Lomonosov Moscow State University,Physics Faculty,
The Moscow Institute of Physics and Technology, Dolgoprudny, Moscow Region,
Skobeltsyn Institute of Nuclear Physics,
NRC «Kurchatov Institute» – ITEP, Moscow

Rare decays of neutral B mesons to four charged leptons in the Standard Model

Danilina A.V., Nikitin N.V.,
anna.danilina@cern.ch, Nikolai.Nikitine@cern.ch,

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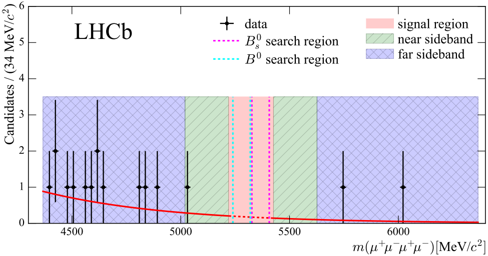
Motivation

Search for $\bar{B}_{d,s} \to \mu^+\mu^-\mu^+\mu^-$ decays



Aaij *et al.* [LHCb Collaboration], «Search for decays of neutral beauty mesons into four muons», JHEP 1703, 001 (2017)

$$\mathrm{Br}_{Exp}\left(\bar{B}^0 \to \mu^+\mu^-\mu^+\mu^-\right) \leq 1.8 \times 10^{-10} \text{ at } 95\%\,\mathrm{CL}.$$

and

$$\mathrm{Br}_{Exp}\left(\bar{B}_s \to \mu^+\mu^-\mu^+\mu^-\right) \leq 8.6 \times 10^{-10} \text{ at } 95\%\,\mathrm{CL}$$

Aaij *et al.* [LHCb Collaboration], «Searches for rare B_s^0 and B^0 decays into four muons», JHEP 03, 109 (2022)

$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+ e^-\right)$: theoretical estimation

- Estimation that represents bremsstrahlung, weak annihilation and ω (782) resonance contributions [A. Kozachuk, D. Melikhov, N. Nikitin, PRD **97**, 053007 (2018) and D. Melikhov, N. Nikitin, PRD **70**, 114028 (2004)]:

$$\begin{aligned} & \mathrm{Br}_{\gamma LL}\left(\bar{B}_d \to \mu^+\mu^- e^+ e^-\right) \approx \\ \approx \quad & \alpha_{em}\Big(\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^-\gamma\right) + \mathrm{Br}\left(\bar{B}_d \to e^+ e^-\gamma\right)\Big) \approx \\ \approx \quad & 4 \times 10^{-12}. \end{aligned}$$

- Estimation under the assumption that the nonperturbative contributions from strong interactions to the decay amplitudes $B^- \to \ell^+\,\ell^-\,\bar{\nu}_{\ell'}\,\ell'^-$ and $B_{d,s} \to \ell^+\,\ell^-\,\ell'^+\,\ell'^-$ are of the same order, i.e. the partial widths differ only due to electroweak factors [Yad. Fiz. 81, no. 3, 331 (2018)] :

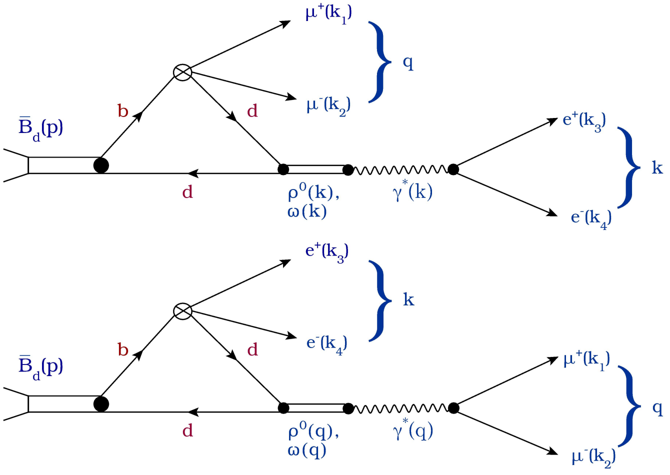
$$Br(\bar{B}_d \to \mu^+\mu^- e^+ e^-) \approx \frac{\tau_{B_d}}{\tau_{B_s}} \frac{|V_{td}|^2}{|V_{ts}|^2} \left(\frac{M_{B_d}}{M_{B_s}}\right)^5 * \ldots$$

