САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

International Conference on Quantum Field Theory, High-Energy Physics, and Cosmology
Bogoliubov Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research (Dubna, Russian Federation)

Energy levels of three - particle muon - electron ions

Fedor Martynenko
Samara University

In collaboration with:
A. E. Dorokhov (BLTP JINR),
V. I. Korobov (BLTP JINR)
R. N. Faustov (FRC CSC RAS),
A. P. Martynenko (Samara University)

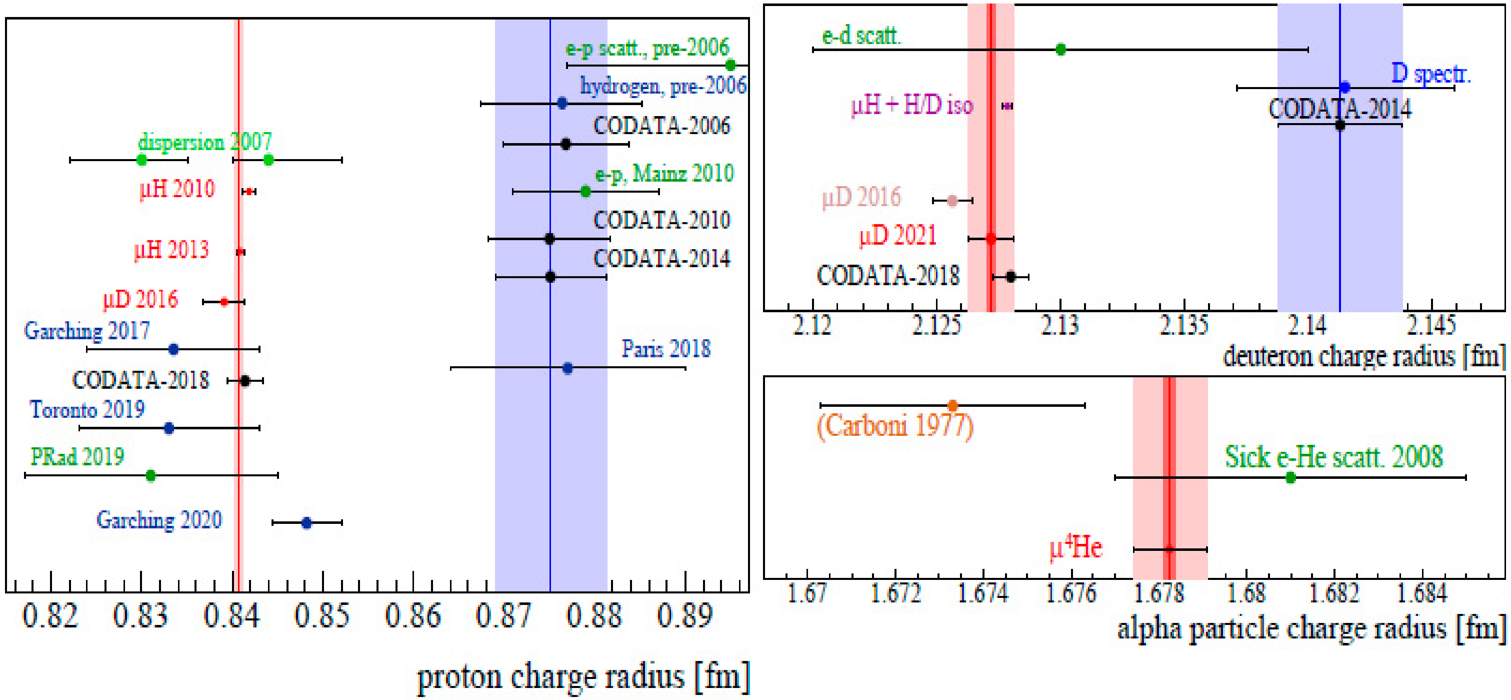
July 18, 2022

- $r_p^{CODATA} = 0.8775(51)\ fm$
  $\rightarrow r_p^{CREMA} = 0.84087(26)\ fm$

- $r_d^{CODATA} = 2.1424(21)\ fm$
  $\rightarrow r_d^{CREMA} = 2.12718(13)\ fm$

- $r_\alpha^{CODATA} = 1.681(4)\ fm$
  $\rightarrow r_\alpha^{CREMA} = 1.67824(13)\ fm$

Antognini A., Kottmann F., Pohl R. Laser spectroscopy of light muonic atoms and the nuclear charge radii //SciPost Physics Proceedings. – 2021. – №. 5. – С. 021.

