A new type of $k_\perp$-dependent (quasi)parton distributions

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Conference QFT-HEP-C
July, 19th 2022
Dubna

$$\boxed{Exclusive\ \ and\ \ Inclusive\ \ Reactions}$$

$$\Big\Downarrow {\scriptstyle Q^2\to\infty}$$

$$\boxed{Hard\ \ Reactions.}$$

$$(asympt.\ \ estimations)\Big\Downarrow Factorization\ \ Theorem$$

$$\boxed{\Big[E(x_1,....)\otimes\Phi(x_1,....)\Big]\oplus\Big[\Phi(y_1,....)\otimes E(x_1,y_1;....)\otimes\Phi(x_1,....)\Big]}$$

Here, $E(....)$ and $\Phi(....)$ denote the hard (perturbative) and soft (non-perturbative) parts, respectively.

Contents II, in a nutshell

$$
\begin{tikzcd}
 & \boxed{\Big[E(....)\otimes\Phi(....)\Big]} \arrow[dl, Rightarrow, bend right, "k_\perp=0"', "\mathbb{S}=\mathbb{I}"] \arrow[d, Rightarrow, "k_\perp\approx0"', "\mathbb{S}\neq\mathbb{I}"] \arrow[dr, Rightarrow, bend left, "\mathbb{S}\neq\mathbb{I}", "k_\perp\neq0"'] & \\
\boxed{\textit{PDs (no EE)}} & \boxed{\textit{PDs (EE)}} & \boxed{\textit{new PDs (EE)}}
\end{tikzcd}
$$

A number of exterior and interior parameters:

- $k_\perp = 0$ and $\mathbb{S} = \mathbb{I}$: $\{P, S\}$- ext.,$\{k^+, s^+\}$-int.;
- $k_\perp \approx 0$ and $\mathbb{S} \neq \mathbb{I}$: $\{P, S\}$- ext., $\{k^+, s^+\}$-int.;
- $k_\perp \neq 0$ and $\mathbb{S} \neq \mathbb{I}$: $\{P, S\}$- ext., $\{k^+, s^+, k_\perp, s_\perp\}$-int.;

We begin with the forward Compton scattering (CS) amplitude which takes the form of

$$\mathcal{A}_{\mu\nu} = \langle P | a^{-}_{\nu}(q)\, \mathbb{S}[\bar{\psi}, \psi, A]\, a^{+}_{\mu}(q) | P \rangle,$$

where $\mathbb{S}$-matrix is given by

$$\mathbb{S}[\psi, \bar{\psi}, A] = \mathrm{T} \exp\Big\{ i \int (d^4 z) \big[\mathcal{L}_{QCD}(z) + \mathcal{L}_{QED}(z) \big] \Big\}.$$

In contrast to the photon Fock states the hadron states cannot be expressed through the relevant operators of creation and annihilation

P.S. The creation and annihilation hadron operators can be introduced with the help of the effective Lagrangian describing the transition of partons onto hadrons. This is the so-called effective quark-hadron Lagrangian of interaction.

Making used the commutation relations of creation (or annihilation) operators with $\mathbb{S}$-matrix

$$\big[a^{\pm}_{\mu}(q),\, \mathbb{S}[\bar{\psi},\psi,A]\big] = \int (d^4\xi) e^{\pm iq\xi} \frac{\delta \mathbb{S}[\bar{\psi},\psi,A]}{\delta A^{\mu}(\xi)} \quad \text{with}$$

$$\frac{\delta \mathbb{S}[\bar{\psi},\psi,A]}{\delta A^{\mu}(\xi)} = \mathrm{T}\Big\{ \int (d^4 z) \frac{\delta \mathcal{L}_{QED}(z)}{\delta A^{\mu}(\xi)}\, \mathbb{S}[\bar{\psi},\psi,A]\Big\},$$

