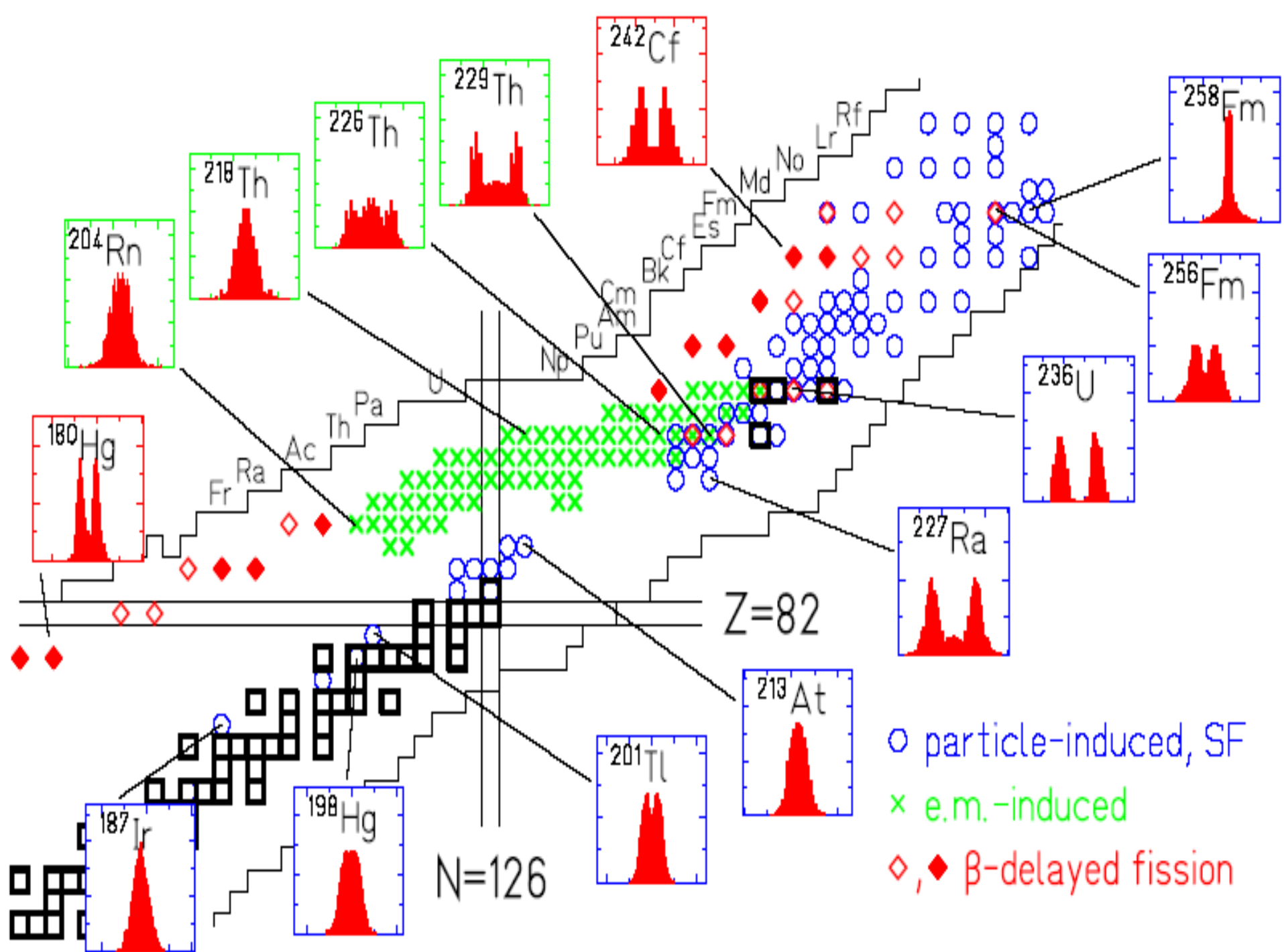
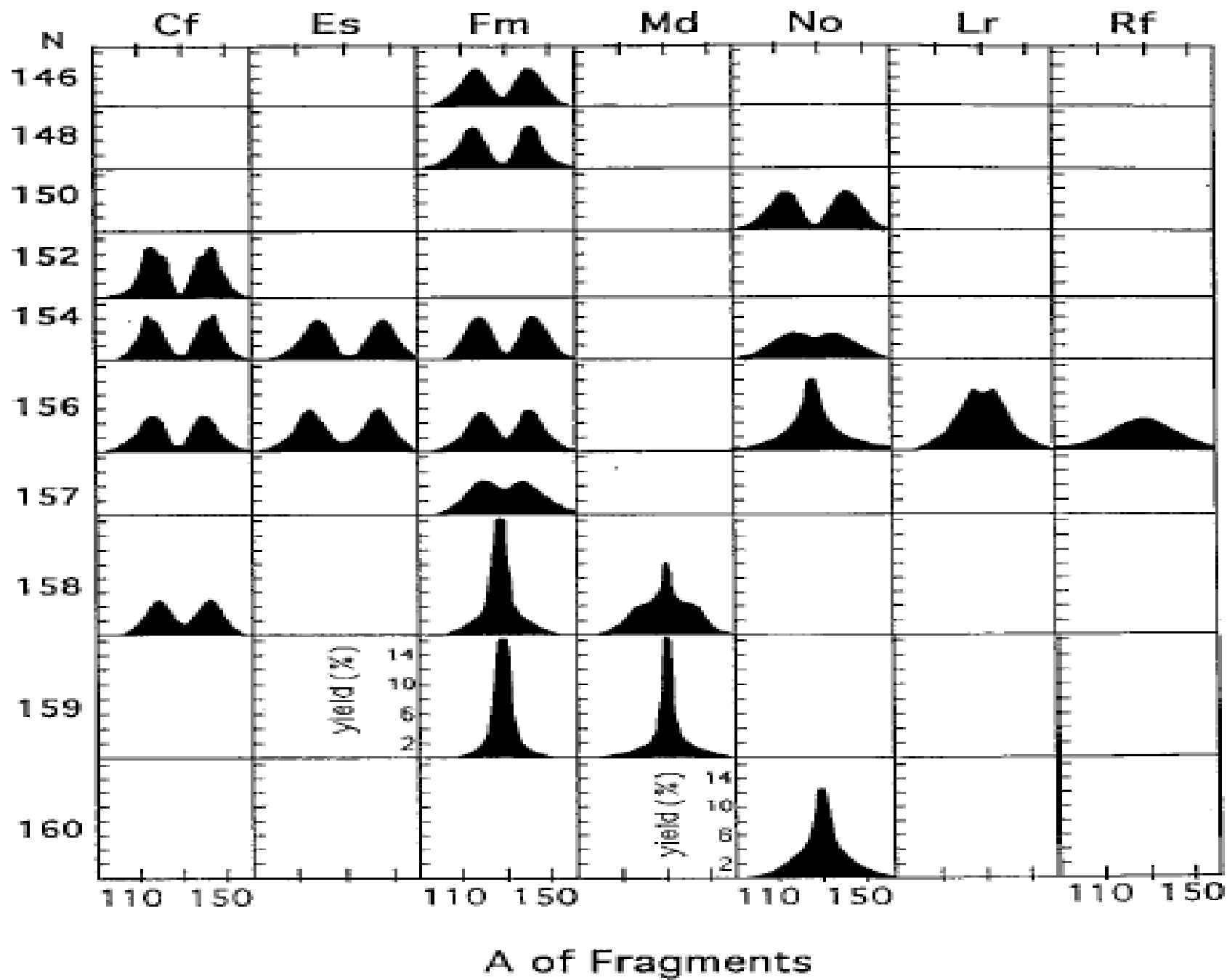


# Описание распределений продуктов деления тяжелых ядер и их корреляция с множественностью нейтронов

H. Pasca, A. Andreev, G. Adamian, N. Antonenko

BLTP, JINR



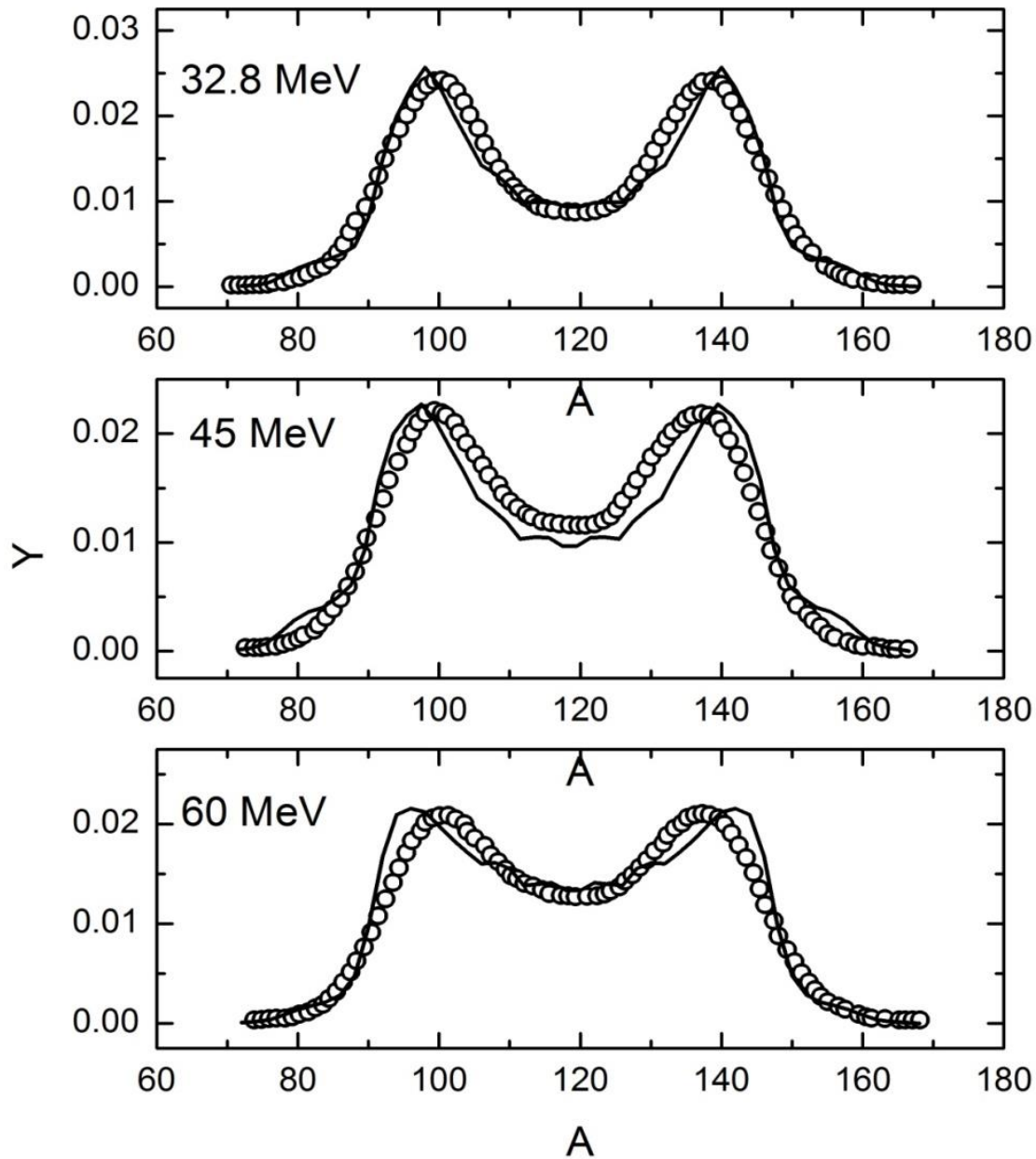


## General opinion:

*The competition between symmetric and asymmetric fission is related to shell effects in deformed fissioning nucleus*

*U.Brosa, S.Grossmann, A.Muller, Phys.Rep.197(1990)167*

**Exper. data for high-energy (60 MeV) neutron-induced fission of  $^{238}\text{U}$**  shows the conservation of asymmetric mass distribution, even though shell effects are supposed to be damped.



**n +  $^{238}\text{U}$**

Exper. asymmetric mass yields result in fission of highly excited ( $\sim 60$  MeV) nuclei  $^{232}\text{Th}$ ,  $^{237-240}\text{U}$ ,  $^{239-242}\text{Np}$ ,  $^{241-244}\text{Pu}$ , produced in transfer reactions  $^{18}\text{O}+^{232}\text{Th}$ ,  $^{238}\text{U}$ ,  $^{237}\text{Np}$  !

K.Hirose PRL **119** (17)222501; PLB **761** (16)125

$^{232}\text{Th}(n,f)$

V.Simukhin NDS **119** (14) 331; J.King EPJA **53** (17) 238

Conservation of asymmetric mass distribution, even though shell effects are supposed to be damped

# *Statistical Scission-point Model*

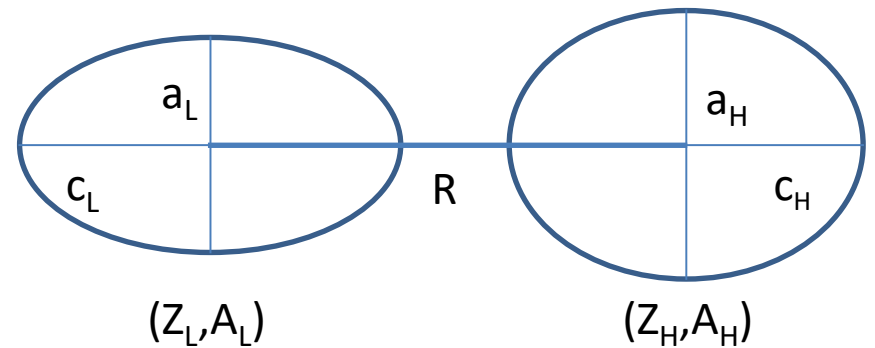
*or*

## *Cluster model of fission*

- Scission-point model relies on assumption that the statistical equilibrium is established at scission where the observable characteristics of fission are formed.
- Scission system - two well-defined fission fragments in contact [**dinuclear system=DNS**].

# Model

Coordinates  $Z_i, A_i, \beta_i$  ( $i = L, H$ ),  $R$  completely describe the geometry of system



## • Total Potential Energy :

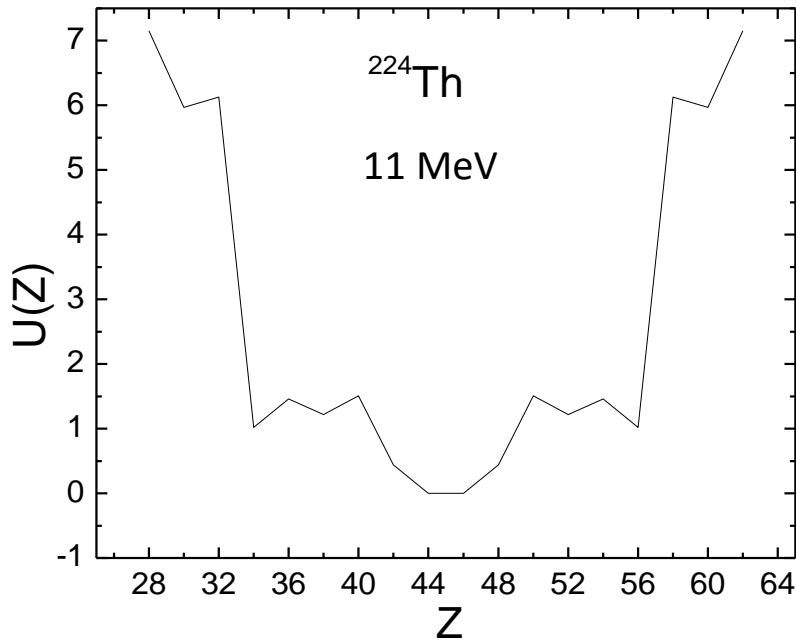
$$U(A_i, Z_i, \beta_i, R)$$

$$\begin{aligned} &= U_L^{\text{LD}}(A_L, Z_L, \beta_L) + \delta U_L^{\text{shell}}(A_L, Z_L, \beta_L, E_H^*) \\ &+ U_H^{\text{LD}}(A_H, Z_H, \beta_H) + \delta U_H^{\text{shell}}(A_H, Z_H, \beta_H, E_H^*) \\ &+ V^C(A_i, Z_i, \beta_i, R) + V^N(A_i, Z_i, \beta_i, R) \end{aligned}$$

$$V = V^C + V^N \quad - \text{interaction potential}$$

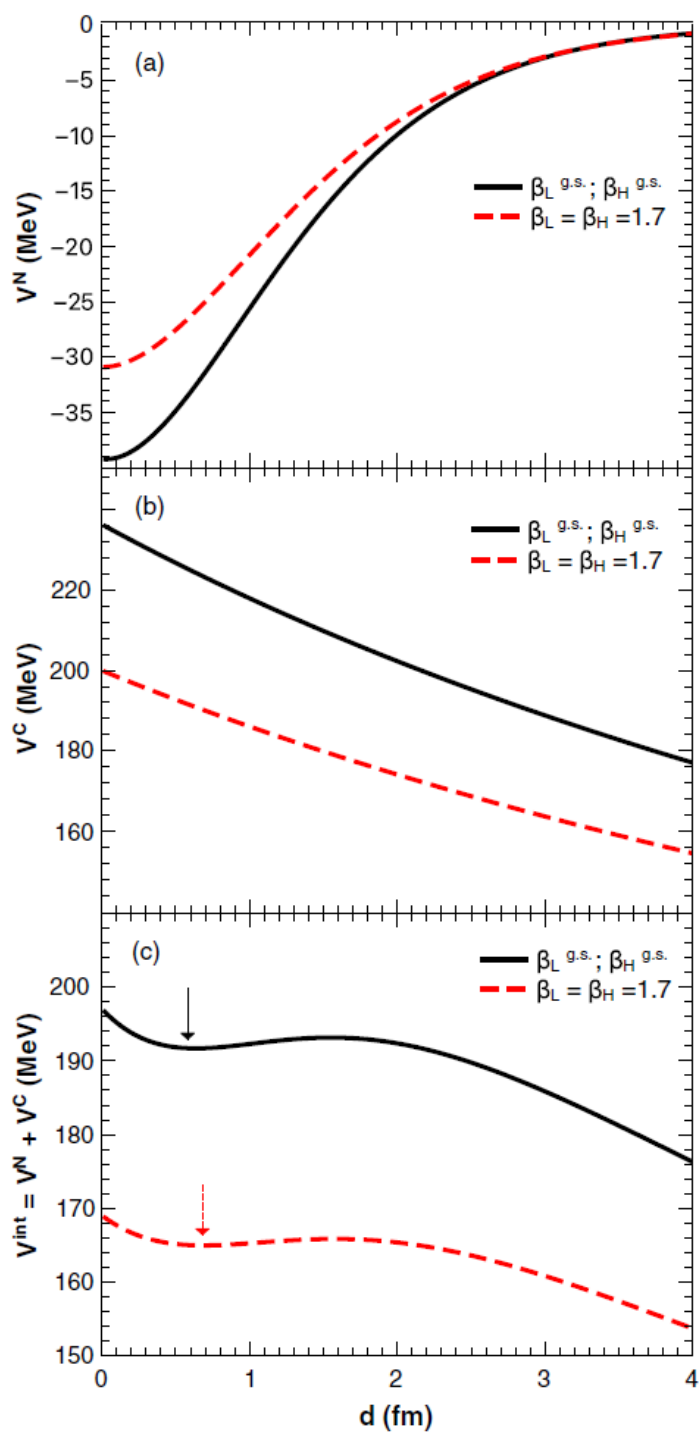






**Minima in potential are result of interplay between liquid-drop, interaction, shell correction energies**

- 1. Liquid-drop energy** globally increases when mass number deviate from symmetry.
- 2. Interaction energy** has the opposite behavior.

