High energy factorization and Monte-Carlo generator PEGASUS

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- TMD-based factorizations in QCD
- TMD parton densities in a proton
- Off-mass shell production amplitudes
- Working with TMDs: some of Monte-Carlo generators and tools
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  - KaTie
- New Monte-Carlo generator PEGASUS
- Summary

Factorization in Quantum Chromodynamics

Conventional (collinear) factorization

$$d\sigma = d\hat{\sigma}(x_1, x_2, \mu^2) \otimes f_a(x_1, \mu^2) \otimes f_b(x_2, \mu^2)$$

where a(x, μ^2) are the conventional parton (quark or gluon) density functions (PDFs) in a proton obeying the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations.

- resums large logarithmic terms ~ ln μ^2/Λ_{QCD} to all higher orders in α_s
- successive parton emissions are strongly ordered in virtuality, so that parton entering the hard scattering subprocess is treated collinear with incoming proton comparing to the large scale μ^2
- good description of most of collider data if NLO + NNLO + … terms are taken into account

Transverse Momentum Dependent (TMD) factorization: account of terms proportional to large logarithms, ln M/q_T or ln s/M^2 ~ ln 1/x

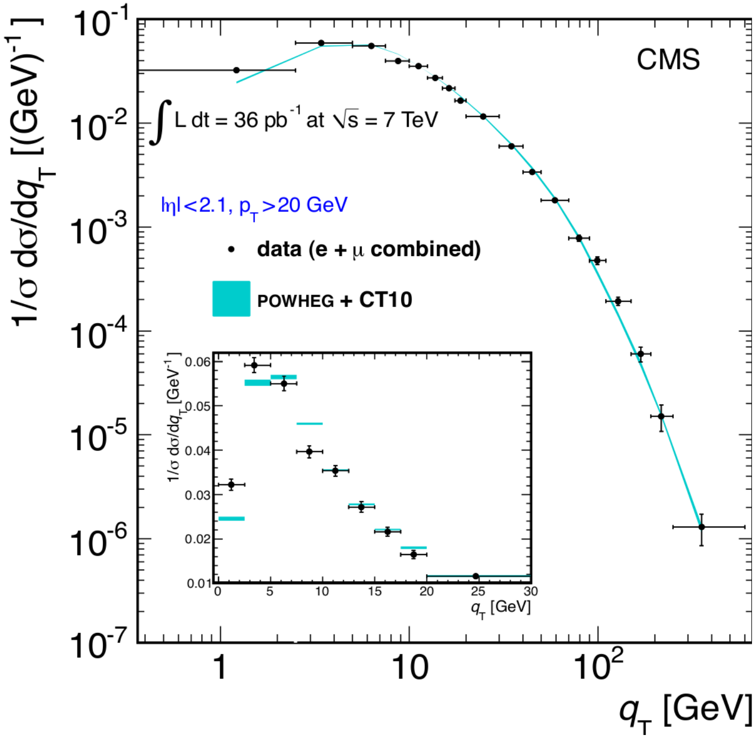
$$d\sigma = d\hat{\sigma}(x_1, x_2, k_{1T}^2, k_{2T}^2, \mu^2) \otimes f_a(x_1, k_{1T}^2, \mu^2) \otimes f_b(x_2, k_{2T}^2, \mu^2)$$

where $f_a(x, k_T^2, \mu^2)$ are the TMD parton densities in a proton

Low q_T-factorization for p_T spectra (Collins-Soper-Sterman)

- valid at $q_T \to 0$ for fixed invariant mass M

  J.C. Collins, D.E. Soper, G.F. Sterman, NPB 223, 381 (1983)
  J.C. Collins, D.E. Soper, G.F. Sterman, NPB 250, 199 (1985)
  J.C. Collins, «Foundation of perturbative QCD», CUP, 2011



Fixed-order (LO, NLO, NNLO, …) pQCD calculations in the collinear factorization:

- reproduce well the DY cross section at high q_T region
- diverge as q_T decreases due to multiparton QCD radiation
- soft gluon resummation technique is needed

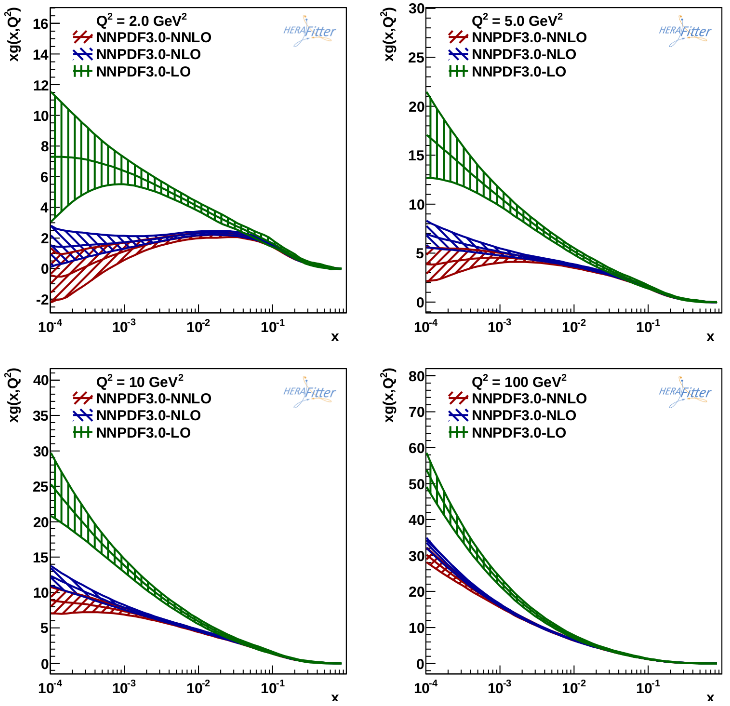
CSS evolution equations:

- resum large logarithmic terms ~ ln M/q_T to all higher orders in α_s
- reproduce the physical behavior of q_T spectra
- can be applied for DY, Higgs, heavy flavors etc

High energy factorization (k_T-factorization)

- valid at $s \to \infty$ at fixed momentum transfer, i.e. at $x \to 0$
- can be applied to many LHC processes which receive contribution from the low x

L.V. Gribov, E.M. Levin, M.G. Ryskin, PR 100, 1 (1983)
E.M. Levin, M.G. Ryskin, Yu.M. Shabelsky, A.G. Shuvaev, Sov. J. Nucl. Phys. 53, 657 (1991)
S. Catani, M. Ciafoloni, F. Hautmann, NPB 366, 135 (1991)
J.C. Collins, R.K. Ellis, NPB 360, 3 (1991)



At small x:

- pQCD corrections are large
- multiple gluon radiation over long interval in rapidity not ordered in transverse momentum k_T

Balitsky-Fadin-Kuraev-Lipatov (BFKL) evolution equation:

E.A. Kuraev, L.N. Lipatov, V.S. Fadin, JETP 44, 443 (1976)
E.A. Kuraev, L.N. Lipatov, V.S. Fadin, JETP 45, 199 (1976)
I.I. Balitsky, L.N. Lipatov, Sov. J. Nucl. Phys. 28, 822 (1978)

- resum large logarithmic terms $\sim \ln s/M^2 \sim \ln 1/x$ to all higher orders in α_s
- off-mass shell (or k_T-dependent) partonic amplitudes

High energy factorization (k_T-factorization)

- provides possibility for taking into account the precise subprocess kinematics already at leading order

- main advantage is that, even with LO amplitudes, one can include a large piece of higher-order pQCD corrections (NLO + NNLO + …, important at high energies) taking them into account in the form of TMD gluon densities in a proton

- at the moment, some methods to account high-order corrections to the LO k_T-factorization approach are discussed in the literature

  M. Hentschinski, PRD 104, 054014 (2021); R. Maciula, A. Szczurek, PRD 100, 054001 (2019); A.V. Karpishkov, M.A. Nefedov, V.A. Saleev, PRD 96, 096019 (2017)

- k_T-factorization approach at LO was already applied to the number of high-energy processes
  - heavy quark production
  - heavy quarkonia (charmonia, bottomonia) production
  - inclusive and/or jet associated prompt photon or gauge boson production
  - inclusive and/or jet associated Higgs production
  - proton SFs

  R. Angeles-Martinez et al., APP B 46, 2501 (2015)

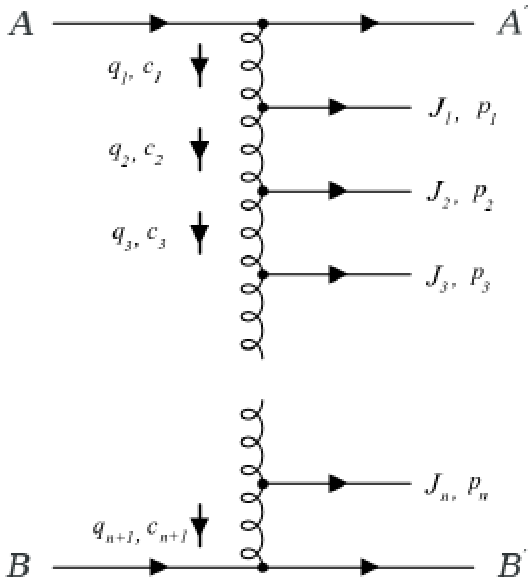
- all processes above are sensitive to the TMDs and can be used to constrain them

Off-shell production amplitudes

- multi-Regge limit: produced particles are well separated in rapidity and have finite transverse momentum

- multiparticle production amplitude $A + B \to A' + J_1 + \ldots + J_n + B'$ can be written as

$$\mathcal{A}(A + B \to A' + J_1 + \cdots + J_n + B') \sim$$
$$\sim \Gamma^{c_1}_{-AA'} \left(\prod_{i=1}^{n} \Gamma^{c_i c_{i+1}}_{+-J_i}(q_i, -q_{i+1}) \left[\frac{s_i}{s_0}\right]^{\omega(t_i)} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left[\frac{s_{n+1}}{s_0}\right]^{\omega(t_{n+1})} \Gamma^{c_{n+1}}_{+BB'}$$

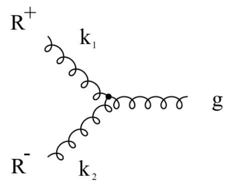


- gauge-invariant effective vertices Γ are connected to each other by t-channel exchange of reggeized gluons $R^\pm$ or quarks $Q^\pm$

- sign «±» denotes large light-cone momentum carried by the reggeized parton having the transverse momentum of the same order

Effective vertices

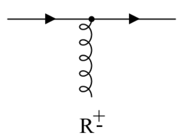
- interaction vertex for two reggeized gluons with Yang-Mills gluon, R^+R^-g:



$$\Gamma^{abc}_{+\mu-}(k_1,k_2) = -2gf^{abc}\left[(k_2-k_1)_\mu + \left(\frac{k_1^2}{k_2^-}+k_1^+\right)n^-_\mu - \left(\frac{k_2^2}{k_1^+}+k_2^-\right)n^+_\mu\right]$$

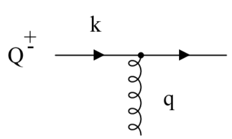
- R^+R^-g vertex can be replaced by the usual 3g vertex with special choice for off-shell gluon polarization tensor: $\sum \epsilon^\mu \epsilon^{*\nu} = \frac{k_T^\mu k_T^\nu}{\mathbf{k}_T^2}$