the CS-amplitude can be rewritten as

$$\mathcal{A}_{\mu\nu} = \int (d^4\xi_1)(d^4\xi_2) e^{-iq(\xi_1-\xi_2)} \langle P| \frac{\delta^2 \mathbb{S}[\bar{\psi},\psi,A]}{\delta A^{\mu}(\xi_1)\delta A^{\nu}(\xi_2)} |P\rangle$$

$$\Rightarrow \int (d^4 z) e^{-iqz} \langle P|\, \mathrm{T}\Big\{ [\bar{\psi}(0)\gamma_{\nu}\psi(0)]\, [\bar{\psi}(z)\gamma_{\mu}\psi(z)]\, \mathbb{S}[\bar{\psi},\psi,A]\Big\} |P\rangle.$$

Using Wick's theorem and calculating the only quark operator contraction, the simplest "hand-bag" diagram contribution to the CS-amplitude reads ($\delta^{(4)}$(momentum conserv.), as a common prefactor, is not shown.)

$$\mathcal{M}_{\mu\nu}^{\text{hand-bag}} = \int (d^4 k)\,\mathrm{tr}\big[E_{\mu\nu}(k)\Phi(k)\big],$$

where (Here, the subscript "c" denotes the connected diagram contributions which we only consider.)

$$E_{\mu\nu}(k) = \gamma_\mu S(k+q)\gamma_\nu + \gamma_\nu S(k-q)\gamma_\mu,$$

$$\boxed{\Phi(k) = \int (d^4 z)\, e^{ikz} \langle P|\,\widetilde{\mathrm{T}}\,\bar{\psi}(0)\psi(z)\mathbb{S}[\bar{\psi},\psi,A]|P\rangle_c.}$$

It is more compact to use, however, the Heisenberg representation of correlators, *i.e.*

$$\Phi(k) = \int (d^4 z)\, e^{ikz} \langle P|\bar{\psi}(0)\psi(z)|P\rangle^H.$$

Notice that the Factorization Procedure is not yet applied !

- Since $\mathcal{M}_{\mu\nu}$ involves the non-perturbative correlator, $\langle P|\mathcal{O}(0,z)|P\rangle$, it cannot be calculated within the Standard Theory. Indeed,

$$|P\rangle = \mathbf{a}_h^{+}(\psi,\bar{\psi}|A)\,|0\rangle \;\text{ with }\; \mathbf{a}_h^{+} \;\text{ being undefined in ST/QCD,}$$
$$\big[\,\psi(0),\, \mathbf{a}_h^{+}(\psi,\bar{\psi}|A)\,\big]_{+} \to \;\text{unknown}$$

- But instead, $\mathcal{M}_{\mu\nu}$ can be estimated with the help of the suitable asymptotical regime, $q^2 = -Q^2 \to \infty$

To illustrate the (typical) factorization procedure, we consider (here, for the sake of brevity, we omit all possible Lorentz indices)

$$A = \int (d^4 k) E(k, q)\, \Phi(k),$$

where $E(k, q)$ is given by the propagator product and

$$\Phi(k) \stackrel{\mathcal{F}}{=} \langle \bar{\psi}(z) \Gamma \psi(0) \rangle,$$

with $\stackrel{\mathcal{F}}{=}$ denoting the Fourier transform.

- We have to choose the dominant directions dictated by the given process kinematics. For CS-amplitude, we deal with the only dominant directions associated with the plus light-cone direction.
- We have to introduce the definitions of the dimensionless parton fractions as

$$d^4k \Rightarrow d^4k \int_{-1}^{+1} dx \delta(x - k^+/P^+).$$

- We expand $E(k,q)$ around the chosen dominant direction.

As a result, we obtain that

$$A^{(0)} = \int (dx)\, E(xP^+; q) \times \Big\{ \int (d^4k)\delta(x - k^+/P^+)\Phi(k) \Big\}$$

if $k_i^\perp$-terms are neglected in the expansion;

and

$$A^{(i)} = \int (dx) \sum_i E^{(i)}(xP^+; q) \times \Big\{ \int (d^4k)\delta(x - k^+/P^+) \prod_{i'=1}^{i} k_{i'}^\perp \,\Phi(k) \Big\}$$

if $k_\perp$-terms are essential in the expansion.