$$\ldots * Br(\bar{B}_s \to \mu^+\mu^- e^+ e^-) \approx 8 \times 10^{-12}$$

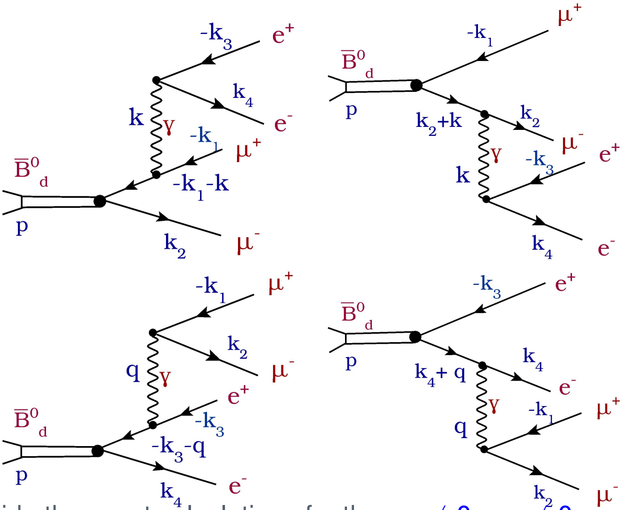
Main contributions to the $\bar{B}_d \to \mu^+\mu^- e^+e^-$ decay

- Resonant contributions from $d\bar{d}$ – pairs: $\rho^0(770)$ and $\omega(782)$ (VMD);
- Bremsstrahlung for $m_e \neq 0$ and $m_\mu \neq 0$, but $x_{34}^{min} > (2m_\mu/M_1)^2$ for the photon pole restricted;
- «Tails» from J/ψ and $\psi(2S)$ resonances (the resonances areas are excluded) and other vector charmonium contributions;
- Non resonant contribution from $b\bar{b}$ – pairs;
- The «weak annihilation» processes, neglected $(m_c/M_1)^2$ and $(m_u/M_1)^2$ corrections

$\rho^0(770)$ and $\omega(782)$ resonant contribution



The bremsstrahlung – I



We provide the exact calculations for the $m_e \neq 0$ $m_\mu \neq 0$.

The bremsstrahlung – II

The amplitude for the $\mu^+\mu^-$– pair emmited by electron and positron in the final state:

$$\begin{aligned}
\mathcal{M}_{fi}^{(\mu)} &= \sqrt{2}\, G_{\mathrm{F}}\, \alpha_{em}^2\, V_{tb}\, V_{ts}^{*} \left(\overline{\mu}(k_2)\gamma^{\mu}\mu(-k_1)\right) \Big[\\
& \quad i\, d^{(\mathrm{VP})}(x_{12},\, x_{123},\, x_{124})\, k_{\mu} \left(\overline{e}(k_4)\gamma^{5} e(-k_3)\right) + \\
&+ \quad f^{(\mathrm{VT})}(x_{12},\, x_{123},\, x_{124})\, \varepsilon_{\mu\nu\alpha\beta}\, p^{\nu} \left(\overline{e}(k_4)\gamma^{\alpha}\gamma^{\beta} e(-k_3)\right)\Big],
\end{aligned}$$

where M_1 is the mass of B_d meson , $x_{12} = q^2/M_1^2$, $x_{34} = k^2/M_1^2$ and $x_{ijn} = (k_i + k_j + k_n)^2/M_1^2$

$$d^{(\mathrm{VP})}(\ldots) = -\frac{4C_{10A}\hat{m}_e\hat{f}_{B_s}}{M_1^2}\, \frac{1}{x_{12}\,(x_{124} - \hat{m}_e^2)\,(x_{123} - \hat{m}_e^2)}\, \frac{(k_3 - k_4,\, q)}{M_1^2},$$

$$f^{(\mathrm{VT})}(\ldots) = -\frac{2C_{10A}\hat{m}_e\hat{f}_{B_s}}{M_1^2}\, \frac{1}{x_{12}\,(x_{124} - \hat{m}_e^2)\,(x_{123} - \hat{m}_e^2)}\, \frac{1 + x_{12} - x_{34}}{2}.$$