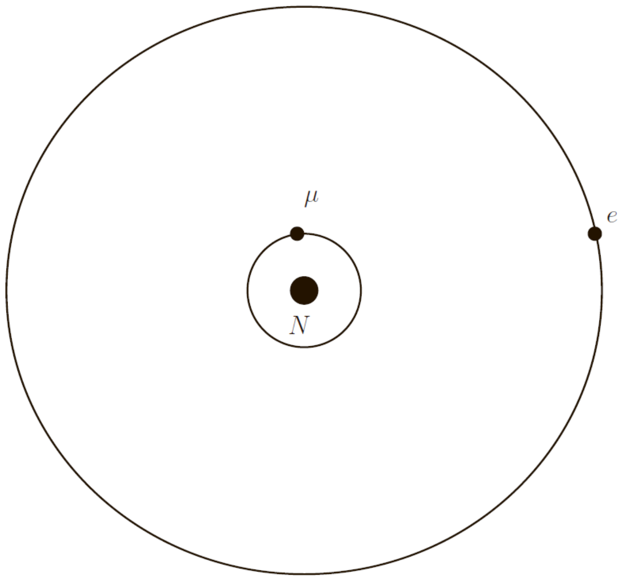


- New measurement of ground state hyperfine structure of muon – electron helium atoms. The aim is to increase precision 1000 times than previous experiments.

- New generation of laser spectroscopy experiments with muonic atoms and ions by CREMA collaboration.

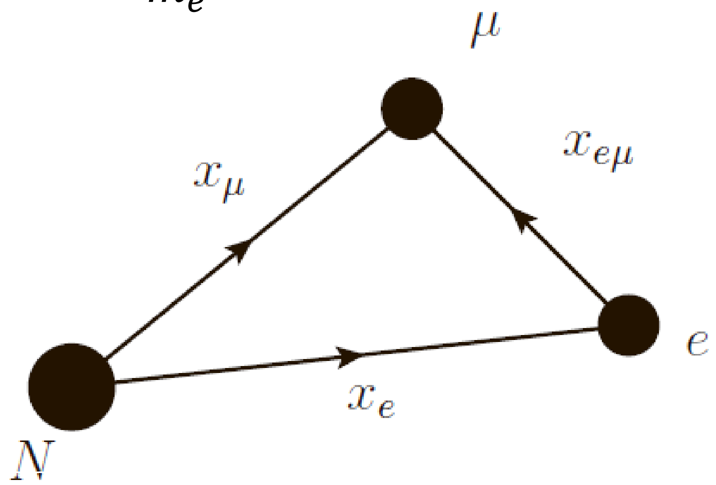
Fukumura S. et al. Proposal for new measurements of muonic helium hyperfine structure at J-PARC //EPJ Web of Conferences. – EDP Sciences, 2022. – Т. 262. – С. 01012.

Schmidt S. et al. The next generation of laser spectroscopy experiments using light muonic atoms //Journal of Physics: Conference Series. – IOP Publishing, 2018. – Т. 1138. – №. 1. – С. 012010.



Bound particles in muon-electron lithium, beryllium, and boron ions have different masses:

$$m_e \ll m_\mu \ll M \Rightarrow a_0^\mu \sim \frac{1}{m_\mu} \ll a_0^e \sim \frac{1}{m_e}$$



The reduced masses of the electron – nucleus and muon – nucleus subsystems are:

$$M_e = \frac{Mm_e}{M + m_e}, \qquad M_\mu = \frac{Mm_\mu}{M + m_\mu}$$

The relative muon and electron coordinates are

$$x_\mu = |\boldsymbol{r}_\mu - \boldsymbol{r}_N|$$

$$x_e = |\boldsymbol{r}_e - \boldsymbol{r}_N|$$

$$x_{\mu e} = |\boldsymbol{x}_\mu - \boldsymbol{x}_e|$$

The muon and the nucleus form a quasinucleus, and in the first approximation, the muon-electron ion can be considered as a two-particle system with Hamiltonian:
S. D. Lakdawala and P. J. Mohr, Phys. Rev. A 29, 1047 (1984).

$$H_0 = -\frac{1}{2M_\mu}\nabla_\mu^2 - \frac{1}{2M_e}\nabla_e^2 - \frac{Z\alpha}{x_\mu} - \frac{(Z-1)\alpha}{x_e},$$
$$\Delta H = \frac{\alpha}{x_{\mu e}} - \frac{\alpha}{x_e}, \qquad \Delta H^{rec} = -\frac{1}{M}\nabla_\mu \cdot \nabla_e, \qquad H = H_0 + \Delta H + \Delta H^{rec} + \Delta H^{vp} + \Delta H^{str} \tag{1}$$

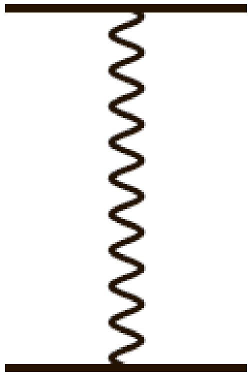
In the initial approximation, the wave function of three particle system is determined in terms of muon and electron wave functions:

$$\Psi_{1S}(\boldsymbol{x}) = \frac{W^{\frac{3}{2}}}{\sqrt{\pi}} e^{Wx}, \qquad \Psi_{2S}(\boldsymbol{x}) = \frac{W^{\frac{3}{2}}}{2\sqrt{2\pi}} e^{-\frac{Wx}{2}}\left(1 - \frac{Wx}{2}\right), \qquad \Psi_{2P}(\boldsymbol{x}) = \frac{W^{\frac{3}{2}}}{2\sqrt{6}} e^{-\frac{Wx}{2}} Wx\,(\boldsymbol{\varepsilon n}) \tag{2}$$