In the similar manner, we can treat the DY-like hadron tensor. Before factorization, it reads

$$W = \int (d^4 k_1)(d^4 k_2) E(k_1, k_2, q)\, \Phi_1(k_1)\bar{\Phi}_2(k_2),$$

where

$$E(k_1, k_2, q) = \delta^{(4)}(k_1 + k_2 - q)\, \mathcal{E}(k_1, k_2, q)$$
$$\Phi_1(k_1) \stackrel{\mathcal{F}_1}{=} \langle \bar{\psi}(z_1)\Gamma_1 \psi(0)\rangle,$$
$$\bar{\Phi}_2(k_2) \stackrel{\mathcal{F}_2}{=} \langle \bar{\psi}(0)\Gamma_2 \psi(z_2)\rangle$$

and $\stackrel{\mathcal{F}_i}{=}$ denotes the corresponding Fourier transforms.

As a result of factorization, we obtain that

$$W^{(0)} = \int (dx_1)(dx_2) E(x_1 P_1^+, x_2 P_2^-; q)$$

$$\times \Big\{ \int (d^4 k_1) \delta(x_1 - k_1^+/P_1^+) \Phi_1(k_1) \Big\}$$

$$\times \Big\{ \int (d^4 k_2) \delta(x_2 - k_2^-/P_2^-) \bar{\Phi}_2(k_2) \Big\} \text{ for } k_\perp = 0$$

and

$$W^{(i,j)} = \int (dx_1)(dx_2) \sum_{i,j} E^{(i,j)}(x_1 P_1^+, x_2 P_2^-; q)$$

$$\times \Big\{ \int (d^4 k_1) \delta(x_1 - k_1^+/P_1^+) \prod_{i'=1}^{i} k^{\perp}_{1\,i'} \, \Phi_1(k_1) \Big\}$$

$$\times \Big\{ \int (d^4 k_2) \delta(x_2 - k_2^-/P_2^-) \prod_{j'=1}^{j} k^{\perp}_{2\,j'} \, \bar{\Phi}_2(k_2) \Big\} \text{ for } k_\perp \neq 0.$$

In contrast to our genuine-factorized forms, the approaches with $q_\perp \neq 0$ and without the δ-function expansion result in

$$\tilde{W}^{(0)} = \int (dx_1)(dx_2)\mathcal{E}(x_1 P_1^+, x_2 P_2^-; q)$$
$$\times \Big\{ \int (d^2\vec{\mathbf{k}}_1^\perp)(d^2\vec{\mathbf{k}}_2^\perp)\delta^{(2)}(\vec{\mathbf{k}}_1^\perp + \vec{\mathbf{k}}_2^\perp - \vec{\mathbf{q}}^\perp)$$
$$\times \int (dk_1^+ dk_1^-)\delta(x_1 - k_1^+/P_1^+)\Phi_1(k_1)$$
$$\times \int (dk_2^- dk_2^+)\delta(x_2 - k_2^-/P_2^-)\bar{\Phi}_2(k_2)\Big\},$$

where $\Phi(k_1)$ and $\bar{\Phi}(k_2)$ cannot be independent each others.
This leads to the factorization breaking effects which should be compensated by, for example, $e^{-S(\vec{\mathbf{k}}_\perp^2/\Lambda^2)}$-multiplication minimizing the non-factorized effects. This way is not in our favour !