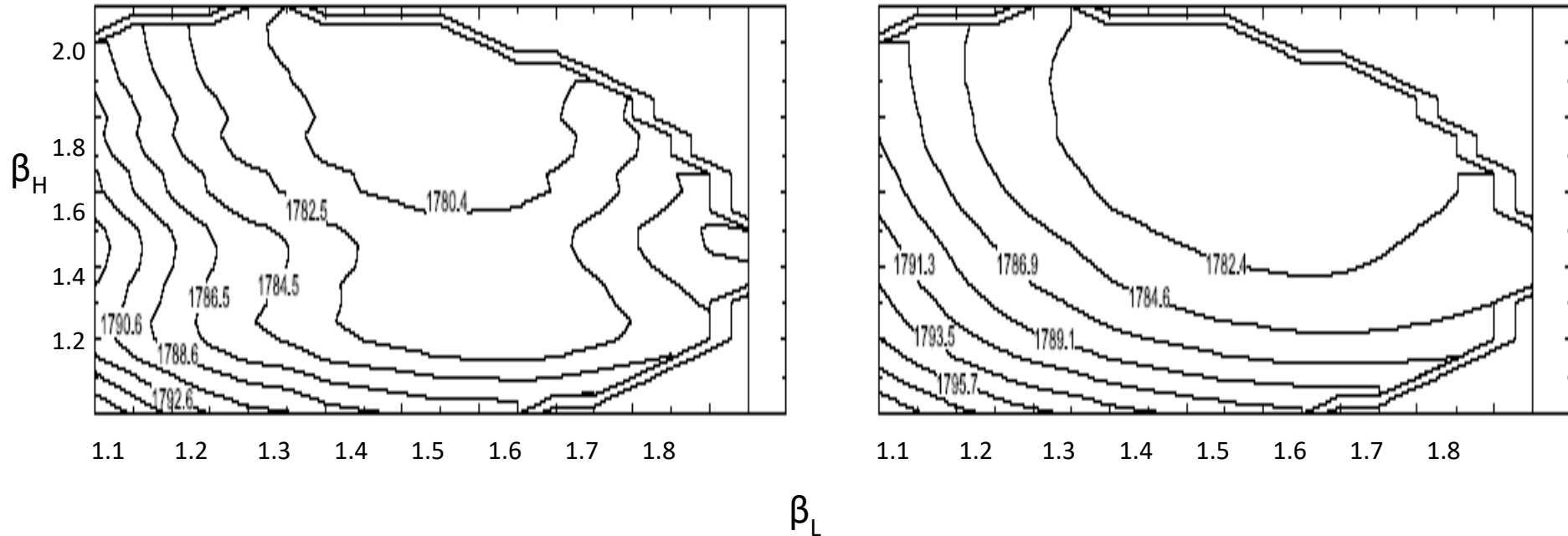


The calculated nuclear  $V^N$  and Coulomb  $V^C$  interaction potentials and their sum  $V^{int} = V^N + V^C$  as a function of the distance  $d$  between the tips of the fragments for the fragmentation  $^{222}\text{Th} \rightarrow ^{110}\text{Ru} + ^{112}\text{Pd}$ . The position of the interaction potential pocket minimum is indicated by the arrow. The deformations  $\beta_{L,H}$  of the DNS nuclei are indicated.

11 MeV

$^{88}\text{Kr}+^{136}\text{Xe}$

60 MeV



Minimum becomes wider, migrates to larger deformations with increasing excitation energy

Ratio of yields of fragments with different charge/mass is governed by difference in energy and width between their potential minima in PES ( $\beta_L, \beta_H$ ).

If two minima are close in energy, higher yield stems from wider-shallower minimum, lower yield emerges from abrupt-narrow minimum.

## Model

Yields:

$$w(A_i, Z_i, \beta_i, E^*) = N_0 \exp \left[ -\frac{U(A_i, Z_i, \beta_i, R_b)}{T} \right]$$

$$Y(A_i, Z_i, E^*) = \int d\beta_L d\beta_H w(A_i, Z_i, \beta_i, E^*)$$

$$Y(A_i, E^*) = \frac{\sum_{Z_i} Y(A_i, Z_i, E^*)}{\sum_{Z_i, A_i} Y(A_i, Z_i, E^*)},$$

$$Y(Z_i, E^*) = \frac{\sum_{A_i} Y(A_i, Z_i, E^*)}{\sum_{Z_i, A_i} Y(A_i, Z_i, E^*)}.$$

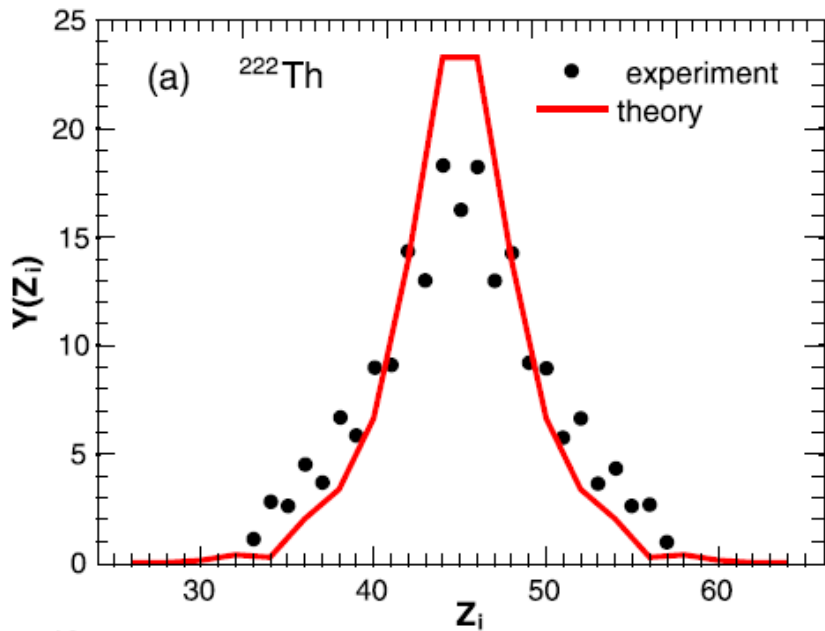
$$\text{TKE}(A_i, Z_i, \beta_i) = V^C(A_i, Z_i, \beta_i, R_b) + V^N(A_i, Z_i, \beta_i, R_b),$$

$$\langle \text{TKE} \rangle (Z_i) = \frac{\sum_{A_i} \int d\beta_L d\beta_H \text{TKE}(A_i, Z_i, \beta_i) w(A_i, Z_i, \beta_i, E^*)}{\sum_{A_i} \int d\beta_L d\beta_H w(A_i, Z_i, \beta_i, E^*)}.$$

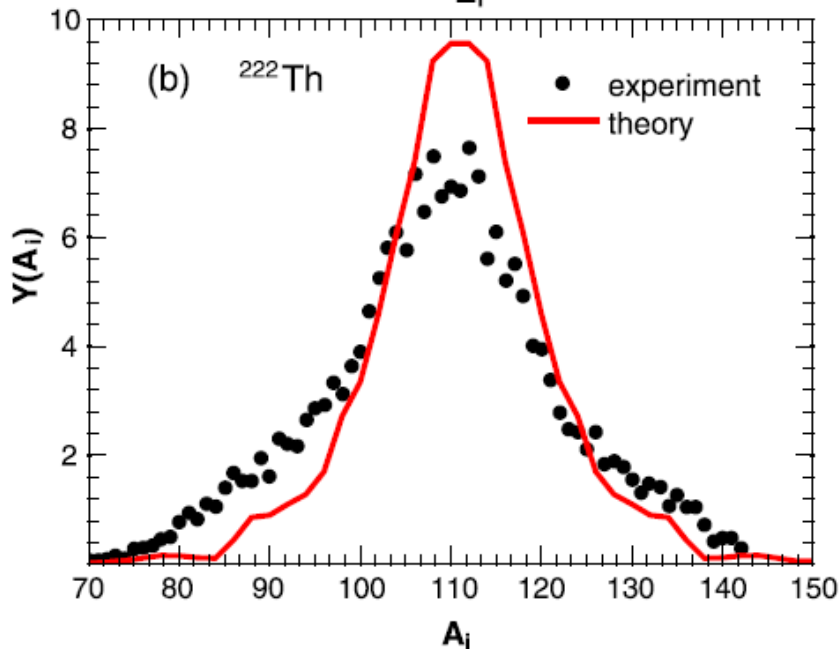
$$\langle n \rangle (Z_i) = \frac{\sum_{A_i} \int d\beta_L d\beta_H n(A_i, Z_i, \beta_i) w(A_i, Z_i, \beta_i, E^*)}{\sum_{A_i} \int d\beta_L d\beta_H w(A_i, Z_i, \beta_i, E^*)},$$

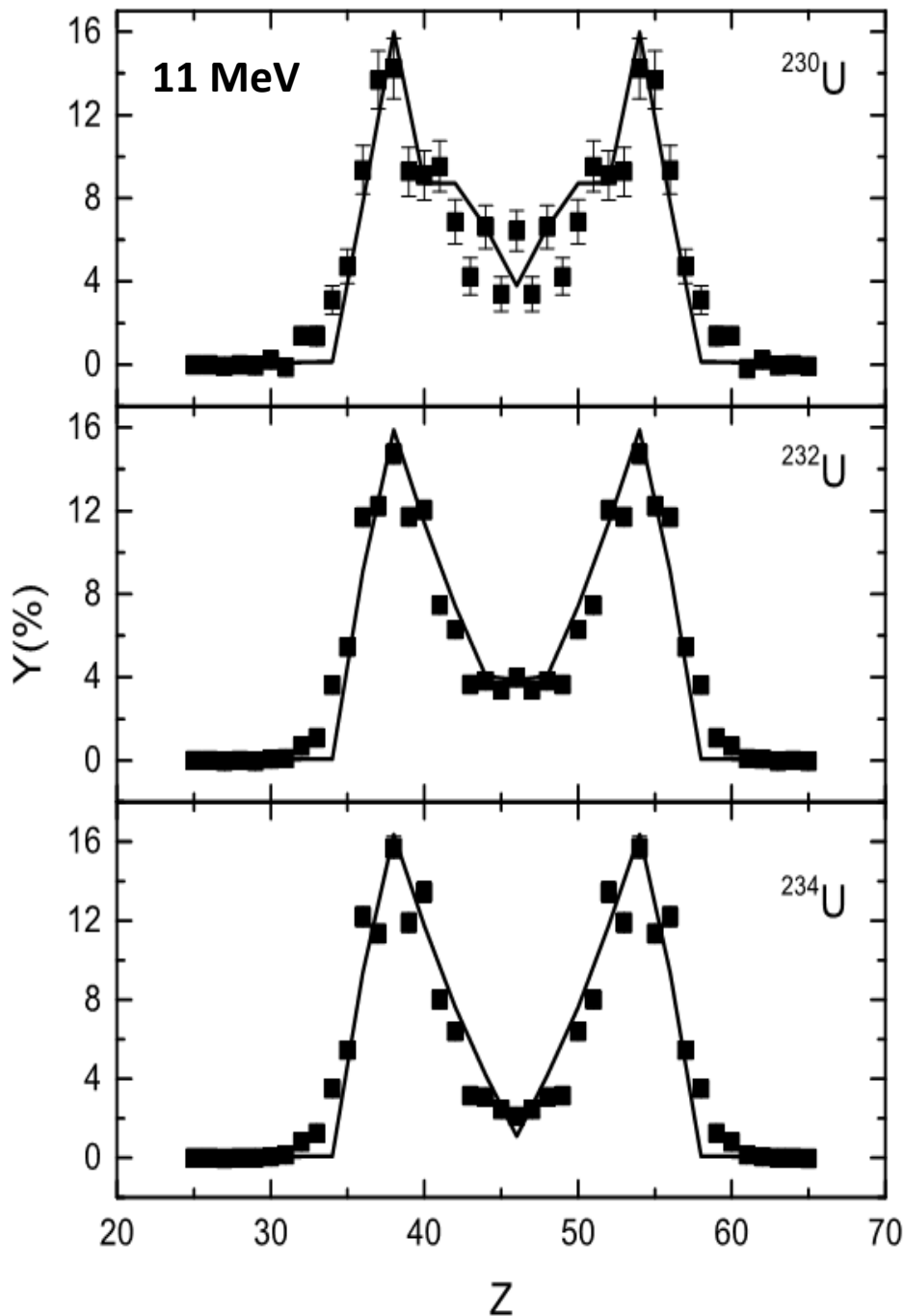
$$n(A_i, Z_i, \beta_i) = \sum_{i=L,H} \frac{E_i^*(A_i, Z_i, \beta_i) + E_i^{\text{def}}(A_i, Z_i, \beta_i)}{S_i^n + 2T_i}.$$

The values of  $S_i^n$  are the average separation energies of the first two neutrons.



The calculated (lines) and experimental (symbols) [NPA **665**, 221 (2000); **693**, 169 (2001); PRL **124**, 202502 (2020)] charge (a) and mass (b) distributions of fission fragments for electromagnetic induced fission of  $^{222}\text{Th}$  at 11 MeV excitation energy. The lines connect the calculated points for even-even fission fragments.





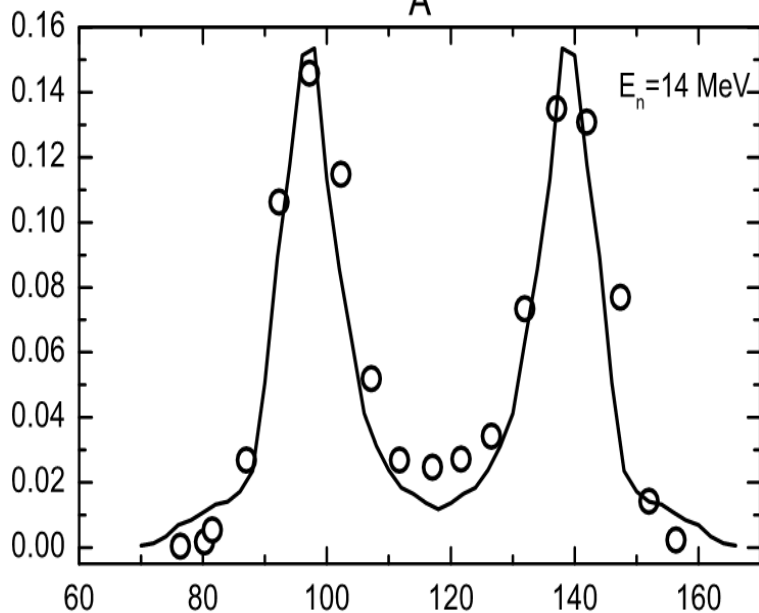
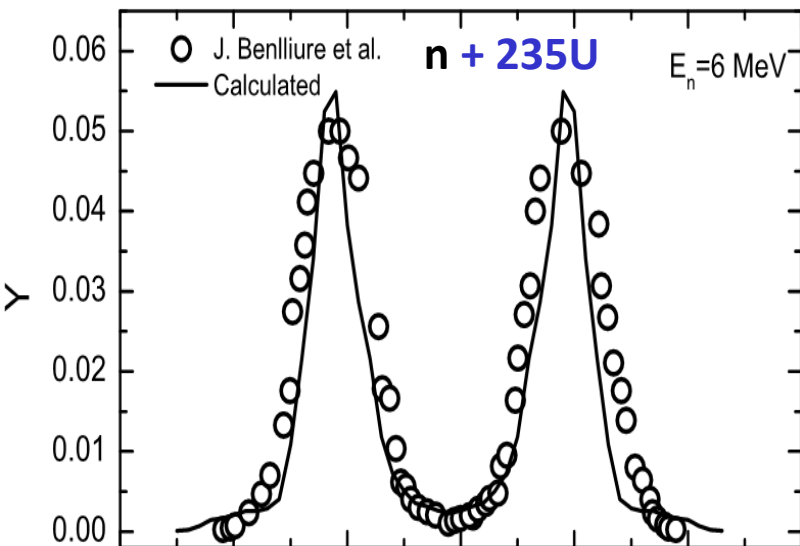
Calculated charge distributions for electromagnetic-induced fission of  $^{230-234}\text{U}$  at exc. energy about 11 MeV

*Exper. : K.-H.Schmidt et al.,  
NPA 665(2000)221.*

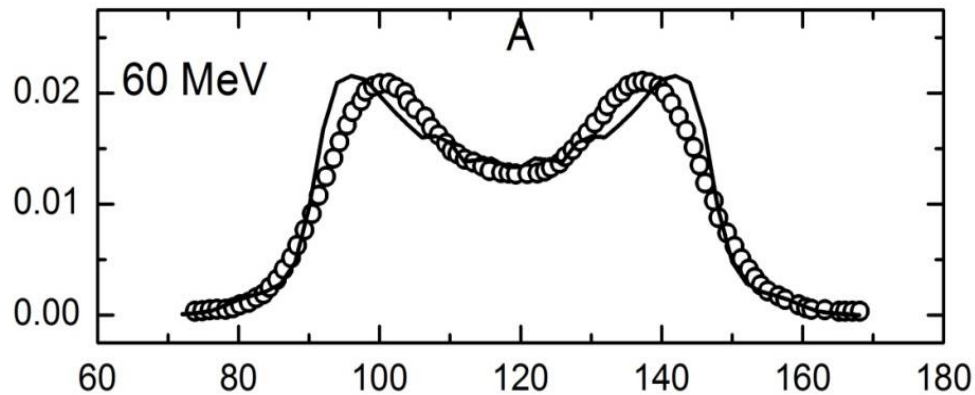
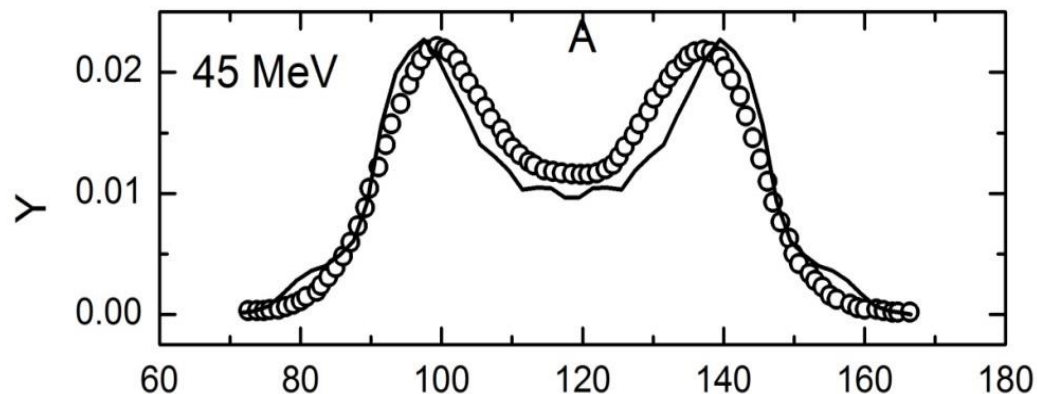
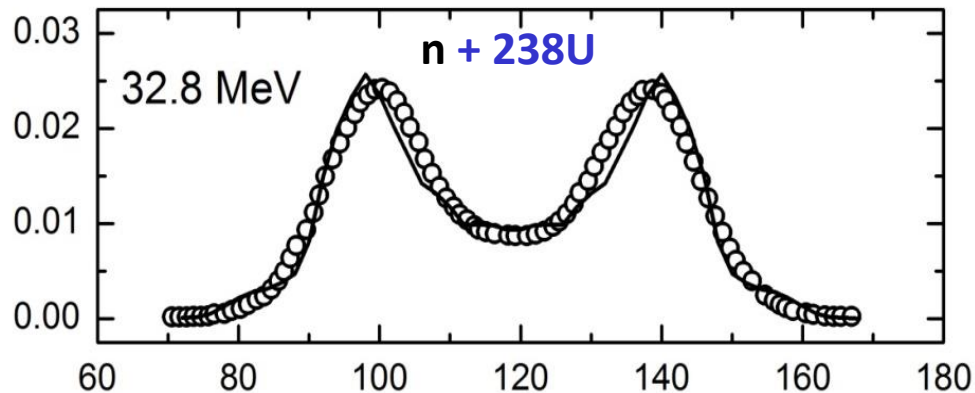
*Model is well suited for describing both asymmetric and symmetric fission distributions as well as transition between two.*