Charmonium vector resonances contributions

The effective Hamiltonian for the transition $b \to d\ell^+\ell^-$:

$$H_{eff}^{b\to d\ell^+\ell^-} = \frac{G_F}{\sqrt{2}}\,\frac{\alpha_{em}}{2\pi}\,V_{tb}V_{td}^*\left[-2\,\frac{C_{7\gamma}(\mu)}{q^2}\left\{m_b\,\left(\bar{d}\,i\sigma_{\mu\nu}\,(1+\gamma_5)\,q^\nu\,b\right)\right.\right.$$
$$\left. + m_d\,\left(\bar{d}\,i\sigma_{\mu\nu}\,(1-\gamma_5)\,q^\nu\,b\right)\right\}\cdot\left(\bar{\ell}\gamma^\mu\ell\right)$$
$$\left. + C_{9V}^{eff}(\mu,q^2)\,(\bar{s}O_\mu b)\cdot\left(\bar{\ell}\gamma^\mu\ell\right) + C_{10A}(\mu)\,\left(\bar{d}O_\mu b\right)\cdot\left(\bar{\ell}\gamma^\mu\gamma_5\ell\right)\right],$$

where $O_\mu = \gamma_\mu(I-\gamma^5)$ and q^ν is the four-momentum of $\ell^+\ell^-$ pair.
J/ψ, $\psi(2S)$ ..., $\rho^0(770)$ and $\omega(782)$ contributions are contained in the coefficient

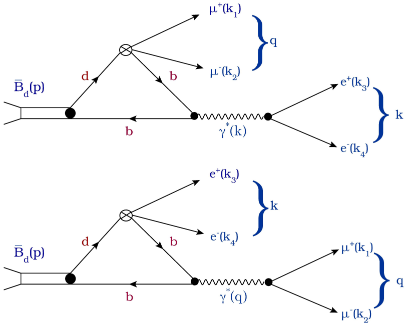
$$C_{9V}^{eff}(\mu,q^2) = C_{9V}(\mu) + (c\bar{c}\text{ and }u\bar{u}\text{ quark loops contribution}) +$$
$$+ (\text{vector resonances contribution}) = C_{9V}(\mu) + \Delta C_{9V}^{c\bar{c}+u\bar{u}}(\mu,s).$$

In the factorization approximation [D.Melikhov, N.Nikitin, S.Simula, PLB430, p.333, 1998]:

$$\Delta C_{9V}^{c\bar{c}}(\mu,q^2) = 3a_1(\mu)\left(h\left(\frac{m_c}{m_b},\frac{q^2}{m_b^2}\right) + \frac{3}{\alpha_{em}^2}\kappa\sum_{V=J/\psi,\psi',\dots}\frac{\pi\Gamma(V\to\ell\ell)M_V}{M_V^2-q^2-iM_V\Gamma_V}\right).$$

We define $C_2(M_W) = -1$, $a_1(5\,\mathrm{GeV}) = -0.13$, $C_{9V}(5\,\mathrm{GeV}) = -4.21$.

Non-resonant contribution from $b\bar{b}$ – pairs

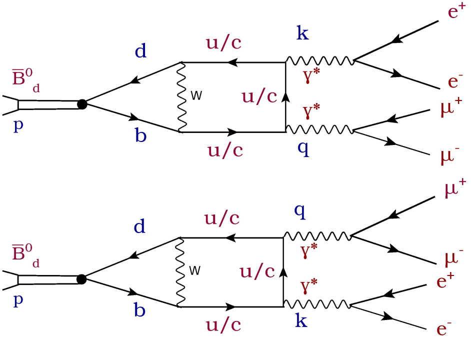


Non-resonant form factors parametrization:

$$F_i(q^2, k^2) = \frac{F_i(q^2 = 0,\, k^2 = 0)}{\left(1 - \frac{q^2}{M^2_{\Upsilon(1S)}}\right)\left(1 - \frac{k^2}{M^2_{R_i}}\right)}, \quad \text{where} \quad i = \{V,\, A,\, TV,\, TA\}.$$

Here $F_i(0, 0)$ and M_{R_i} are taken from A. Kozachuk, D. Melikhov, N. Nikitin, PRD **97**, 053007 (2018).