The ground state wave function is:

$$\Psi_{1S}(\boldsymbol{x}_\mu, \boldsymbol{x}_e) = \Psi_{1S}(\boldsymbol{x}_\mu) \cdot \Psi_{1S}(\boldsymbol{x}_e) = \frac{1}{\pi}(W_\mu W_e)^{\frac{3}{2}} e^{-W_e x_e} e^{-W_\mu x_\mu}$$

$$W_e = M_e(Z-1)\alpha, \qquad W_\mu = M_\mu Z\alpha$$



The hyperfine structure Hamiltonian has the form:

$$\Delta H_0^{hfs} = -\frac{8\pi}{3}\boldsymbol{\mu}_N\boldsymbol{\mu}_\mu\delta(\boldsymbol{x}_\mu) - \frac{8\pi}{3}\boldsymbol{\mu}_e\boldsymbol{\mu}_\mu\delta(\boldsymbol{x}_\mu - \boldsymbol{x}_e) - \frac{8\pi}{3}\boldsymbol{\mu}_N\boldsymbol{\mu}_e\delta(\boldsymbol{x}_e). \quad (3)$$

Magnetic moments of nucleus, muon and electron are:

$$\boldsymbol{\mu}_N = \frac{g_N e}{2m_p}\boldsymbol{S}_N, \qquad \boldsymbol{\mu}_\mu = -\frac{g_\mu e}{2m_\mu}\boldsymbol{S}_\mu, \qquad \boldsymbol{\mu}_e = -\frac{g_e e}{2m_e}\boldsymbol{S}_e \quad (4)$$

The hyperfine structure Hamiltonian can be presented in the form:

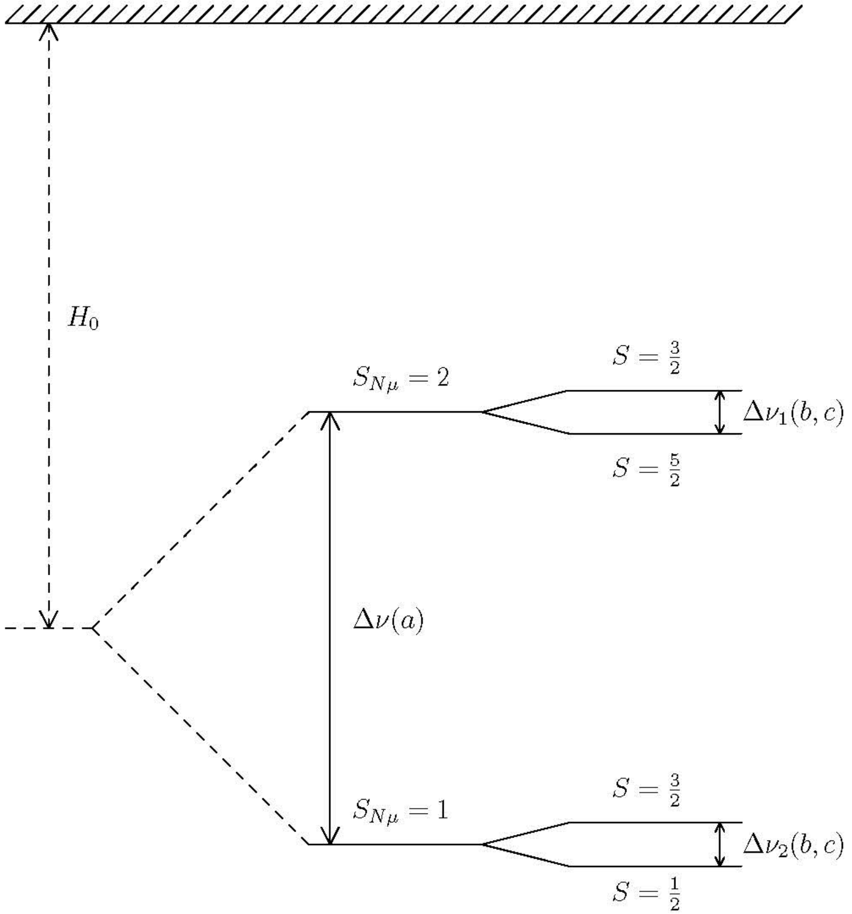
$$\Delta H_0^{hfs} = \tilde{a}(\boldsymbol{S}_N\boldsymbol{S}_\mu) - \tilde{b}(\boldsymbol{S}_e\boldsymbol{S}_\mu) + \tilde{c}(\boldsymbol{S}_N\boldsymbol{S}_e), \quad (5)$$

$$\tilde{a} = \frac{2\pi\alpha}{3}\frac{g_N g_\mu}{m_p m_\mu}\delta(\boldsymbol{x}_\mu), \qquad \tilde{b} = \frac{2\pi\alpha}{3}\frac{g_e g_\mu}{m_e m_\mu}\delta(\boldsymbol{x}_\mu - \boldsymbol{x}_e), \qquad \tilde{c} = \frac{2\pi\alpha}{3}\frac{g_N g_e}{m_p m_e}\delta(\boldsymbol{x}_e). \quad (6)$$