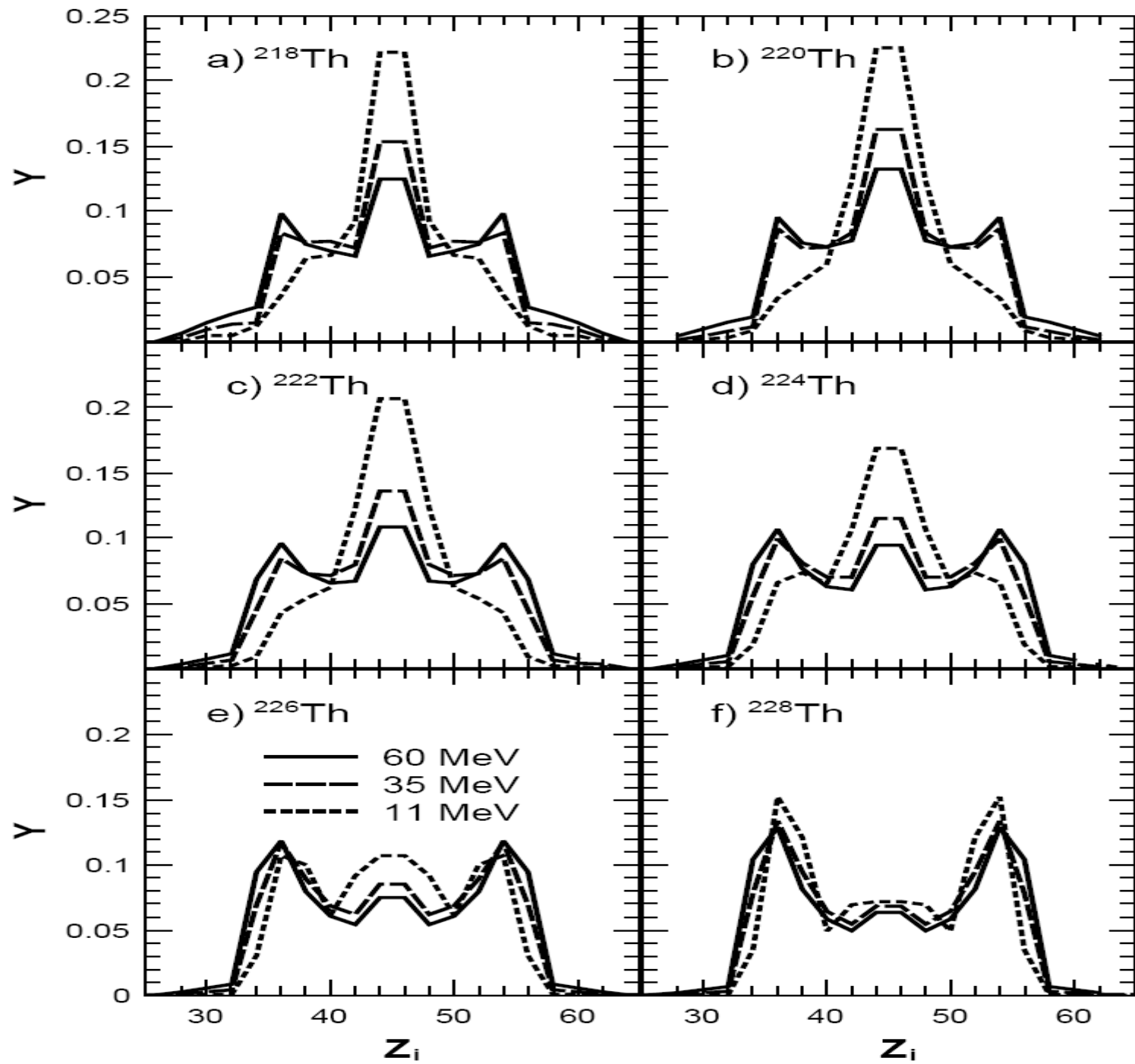
# High excitation energy of fissioning nucleus

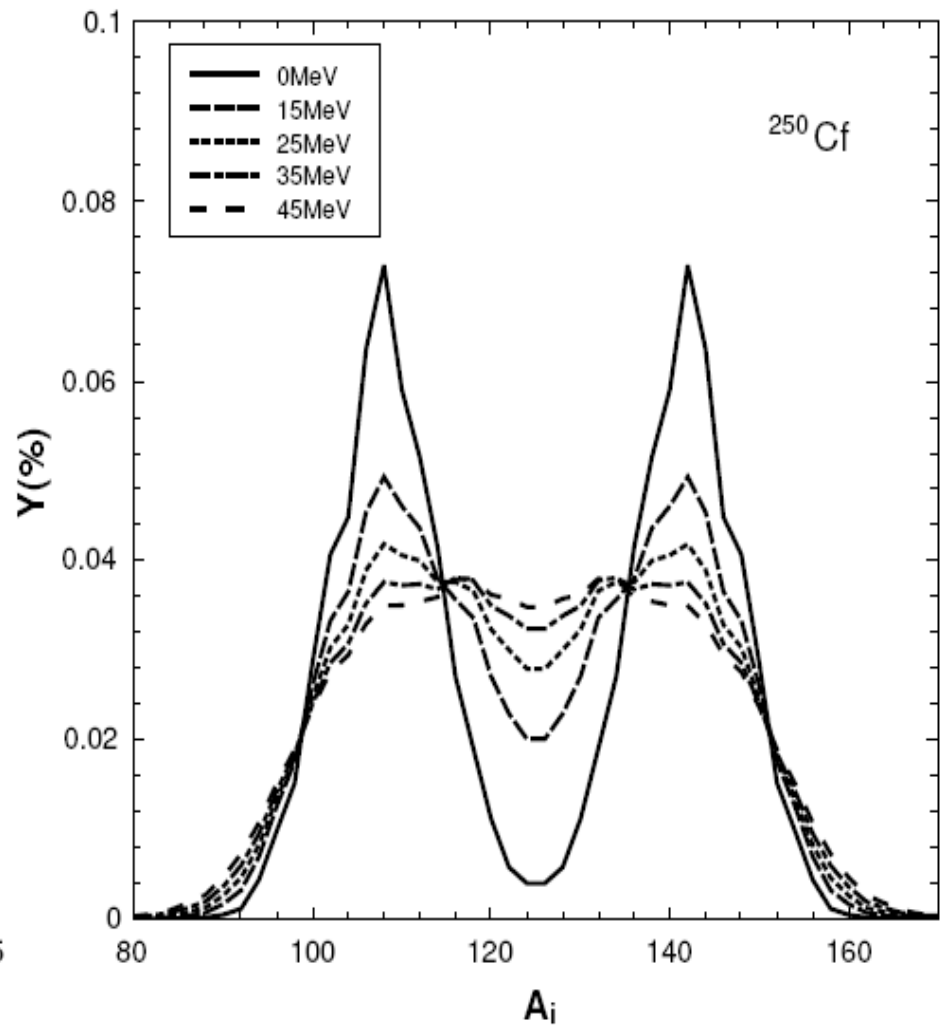
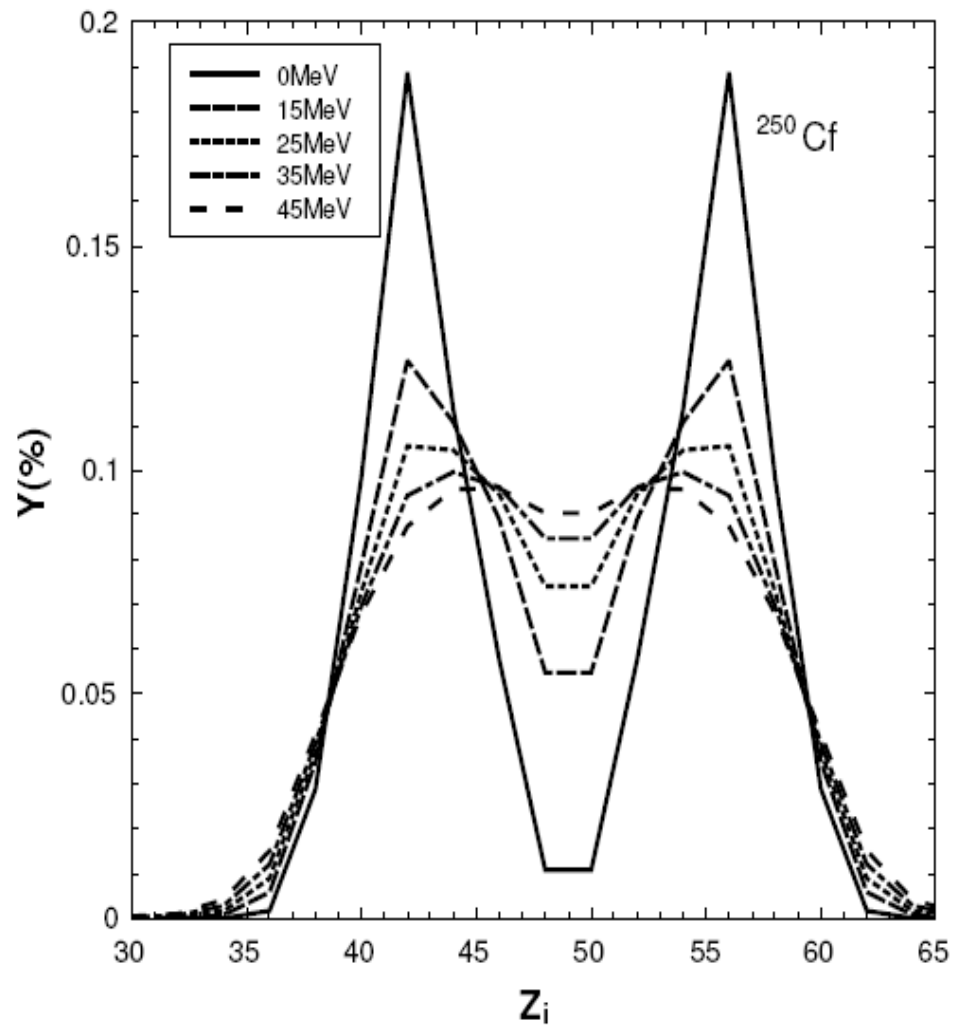