Weak annihilation



We take into account axial anomaly neglecting $(m_c/M_1)^2$ and $(m_u/M_1)^2$ corrections.

$$\mathcal{M}_{fi}^{(WA)} = \frac{32\sqrt{2}}{3\pi}\,\frac{G_{\rm F}}{M_1^3}\,\alpha_{em}^2\,\left(V_{ub}\,V_{ud}^* + V_{cb}\,V_{cd}^*\right)\,a_1(\mu)\,\hat{f}_{B_d}$$

$$\frac{1}{x_{12}\,x_{34}}\,\varepsilon_{\mu\alpha kq}\left(\overline{\mu}(k_2)\,\gamma^{\mu}\,\mu(-k_1)\right)\,\left(\overline{e}(k_4)\,\gamma^{\alpha}\,e(-k_3)\right).$$

EvtGen model for rare four-leptonic B_d decay

We prepare the new EvtGen model BD2MUMUEE for rare four-leptonic B_d meson decays. In this model:

- decay channel of $B_d/\bar{B}_d \to \mu^+\mu^- e^+ e^-$ mesons included;
- contributions of $\rho^0(770)$ and $\omega(782)$, J/ψ, $\psi(2S)$, $\psi(3770)$, $\psi(4040)$, $\psi(4160)$, $\psi(4415)$ resonances in the SM are considered;
- the form factors are calculated using the dispersion relativistic constituent quark model;
- A, λ, $\bar{\rho}$ and $\bar{\eta}$ CKM matrix parameters are variable;

$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+ e^-\right)$

For the partial width $\bar{B}_d \to \mu^+\mu^- e^+ e^-$ decay

$$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+ e^-\right) = (3.2 \pm 0.9) * 10^{-11}.$$

If we excl. $\omega(782)$ – resonant contribution

$$\mathrm{Br}_{Ex\omega}\left(\bar{B}_d \to \mu^+\mu^- e^+ e^-\right) = (1.1 \pm 0.1) * 10^{-11}.$$

In accordance with Y. Dincer, I. M. Sehgal, PLB **556**, p. 169 (2003),

$$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+ e^-\right) : \mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- \mu^+\mu^-\right) = 3 : 1.$$

For the $\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- \mu^+\mu^-\right)$ we obtain

$$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- \mu^+\mu^-\right) \sim 10^{-11}.$$

It is in agreement with the experimental upper limit

$$\mathrm{Br}_{Exp}\left(\bar{B}_d \to \mu^+\mu^- \mu^+\mu^-\right) \leq 1.8 * 10^{-10}$$

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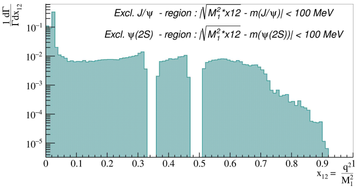
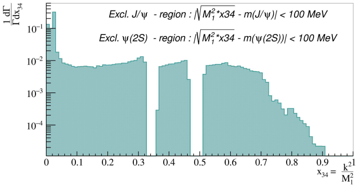
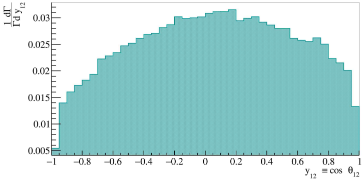
Dilepton invariant mass and angular distributions from EvtGen model



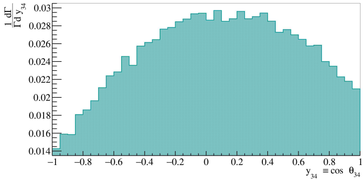
1)



2)