The contribution to hyperfine structure is determined by matrix elements:

$$\nu = <\Delta H_0^{hfs}> = a < (\boldsymbol{S}_N\boldsymbol{S}_\mu) > -\mathrm{b} < (\boldsymbol{S}_e\boldsymbol{S}_\mu) > +\mathrm{c} < (\boldsymbol{S}_N\boldsymbol{S}_e) > \quad (7)$$

The hyperfine structure levels correspond to different values of the total spin and the spin of the muon-nucleus subsystem ($\boldsymbol{S}_{N\mu} = \boldsymbol{S}_N + \boldsymbol{S}_\mu, \boldsymbol{S} = \boldsymbol{S}_e + \boldsymbol{S}_{N\mu}$). Спин ядра $S_N = \frac{3}{2}$. The total spin of a three-particle system can take the values $\boldsymbol{S} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$.



The average values of the spin-spin interaction operators can be calculated using the transformation of the basis wave functions :

$$\Psi_{S_{N\mu},S,S_z} = \sum_{S_{Ne}} (-1)^{S_\mu+S_N+S_e+S} \sqrt{(2S_{N\mu}+1)(S_{Ne}+1)} \begin{Bmatrix} S_e & S_N & S_{Ne} \\ S_\mu & S & S_{N\mu} \end{Bmatrix} \Psi_{S_{Ne},S,S_z} \qquad (8)$$

$$a \sim < \Psi_{S_{N\mu},S,S_z} | \boldsymbol{S}_N \boldsymbol{S}_\mu | \Psi_{S_{N\mu},S,S_z} >$$

$$\boldsymbol{S}_N \boldsymbol{S}_\mu = \frac{1}{2}\left[\boldsymbol{S}_{N\mu}^2 - \boldsymbol{S}_N^2 - \boldsymbol{S}_\mu^2\right], \qquad \boldsymbol{S}_{N\mu}^2 |\Psi_{S_{N\mu},S,S_z} > = S_{N\mu}(S_{N\mu}+1) |\Psi_{S_{N\mu},S,S_z} >$$

$$c \sim < \Psi_{S_{Ne},S,S_z} | \boldsymbol{S}_N \boldsymbol{S}_e | \Psi_{S_{Ne},S,S_z} >$$

$$b \sim < \Psi_{S_{e\mu},S,S_z} | \boldsymbol{S}_e \boldsymbol{S}_\mu | \Psi_{S_{e\mu},S,S_z} >$$

The matrix elements of the hyperfine splitting Hamiltonian ΔH_0^{hfs} give 4×4 matrix, corresponding to different values $S_{N\mu}$ and S :

$$\nu = \begin{pmatrix} & \Psi_{1,\frac{1}{2},S_z} & \Psi_{1,\frac{3}{2},S_z} & \Psi_{2,\frac{3}{2},S_z} & \Psi_{2,\frac{5}{2},S_z} \\ \Psi_{1,\frac{1}{2},S_z} & -\frac{5}{4}a-\frac{1}{4}b-\frac{5}{4}c & 0 & 0 & 0 \\ \Psi_{1,\frac{3}{2},S_z} & 0 & -\frac{5}{4}a+\frac{1}{8}b+\frac{5}{8}c & -\frac{\sqrt{15}}{8}b+\frac{\sqrt{15}}{8}c & 0 \\ \Psi_{2,\frac{3}{2},S_z} & 0 & -\frac{\sqrt{15}}{8}b+\frac{\sqrt{15}}{8}c & \frac{3}{4}a+\frac{3}{8}b-\frac{9}{8}c & 0 \\ \Psi_{2,\frac{5}{2},S_z} & 0 & 0 & 0 & \frac{3}{4}a-\frac{1}{4}b+\frac{3}{4}c \end{pmatrix} \qquad (9)$$

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

After diagonalizing the matrix, we obtain 4 eigenvalues that determine the energy levels of the hyperfine structure of the spectrum:

$$\nu_1\left(S_{N\mu}=1,\qquad S=\frac{1}{2}\right)=-\frac{5}{4}a-\frac{1}{4}b-\frac{5}{4}c$$

$$\nu_2\left(S_{N\mu}=1,\qquad S=\frac{3}{2}\right)=\frac{1}{4}\left(-a+b-c-\sqrt{16a^2+4b^2+16c^2+4ab-28ac-11bc}\right)$$

$$\nu_3\left(S_{N\mu}=2,\qquad S=\frac{3}{2}\right)=\frac{1}{4}\left(-a+b-c+\sqrt{16a^2+4b^2+16c^2+4ab-28ac-11bc}\right)$$

$$\nu_4\left(S_{N\mu}=2,\qquad S=\frac{5}{2}\right)=\frac{3}{4}a-\frac{1}{4}b+\frac{3}{4}c$$