A

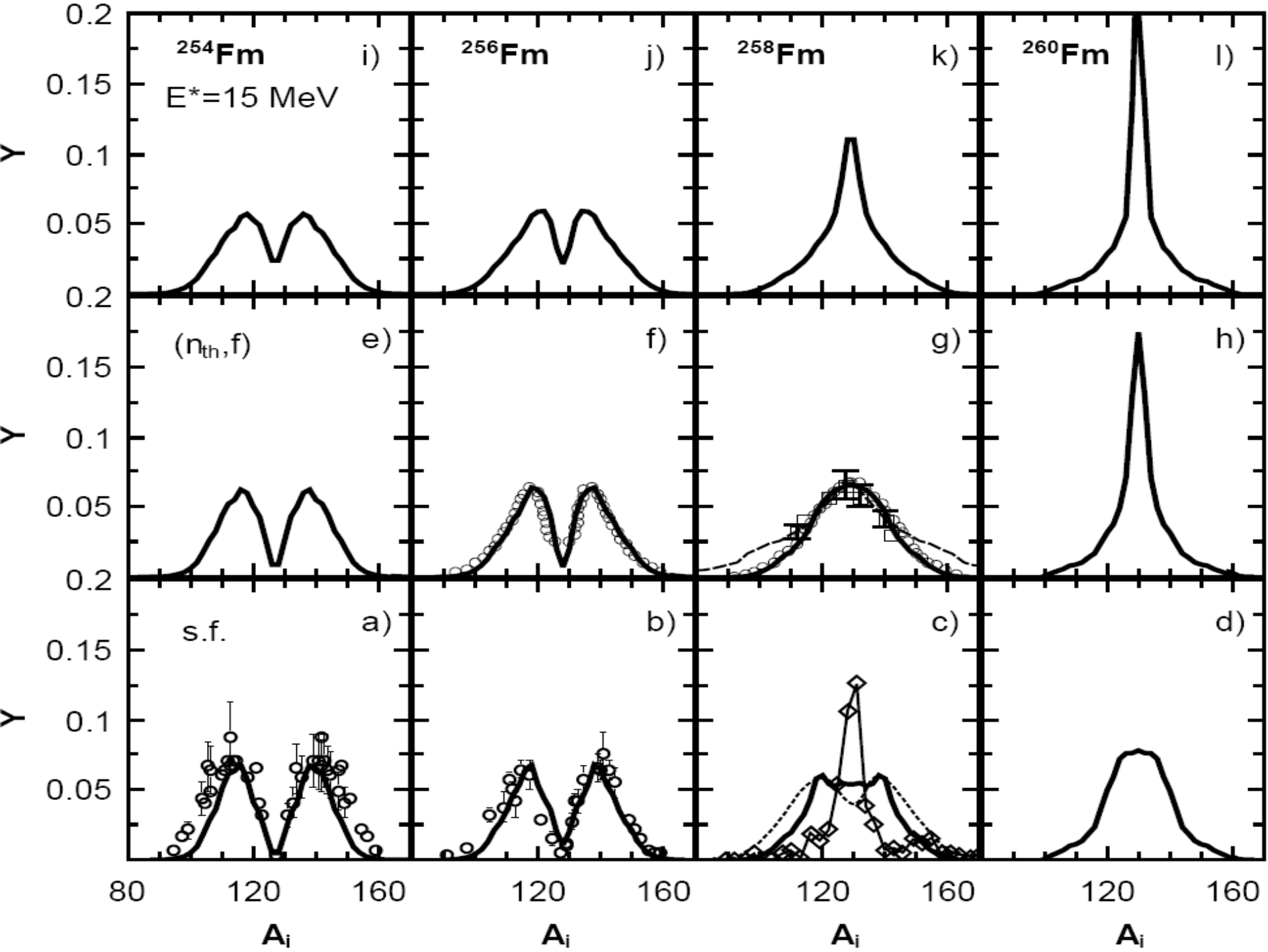


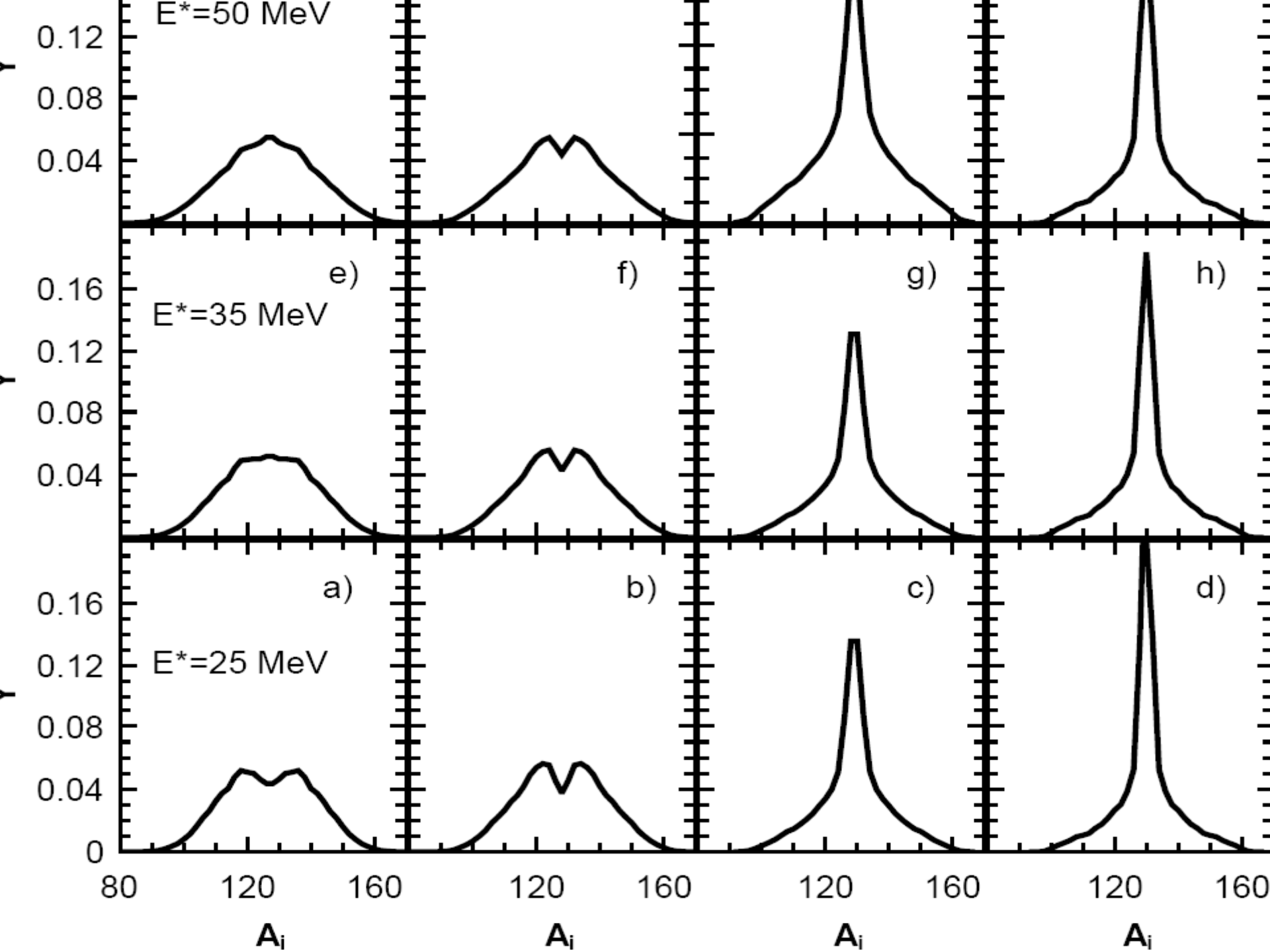
A

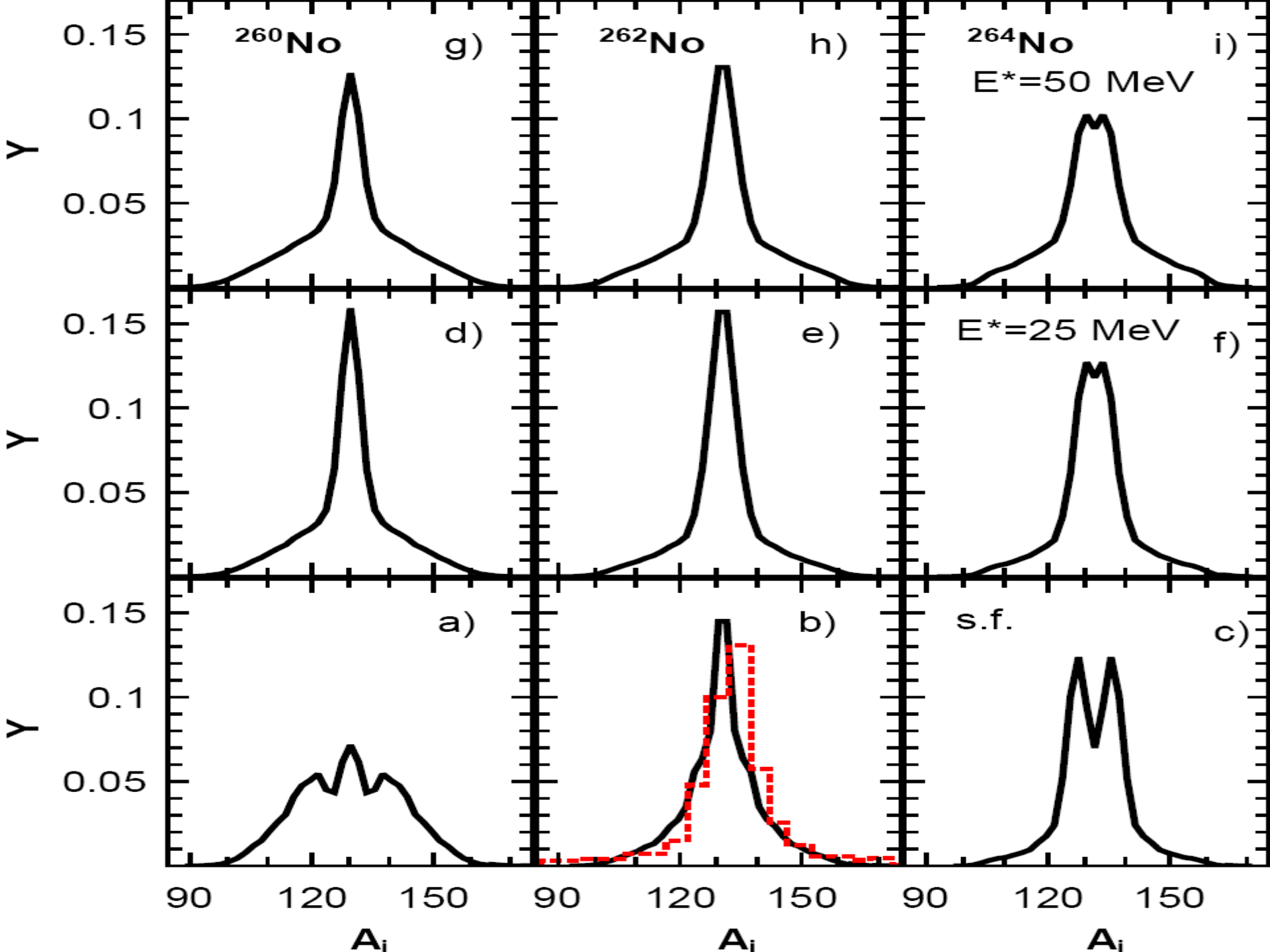


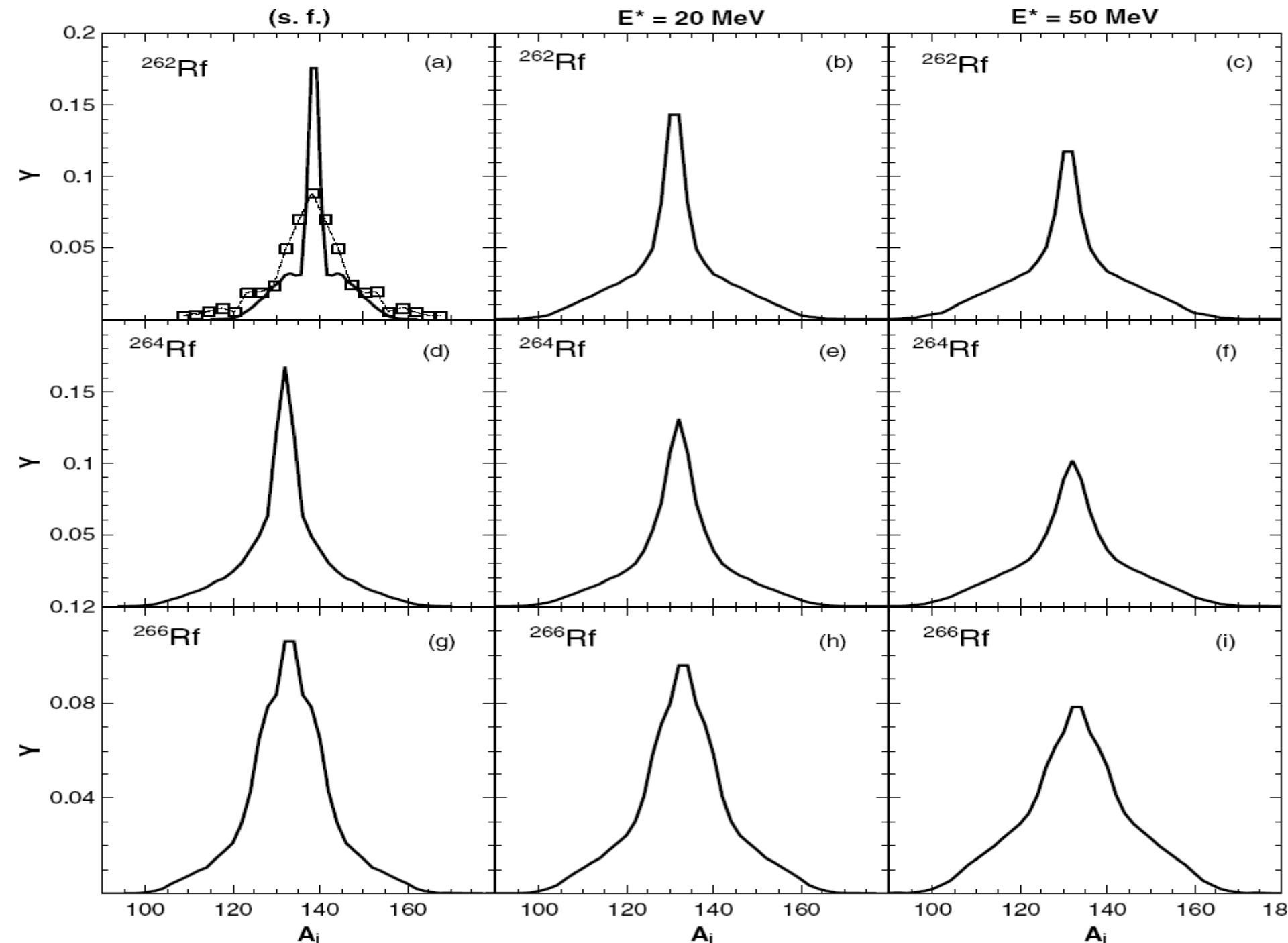


# Fission of heavy actinides

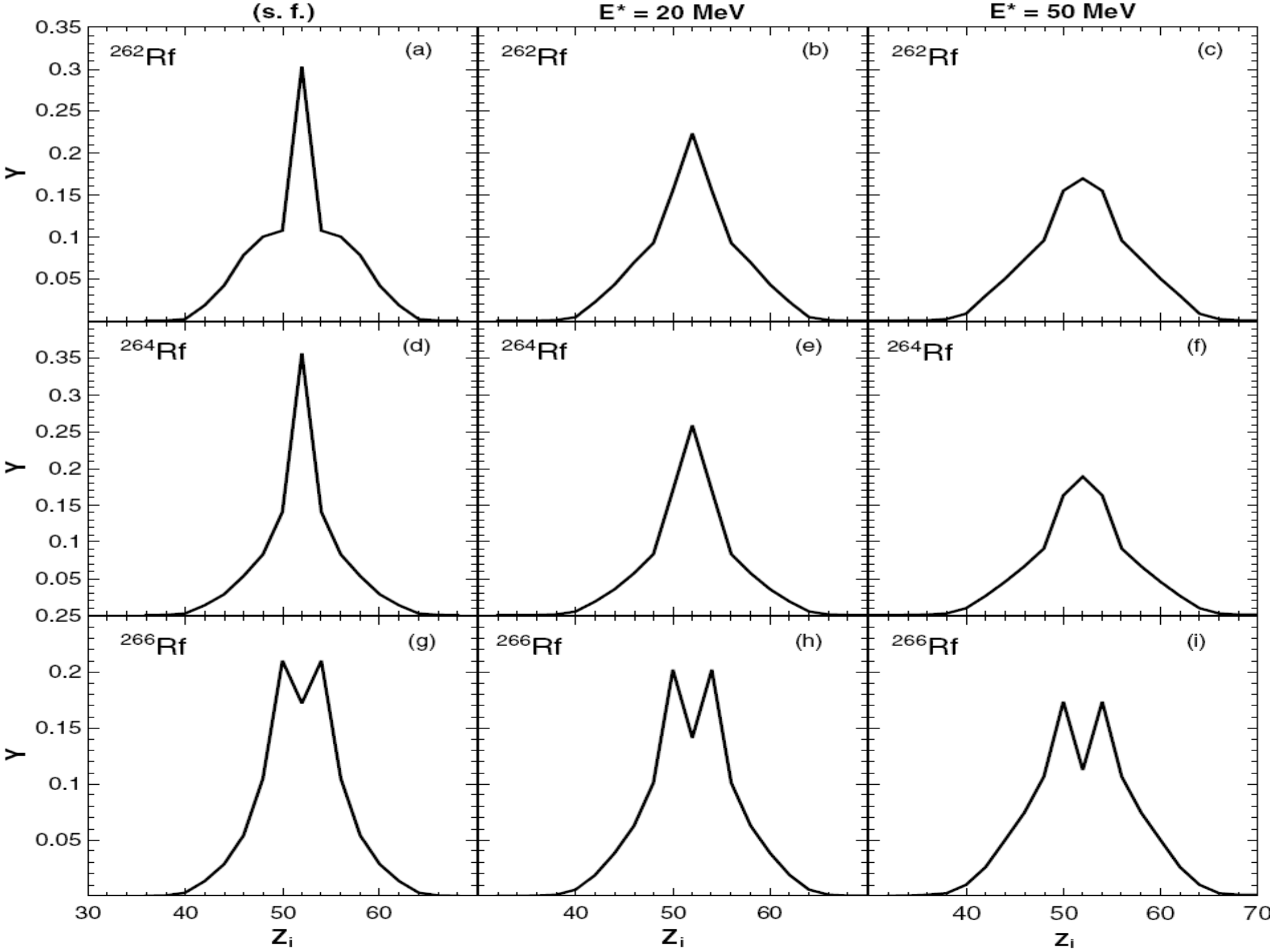












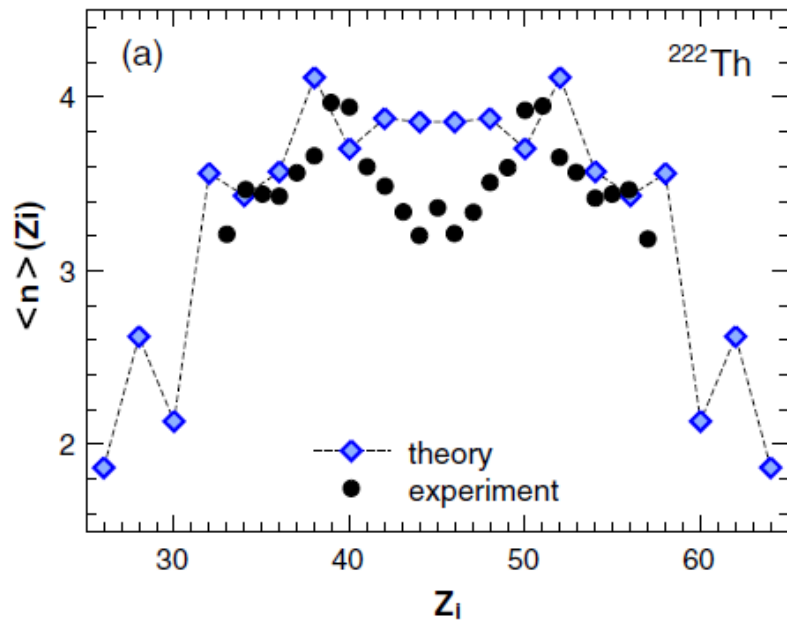
**Experimental verifications**

**of this unexpected difference between mass and charge distributions are desirable**

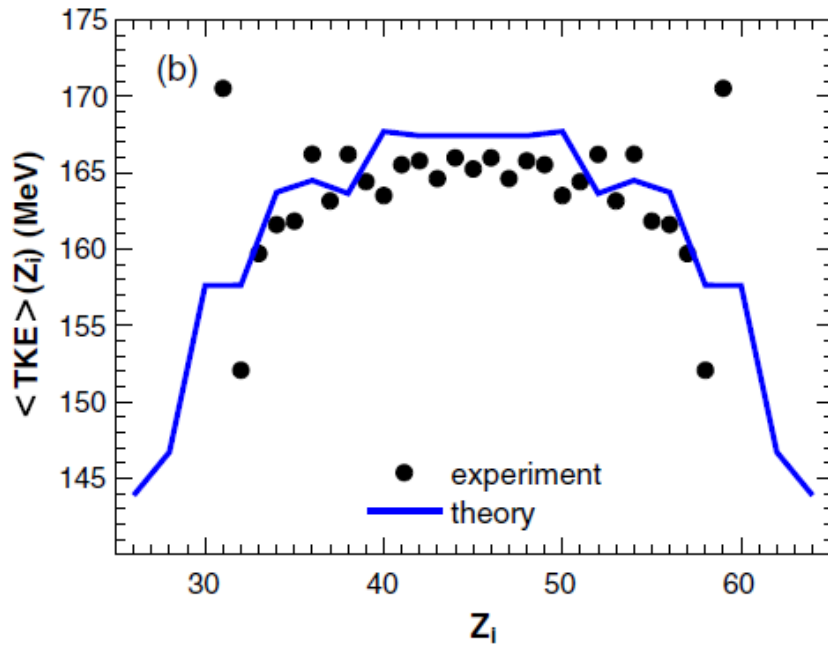
**The change of charge/mass-yields with increasing isospin or excitation energy is related to the change of PES at scission point**

# Potential energy at scission is main ingredient

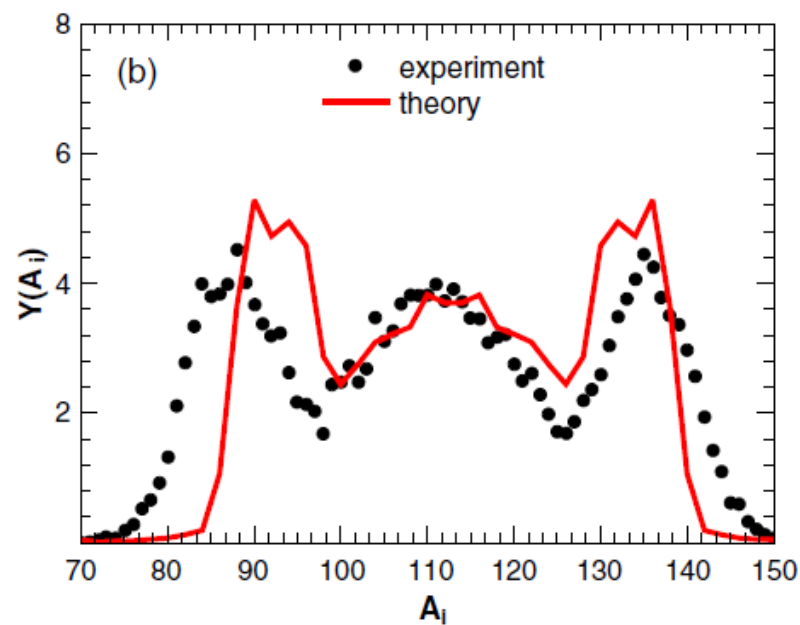
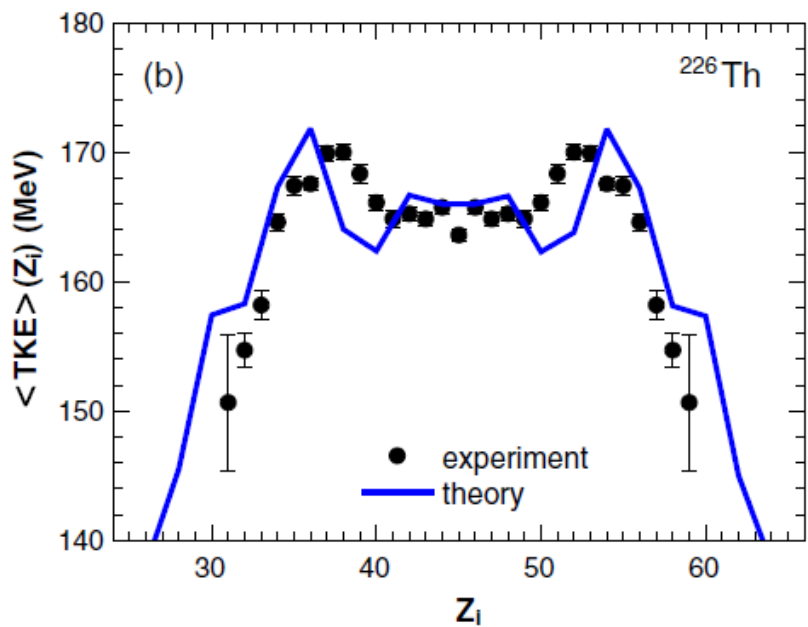
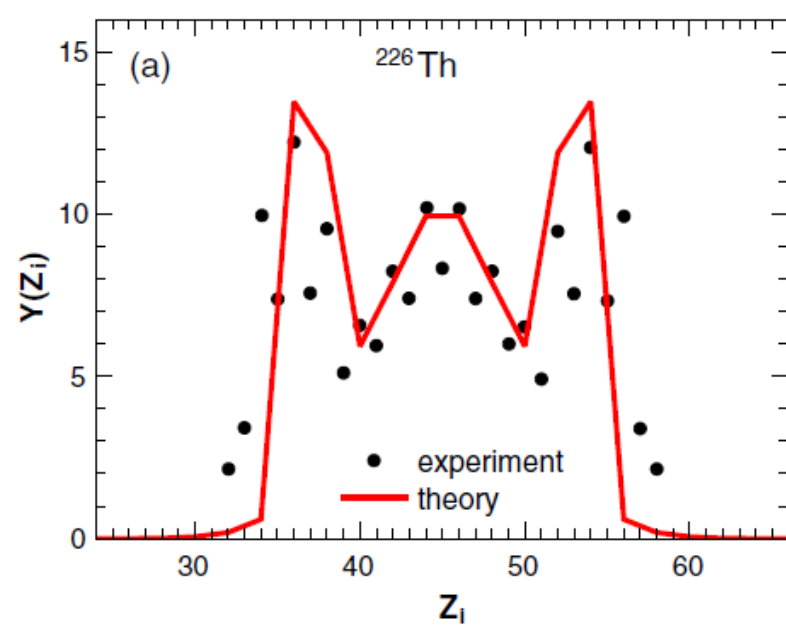
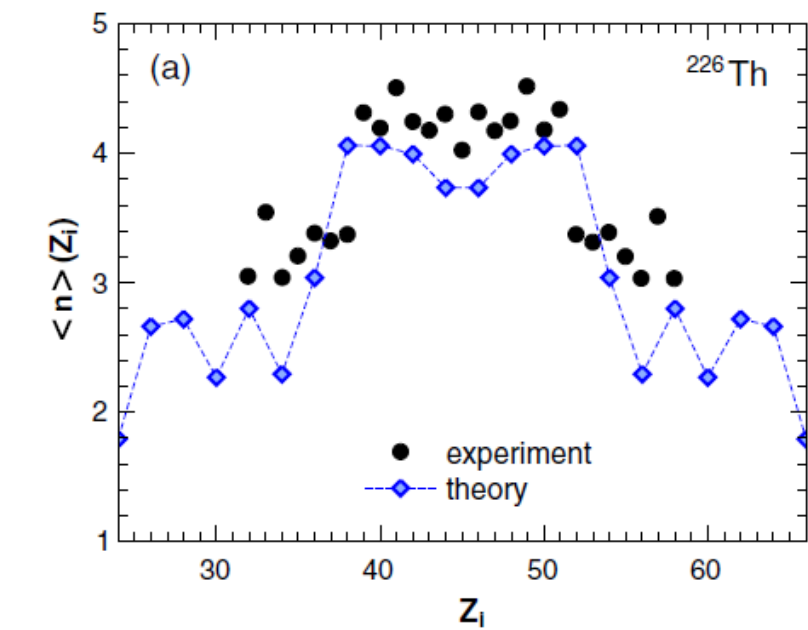
- 1) **Liquid-drop energy** globally increases when mass number deviate from symmetry.
- 2) **Interaction energy** has the opposite behavior.
- 3) **Both** depend on deformations of nuclei: larger deformations result in smaller interaction energy, larger liquid-drop energy.
- 4) **Deformations** depend on shell effect: **magic** nuclei are expressed in small deformations.
- 5) **Shell correction energy**



