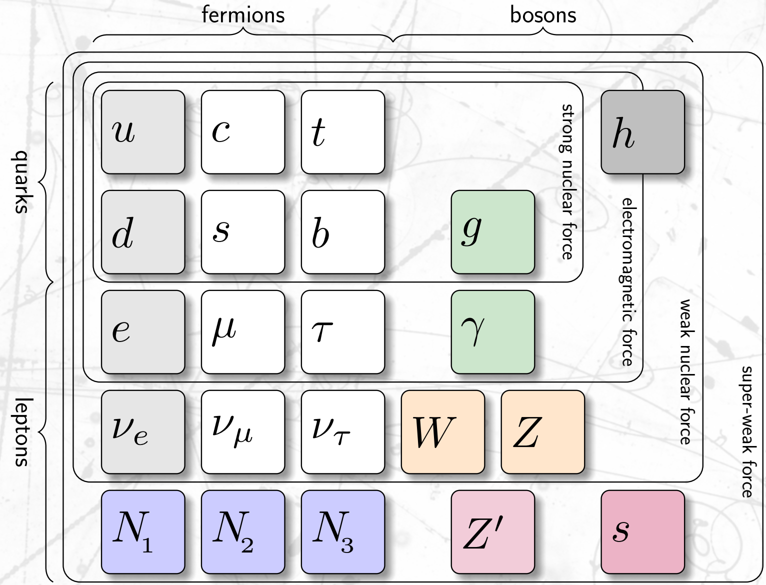

Particle content of the standard model with the super-weak extension.

Group representations and charges of the fermions and scalars in the Super-weak Standard Model (SWSM)

field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$U(1)_z$
Q_L	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	$\frac{1}{6}$
u_R	$\mathbf{3}$	$\mathbf{1}$	$\frac{2}{3}$	$\frac{7}{6}$
d_R	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$	$-\frac{5}{6}$
L_L	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
ℓ_R	$\mathbf{1}$	$\mathbf{1}$	-1	$-\frac{3}{2}$
N_R	$\mathbf{1}$	$\mathbf{1}$	0	$\frac{1}{2}$
ϕ	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	1
χ	$\mathbf{1}$	$\mathbf{1}$	0	-1

U(1) charge operators, third $SU(2)_L$ generator and Q

f	y	z	T_3	Q
u_L	$-\frac{1}{6}$	$\frac{1}{6}$	$-\frac{1}{2}$	$\frac{2}{3}$
u_R	$-\frac{2}{3}$	$\frac{7}{6}$	0	$\frac{2}{3}$
d_L	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{3}$
d_R	$\frac{1}{3}$	$-\frac{5}{6}$	0	$-\frac{1}{3}$
ν_L	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0
N_R	0	$\frac{1}{2}$	0	0
ℓ_L	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	-1
ℓ_R	1	$-\frac{3}{2}$	0	-1

Gauge extension

The gauge group of the super-weak model:

$$\mathrm{SU}(3)_\mathrm{c} \otimes \mathrm{SU}(2)_\mathrm{L} \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_z$$

The SM group is extended by an extra U(1) group

The kinetic terms of the $\mathrm{U}(1)_Y \otimes \mathrm{U}(1)_z$ sector of the group can be described with the Lagrangian density:

$$\mathcal{L}^{\mathrm{U}(1)} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{4} F'^{\mu\nu} F'_{\mu\nu} - \frac{\varepsilon}{2} F^{\mu\nu} F'_{\mu\nu}$$

where $F_{\mu\nu}$ and $F'_{\mu\nu}$ correspond to the field strength tensors of $\mathrm{U}(1)_Y$ and $\mathrm{U}(1)_z$

Gauge extension

The covariant derivative acting on the fermion field f:

$$\mathcal{D}_\mu^{U(1)} = \partial_\mu - \mathrm{i}(y^f g_y B_\mu + z^f g_z B'_\mu)$$

The y^f and z^f are the hypercharge and $\mathrm{U}(1)_z$ charge of f

The mass eigenstates (A_μ, Z_μ, Z'_μ) with a rotation:

$$\begin{pmatrix} \hat{B}_\mu \\ W_\mu^3 \\ \hat{B'}_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\cos\theta_Z \sin\theta_W & -\sin\theta_Z \sin\theta_W \\ \sin\theta_W & \cos\theta_Z \cos\theta_W & \cos\theta_W \sin\theta_Z \\ 0 & -\sin\theta_Z & \cos\theta_Z \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_\mu \\ Z'_\mu \end{pmatrix}$$

θ_W is the weak mixing angle and θ_Z is the $Z - Z'$ mixing angle

Scalar extension

The scalar sector of the SM Higgs SU(2) doublet ϕ with charges $(y_\phi, z_\phi) = (1/2, 1)$ and a complex singlet scalar χ with charges $(y_\chi, z_\chi) = (0, -1)$. The relevant Lagrangian:

$$\mathcal{L}_{\text{scalar}} = |D_\mu \phi|^2 + |D_\mu \chi|^2 - \mu_\phi^2 |\phi|^2 - \mu_\chi^2 |\chi|^2 - \lambda_\phi |\phi|^4 - \lambda_\chi |\chi|^4 - \lambda |\phi|^2 |\chi|^2$$