- interaction vertex for reggeized gluon with two quarks, $R^\pm qq$:



$$\Gamma^a_\pm = gt^a \hat{n}^\mp$$

- interaction vertex for reggeized quark with Yang-Mills gluon and quark, $Q^\pm qg$:



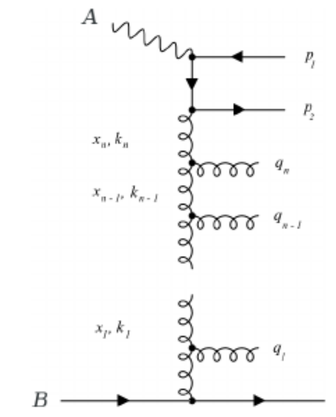
$$\Gamma^a_{\pm\mu}(k,q) = gt^a\left(\gamma_\mu + n^\mp_\mu \frac{\hat{k}}{q^\mp}\right) \qquad n^\pm = (1,0,0,\mp 1)$$

E.N. Antonov, I.O. Cherednikov, E.A. Kuraev, L.N. Lipatov, NPB 721, 111 (2005)
A.V. Bogdan, V.S. Fadin, NPB 740, 36 (2006)

CCFM evolution equation (Catani-Ciafoloni-Fiorani-Marchesini)

- resums large logarithmic terms ~ ln 1/x and ~ ln 1/(1-x)
- valid at both small and large x

M. Ciafaloni, NPB 296, 49 (1988)
S. Catani, F. Fiorani, G. Marchesini, PLB 234, 339 (1990)
S. Catani, F. Fiorani, G. Marchesini, NPB 336, 18 (1990)
G. Marchesini, NPB 445, 49 (1995)



In the LL approximation

$$f_g(x, \mathbf{k}_T^2, \mu^2) = f_g^{(0)}(x, \mathbf{k}_T^2, \mu_0^2)\Delta_s(\mu^2, \mu_0^2) +$$
$$+ \int \frac{dz}{z} \int \frac{dq^2}{q^2} \theta(\mu - zq)\Delta_s(\mu^2, z^2q^2) P_{gg}(z, q^2, \mathbf{k}_T^2) f_g(x/z, \mathbf{k'}_T^2, q^2)$$

$$P_{gg}(z, \mathbf{q}_T^2, \mathbf{k}_T^2) = \frac{N_c}{\pi}\alpha_s\left(\mathbf{q}_T^2(1-z)^2\right)\left[\frac{1}{1-z} - 1 + \frac{z(1-z)}{2}\right] +$$
$$+ \frac{N_c}{\pi}\alpha_s(\mathbf{k}_T^2)\left[\frac{1}{z} - 1 + \frac{z(1-z)}{2}\right]\Delta_{ns}(z, \mathbf{q}_T^2, \mathbf{k}_T^2)$$

$$p = p_1 + p_2 = \Upsilon(P_A + \Xi P_B) + Q_T$$
$$q_i = v_i(P_A + \xi_i P_B) + q_{iT}, \quad \xi_i = \frac{q_{iT}^2}{s v_i^2}$$
$$v_i = (1 - z_i)x_{i-1} \text{ и } x_i = z_i x_{i-1}$$
$$\xi_i = \cos\theta_i$$

Anglular ordering condition:

$$\xi_1 < \xi_2 < \ldots < \xi_n < \Xi$$

or

$$z_{i-1}\bar{q}_{i-1} < \bar{q}_i$$
$$\bar{q}_i = x_{i-1}\sqrt{\xi_i s} = \frac{q_{iT}}{1 - z_i}$$

Sudakov form-factor: $\ln \Delta_s(p^2, q^2) = -\frac{N_c}{\pi}\int\limits_{q^2}^{p^2}\frac{dk^2}{k^2}\int\limits_0^{1-\Delta}\frac{\alpha_s(k^2(1-z)^2)}{1-z}$

Non-Sudakov form-factor: $\ln \Delta_{ns}(z, \mathbf{q}_T^2, \mathbf{k}_T^2) = -\frac{N_c}{\pi}\alpha_s(\mathbf{k}_T^2)\int\limits_z^1 \frac{dz'}{z'}\int\limits_{z'|\mathbf{q}_T|}^{|\mathbf{k}_T|}\frac{dp^2}{p^2}$

Relation $\bar{q}^2 \simeq x^2 \Xi s = \hat{s} + \mathbf{Q}_T^2$ determines the choice of factorization scale

CCFM-evolved gluon densities in a proton

- uPDFevolve: evolution code for TMD gluon density from CCFM equation *https://updfevolv.hepforge.org/*

  F. Hautmann, H. Jung, S. Taheri Monfared, EPJC 74, 3082 (2014)

Latest TMD gluons from CCFM equation:

- JH 2013 set 1 and set 2 (H. Jung, F. Hautmann)
  - phenomenological parameters of initial (starting) gluon density were determined from the fit to the precise HERA data on proton SFs $F_2(x, Q^2)$ and/or $F_2(\text{charm})(x, Q^2)$ at low $x < 0.005$

  F. Hautmann, H. Jung, NPB 883, 1 (2014)

- MD 2018 (Moscow — Dubna)
  - expression for starting gluon distribution is derived in a framework of quark-gluon string model with account of saturation effects. Phenomenological parameters are fitted to the RHIC and LHC data on charged hadron production in pp and AA collisions

  N.A. Abdulov, H. Jung, A.V. Lipatov, G.I. Lykasov, M.A. Malyshev, PRD 98, 054010 (2018)

KMR approach (Kimber-Martin-Ryskin)