Small intervals of the hyperfine structure are represented with good accuracy in the form ($a \gg b, a \gg c$):

$$\Delta\nu_1=[\nu_2-\nu_1]=\frac{3b}{8}+\frac{15c}{8}+O\left(\frac{b}{a},\frac{c}{a}\right),\qquad \Delta\nu_2=[\nu_3-\nu_4]=\frac{5b}{8}-\frac{15c}{8}+O\left(\frac{b}{a},\frac{c}{a}\right). \tag{10}$$

The parameters a, b, and c, which determine the levels and intervals of the hyperfine structure, are calculated within the perturbation theory:

$$a = a_0 + a_0^{vp} + a_1 + a_1^{vp} + \cdots, \qquad b = b_0 + b_0^{vp} + b_1 + b_1^{vp} + \cdots, \qquad c = c_0 + c_0^{vp} + c_1 + c_1^{vp} + \cdots$$

The main contribution to the parameters a, b, and c in the first order of perturbation theory has the form:

$$a_0 = \frac{2\pi\alpha}{3}\frac{g_N g_\mu}{m_p m_\mu} < \Psi_{1S}(\boldsymbol{x}_\mu, \boldsymbol{x}_e)|\delta(\boldsymbol{x}_\mu)|\Psi_{1S}(\boldsymbol{x}_\mu, \boldsymbol{x}_e) > = \nu_F \frac{g_\mu g_N}{4\left(\frac{W_e}{W_\mu}\right)^3}\frac{m_e}{m_p} \tag{11}$$

$$b_0 = \frac{2\pi\alpha}{3}\frac{g_e g_\mu}{m_e m_\mu} < \delta(\boldsymbol{x}_\mu - \boldsymbol{x}_e) > = \nu_F \frac{g_\mu g_e}{4\left(1 + \frac{W_e}{W_\mu}\right)^3}, \qquad c_0 = \frac{2\pi\alpha}{3}\frac{g_N g_e}{m_p m_e} < \delta(\boldsymbol{x}_e) > = \nu_F \frac{g_N g_e}{4}\frac{m_\mu}{m_p}$$

where $W_e = M_e(Z-1)\alpha$, $W_\mu = M_\mu Z\alpha, \nu_F = \frac{8W_e^3\alpha}{3m_e m_\mu}$ – Fermi energy. Gyromagnetic factors of the nucleus $g_N = \frac{\mu_N}{s_N}$, muon $g_\mu = 2(1+\kappa_\mu)$, and electron $g_e = 2(1+\kappa_e)$.

$$a_0 = \begin{pmatrix} {}^7_3Li: 6.08349 * 10^8\ MHz \\ {}^9_4Be: -5.26948 * 10^8 MHz \\ {}^{11}_5B: 23.66123 * 10^8 MHz \end{pmatrix}, b_0 = \begin{pmatrix} {}^7_3Li: 355830.53\ MHz \\ {}^9_4Be: 120791.03\ MHz \\ {}^{11}_5B: 286127.04\ MHz \end{pmatrix}, c_0 = \begin{pmatrix} {}^7_3Li: 4422.90\ MHz \\ {}^9_4Be: -5397.57\ MHz \\ {}^{11}_5B: 29216.41\ MHz \end{pmatrix} \tag{12}$$

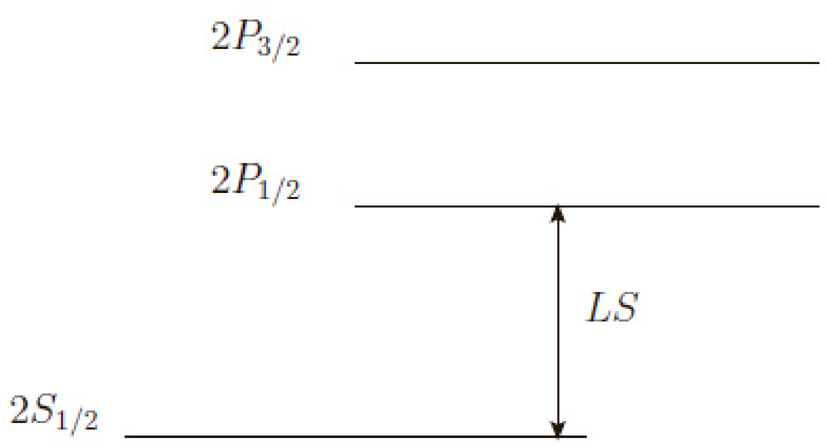
Numerical values of intervals of hyperfine structure of muon-electron lithium $(\mu e^7_3Li)^+$, berillium $(\mu e^9_4Be)^{2+}$, boron $\left(\mu e^{11}_5B\right)^{3+}$

Мюон-электронный ион	$\Delta\nu_1, MHz$	$\Delta\nu_2, MHz$
$(\mu e^7_3Li)^+$	21731.04(41)	13994.76(41)
$(\mu e^9_4Be)^{2+}$	35067.07(1.74)	85539.16(1.74)
$\left(\mu e^{11}_5B\right)^{3+}$	162228.31(4.68)	123767.56(4.68)

Faustov R. N. et al. Ground-state hyperfine structure of light muon-electron ions //Physical Review A. – 2022. – T. 105. – №. 4. – C. 042816.