Consider the case of $\mathbb{S} = \mathbb{I}$ (no QCD interactions), for the hand-bag diagram we have the following

$$\mathcal{A}_{\mu\nu}\Big|_{\mathbb{S}=\mathbb{I}} = \int (d^4 z_1\, d^4 z_2)\, e^{-iq(z_1 - z_2)} \langle P|\, :\bar{\psi}(z_1)\, E_{\mu\nu}(z_1 - z_2)\, \psi(z_2): |P\rangle$$

or, focusing on γ^+-projection in the correlator (within the momentum repres.),

$$\mathcal{M}_{\mu\nu}\Big|_{\mathbb{S}=\mathbb{I}} = \int (d^4 k)\, \mathrm{tr}\big[E_{\mu\nu}(k)\, \gamma^-\big] \underbrace{\int (d^4 z)\, e^{ikz} \langle P|\, :\bar{\psi}(0)\gamma^+\psi(z): |P\rangle}_{\Phi^{[\gamma^+]}(k)} \Big].$$

Then, we use the Fourier transforms for the operators in the correlator, we have

$$\langle P|\,:\bar{\psi}(0)\gamma^+\psi(z):\,|P\rangle = \int (d^4k_1 d^4k_2)e^{-ik_1z}$$
$$\times \underbrace{\left[\bar{u}(k_2)\gamma^+u(k_1)\right]}_{L^{[\gamma^+]}(k_2,k_1)}\ \underbrace{\langle P|b^+(k_2)b^-(k_1)|P\rangle}_{\delta^{(4)}(k_1-k_2)\mathcal{M}(k_2,k_1|P)},$$

where

$L^{[\gamma^+]}(k_2, k_1)$ gives the Lorentz structure (Lorentz parametrization);
$\mathcal{M}(k_2, k_1|P)$ is the quark-hadron $\mathcal{M}$-amplitude.

After factorization in LO (Collinear Limit), we finally derive that

$$\boxed{\mathcal{M}_{\mu\nu}\Big|_{\mathbb{S}=\mathbb{I}} = \int (dx)\,\mathrm{tr}\left[E_{\mu\nu}(xP^+)\,\gamma^-\right]\Phi^{[\gamma^+]}(x)}$$

where $(k = (k^+, k^-, \vec{\mathbf{k}}_\perp))$

$$\Phi^{[\gamma^+]}(x) = \int (d^4k)\delta(x - k^+/P^+)\Phi^{[\gamma^+]}(k) \stackrel{\mathcal{F}}{=}$$

$$\underbrace{\langle P|\,:\bar{\psi}(0)\gamma^+\psi(0^+, z^-, \vec{\mathbf{0}}_\perp):|P\rangle}_{\sim\ \text{math. probability to find parton inside hadron}} \quad .$$

Up to now, we deal with the standard case !

Now, we take into account the interaction and we still adhere the (almost) collinear case of $k_\perp \approx 0$ ($k_\perp$-integrated functions):

$$\Phi^{[\gamma^+]}(x) = \int (d^4k)\delta(x - k^+/P^+)\Phi^{[\gamma^+]}(k) \overset{\mathcal{F}}{=}$$

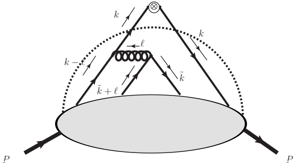
$$\boxed{\langle P|\,\widetilde{\mathrm{T}}\,\bar{\psi}(0)\gamma^+\psi(0^+, z^-, \vec{\mathbf{0}}_\perp)\mathbb{S}[\bar{\psi}, \psi, A]|P\rangle_c}$$

"Evolution" "Structure"

"Evolution" implies the explicit loop integrations, while "Structure" – the implicit loop integration.

In the standard way, we first consider free operators to parametrize the given correlator and $\mathbb{S}$-matrix has been included to derive the evolution of the already-introduced parametrizing functions;

In this case, we do not expect any new functions !



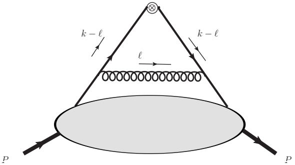


Figure: The types of loop integrations in the corresponding correlators: the left panel corresponds to the demonstration of the implicit loop integrations defined the Lorentz structure; the right panel – to the explicit lop integrations contributing to the evolution integration kernels.