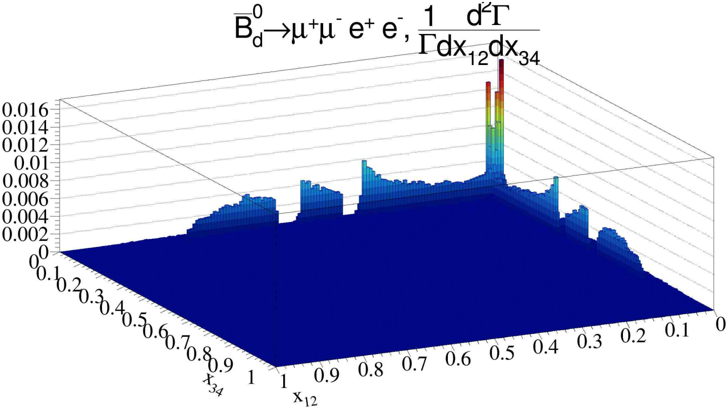
3)



4)

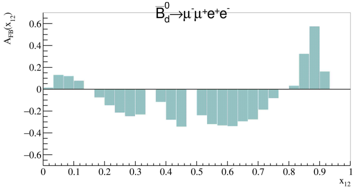
Figure: 1) x_{12} - distribution for the $\mu^+\mu^-$ - pair; 2) x_{34} - distribution for the e^+e^- - pair ; 3) $cos(\theta_{12})$ - distribution; 4) $cos(\theta_{34})$ - distribution.

Double differential distribution

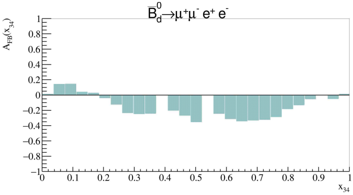


The $J/\psi-$ and $\psi(2S)-$ resonant contributions are excluded from the estimations according to experimental procedure for obtaining the $\mathrm{Br}\,\left(\bar{B}_d \to \ell^+\ell^-\ell^+\ell^-\right)$.

Forward–backward leptonic asymmetries



1)



2)

Figure: 1)The forward–backward leptonic asymmetry in the $\mu^+\mu^-$ - chanel; 2) The forward–backward leptonic asymmetry in the e^+e^- - chanel.

Summary

- In the framework of the Standard Model we present prediction for the $\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+e^-\right)$ taking into account resonant and non resonant contributions, bremsstrahlung and the «weak annihilation»:

$$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+e^-\right) = (3.2 \pm 0.9) * 10^{-11};$$

- The estimation for the $\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^-\mu^+\mu^-\right)$ based on the prediction for $\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^- e^+e^-\right)$:

$$\mathrm{Br}\left(\bar{B}_d \to \mu^+\mu^-\mu^+\mu^-\right) \sim 10^{-11}.$$

is in agreement with experimental result:

$$\mathrm{Br}_{Exp}\left(\bar{B}_d \to \mu^+\mu^-\mu^+\mu^-\right) \leq 1.8 * 10^{-10}$$

[LHCb Collaboration, JHEP 03, 109 (2022)]

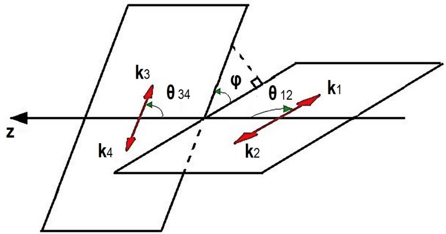
- We provide set of differential distributions for the $\bar{B}_d \to \mu^+\mu^- e^+e^-$ decay, using the new EvtGen-based Monte-Carlo generator model.

Acknowledgments

- The work was supported by grant 22-22-0029 of the Russian Science Foundation. The authors express their gratitude for this support.

Kinematics of $\bar{B}_d \to \mu^+\mu^- e^+ e^-$ decay

We conside the decay $\bar{B}_s(p, M_1) \to \mu^+(k_1)\mu^-(k_2)e^+(k_3)e^-(k_4)$



$$q = k_1 + k_2, \quad k = k_3 + k_4,$$

$$x_{12} = \frac{q^2}{M_1^2}, \quad x_{34} = \frac{k^2}{M_1^2},$$

$$\cos\theta_{12} = -\frac{2}{M_1^2}\frac{(k_1 k) - (k_2 k)}{\sqrt{\lambda(1, x_{12}, x_{34})}},$$

$$\cos\theta_{34} = -\frac{2}{M_1^2}\frac{(k_3 q) - (k_4 q)}{\sqrt{\lambda(1, x_{12}, x_{34})}}.$$