- formalism to construct TMDs from conventional PDFs
- parton k_T arises at the last step of the evolution

$$f_q(x, \mathbf{k}_T^2, \mu^2) = T_q(\mathbf{k}_T^2, \mu^2) \frac{\alpha_s(\mathbf{k}_T^2)}{2\pi} \times$$

$$\times \int\limits_x^1 dz \left[P_{qq}(z) \frac{x}{z} q\left(\frac{x}{z}, \mathbf{k}_T^2\right) \Theta\left(\Delta - z\right) + P_{qg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mathbf{k}_T^2\right) \right],$$

$$f_g(x, \mathbf{k}_T^2, \mu^2) = T_g(\mathbf{k}_T^2, \mu^2) \frac{\alpha_s(\mathbf{k}_T^2)}{2\pi} \times$$

$$\times \int\limits_x^1 dz \left[\sum_q P_{gq}(z) \frac{x}{z} q\left(\frac{x}{z}, \mathbf{k}_T^2\right) + P_{gg}(z) \frac{x}{z} g\left(\frac{x}{z}, \mathbf{k}_T^2\right) \Theta\left(\Delta - z\right) \right]$$

- currently developed at LO and NLO level

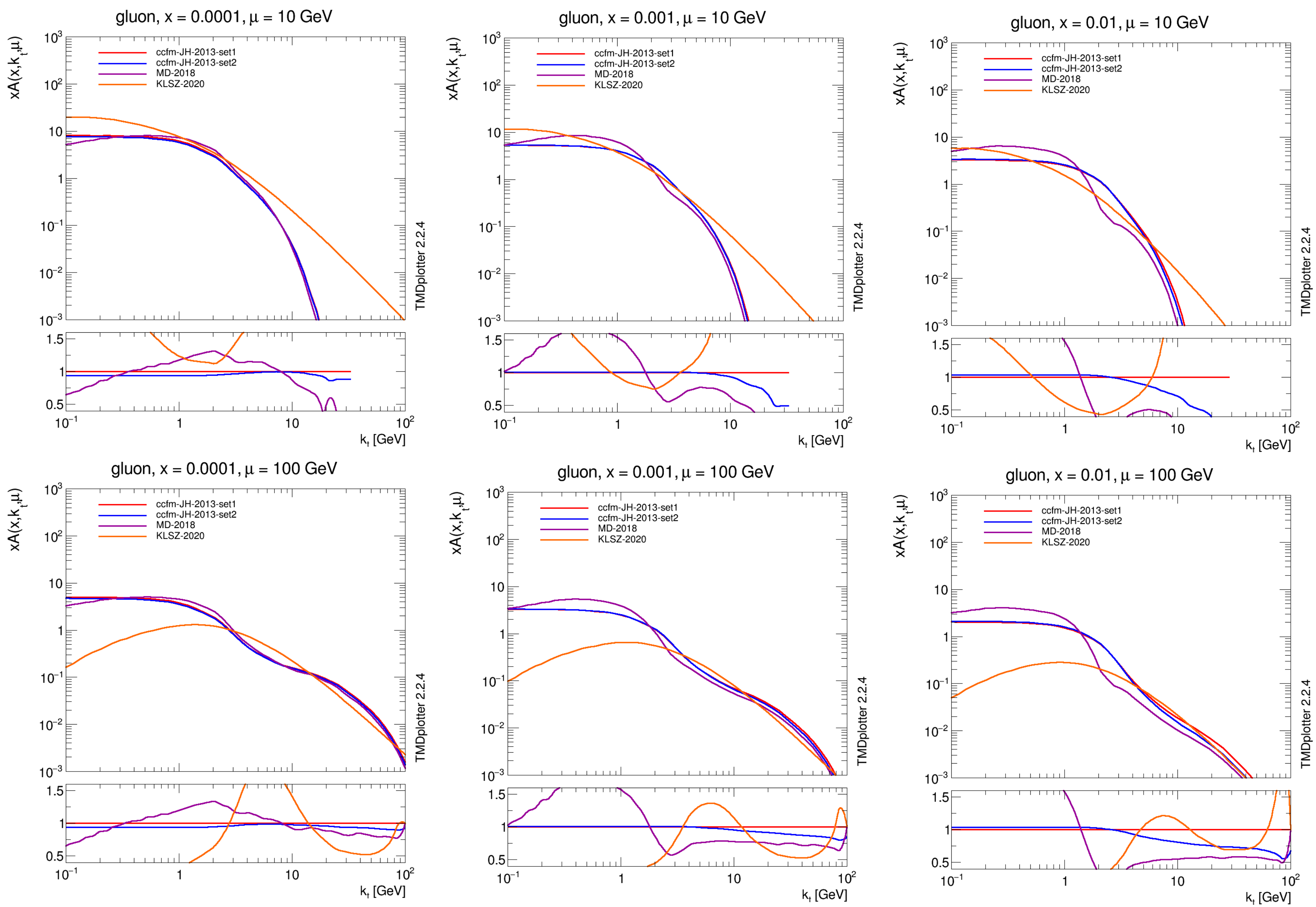
M.A. Kimber, A.D. Martin, M.G. Ryskin, PRD 63, 114027 (2001)
G. Watt, A.D. Martin, M.G. Ryskin, EPJC 31, 73 (2003)
A.D. Martin, M.G. Ryskin, G. Watt, EPJC 66, 163 (2010)

- KLSZ 2020 set: first analytical expressions for gluon and quark TMDs

A.V. Kotikov, A.V. Lipatov, B.G. Shaikhatdenov, P. Zhang, JHEP 02, 028 (2020)