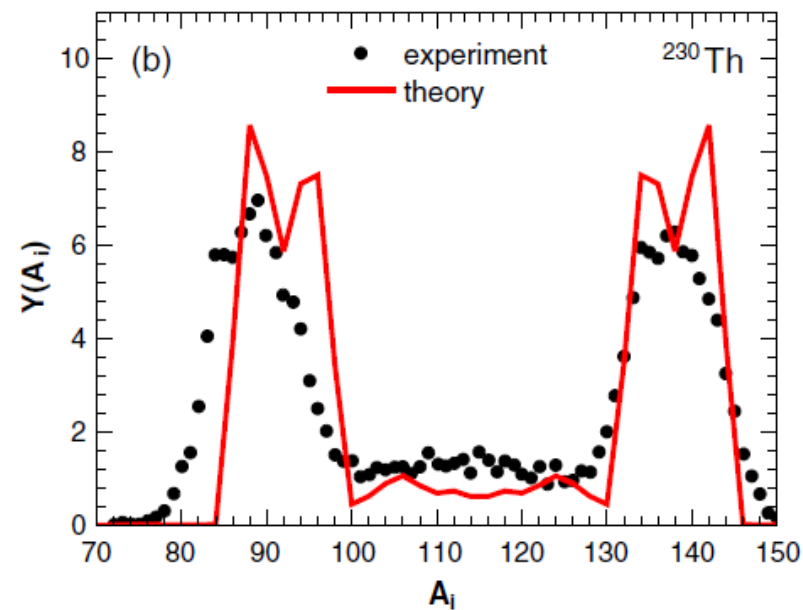
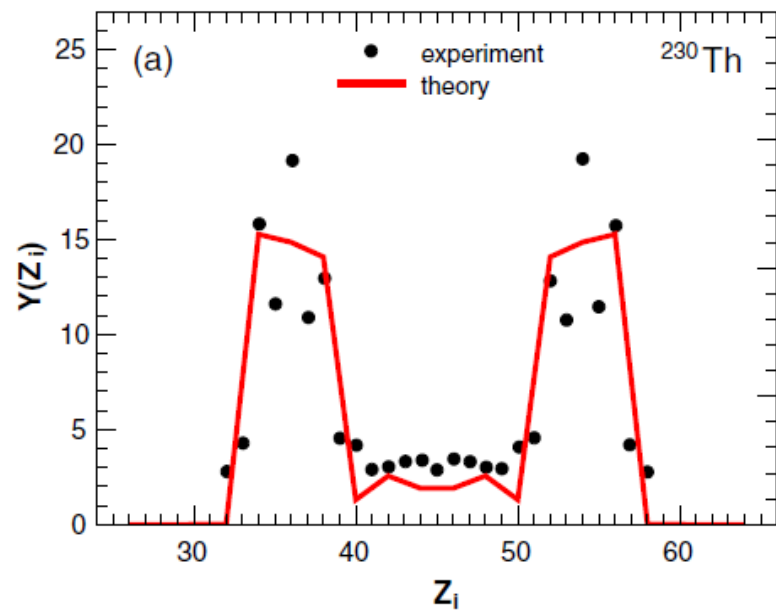
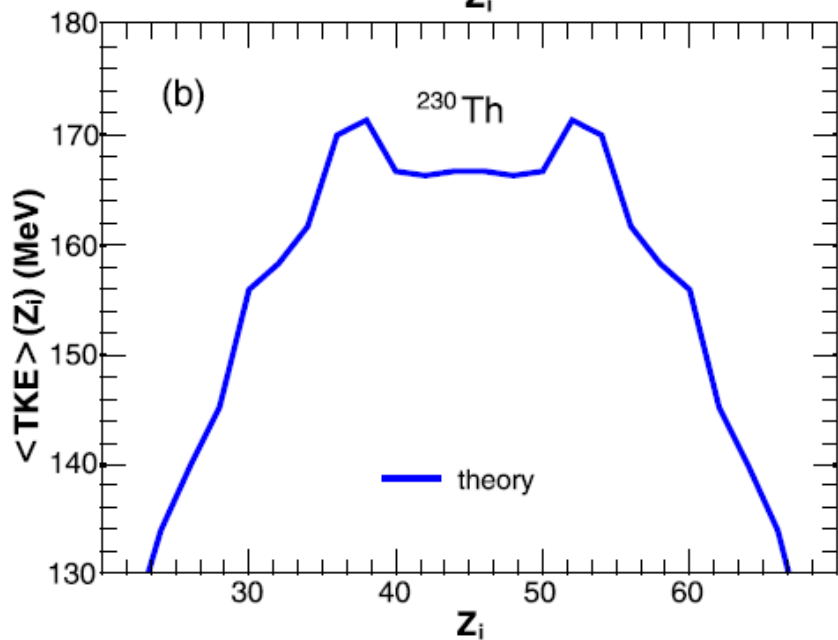
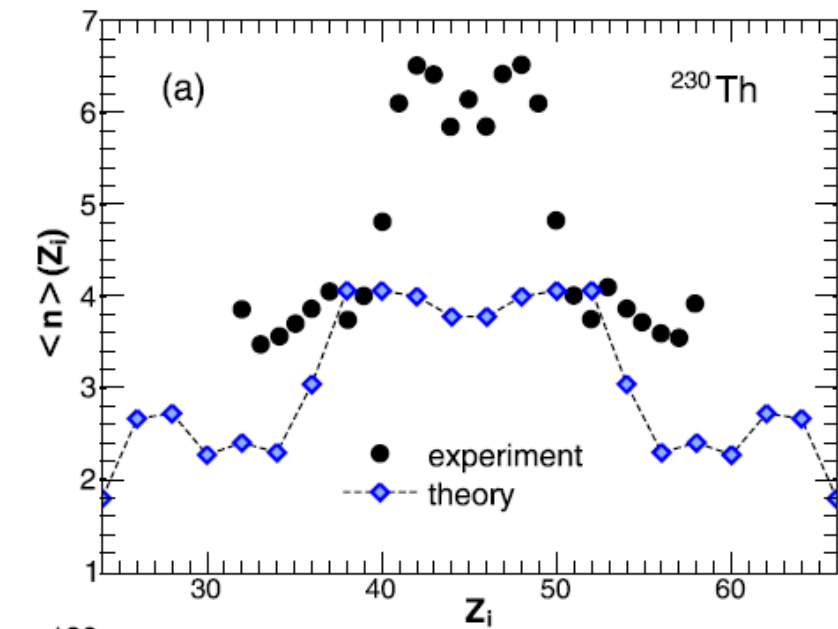
The calculated (lines) and experimental (closed symbols) neutron multiplicity (a) and TKE (b) distributions for electromagnetic induced fission of  $^{222}\text{Th}$  at 11 MeV excitation energy.



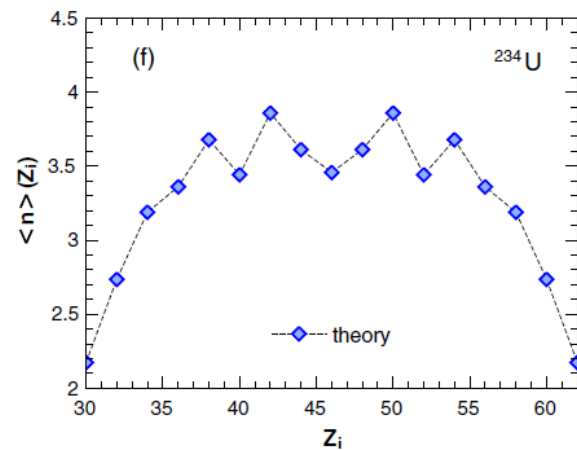
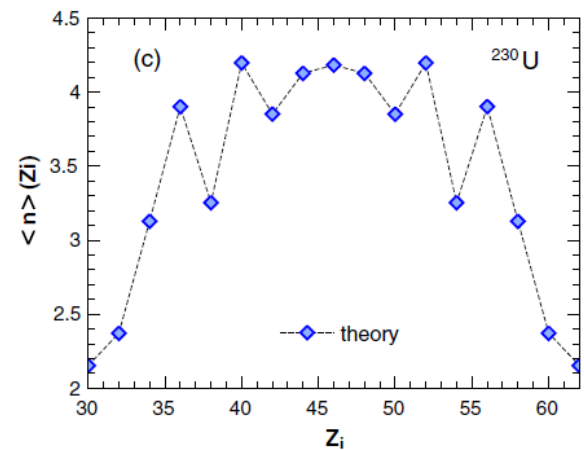
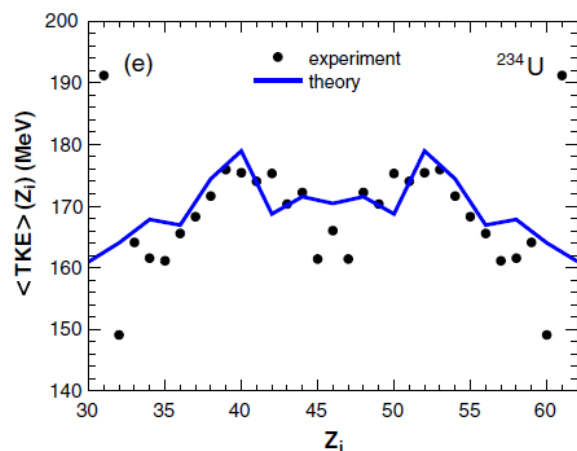
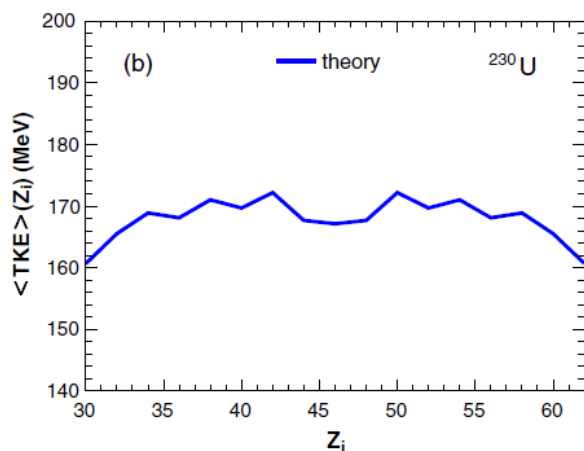
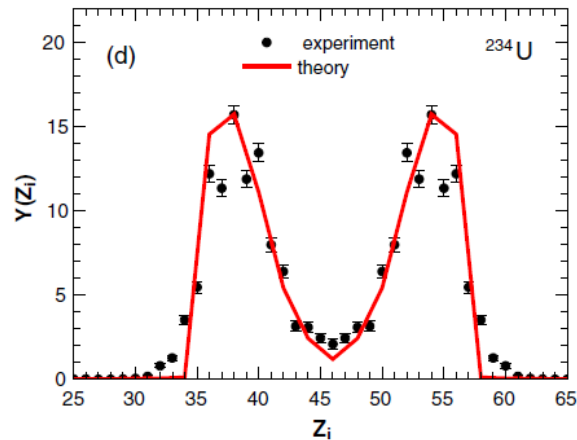
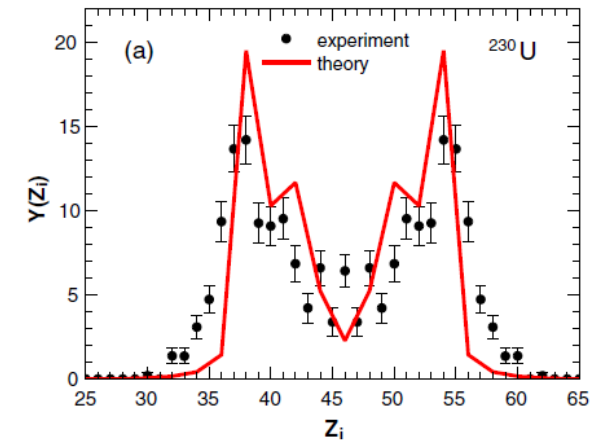
$$Z_1 \times Z_2$$



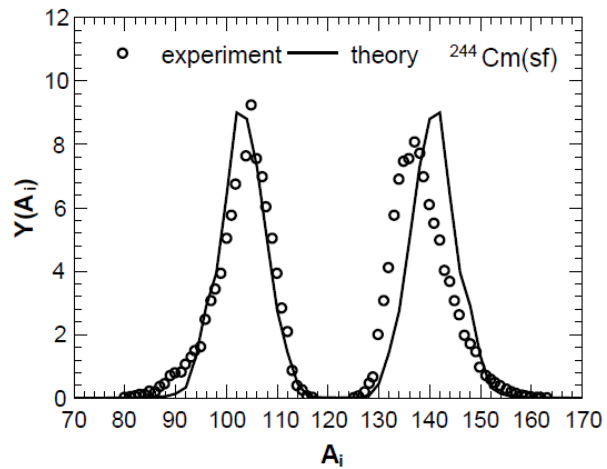
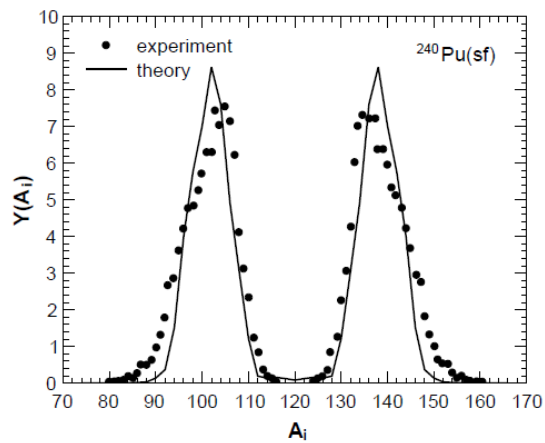
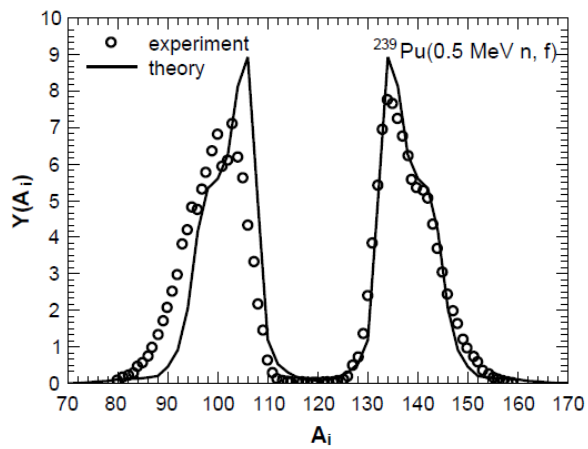
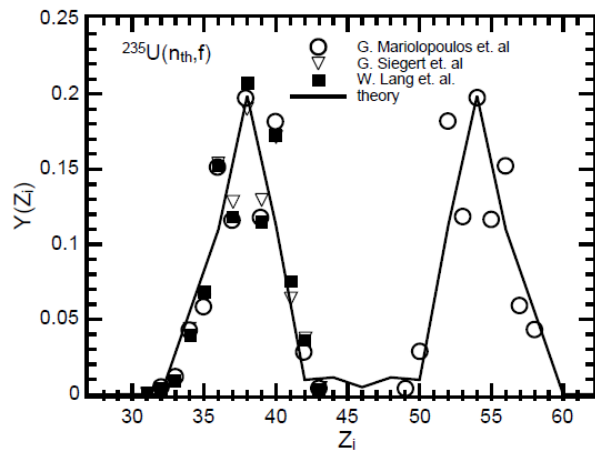
Electromagnetic induced fission of  $^{226}\text{Th}$  at 11 MeV excitation energy.



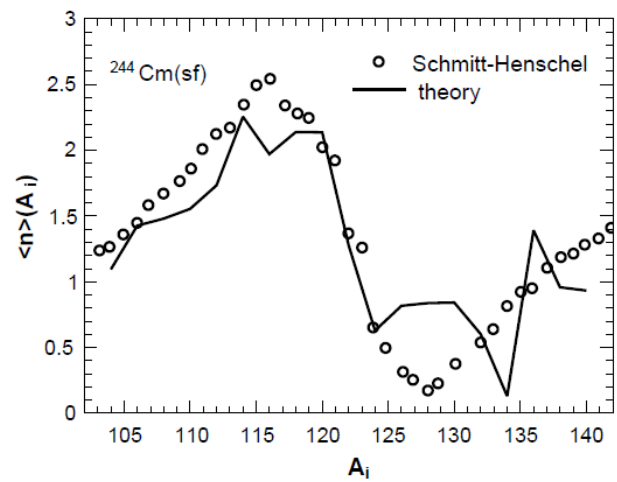
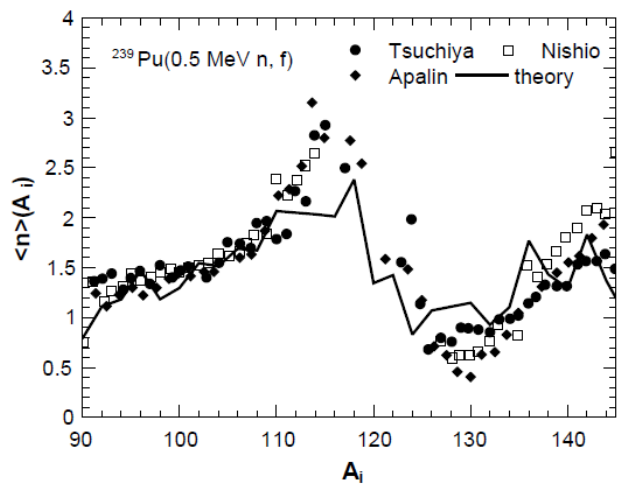
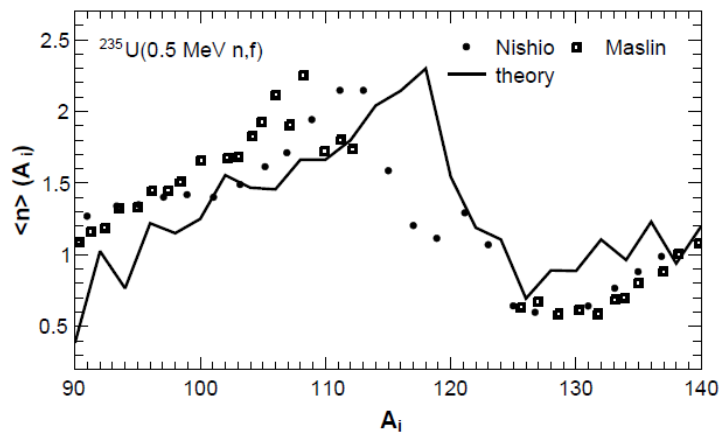
Electromagnetic induced fission of  $^{230}\text{Th}$  at 11 MeV excitation energy.



The calculated (lines) and experimental (closed symbols) [NPA 665, 221 (2000); 693, 169 (2001); 368, 319 (1981)] charge [(a), (d)], TKE [(b), (e)], and neutron multiplicity [(c), (f)] distributions of fission fragments for electro-magnetic induced fission of  $^{230,234}\text{U}$  at 11 MeV excitation energy. The lines connect the calculated points for even-even fission fragments.