Our estimates of hyperfine structure intervals for muon-electron helium atoms are in good agreement with experimental and theoretical results:

Наш результат	$\Delta\nu_{hfs}(\mu e^3_2He)$=**4166.**089**(29)**, MHz	$\Delta\nu_{hfs}(\mu e^4_2He)$=**44**6**4.**504**(29)**, MHz
D. T. Aznabayev, A. K. Bekbaev, and V. I. Korobov, Phys. Part. Nucl. Lett. 15, No. 3, 236 (2018).	$\Delta\nu_{hfs}(\mu e^3_2He)$=4166.39(58), MHz	$\Delta\nu_{hfs}(\mu e^4_2He)$=4464.55(60), MHz



The lamb shift in the case of two-particle hydrogenlike muonic atoms and ions appears in the leading order due to vacuum polarization and nuclear structure effects. The leading contribution has the order $\alpha(Z\alpha)^2$.

$$\Delta E_{LS}(\mu N) = (Z\alpha)^2[a_1\alpha + a_2\alpha^2 + a_3\alpha(Z\alpha) + \cdots]$$

When we consider three-particle bound state, lamb shift arises already due to coulomb interaction ΔH. In the case of electronic lamb shift, the general structure has the form:

$$H_0 = -\frac{1}{2M_\mu}\nabla_\mu^2 - \frac{1}{2M_e}\nabla_e^2 - \frac{Z\alpha}{x_\mu} - \frac{(Z-1)\alpha}{x_e},$$

$$\Delta H = \frac{\alpha}{x_{\mu e}} - \frac{\alpha}{x_e}, \qquad \Delta H^{rec} = -\frac{1}{M}\nabla_\mu \cdot \nabla_e,$$

$$H = H_0 + \Delta H + \Delta H^{rec} + \Delta H^{vp} + \Delta H^{str}$$

Lamb shift in three-particle muon – electron ions is determined in leading order by matrix elements in first order of perturbation theory:

$$\Delta E^{(1)}(2S) =< \Psi_{2S}(\boldsymbol{x}_\mu,\boldsymbol{x}_e)|\Delta H\,(\boldsymbol{x}_\mu,\boldsymbol{x}_e)|\Psi_{2S}(\boldsymbol{x}_\mu,\boldsymbol{x}_e) > \,= \int d\boldsymbol{x}_\mu d\boldsymbol{x}_e\, \Psi_{2S}(\boldsymbol{x}_\mu,\boldsymbol{x}_e)\left(\frac{\alpha}{x_{\mu e}} - \frac{\alpha}{x_e}\right)\Psi_{2S}(\boldsymbol{x}_\mu,\boldsymbol{x}_e) =$$
$$= \alpha W_e\left[-\frac{1}{4} + \frac{8 + a_1\big(20 + a_1(12 + a_1(10 + a_1))\big)}{(2+a_1)^5}\right], \qquad a_1 = \frac{W_e}{W_\mu} = \frac{(Z-1)}{Z}\frac{M_e}{M_\mu} \approx \frac{m_e}{m_\mu} \tag{13}$$

$$\Delta E^{(1)}(2P) =< \Psi_{2P}(\boldsymbol{x}_\mu,\boldsymbol{x}_e)|\Delta H\,(\boldsymbol{x}_\mu,\boldsymbol{x}_e)|\Psi_{2P}(\boldsymbol{x}_\mu,\boldsymbol{x}_e) > = \alpha W_e\left[-\frac{1}{4} + \frac{[(2+a_1)^5 - 6a_1^4 - a_1^5]}{4(2+a_1)^5}\right] \tag{14}$$

Lamb shift in three-particle muon – electron ions is determined in leading order by matrix elements in first order of perturbation theory:

$$\Delta E_{eLS}{}^{(1)} = \alpha W_e\frac{8a_1^2}{(2+a_1)^5} = \begin{cases} 35.018\ GHz, (\mu e_3^7Li)^+ \\ 65.951\ GHz, (\mu e_4^9Be)^{2+} \\ 99.743\ GHz, \left(\mu e_5^{11}B\right)^{3+} \end{cases} \qquad \Delta E_{\mu LS}{}^{(1)} = \alpha W_e\frac{8a_1^2}{(1+2a_1)^5} = \begin{cases} 1098.515\ GHz, (\mu e_3^7Li)^+ \\ 2053.353\ GHz, (\mu e_4^9Be)^{2+} \\ 3099.942\ GHz, \left(\mu e_5^{11}B\right)^{3+} \end{cases}$$