However, if the interaction encoded in the correlator and we together with the essential $k_\perp$-dependence, $k_\perp$-unintegrated functions, we have

$$\Phi^{[\gamma^+]}(x, k_\perp) = \int (dk^+ dk^-)\delta(x - k^+/P^+)\Phi^{[\gamma^+]}(k) \stackrel{\mathcal{F}}{=}$$

$$\boxed{\langle P|\,\widetilde{\mathrm{T}}\,\bar\psi(0)\gamma^+\psi(0^+, z^-, \vec{\mathbf{z}}_\perp)\mathbb{S}[\bar\psi, \psi, A]|P\rangle_c}$$

“Evolution” “Structure”

In the nonstandard way, we parametrize the given correlator where $\mathbb{S}$-matrix has been presented from the very beginning.

In this case, we do have a possibility for the new functions !

Let us begin with the well-know $k_\perp$-dependent function f_1:

$$\Phi^{[\gamma^+]}(k) = P^+ f_1\left(x;\, k_\perp^2, (k_\perp P_\perp)\right) =$$
$$P^+ (k_\perp P_\perp) f_1^{(1)}(x;\, k_\perp^2) + \left\{\text{terms of } (k_\perp P_\perp)^n \,|\, n = 0, n \geq 2\right\},$$

where $k = (xP^+, k^-, \vec{\mathbf{k}}_\perp)$.

$f_1\left(x;\, k_\perp^2, (k_\perp P_\perp)\right)$ has been decomposed into the powers of $(k_\perp P_\perp)$. Keeping the term of decomposition with $n = 1$ represents the minimal necessary requirement for the manifestation of new functions.

Consider the second order of strong interactions, $\mathbb{S}^{(2)}_{QCD}$, in the correlator:

$$\langle \mathcal{O}^{[\gamma^+]} \rangle^{(2)} \equiv \langle P, S | T \bar{\psi}(0) \gamma^+ \psi(z) \, \mathbb{S}^{(2)}_{QCD}[\psi, \bar{\psi}, A] \, | P, S \rangle =$$
$$\int (d^4 k) e^{-i(kz)} \Delta(k^2) \int (d^4 \ell) \Delta(\ell^2) \int (d^4 \tilde{k}) \mathcal{M} \left(k^2, \ell^2, \tilde{k}^2, ... \right)$$
$$\times \left[\bar{u}(k) \gamma^+ \hat{k} \gamma^\perp_\alpha u(k - \ell) \right] \left[\bar{u}(\tilde{k}) \gamma^\perp_\alpha u(\tilde{k} + \ell) \right],$$

where

$$S(k) = \hat{k} \Delta(k^2), \; D^\perp_{\mu\nu}(\ell) = g^\perp_{\mu\nu} \Delta(\ell^2), \; \Delta(k^2) = \frac{1}{k^2 + i\epsilon}, \; \hat{k} = (k\gamma)$$

and $\mathcal{M}$-amplitude is given by

$$\mathcal{M} \left(k_i^2, (k_i k_j), ... \right) \delta^{(4)}(k_1 + k_3 - k_2 - k_4) =$$
$$\langle P, S | b^+(k_1) b^-(k_2) b^+(k_3) b^-(k_4) | P, S \rangle.$$

We single out the region where $|\ell| \ll \{|k|, |\tilde{k}|\}$ and as a result we obtain that

$$\langle \mathcal{O}^{[\gamma^+]}\rangle^{(2)} \sim \int (d^4k) e^{-i(kz)} \Delta(k^2) \big[\bar{u}(k)\gamma^+ \hat{k}\gamma^\perp_\alpha u(k)\big]$$
$$\times \int (d^4\tilde{k}) \big[\bar{u}(\tilde{k})\gamma^\perp_\alpha u(\tilde{k})\big] \int (d^4\ell)\Delta(\ell^2)\mathcal{M}\left(k^2, \ell^2, \tilde{k}^2, ...\right).$$