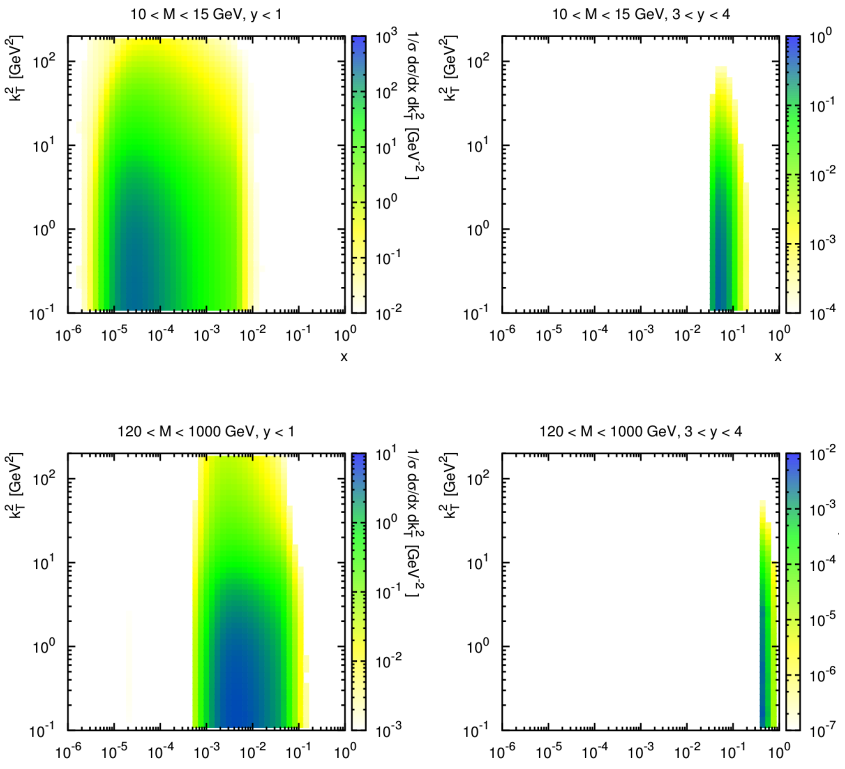
TMDPlotter: on-line plotting tool

https://tmdplotter.desy.de

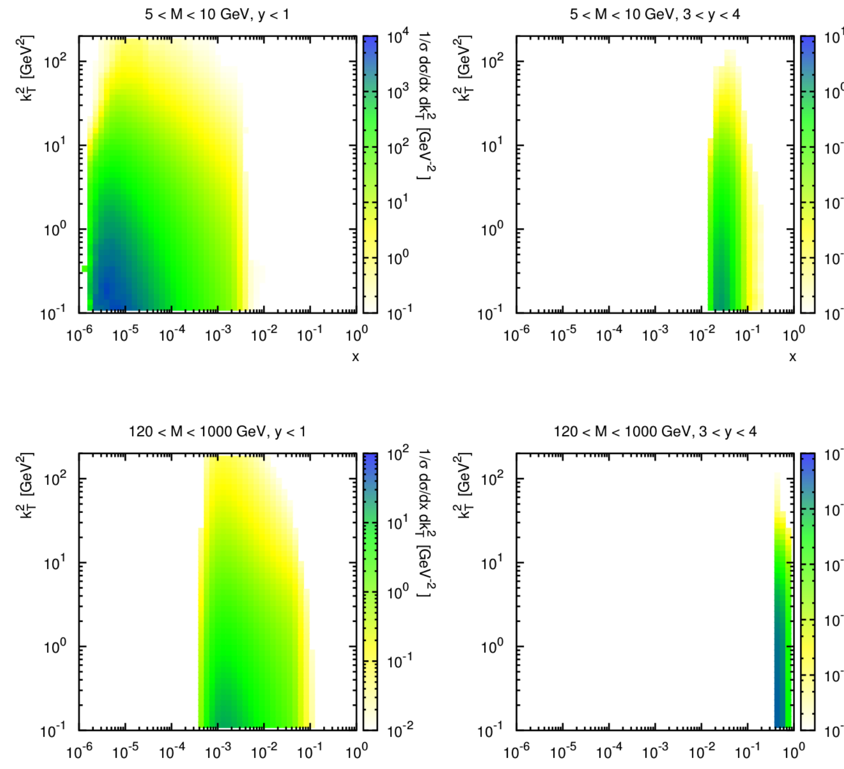


Testing the TMDs evolution using the LHC processes

bb pair



DY pair



One can map the TMDs at the scale M by applying different cuts on the final state

- useful variables: transverse momentum p_T, rapidity y, invariant mass M, azimuthal angle difference Δφ

CASCADE, KaTie and TMDlib

- CASCADE is the hadron level MC event generator for *ep, γp, pp* processes
  - uses the CCFM equation for the initial state gluon cascade
  - simulates events according to implemented off-shell production amplitudes
  - generates uweighted events according to the HEP common standards
  - latest version (CASCADE3) covers full kinematical range (not only small x) by applying the Parton Branching method to evaluate the TMD quark and gluon densities in a proton