The probability of evaporating exactly  $x$  neutrons from the excited fragment " $i$ " with excitation energy  $E_i^*$

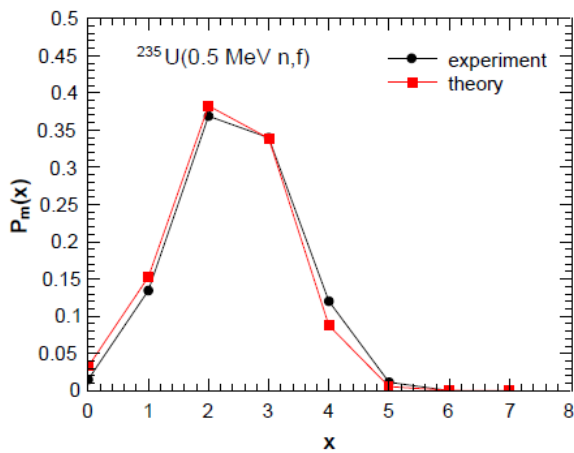
$$P_{xn}(E_i^*) = P(x) - P(x + 1),$$

$$P(x) = 1 - e^{-\Delta_x} \left( 1 + \sum_{k=1}^{2x-3} \frac{(\Delta_x)^k}{k!} \right),$$

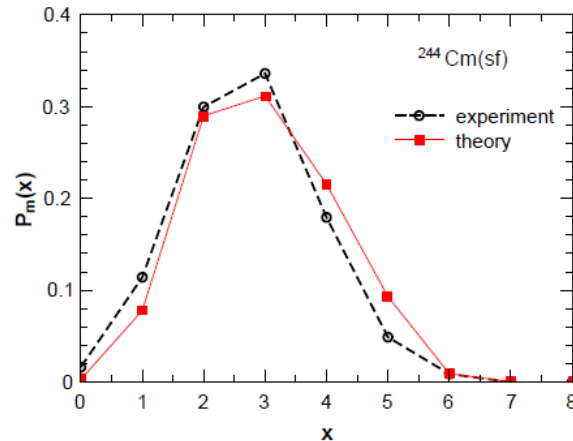
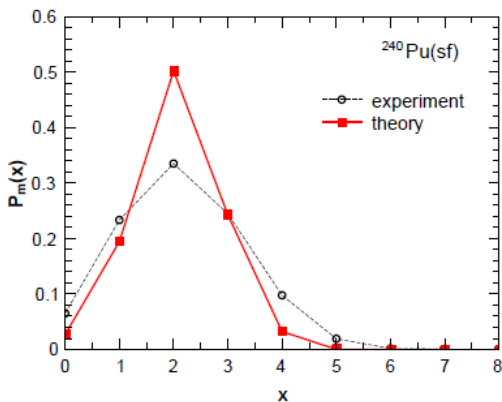
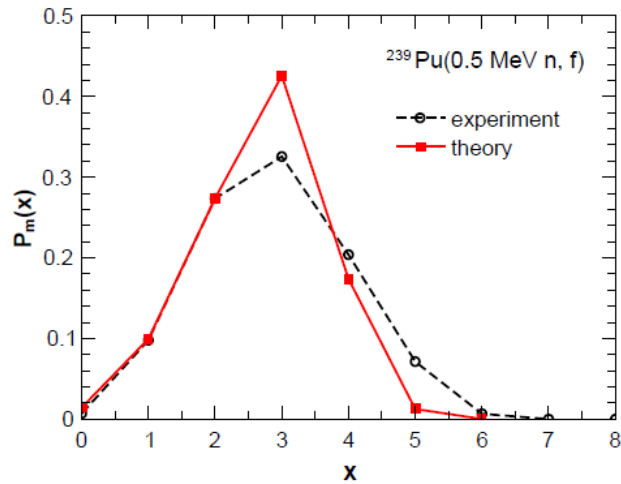
$$P(x + 1) = 1 - e^{-\Delta_{x+1}} \left( 1 + \sum_{k=1}^{2x-1} \frac{(\Delta_{x+1})^k}{k!} \right),$$

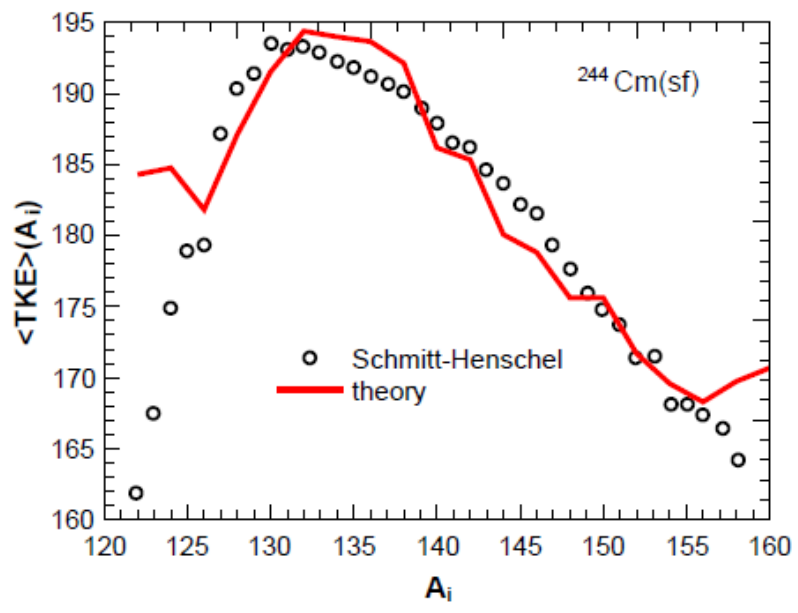
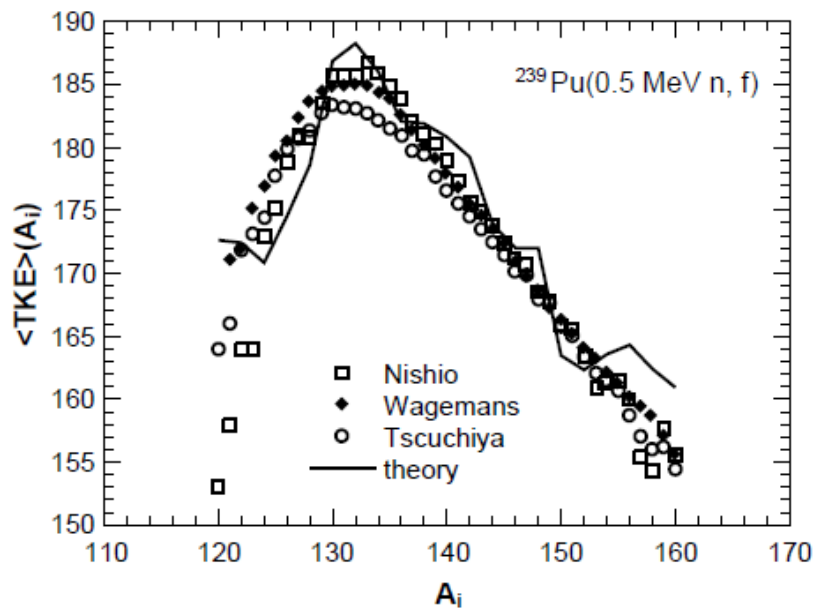
$\Delta_x = (E_i^* - \sum_{k=1}^x B_k) / T_i$ , where  $B_k$  is the experimental

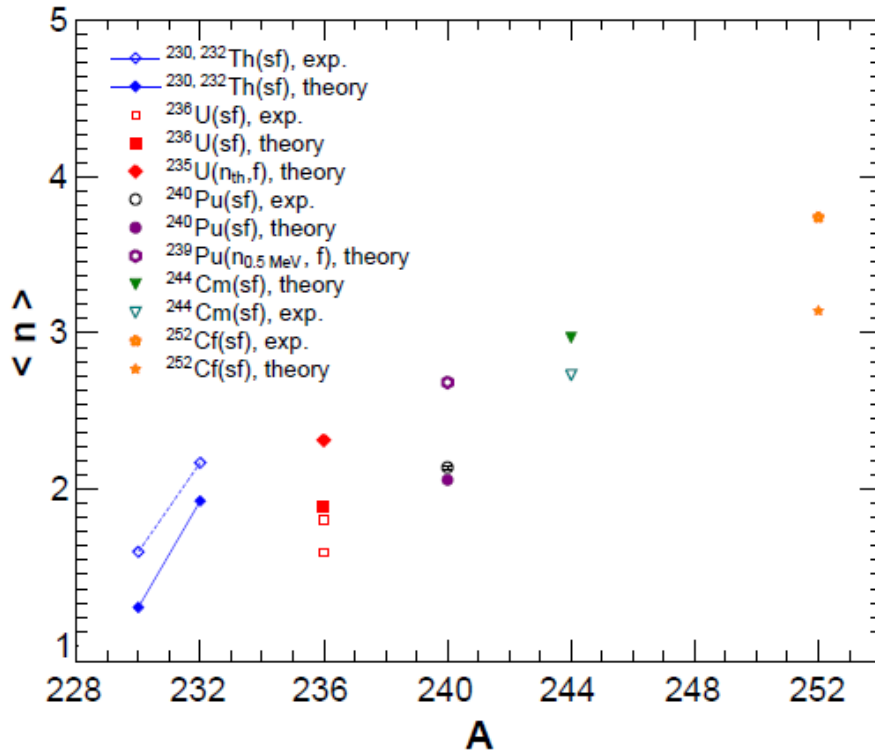
neutron binding energy at the  $k$ -th evaporation step and  $T_i = (E_i^* / a_i)^{1/2}$  is the temperature.



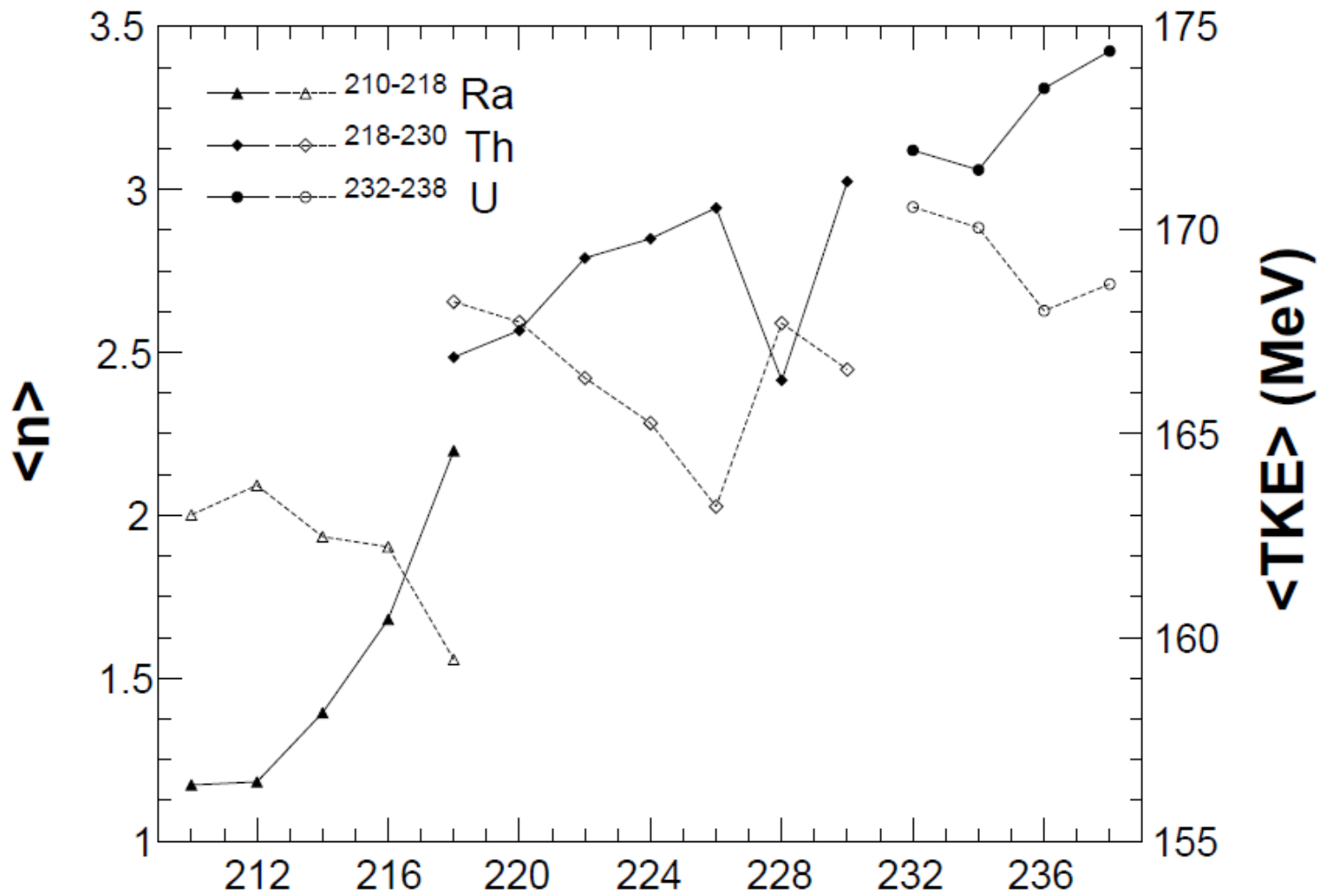
The calculated (squares connected by lines) and experimental (open circles connected by lines) [PR **101**, 1012 (1956); BNL-NCS-35513 (1985); Nucl. Sci. Eng. **86**, 315 (1984)] probabilities  $P_m(x)$  as functions of multiplicity  $x$  for the indicated fission reactions.







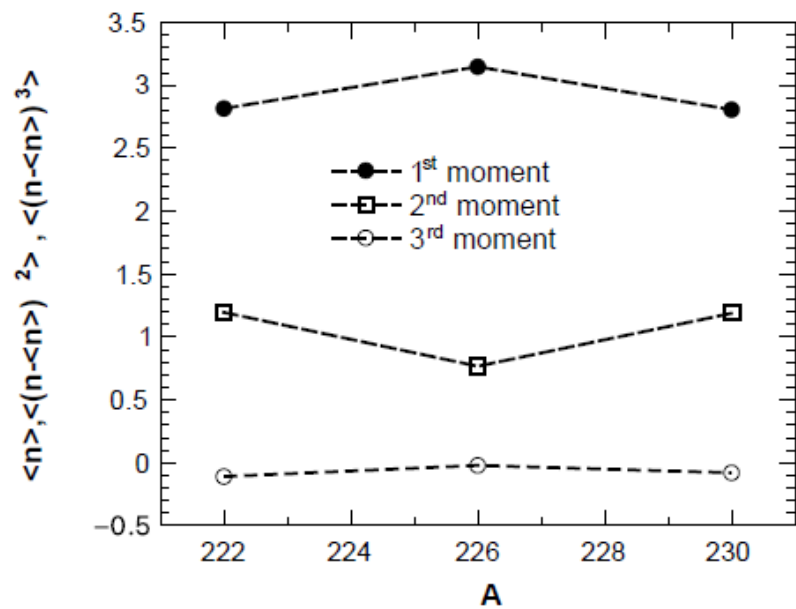
The calculated (open symbols) and experimental (closed symbols) average numbers of neutrons  $\langle n \rangle$  emitted per act of fission as functions of the mass number  $A$  of indicated fissioning nucleus.



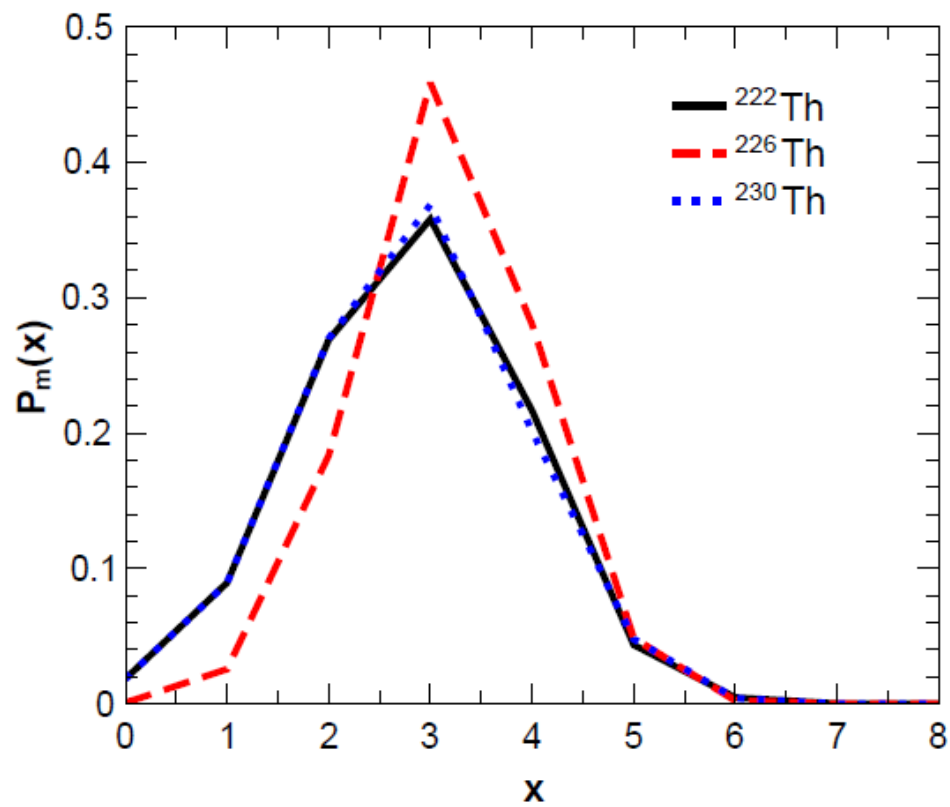
open symbols

**A**

closed symbols



$^{222,226,230}\text{Th}$  ( $E^* = 11$  MeV)



**Minima in potential (maxima in yields) are direct result of interplay between liquid-drop, interaction, shell correction energies.**

- **Shell effects** affect **indirectly** (through deformations) appearance of minima of PES, facilitation of large number of magic fragments.

**As  $E^*$  increases**, shell & stiffness diminish, **shifting minima** on PES towards larger deformations and **widening minima**.

**Direct role of shell effects** is expressed by their ability to enhance or suppress formation of minima of PES.



# Saturation effect

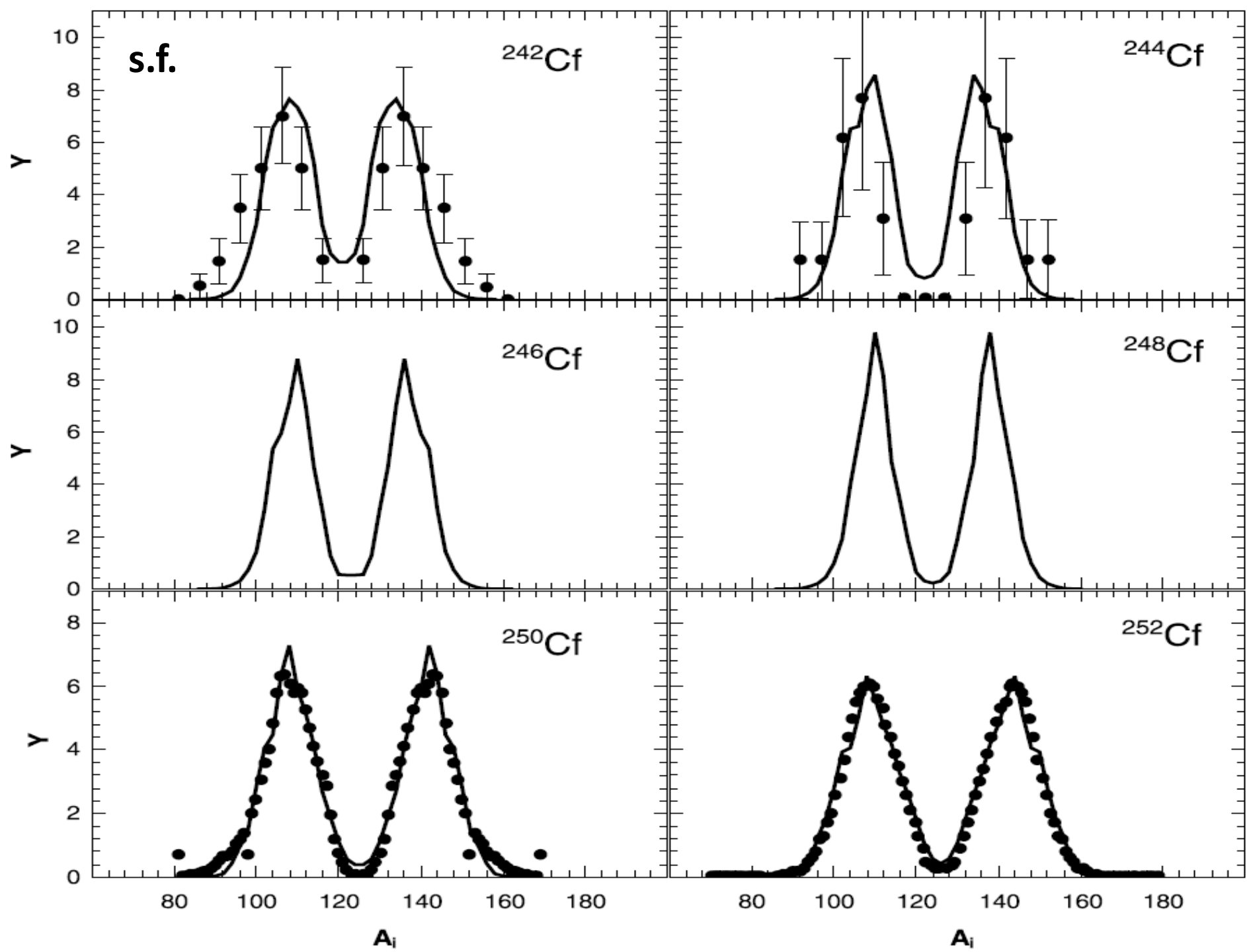
At some critical excitation energy saturation of symmetric yields occurs.