The next stage is to transform the spinor lines of this expression. For the first spinor line, we write

$$\big[\bar{u}(k)\gamma^+ \hat{k}\gamma^\alpha_\perp u(k)\big] = \mathcal{S}^{+k\alpha\beta}\big[\bar{u}(k)\gamma^\beta u(k)\big] + (\text{axial})$$
$$\Longrightarrow k^\alpha_\perp \big[\bar{u}(k)\gamma^+ u(k)\big] + (\text{other terms}),$$

where the following notation has been used

$$\mathcal{S}^{\mu_1\mu_2\mu_3\mu_4} = \frac{1}{4}\text{tr}\big[\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\big].$$

The second spinor line can be considered with the help of the covariant (invariant) integration given by

$$
\begin{aligned}
&k_\perp^\alpha \int (d^4\tilde{k})\, \left[\bar{u}(\tilde{k})\gamma^\alpha u(\tilde{k})\right] \mathcal{M}\left(\tilde{k}^2, (\tilde{k}P), ...\right) = \\
&k_\perp^\alpha \int (d^4\tilde{k})\, \tilde{k}^\alpha\, \mathcal{M}\left(\tilde{k}^2, (\tilde{k}P), ...\right) = \\
&(k_\perp P_\perp) \int (d^4\tilde{k})\, \frac{(\tilde{k}_\perp P_\perp)}{P_\perp^2}\, \mathcal{M}\left(\tilde{k}^2, (\tilde{k}P), ...\right).
\end{aligned}
$$

One can observe that

$$P^{+}(k_{\perp}P_{\perp})f_{1}^{(1)}(x;\,k_{\perp}^{2})\sim$$
$$\left[\bar{u}(k)\gamma^{+}u(k)\right]\int(d^{4}\tilde{k})\left[\bar{u}(\tilde{k})\hat{k}_{\perp}u(\tilde{k})\right]$$
$$\times\int(d^{4}\ell)\Delta(\ell^{2})\Delta(k^{2})\mathcal{M}\left(k^{2},\ell^{2},\tilde{k}^{2},...\right).$$

In other words, this parametrization with the Lorentz combination $P^{+}(k_{\perp}P_{\perp})$ is related to the two spinor lines

$$\left[\bar{u}(k)\gamma^{+}u(k)\right]\Rightarrow k^{+}\sim P^{+},\quad\left[\bar{u}(\tilde{k})\hat{k}_{\perp}u(\tilde{k})\right]\Rightarrow(k_{\perp}P_{\perp})$$

at g^2-order.

In the region of $|\tilde{k}| \sim |k|$, two spinor lines can be transformed into the other spinor lines with the help of Fierz transformations:

$$\big[\bar{u}^{(a)} O_1 u^{(b)}\big]\big[\bar{u}^{(c)} O_2 u^{(d)}\big] = \frac{1}{4}\sum_{A,R_1,R_2} \Big\{\frac{1}{4}\mathrm{tr}\big[\Gamma_A O_1 \Gamma_{R_1}\big]\Big\}\Big\{\frac{1}{4}\mathrm{tr}\big[\Gamma^A O_2 \Gamma_{R_2}\big]\Big\}$$
$$\times\big[\bar{u}^{(c)}\Gamma^{R_1} u^{(b)}\big]\big[\bar{u}^{(a)}\Gamma^{R_2} u^{(d)}\big]$$

with $O_1 = \gamma^+\gamma_j^\perp\gamma_5$, $O_2 = \mathbf{1}$, $\Gamma^A = \gamma_i^\perp$, $\Gamma^{R_1} = \gamma^+$, $\Gamma^{R_2} = \gamma_i^\perp$.