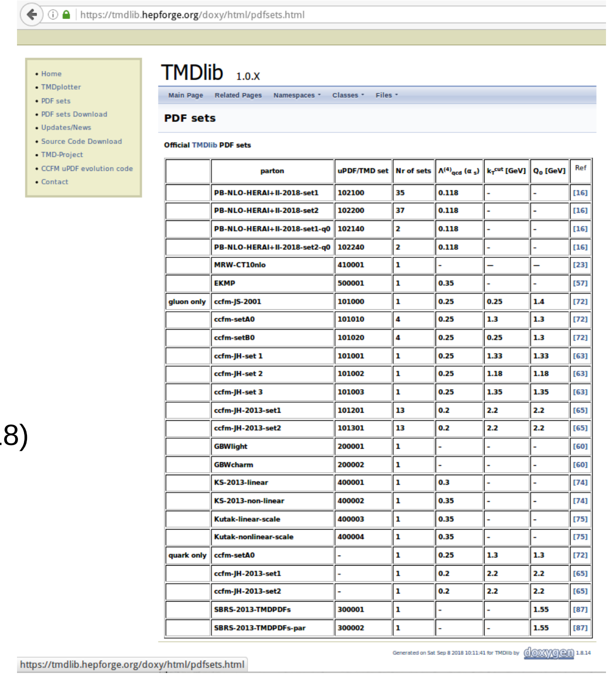
  https://cascade.hepforge.org H. Jung et al, EPJC 81, 425 (2021)

- KaTie is the parton level MC generator
  - automatically calculates the tree-level off-shell production amplitudes for arbitrary SM processes (up to 4 particles in a final state)
  - uses TMDLib to access the TMDs
  - produces weighted events

  https://bitbucket.org/hamereb/katie A. van Hameren, CPC 224, 371 (2018)

- TMDlib: library for TMD parton distributions
  - C++ library which provides a framework and an interface to a collection of different TMDs

  https://tmdlib.hepforge.org H. Jung et al, EPJC 81, 752 (2021)



https://tmdlib.hepforge.org/doxy/html/pdfsets.html

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- CCFM uPDF evolution code
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TMDlib 1.0.X

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PDF sets

Official TMDlib PDF sets

	parton	uPDF/TMD set	Nr of sets	$\Lambda^{(4)}_{qcd}$ (α_s)	k_T^{cut} [GeV]	Q_0 [GeV]	Ref
	PB-NLO-HERAI+II-2018-set1	102100	35	0.118	-	-	[16]
	PB-NLO-HERAI+II-2018-set2	102200	37	0.118	-	-	[16]
	PB-NLO-HERAI+II-2018-set1-q0	102140	2	0.118	-	-	[16]
	PB-NLO-HERAI+II-2018-set2-q0	102240	2	0.118	-	-	[16]
	MRW-CT10nlo	410001	1	-	—	—	[23]
	EKMP	500001	1	0.35	-	-	[57]
gluon only	ccfm-JS-2001	101000	1	0.25	0.25	1.4	[72]
	ccfm-setA0	101010	4	0.25	1.3	1.3	[72]
	ccfm-setB0	101020	4	0.25	0.25	1.3	[72]
	ccfm-JH-set 1	101001	1	0.25	1.33	1.33	[63]
	ccfm-JH-set 2	101002	1	0.25	1.18	1.18	[63]
	ccfm-JH-set 3	101003	1	0.25	1.35	1.35	[63]
	ccfm-JH-2013-set1	101201	13	0.2	2.2	2.2	[65]
	ccfm-JH-2013-set2	101301	13	0.2	2.2	2.2	[65]
	GBWlight	200001	1	-	-	-	[60]
	GBWcharm	200002	1	-	-	-	[60]
	KS-2013-linear	400001	1	0.3	-	-	[74]
	KS-2013-non-linear	400002	1	0.35	-	-	[74]
	Kutak-linear-scale	400003	1	0.35	-	-	[75]
	Kutak-nonlinear-scale	400004	1	0.35	-	-	[75]
quark only	ccfm-setA0	-	1	0.25	1.3	1.3	[72]
	ccfm-JH-2013-set1	-	1	0.2	2.2	2.2	[65]
	ccfm-JH-2013-set2	-	1	0.2	2.2	2.2	[65]
	SBRS-2013-TMDPDFs	300001	1	-	-	1.55	[87]
	SBRS-2013-TMDPDFs-par	300002	1	-	-	1.55	[87]

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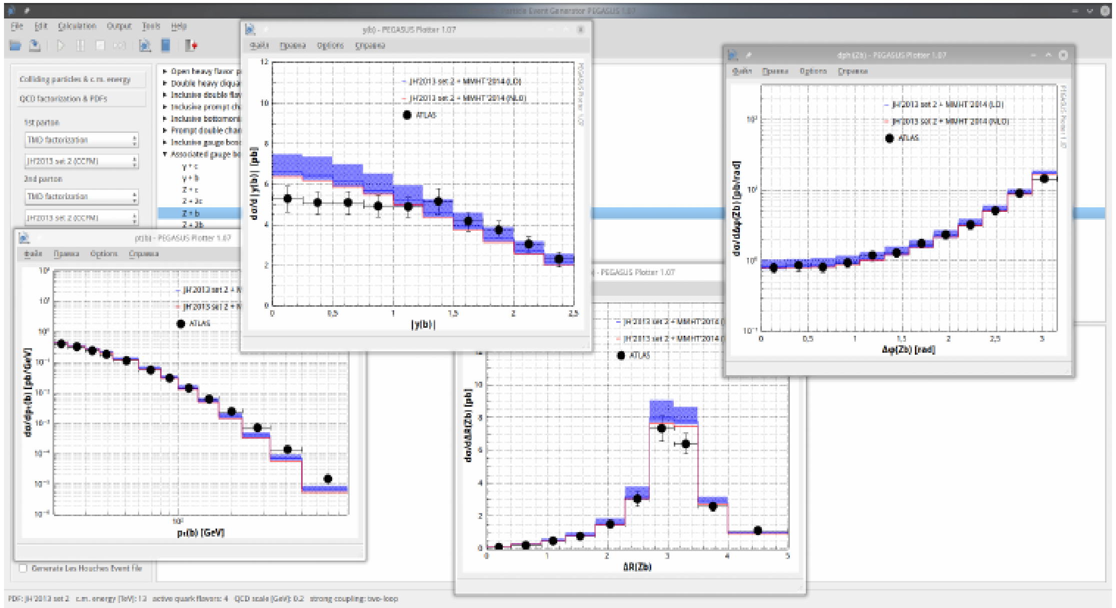
https://tmdlib.hepforge.org/doxy/html/pdfsets.html