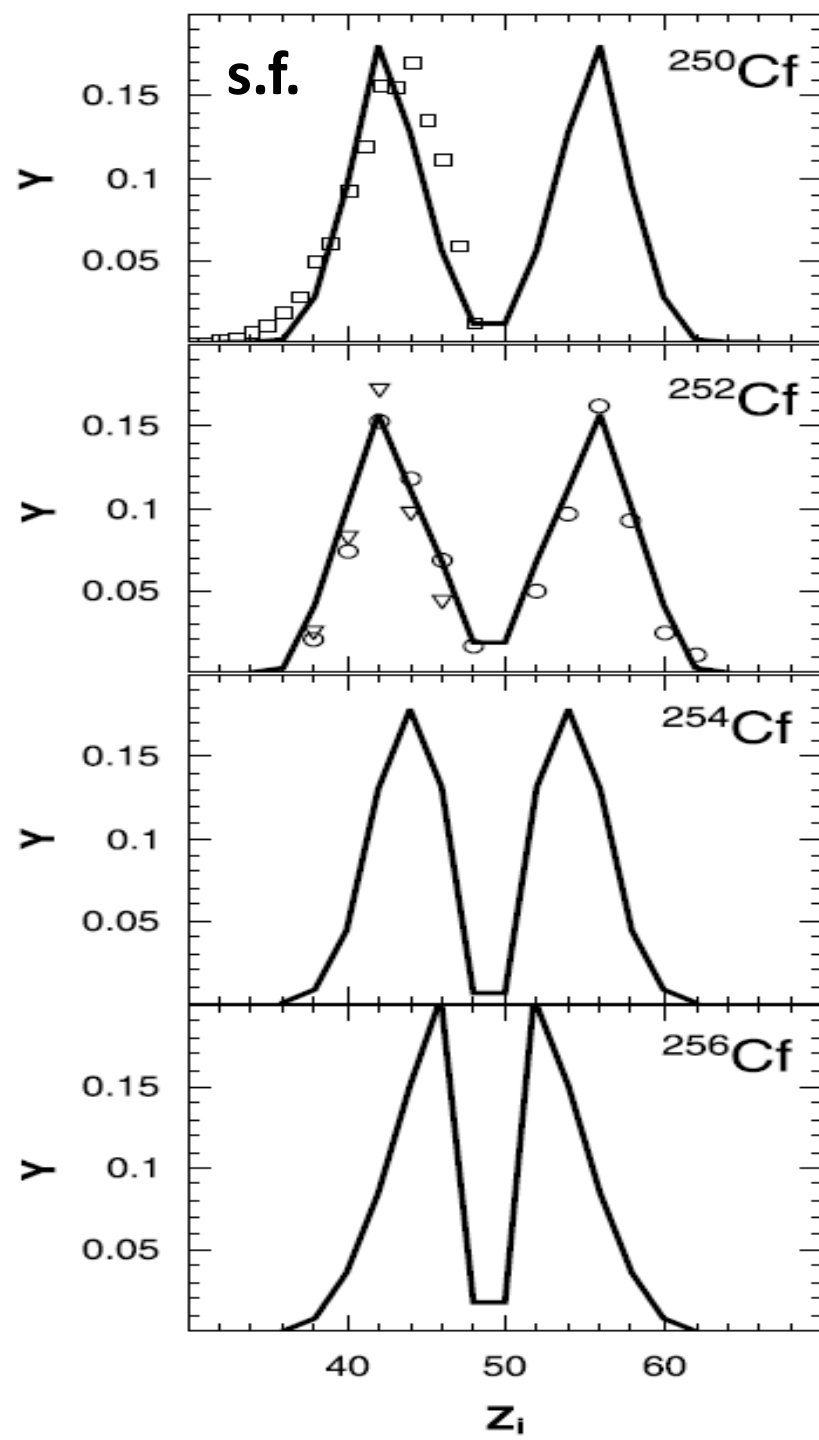
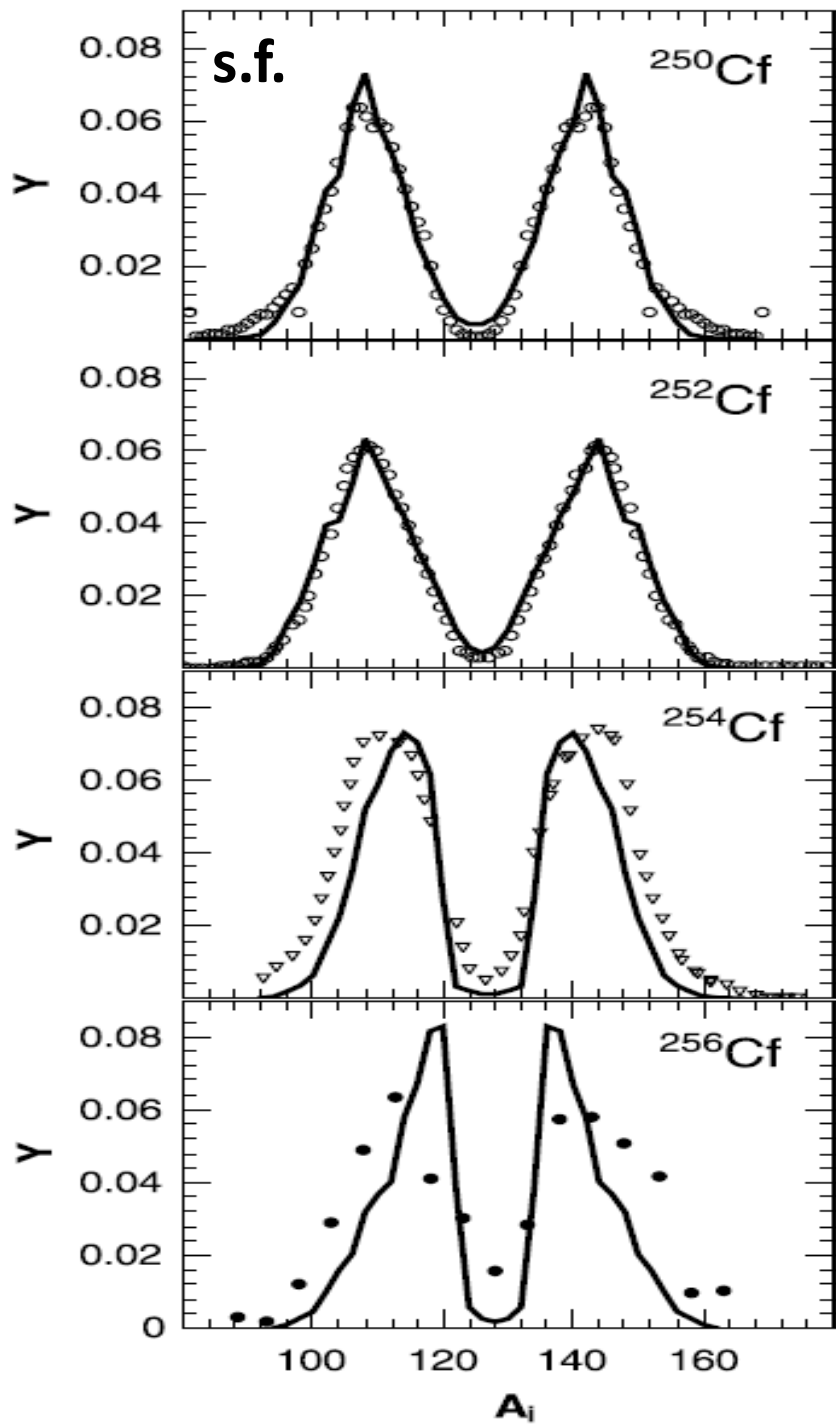
Further increase of  $E^*$  leads only to population of more asymmetric accessible configurations.

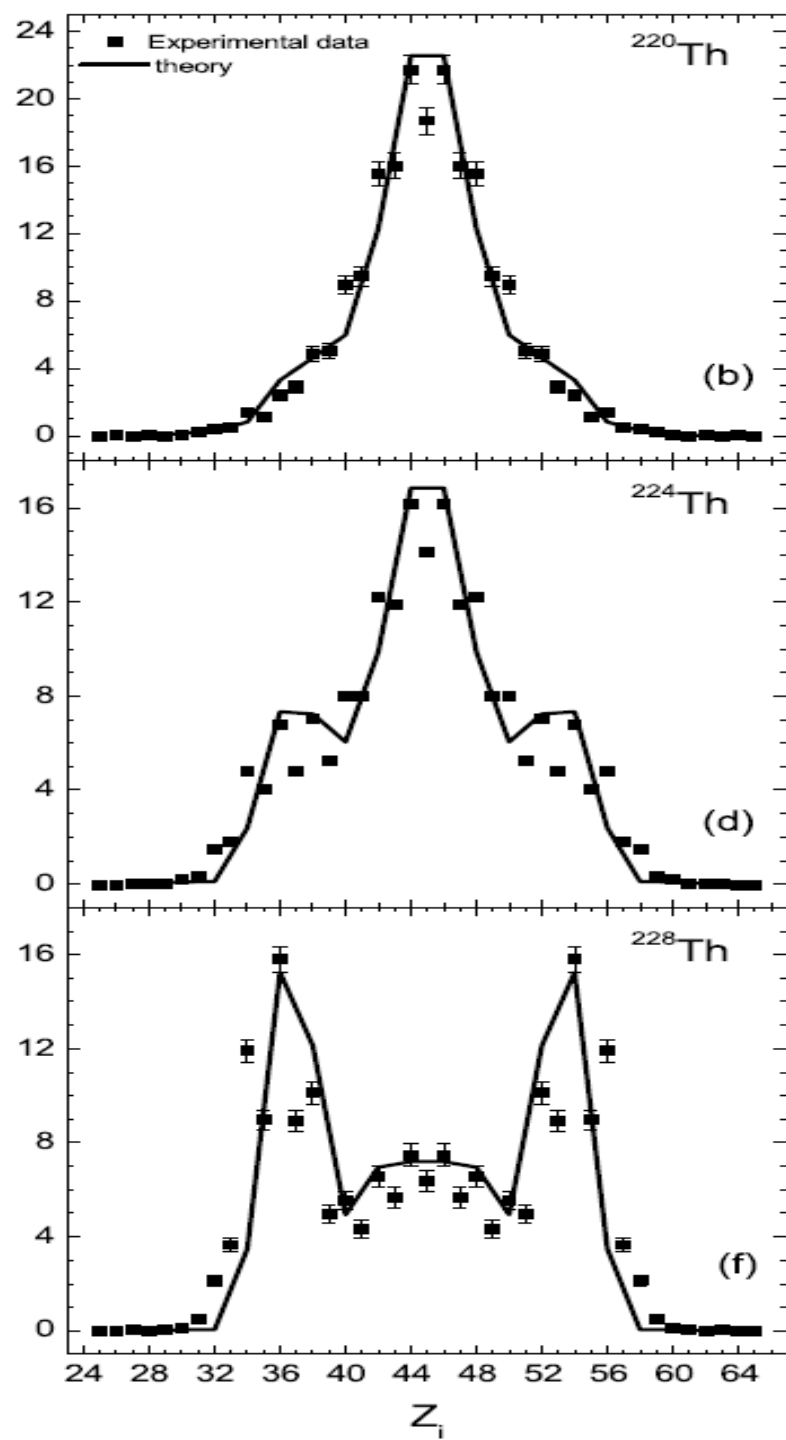
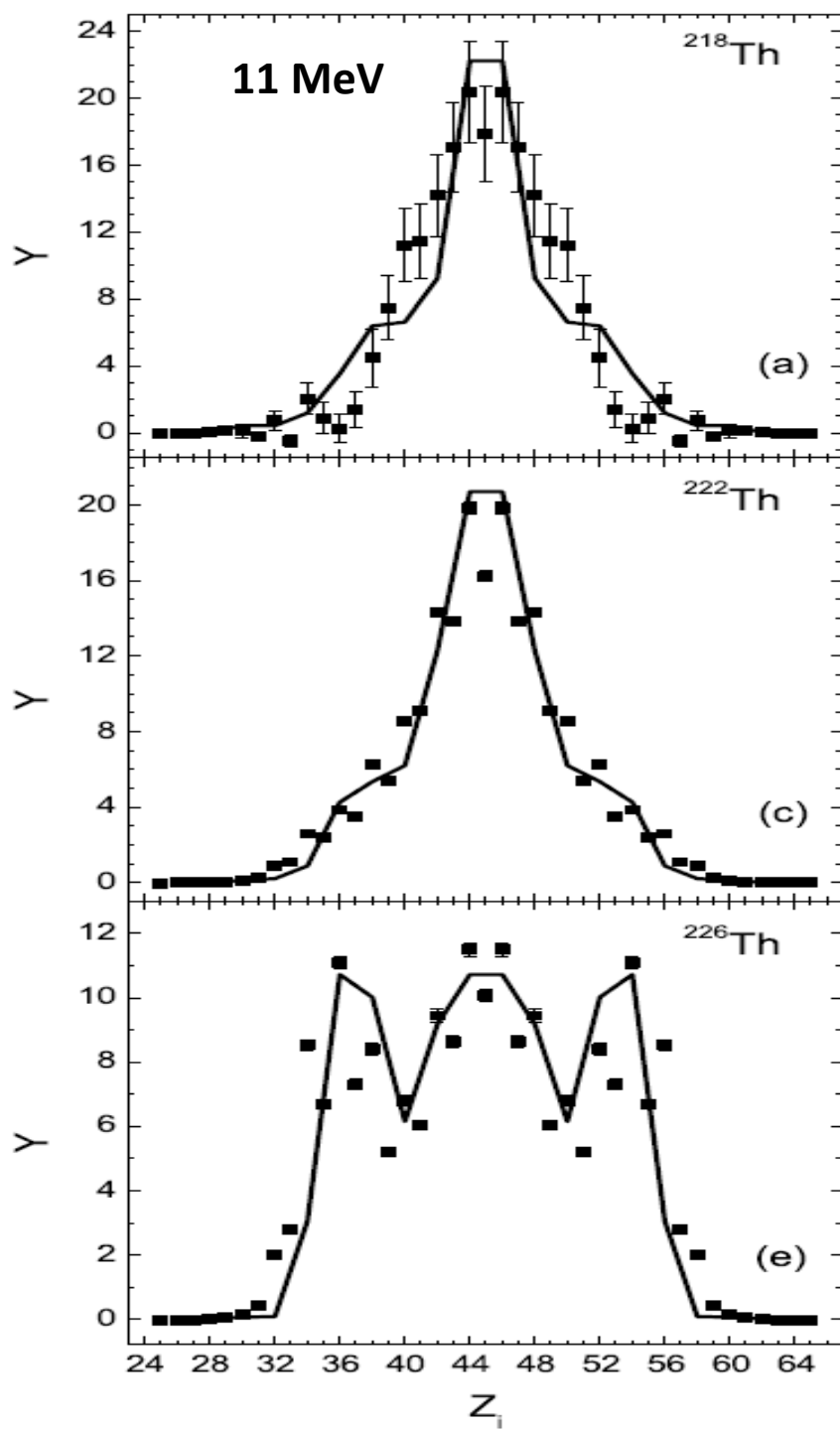
It is worth to be studied experimentally

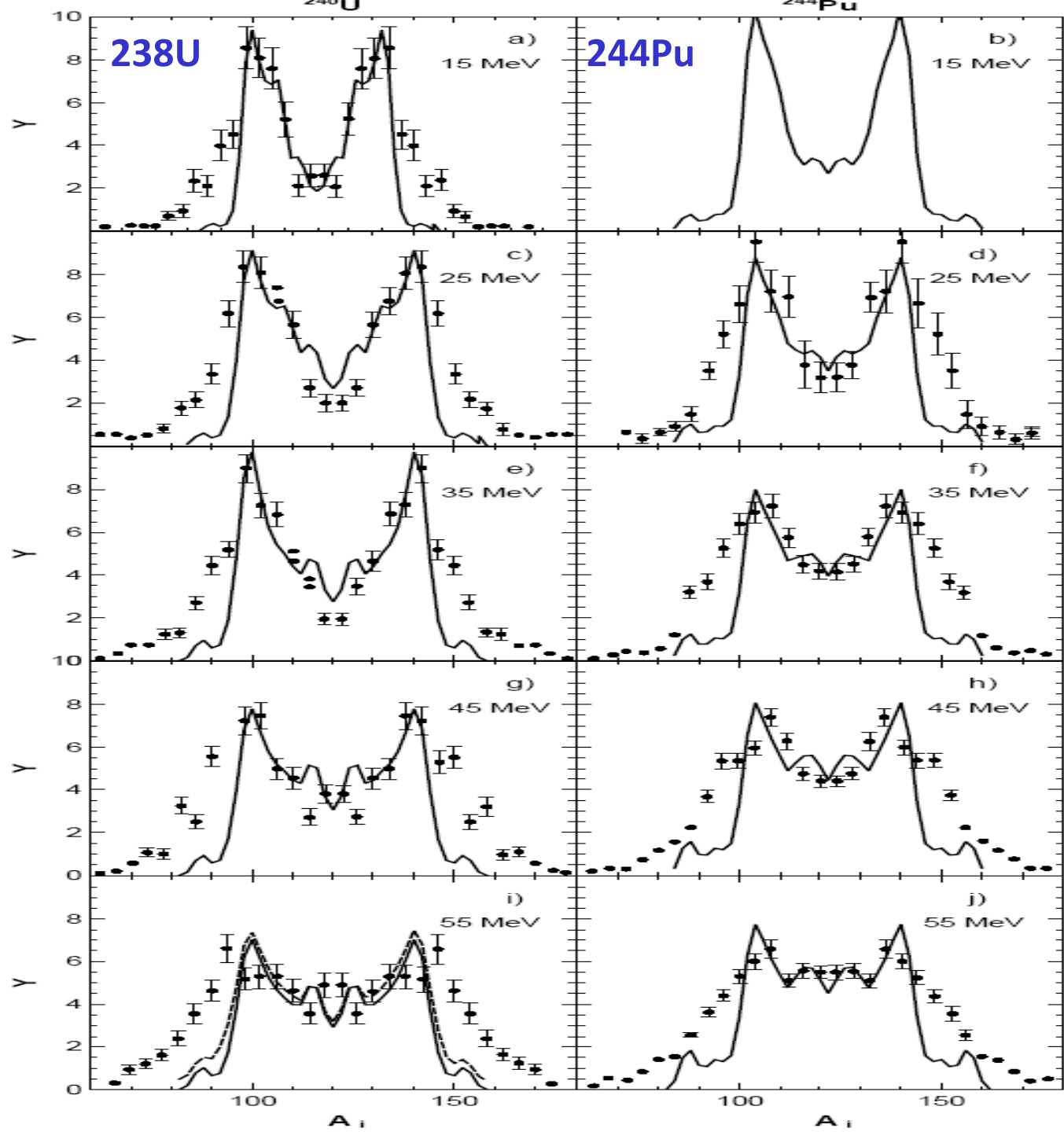
- **Transition from the symmetric to the asymmetric fission in the Th isotopes influences  $\langle n \rangle$ ,  $\langle TKE \rangle$ , and  $P_m(x)$ . These values can serve as markers to identify such transitions.**
- **The same can be observed in the transition from asymmetric fission of  $^{254,256}\text{Fm}$  ( $^{252}\text{No}$ ) to symmetric one of  $^{258,260}\text{Fm}$  ( $^{258,262}\text{No}$ ).**

Thank You For  
Your Attention !