Thus, we obtain that

$$\big[\bar{u}^{(\uparrow_x)}(k)\gamma^+\gamma_j^\perp\gamma_5 u^{(\uparrow_x)}(k)\big]\big[\bar{u}^{(\uparrow_x)}(k) u^{(\uparrow_x)}(k)\big] =$$
$$C\,\big[\bar{u}^{(\uparrow_x)}(k)\gamma^+ u^{(\uparrow_x)}(k)\big]\big[\bar{u}^{(\uparrow_x)}(k)\gamma_i^\perp u^{(\uparrow_x)}(k)\big],$$

where, for the fixed indices, $i \neq j$, the coefficient C is given by

$$C = \frac{1}{16}\mathrm{tr}\big[\gamma_i^\perp\gamma^+\gamma_j^\perp\gamma_5\gamma^-\big]\mathrm{tr}\big[\gamma_i^\perp\gamma_i^\perp\big].$$

It can be readily inverted in order to get the following representation

$$\left[\bar{u}^{(\uparrow_x)}(k)\gamma^+\gamma_1^\perp\gamma_5 u^{(\uparrow_x)}(k)\right]\int(d^4\tilde{k})\int(d^4\ell)$$
$$\times\Delta(\ell^2)\Delta(k^2)\mathcal{M}\left(k^2,\ell^2,\tilde{k}^2,...\right)\Rightarrow$$
$$k^+ i\epsilon^{+-P_\perp s_\perp}\tilde{f}_1^{(1)}(x;\;k_\perp^2).$$

- For the existence of Lorentz vector defined as $\epsilon^{+-P_\perp s_\perp}$, it is necessary to assume that the quark spin $s_\perp$ is not a collinear vector to the hadron transverse momentum $P_\perp$.

Within the Collins-Soper frame, the hadron transverse momentum can be naturally presented as

$$\vec{\mathbf{P}}_\perp = (P_1^\perp, 0).$$

Since the hadron spin vector S can be decomposed on the longitudinal and transverse components as

$$S^L + S^\perp = \lambda P^+/m_N + S^\perp,$$

we get $P \cdot S = \vec{\mathbf{P}}^\perp \vec{\mathbf{S}}^\perp = 0$. Hence, it is natural to suppose that quark $s^\perp$ and hadron $S^\perp$ are collinear ones.

- This is a kinematical constraint (or evidence) for the nonzero Lorentz combination $\epsilon^{+-P_\perp s_\perp}$ and, therefore, for the existence of a new function $\tilde{f}_1^{(1)}(x; k_\perp^2)$.

So, it explicitly shows that the function $\tilde{f}_1^{(1)}(x;\, k_\perp^2)$ and its analogues must appear in the parametrization of the hadron matrix element, *i.e.*

$$\begin{aligned}
\Phi^{[\gamma^+]}(k) &\equiv \int (d^4 z) e^{+i(kz)} \langle P,S|\bar{\psi}(0)\,\gamma^+\,\psi(z)\,\mathbb{S}[\psi,\bar{\psi},A]\,|P,S\rangle\Big|^{k^-=0,k_\perp\neq 0}_{k^+=xP^+} \\
&= i\epsilon^{+-P_\perp s_\perp}\tilde{f}_1^{(1)}(x;\,k_\perp^2) + i\epsilon^{+-k_\perp s_\perp} f_{(2)}(x;\,k_\perp^2) + ...,
\end{aligned}$$

where the ellipse denotes the other possible terms of parametrization.

We also observe that Lorentz structure tensor, $\epsilon^{+-P_\perp s_\perp}$, associated with our function resembles the Sivers structure, $\epsilon^{+-P_\perp S_\perp}$ in which the nucleon spin vector $S_\perp$ is replaced by the quark spin vector $s_\perp$. However, despite this similarity the Sivers function and the introduced function $\tilde{f}_1^{(1)}(x;\, k_\perp^2)$ have totally different physical meaning.

The simplest example of application is related to the well-known unpolarized Drell-Yan (DY) process, *i.e.* the lepton-production in nucleon-nucleon collision:

$$
\begin{aligned}
N(P_1) + N(P_2) &\to \gamma^*(q) + X(P_X) \\
&\to \ell(l_1) + \bar{\ell}(l_2) + X(P_X),
\end{aligned}
$$

with the initial unpolarized nucleons N.