Particle Event Generator: A Simple-in-Use System

- PEGASUS is the parton level Monte-Carlo event generator newly developed in the Moscow State University

  A.V. Lipatov, M.A. Malyshev, S.P. Baranov,
  EPJC 80, 330 (2020)



- underlying physics in PEGASUS, CASCADE and KaTie is basically the same. However, there are features which make PEGASUS more flexible and better adjustable to the user's need

- extremely user-friendly graphical interface
  - allows one to easily implement different kinematical cuts
  - built-in plotting tool PEGASUS Plotter

- number of implemented processes

- compatible with HEPData repository *https://www.hepdata.net*

Particle Event Generator: A Simple-in-Use System

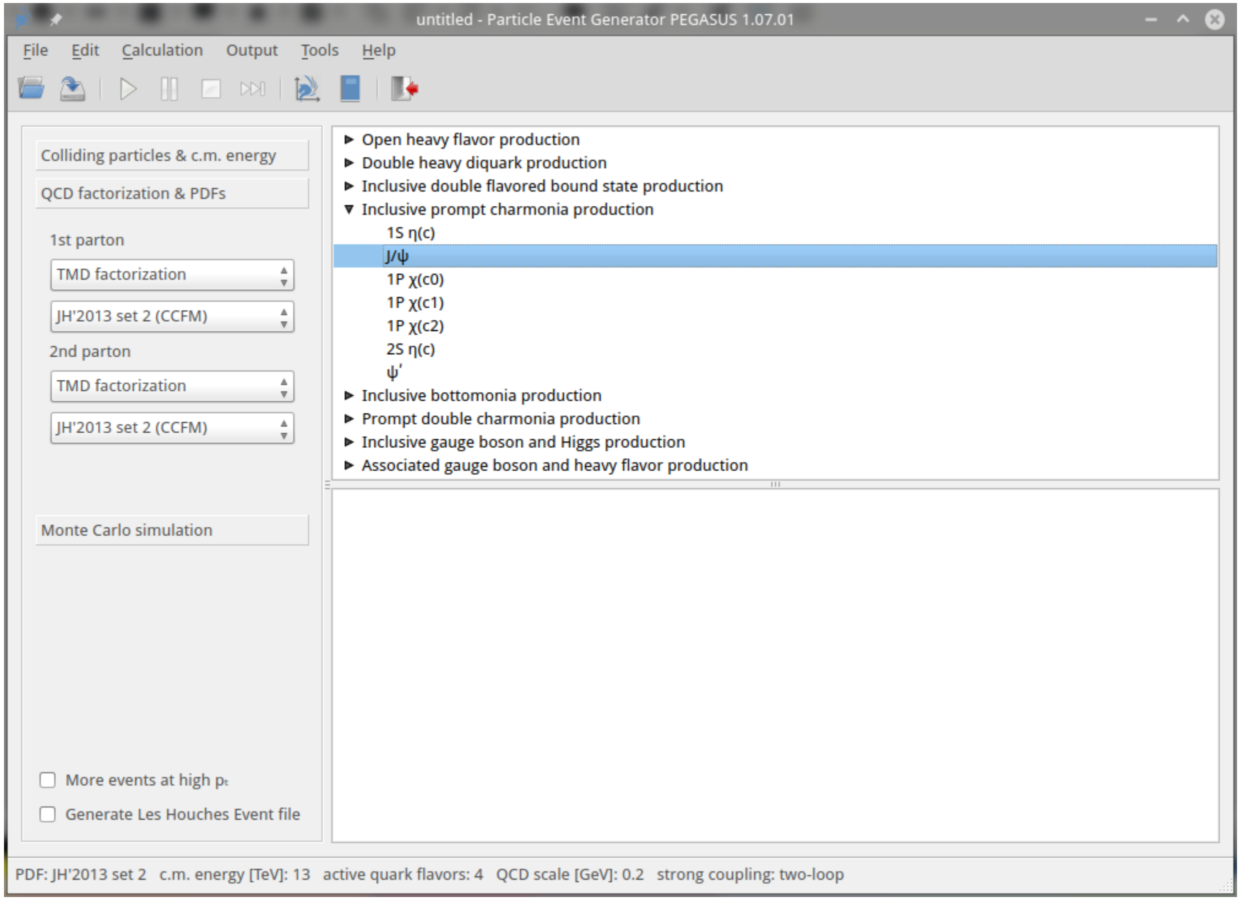
Other important features of PEGASUS:

- provides all necessary components, including off-shell production amplitudes and grid files for implemented TMDs, interpolated automatically
- produces weighted or unweighted events
- possibility to switch between the TMD or collinear mode
- generated events can be stored in the *.lhe file or presented «on the fly» with convenient bulit-in PEGASUS Plotter tool
- no any programming skills are needed
- no experience with other MC generators is required
- no special installation procedure
- there is console version if a long-time calculation is needed

Particle Event Generator: A Simple-in-Use System

Processes available to simulation in PEGASUS 1.07:

- open heavy quark production (c, b, t)
- double heavy di-quark production
- double heavy meson (B_c, B_c*) and barion (Ξ, Ω) production
- inclusive prompt S- and P-wave charmonia (J/ψ, ψ(2S), η_c, χ_c) production
- inclusive S- and P-wave bottomonia (Y, χ_b) production
- prompt double charmonia (J/ψ + J/ψ) production
- Inclusive Higgs production
- Associated prompt photon/gauge boson and heavy quark (c, b) production