Parametrizing the fields after spontaneous symmetry breaking (SSB):

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\sqrt{2}\sigma^+ \\ v + h' + i\sigma_\phi \end{pmatrix}, \quad \chi = \frac{1}{\sqrt{2}} (w + s' + i\sigma_\chi)$$

$v \simeq 246.22$ GeV and w are the vacuum expectation values and the fields h', s', σ_χ and σ_ϕ are real

The fields σ_ϕ and σ_χ correspond to the Goldstone bosons

Scalar extension

Scalar and Goldstone mixing angles:

$$\sin\theta_S = -\frac{\lambda v w}{\lambda_\phi v^2 - \lambda_\chi w^2}, \ \tan\theta_G = \frac{M_{Z'}}{M_Z}\tan\theta_Z$$

Fermion extension

The fermion sector of the super-weak model is extended with three sterile massive Majorana neutrinos $N_R = (\nu_4, \nu_5, \nu_6)$

The gauge invariant Yukawa interactions of the neutrinos are given by Lagrangian density

$$\mathcal{L}_Y^\nu = -\overline{N_R} Y_\nu \varepsilon_{\alpha\beta} L_{L\alpha} \phi_\beta - \frac{1}{2} \overline{N_R} Y_N (N_R)^c \chi + \text{h.c.}$$

α and β are $\mathrm{SU}(2)_L$ indices and $\varepsilon_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

The 3×3 Dirac and Majorana mass matrices:

$$M_D = \frac{v}{\sqrt{2}} Y_\nu, \quad M_N = \frac{w}{\sqrt{2}} Y_N$$

Fermion extension

The light neutrino mass matrix $M_L = -M_D M_N^{-1} M_D^\dagger + \text{h.c.}$ can be obtained by block-diagnonalizing the full 6×6 neutrino mass matrix M via a unitary matrix U:

$$U^T M U = U^T \begin{pmatrix} 0 & M_D^T \\ M_D & M_N \end{pmatrix} U = M_{\text{diag}} = \text{diag}(m_1, \ldots, m_6)$$

Active-sterile mixing

The 3×3 active-sterile mixing matrix:

$$U_{\rm as} = \begin{pmatrix} U_{e4} & U_{e5} & U_{e6} \\ U_{\mu4} & U_{\mu5} & U_{\mu6} \\ U_{\tau4} & U_{\tau5} & U_{\tau6} \end{pmatrix} = m_D^* m_R^{-1\dagger} = U^{\rm PMNS}\sqrt{M_L^{\rm diag}}\, iR^T\sqrt{M_R^{-1}}$$

R is real, parametrized as:

$$R = \begin{pmatrix} c_{12} & -s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & s_{23} & c_{23} \end{pmatrix}$$

$c_{ij} = \sqrt{1 - s_{ij}^2}$ and $s_{ij} \in [0, 1]$

RGE for neutrino mass eigen values and mixing angles:

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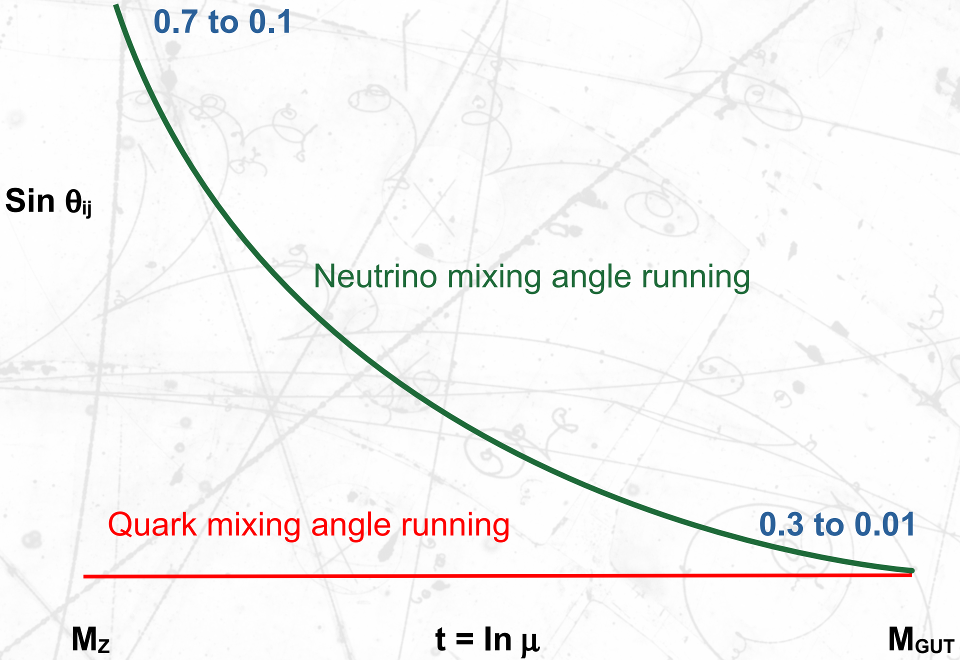
0.7 to 0.1
Sin θij
Neutrino mixing angle running
Quark mixing angle running
0.3 to 0.01
MZ
t = ln μ
MGUT