The importance of the unpolarized DY differential cross section is due to the fact that it has been involved in the denominators of any spin asymmetries.

At the leading order, the hadron tensor which describes the unpolarized DY-process takes the following form:

$$\mathcal{W}^{(0)}_{\mu\nu} = \quad \delta^{(2)}(\vec{\mathbf{q}}_\perp) \int (dx)(dy)\delta(xP_1^+ - q^+)\delta(yP_2^- - q^-)$$
$$\times \mathrm{tr}\big[\gamma_\nu\,\gamma^+\,\gamma_\mu\,\gamma^-\big]\,\Phi^{[\gamma^-]}(y)\Big\{\int (d^2\vec{\mathbf{k}}_1^\perp)\bar{\Phi}^{[\gamma^+]}(x, k_1^{\perp\,2})\Big\},$$

where

$$\Phi^{[\gamma^-]}(y) = P_2^-\,f(y), \quad \bar{\Phi}^{[\gamma^+]}(x, k_1^{\perp\,2}) = i\epsilon^{+-k_1^\perp s^\perp} f_{(2)}(x;\,k_1^{\perp\,2}).$$

Calculating the contraction of hadron tensor with the unpolarized lepton tensor $\mathcal{L}^{U}_{\mu\nu}$, we derive that

$$d\sigma^{unpol.} \sim \int (d^2\vec{\mathbf{q}}_\perp)\mathcal{L}^{U}_{\mu\nu}\mathcal{W}^{(0)}_{\mu\nu} =$$
$$\int (dx)(dy)\delta(xP_1^+ - q^+)\delta(yP_2^- - q^-)$$
$$\times(1+\cos^2\theta)f(y)\int (d^2\vec{\mathbf{k}}_1^\perp)\epsilon^{P_2 - k_1^\perp s^\perp}\Im m f_{(2)}(x;\, k_1^{\perp\,2}),$$

where

$$\epsilon^{+-k_1^\perp s^\perp} = \vec{\mathbf{k}}_1^\perp \wedge \vec{\mathbf{s}}^\perp \sim \sin(\phi_k - \phi_s)$$

with ϕ_A, for $A = (k, s)$, denoting the angles between $\vec{\mathbf{A}}_\perp$ and $O\hat{x}$-axis in the Collins-Soper frame.

Thus, the new $k_\perp$-dependent function $f_{(2)}(x;\, k_1^{\perp\, 2})$ gives the additional and additive contribution to the depolarization factor $D_{[1+\cos^2\theta]}$ appeared in the differential cross section of unpolarized DY process.

Conclusions

- We have introduced the new $k_\perp$-dependent function $\tilde{f}_1^{(1)}(x;\, k_\perp^2)$ and $f_{(2)}(x;\, k_\perp^2)$ which describe the transverse quark motion by the quark alignment along the fixed transverse direction. The introduced functions can be considered as a "in-between" functions of the Sivers and Boer-Mulders functions.
- We have shown that, to the second order of strong interactions, the new parametrizing function $\tilde{f}_1^{(1)}(x;\, k_\perp^2)$ can be related to the function $f_1^{(1)}(x;\, k_\perp^2)$ of (1) imposing the condition $\ell \ll |\tilde{k}| \sim |k|$ which corresponds to the regime where the appeared two spinor lines are interacting by exchanging of soft gluon. Moreover, the occurred four spinors generated by two spinor lines have the polarizations aligned along the same transverse direction. In physical terms, the $k_\perp$-dependent function $\tilde{f}_1^{(1)}(x;\, k_\perp^2)$ which describes the regime where $k_\perp$-dependence (or the transverse motion of quarks inside the hadron) has been entirely generated by the quark spin alignment.

Conclusions

- As a practical application of the new functions, we have illustrated that the function $f_{(2)}(x; k_\perp^2)$ provides the additional contribution to the depolarization factor $D_{[1+\cos^2\theta]}$ which is associated with the differential cross section of unpolarized DY process.

Thank you for your attention !