Particle Event Generator: A Simple-in-Use System

Production amplitudes implemented into PEGASUS 1.07:

off-shell

- $g^* + g^* \to Q + \bar{Q}$, где $Q = c$ или b
- $g^* + g^* \to \mathcal{Q} + b + \bar{c}$, где $\mathcal{Q} = B_c$ или B_c^*
- $g^* + g^* \to \mathcal{Q} + \bar{Q} + \bar{Q}'$, где $\mathcal{Q} = (QQ')_0$ или $(QQ')_1$ и $Q = c$ или b
- $g^* + g^* \to Q\bar{Q}\left[{}^3S_1^{(1)}\right] + g \to \mathcal{Q} + g$, где $\mathcal{Q} = \psi'$, J/ψ или $\Upsilon(nS)$
- $g^* + g^* \to Q\bar{Q}\left[{}^1S_0^{(8)},\ {}^3S_1^{(8)},\ {}^3P_J^{(8)}\right] \to \mathcal{Q}$, где $\mathcal{Q} = \psi'$, J/ψ или $\Upsilon(nS)$
- $g^* + g^* \to Q\bar{Q}\left[{}^3P_J^{(1)},\ {}^3S_1^{(8)},\ {}^1P_1^{(8)}\right] \to \mathcal{Q}$, где $\mathcal{Q} = \chi_{cJ}(1P)$ или $\chi_{bJ}(mP)$
- $g^* + g^* \to Q\bar{Q}\left[{}^1S_0^{(1)},\ {}^1S_0^{(8)},\ {}^3S_1^{(8)},\ {}^1P_1^{(8)}\right] \to \mathcal{Q}$, где $\mathcal{Q} = \eta_c(1S)$, $\eta_c(2S)$ или $\eta_b(nS)$
- $g^* + g^* \to Q\bar{Q}\left[{}^3S_1^{(1)}\right] + Q\bar{Q}\left[{}^3S_1^{(1)}\right] \to \mathcal{Q} + \mathcal{Q}$, где $\mathcal{Q} = \psi'$, J/ψ или $\Upsilon(nS)$
- $g^* + g^* \to H^0 \to \gamma\gamma$
- $g^* + g^* \to H^0 \to ZZ^* \to 4l$
- $g^* + g^* \to H^0 \to W^+W^- \to e^\pm\mu^\mp\nu\bar{\nu}$
- $g^* + g^* \to V + Q + \bar{Q}$, где $V = \gamma$ или Z/γ^* и $Q = c$ или b

on-shell

- $q + \bar{q} \to Q + \bar{Q}$, где $Q = c$ или b
- $g + g \to Q\bar{Q}\left[{}^1S_0^{(8)},\ {}^3S_1^{(8)},\ {}^3P_J^{(8)}\right] + g \to \mathcal{Q} + g$, где $\mathcal{Q} = \psi'$, J/ψ или $\Upsilon(nS)$
- $g + g \to Q\bar{Q}\left[{}^3P_J^{(1)},\ {}^3S_1^{(8)},\ {}^1P_1^{(8)}\right] + g \to \mathcal{Q} + g$, где $\mathcal{Q} = \chi_{cJ}(1P)$ или $\chi_{bJ}(mP)$
- $g + g \to Q\bar{Q}\left[{}^1S_0^{(1)},\ {}^1S_0^{(8)},\ {}^3S_1^{(8)},\ {}^1P_1^{(8)}\right] + g \to \mathcal{Q} + g$, где $\mathcal{Q} = \eta_c(1S)$, $\eta_c(2S)$ или $\eta_b(nS)$
- $q + g \to V + q$, где $V = \gamma$ или Z/γ^*
- $q + Q \to V + q + Q$, где $V = \gamma$ или Z/γ^* и $Q = c$ или b
- $q + \bar{q} \to V + Q + \bar{Q}$, где $V = \gamma$ или $V = Z/\gamma^*$ и $Q = c$ или b
- $q + g \to q + V + Q + \bar{Q}$, где $V = \gamma$ или $V = Z/\gamma^*$ и $Q = c$ или b

Particle Event Generator: A Simple-in-Use System

- normally, no special installation procedure is needed.
  PEGASUS can be downloaded from
  https://theory.sinp.msu.ru/dokuwiki/doku.php/pegasus/download
  as a precompiled executive file for Linux machines

- executive file can be just run from a terminal as ./PEGASUS

- generated *.lhe file can be processed with CASCADE or PYTHIA
  for parton showering, hadronization etc

- PEGASUS has been already used by the ALICE Collaboration in their
  recent analysis of prompt charmonia production at 5.02 and 13 TeV

  ALICE Collaboration, EPJC 81, 1121 (2021) [arXiv:2108.01906]
  ALICE Collaboration, arXiv:2108.02523

Summary

- Using the TMD-based factorization and TMD parton densities one can resum logarithmically enhanced pQCD corrections to all orders
- Resummation of terms ~ ln M/q_T can be done using Collins-Soper-Sterman approach, whereas large terms ~ ln s/M^2 ~ ln $1/x$ can be resummed by the Balitsky-Fadin-Kuraev-Lipatov or Catani-Ciafaloni-Fiorani-Marchesini evolution equations using high energy, or k_T-factorization approach
- Evaluation of gauge-invariant off-shell partonic amplitudes can be done using the effective field theory. Appropriate automatical tools (such as KaTie) are developed
- TMD parton densities in a proton can be obtained from the numerical solution of CCFM equation or within the Kimber-Martin-Ryskin approach
- Several Monte-Carlo event generators are available
  - CASCADE
  - KaTie
  - PEGASUS

Thank you for attention!