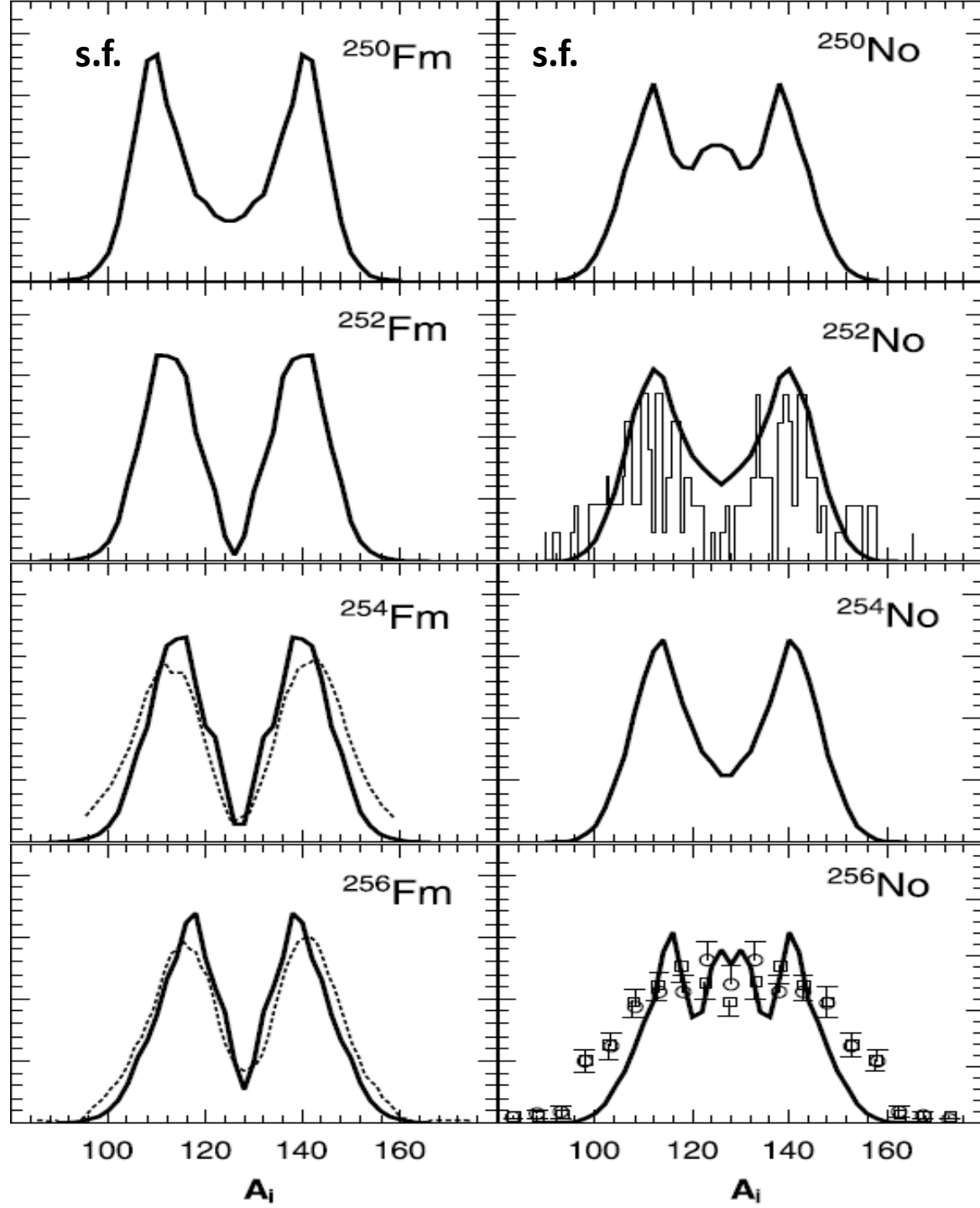




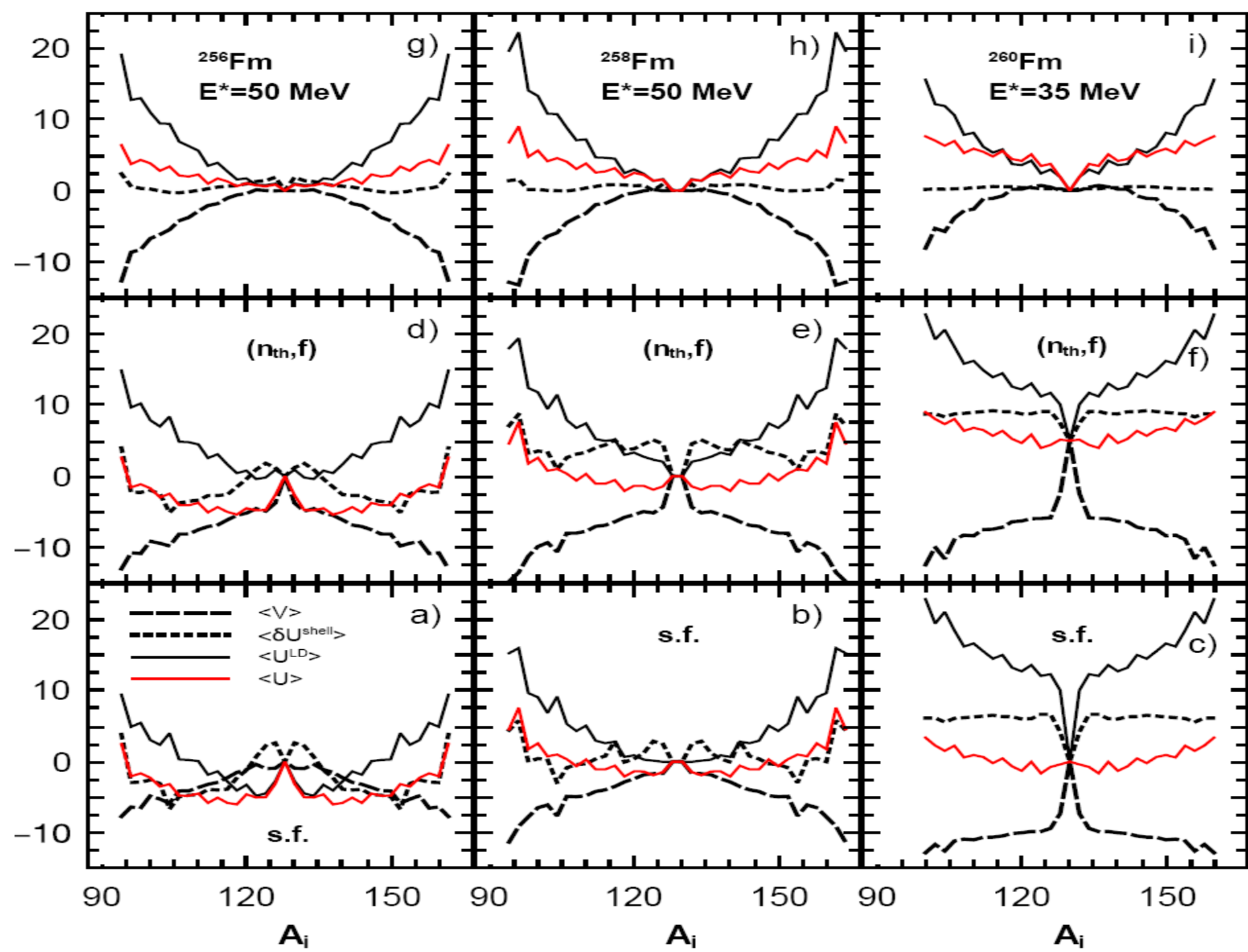




Exper.:K.Hirose et al.  
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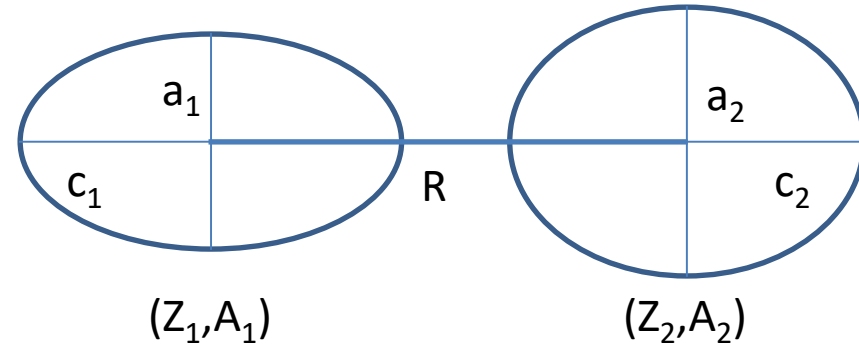






# Model

Coordinates  $Z_i, A_i, \beta_i$  ( $i=1,2$ ),  $R$  completely describe the geometry of system.



The interaction potential between fragments is:

$$V(R, Z, A, J, \beta_1, \beta_2) = V_N + V_C$$

$$V_c(R, Z_1, Z_2, \beta_1, \beta_2) = \frac{e^2 Z_1 Z_2}{R} + \left(\frac{9}{20\pi}\right)^{1/2} \frac{e^2 Z_1 Z_2}{R^3} \sum_{i=1}^2 R_i^2 \beta_i \left[ 1 + \frac{2}{7} \left(\frac{5}{\pi}\right)^{1/2} \beta_i \right] P_2(\cos\theta_i)$$

$$V_N = \int \rho_1(r_1) \rho_2(R - r_2) F(r_1 - r_2) dr_1 dr_2$$

$$F(r_1 - r_2) = C_0 \left[ F_{in} \frac{\rho_1(r_1)}{\rho_{00}} + F_{ex} \left( 1 - \frac{\rho_1(r_1)}{\rho_{00}} \right) \right] \delta(r_1 - r_2)$$

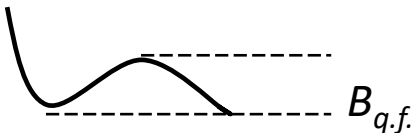
$$\rho_0(r) = \rho_1(r) + \rho_2(R - r)$$

$$F_{in,ex} = f_{in,ex} + f'_{in,ex} \frac{(N - Z)(N_2 - Z_2)}{(N + Z)(N_2 + Z_2)}$$

$$C_0 = 300 \text{ MeV fm}^3$$

$$f_{in} = 0.09, f_{ex} = -2.59$$

$$\rho_{00} = 0.17 \text{ fm}^{-3}, a = 0.51 - 0.56 \text{ fm}$$



# Model

- The total energy:

$$\begin{aligned} U(A_i, Z_i, \beta_i, d) &= \\ &= U_{macro}(A_i, Z_i, \beta_i, d) + \delta U^{shell}(A_i, Z_i, \beta_i) = \\ &= \sum_{i=1,2} U_i^{LD}(A_i, Z_i, \beta_i) + \sum_{i=1,2} \delta U_i^{shell}(A_i, Z_i, \beta_i, E_i^*) + \\ &+ V_N(A_i, Z_i, \beta_i, d) + V_C(A_i, Z_i, \beta_i, d). \end{aligned}$$

$$U_i^{L.D.}(A_i, Z_i, \beta_i) = U_i^{Surface}(A_i, Z_i, \beta_i) + U_i^C(A_i, Z_i, \beta_i) + U_i^{Sym}(A_i, Z_i)$$

- Liquid drop terms:

$$U_i^{sym}(A_i, Z_i) = 27.612 \frac{(N_i - Z_i)^2}{A_i}$$

$$U_i^C(A_i, Z_i, \beta_i) = \frac{3}{5} \frac{(Z_i e)^2}{R_{0,i}} \frac{\beta_i^{1/3}}{\sqrt{\beta_i^2 - 1}} \ln(\beta_i + \sqrt{\beta_i^2 - 1})$$

# Model

Surface energy with variable surface tension:

$$U_i^{Surface}(A_i, Z_i, \beta_i) = \sigma_i S_i$$

$$k_i = \frac{1}{1 + \exp[-0.063(C_{vib}^i(Z_i, A_i) - 67)]}$$

$$\sigma_i = \sigma_{0,i}(1 + k_i(\beta_i - \beta_i^{g.s.})^2)$$

$$C_{vib}^i(Z_i, A_i) = \frac{\hbar\omega_{vib}^i(3Z_i e R_{0,i}^2 / (4\pi))^2}{2B(E2)_{vib}^i}$$

$$\sigma_{0,i} = 0.9517(1 - 1.7826((N_i - Z_i)^2 / A_i)^2)$$

$$B(E2)_{vib}^i = E_{2+}^i B(E2)_{rot}^i / (\hbar\omega_{vib}^i)$$

Excitation energy: of the scission configuration can be calculated as a sum of the initial excitation energy of the fissioning nucleus and the difference of the potential energies of the fissioning nucleus and scission configuration:

$$E^*(A_i, Z_i, \beta_i, R_m) = E_{CN}^* + [U_{CN}(A, Z, \beta) - U(A_i, Z_i, \beta_i, R_m)].$$

$$T_{DNS}(E^*) = \sqrt{E^*/a}, a = A/12 \text{ MeV}^{-1}$$

Temperature dependence of LD terms:

$$U_i^{sym}(A_i, Z_i, T) = U_i^{sym}(A_i, Z_i, T = 0)(1 + 0.5 * 10^{-4} T^2),$$

$$U_i^C(A_i, Z_i, \beta_i, T) = U_i^C(A_i, Z_i, \beta_i, T = 0)(1 - 10^{-2} T^2)$$

$$U_i^{Surf}(A_i, Z_i, \beta_i, T) = U_i^{Surf}(A_i, Z_i, \beta_i, T = 0)(1 + 8.5 * 10^{-3} T^2).$$

$$k_i(E_i^*) = k_i * \exp[-E_i^*/E_k]$$

Shell damping:

$$\delta U_i^{shell}(A_i, Z_i, \beta_i', E_i^*) = \delta U_i^{shell}(A_i, Z_i, \beta_i', E_i^* = 0) \exp[-E_i^*/E_D]$$

# Model

## Yields:

Using  $P_{Z,A}(E_{CN}^*, \beta_1, \beta_2) \sim \exp\{-U(R_m, Z, A, \beta_1, \beta_2)/T\}$

$$P_{Z,A,\beta_1,\beta_2}^{decay} \sim \exp\{-B_{qf}(Z, A, \beta_1, \beta_2)/T\}$$

$$w(A_i, Z_i, \beta_i, E^*) = N_0 \exp\left[-\frac{U(A_i, Z_i, \beta_i, R_m) + B_{qf}(A_i, Z_i, \beta_i)}{T}\right]$$

The different yields can be calculate by integrating over the deformations:

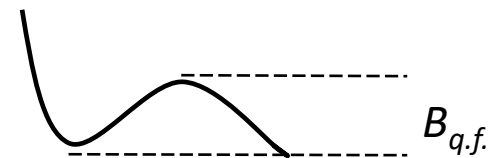
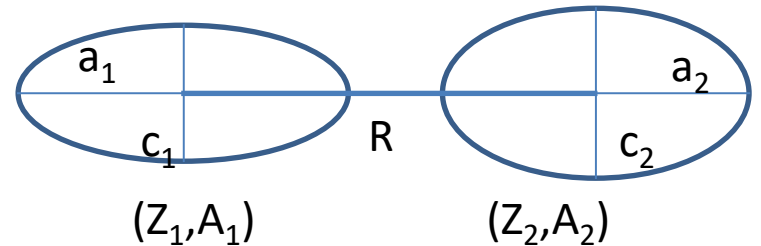
$$Y(A_i, Z_i) = N_0 \int \int w(A_i, Z_i, \beta_1, \beta_2, E^*) d\beta_1 d\beta_2,$$

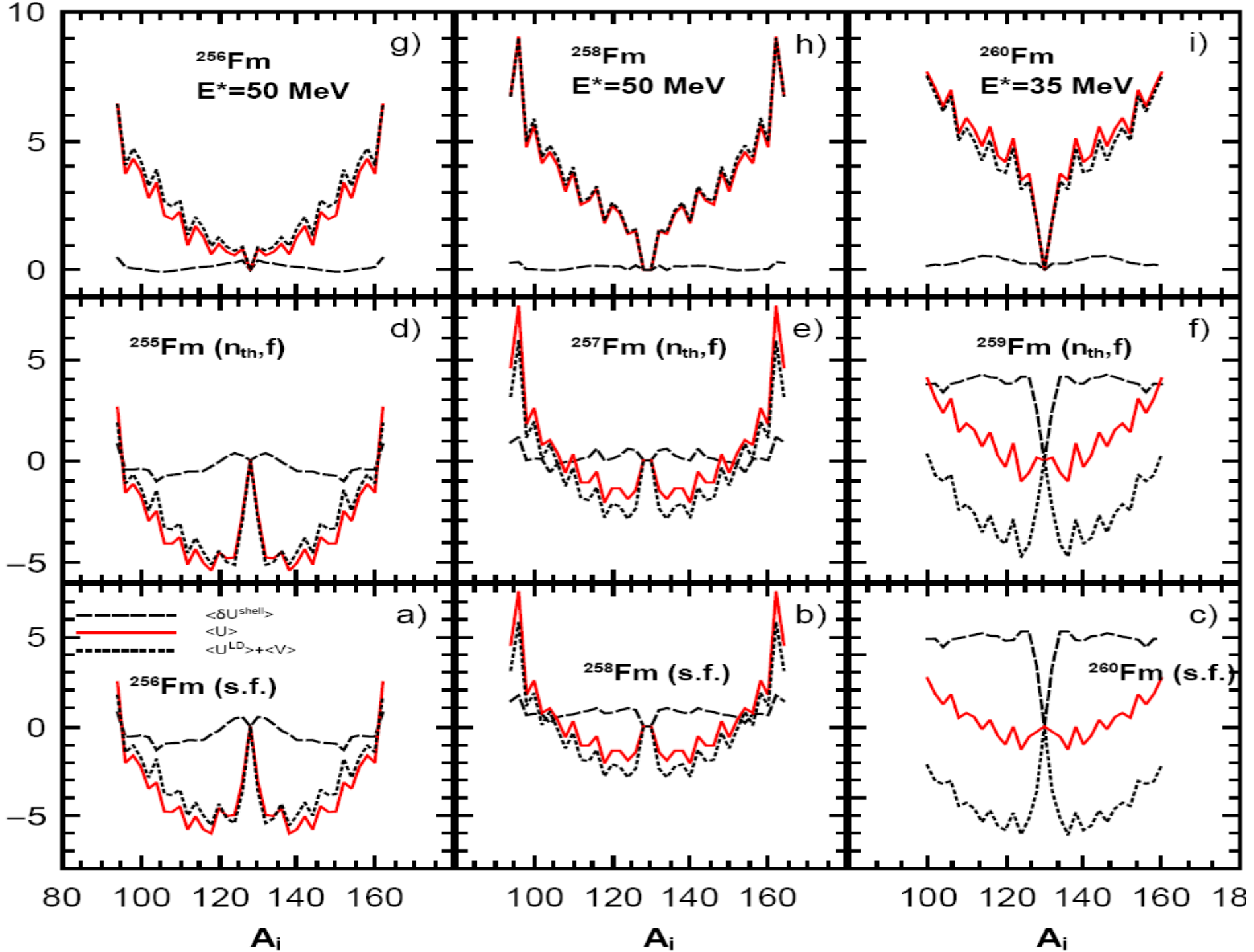
$$Y(A_i) = N_0 \sum_{Z_i} \int \int w(A_i, Z_i, \beta_1, \beta_2, E^*) d\beta_1 d\beta_2,$$

$$Y(Z_i) = N_0 \sum_{A_i} \int \int w(A_i, Z_i, \beta_1, \beta_2, E^*) d\beta_1 d\beta_2,$$

$$TKE(A_i, Z_i) = V_c(A_i, Z_i, \beta_1, \beta_2) + V_n(A_i, Z_i, \beta_1, \beta_2)$$

$$\langle TKE \rangle (A_i) = \frac{\sum_{Z_i} \int TKE(A_i, Z_i, \beta_1, \beta_2) w(A_i, Z_i, \beta_1, \beta_2, E^*) d\beta_1 d\beta_2}{\sum_{Z_i} \int w(A_i, Z_i, \beta_1, \beta_2, E^*) d\beta_1 d\beta_2}$$





**For 260Fm(sf)**, for **130Sn+130Sn**, interaction and liquid-drop energies have opposite and almost canceling effects. Strong shell in **130Sn+130Sn** is seen. This also affects macro parts of potential by fixing minima at small deformations.

**As  $E^*$  increases**, shell & stiffness diminish, shifting minima on PES [**130Sn+130Sn**] towards larger deformations. With increasing deformations **V** & **U** considerably decrease, mass yield becomes narrow.

Symmetric mode is already saturated at  **$E^*=15$  MeV**, at larger  $E^*$  distribution only becomes wider, minimum at  **$A/2$**  is given by competition between interaction & liquid-drop energies.

**For  $^{258}\text{Fm}(sf)$ ,** potential shows two wide asymmetric minima separated by small local maximum around symmetry. Shell corrections establish position of maxima in yield.

With increasing  $E^*$ , shell starts to be washed out but remains rather strong for  **$^{128}\text{Sn}+^{130}\text{Sn}$** .

**For  $^{258}\text{Fm}(n_{th},f)$ ,** at symmetry strong shell fixes minima of PES at small deformations which lead to large [small] interaction potential [liquid-drop energy] compared to neighboring DNS. Damping of shell for asymmetric DNS reduces asymmetric yields, dominant symmetric mode is enhanced due to shell closure of **Sn** nuclei.

**At  $E^*=50$  MeV,** shell effects are completely damped, stiffness of nuclear surface reaches its minimal value, mass yield becomes wider with further increasing  $E^*$ .