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

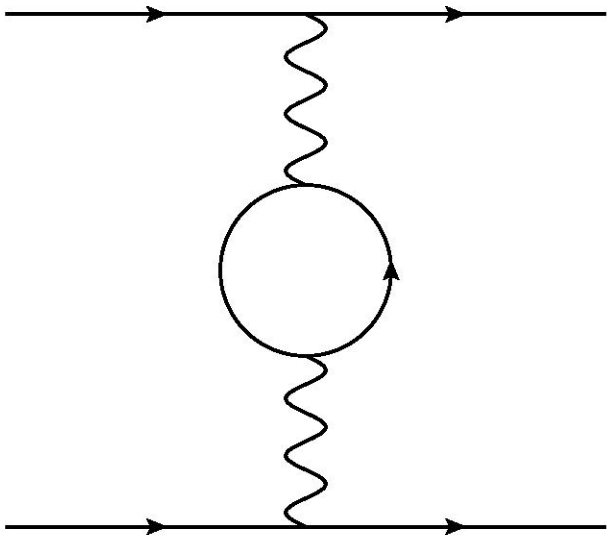
Vacuum Polarization Effects

Effects of vacuum polarization are described by following interaction potentials:

$$\Delta V_{vp}(x_e) = \frac{\alpha}{3\pi}\int_1^\infty d\xi\,\rho(\xi)e^{-2m_e\xi x_e}\left(-\frac{Z\alpha}{x_e}\right), \qquad \rho(\xi) = \frac{\sqrt{\xi^2-1}(2\xi^2+1)}{\xi^4} \tag{15}$$

$$\Delta V_{vp}(x_\mu) = \frac{\alpha}{3\pi}\int_1^\infty d\xi\,\rho(\xi)e^{-2m_e\xi x_\mu}\left(-\frac{Z\alpha}{x_\mu}\right), \qquad \Delta V_{vp}(x_{\mu e}) = \frac{\alpha}{3\pi}\int_1^\infty d\xi\,\rho(\xi)e^{-2m_e\xi x_{\mu e}}\left(-\frac{\alpha}{x_{\mu e}}\right)$$

In first order of perturbation theory the contributions have the form



$$\Delta E^{(1)}_{\mu LS,eN\,VP} = 0, \qquad a_1 = \frac{W_e}{W_\mu}, a_2 = \frac{m_e}{W_\mu}$$

$$\Delta E^{(1)}_{\mu LS,\mu N\,VP} = \frac{W_e\alpha(Z\alpha)}{18\pi}\frac{1}{a_1} = \begin{Bmatrix} 1.132195393\ 10^6\text{GHz}, (\mu e^7_3Li) \\ 2.238039356\ 10^6\ \text{GHz}, (\mu e^9_4Be) \\ 3.713166335\ 10^6\ \text{GHz}, (\mu e^{11}_5B) \end{Bmatrix} \tag{16}$$

$$\Delta E^{(1)}_{eLS,eN\,VP} = \frac{W_e\alpha(Z\alpha)}{30\pi}\frac{a_1^2}{a_2^2} = \begin{Bmatrix} 0.639\ \text{GHz}, (\mu e^7_3Li) \\ 2.852\ \text{GHz}, (\mu e^9_4Be) \\ 8.377\ \text{GHz}, (\mu e^{11}_5B) \end{Bmatrix}, \tag{17}$$

$$\Delta E^{(1)}_{eLS,\mu N\,VP} = 0$$

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

$$\Delta E^{(1)}_{eLS,e\mu\, VP} = -\frac{W_e \alpha^2}{3\pi} a_1^2 \times \tag{18}$$

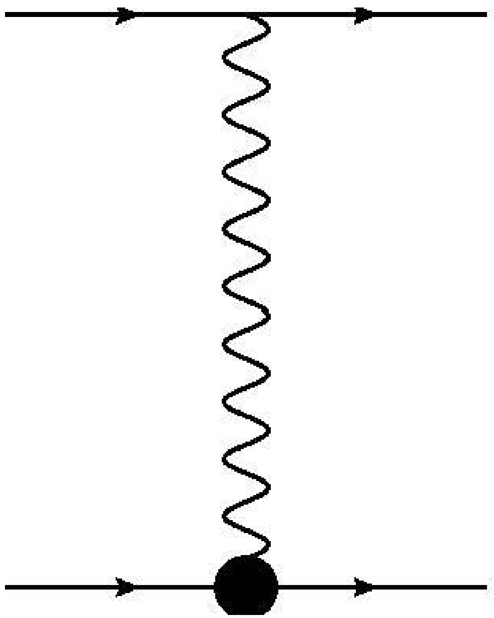
$$\left[\frac{2}{5a_2^2} + \frac{a_1}{\sqrt{4-a_2^2}}\left[\sqrt{4-a_2^2}\left(-620a_2^4 - 120a_2^6 - 150\pi a_2 - 270\,\pi a_2^3 + 135\pi a_2^5 + 30\pi a_2^7\right) + 30a_2^4\left(-40 + 5a_2^2 + 2a_2^4\right)\ln\frac{2-\sqrt{4-a_2^2}}{a_2}\right]\right]$$

$$\Delta E^{(1)}_{\mu LS,e\mu\, VP} = \begin{Bmatrix} -1.031\text{ GHz}, (\mu e^7_3 Li) \\ -2.736\text{ GHz}, (\mu e^9_4 Be) \\ -4.795\text{ GHz}, (\mu e^{11}_5 B) \end{Bmatrix},$$

$$\Delta E^{(1)}_{eLS,e\mu\, VP} = \begin{Bmatrix} 0.212\text{ GHz}, (\mu e^7_3 Li) \\ 0.711\text{ GHz}, (\mu e^9_4 Be) \\ 1.670\text{ GHz}, (\mu e^{11}_5 B) \end{Bmatrix},$$

Effects of vacuum polarization are described by following interaction potentials:

$$\Delta V_{str}(k) = -4\pi Z\alpha \frac{1}{k^2}\left(G_E(k^2) - 1\right) \cong -\frac{4\pi Z\alpha}{k^2}\left(1 - \frac{k^2}{6} r_N^2 + \cdots - 1\right) = \frac{2}{3}\pi Z\alpha r_N^2 \quad (19)$$



$$\Delta V_{str}^{\mu N} = \frac{2}{3}\pi (Z\alpha) r_N^2 \delta(\boldsymbol{x_\mu}), \qquad \Delta V_{str}^{eN} = \frac{2}{3}\pi (Z\alpha) r_N^2 \delta(\boldsymbol{x_e})$$

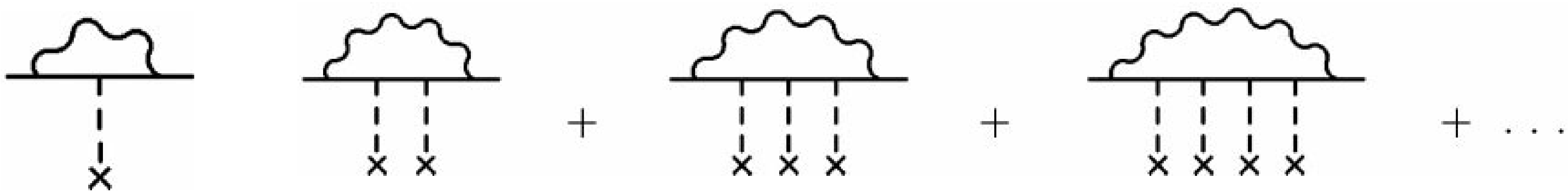
In first order of perturbation theory the contributions have the form

$$\Delta E^{(1)}_{\mu LS, str\ \mu N} = -\frac{(Z\alpha)W_\mu^3}{48\pi} r_N^2 = \begin{Bmatrix} -798105.432 \text{ GHz}, (\mu e^7_3 Li) \\ -2708154.972 \text{ GHz}, (\mu e^9_4 Be) \\ -6164059.905 \text{ GHz}, (\mu e^{11}_5 B) \end{Bmatrix}, \qquad \Delta E^{(1)}_{\mu LS, str\ eN} = 0 \quad (20)$$

$$\Delta E^{(1)}_{eLS, str\ eN} = -\frac{(Z\alpha)W_e^3}{48\pi} r_N^2 = \begin{Bmatrix} -0.028 \text{ GHz}, (\mu e^7_3 Li) \\ -0.134 \text{ GHz}, (\mu e^9_4 Be) \\ -0.369 \text{ GHz}, (\mu e^{11}_5 B) \end{Bmatrix}, \qquad \Delta E^{(1)}_{eLS, str\ \mu N} = 0 \quad (21)$$

There is another important contribution, that must be considered: QED correction, that is the main one for hydrogen atom lamb shift.

Eides M. I., Grotch H., Shelyuto V. A. Theory of light hydrogenic bound states. – Springer Science & Business Media, 2007. – T. 222.



$$\Delta E_{QED}(nS) = \frac{\alpha[(Z-1)\alpha]^4}{\pi n^3}\frac{M_e^3}{m_e^2}\left[\frac{4}{3}\ln\frac{m_e}{M_e(Z-1)^2\alpha^2} - \frac{4}{3}\ln k_0(nS) + \frac{10}{9}\right] \quad (22)$$

$$\Delta E_{QED}(2P) = \frac{\alpha[(Z-1)\alpha]^4}{8\pi}\frac{M_e^3}{m_e^2}\left[-\frac{4}{3}\ln k_0(2P) - \frac{m_e}{6M_e}\right] \quad (23)$$

ул. Московское шоссе, д.34, г.Самара, 443086, тел.: +7 (846) 335-18-26, факс: +7 (846) 335-18-36, сайт: www.ssau.ru, e-mail: ssau@ssau.ru

Numerical values of the contributions to electronic lamb shift in muon – electron ions of lithium $(\mu e^7_3 Li)^+$, beryllium $(\mu e^9_4 Be)^{2+}$, boron $(\mu e^{11}_5 B)^{3+}$ in GHz

Dorokhov A.E. et al. Low-lying electron energy levels in three-particle electron-muon ions of Li, Be, and B//Physical Review A.–2021.–V.103.–№.5.–P.052806.

Contribution	$(\mu e^7_3 Li)$	$(\mu e^9_4 Be)$	$(\mu e^{11}_5 B)$
Contribution from ΔH	34.837	65.684	99.214
Effects of VP	0.306	1.916	6.491
Nuclear structure corrections	-0.027	-0.075	-0.184
QED corrections	-14.257	-63.273	-180.004
Summary	20.859	4.252	-74.483

САМАРСКИЙ УНИВЕРСИТЕТ
SAMARA UNIVERSITY

Thank You!