The chaoticity parameter in two-pion correlation functions

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MPD Cross PWG Meeting
June 28, 2022

The two-particle correlation function

$$C_2(p_1, p_2) = \frac{N_2(p_1, p_2)}{N_1(p_1)N_1(p_2)}$$

where

$$N_1(p_1),\ N_1(p_2) \text{ and } N_2(p_1, p_2)$$

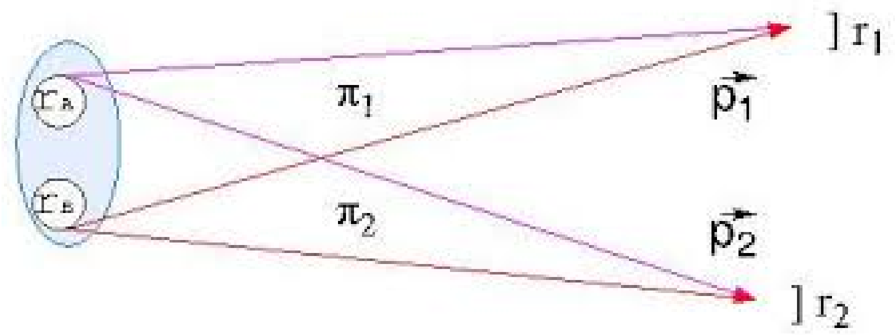
are the one- and two-particle invariant momentum distributions as functions of the single particle four-momenta p_1 and p_2.
The two-particle correlation analysis is performed as a function of the **relative four-momentum** q for a fixed **average four-momentum** K

$$q = p_1 - p_2 = (q_o, \vec{q}), \quad K = \frac{p_1 + p_2}{2} = (k_0, \vec{k})$$

Origin of two-particle correlations

- **Mainly Bose-Einstein quantum statistics for identical particles**
- Conservation laws
- Collective flow
- Jets
- Resonance decays
- ...

Bose-Einstein quantum statistics for identical particles



Bose-Einstein quantum statistics for identical particles

- **For identical bosons, the the correlation for small relative momentum grows as the mean number of pairs which is approximately proportional to the mean multiplicity squared**
- Other sources of correlation (for example pair production due to resonance decays) increase only linearly with the mean multiplicity
- For large multiplicities, such as is the case of pions produced in a heavy-ion environment, Bose-Einstein correlations dominate the correlation function for small relative momentum

Two-pion correlation and space-time source

Neglecting dynamical correlations, the pair momentum distribution is related to the particle emitting source $S(x,p)$ by

$$N_2(p_1,p_2)=\int d^4x_1 d^4x_2 S(x_1,p_1)S(x_2,p_2)\left|\Psi_{p_1,p_2}(x_1,x_2)\right|^2$$

where $\Psi_{p_1,p_2}(x_1,x_2)$ is the **symmetrized pair wave function.**

$$C_2(p_1,p_2)=1+\mathrm{Re}\left[\frac{\widetilde{S}(q,p_1)\widetilde{S}^*(q,p_2)}{\widetilde{S}(0,p_1)\widetilde{S}^*(0,p_2)}\right]$$

where $\widetilde{S}$ is the Fourier Transform of S.

Two-pion correlation and space-time source

For typical sources and kinematical domains found in heavy-ion collisions the function $S(q, p)$ **does not change much as a function of p.** It is thus customary to use the approximation $p_1 \simeq p_2 \simeq K$ to get

$$C_2(q, K) = 1 + \frac{\left|\widetilde{S}(q, K)\right|^2}{\left|\widetilde{S}(0, K)\right|^2}$$

The space region of particle emission is sometimes parametrized in a Gaussian form

$$S(R, r) = \frac{1}{(2\pi)^3} \int d^3 q e^{i\vec{q}\cdot\vec{r}} e^{-\frac{1}{2}\left|\vec{q}\mathbf{R}^2\vec{q}\right|}$$

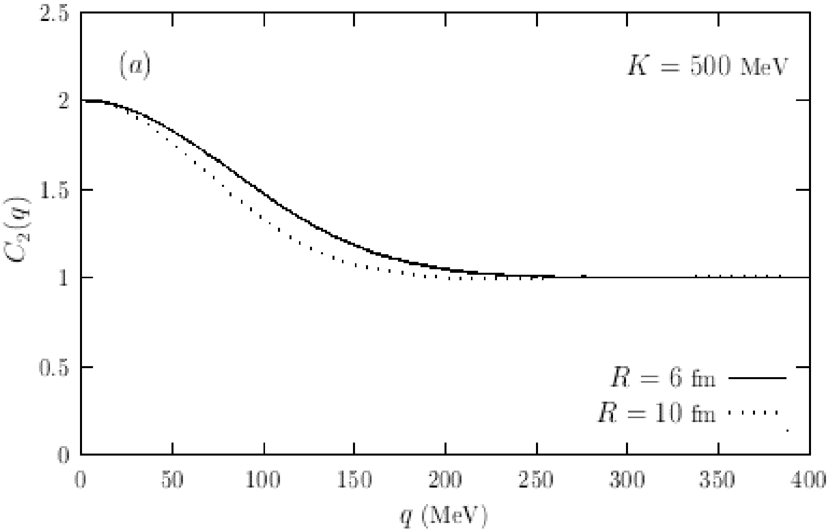
Source size

- $\mathbf{R}^2$ is the **matrix of homogeneity lengths or femtoscopy radii**
- For a spherically symmetric source the radius is given by the width of the correlation function

$$R \sim \frac{1}{q}$$

- This is a direct consequence of Heisenberg's uncertainty relation
$$(\Delta R)(\Delta p) \sim 1$$
- Correlated pairs are emitted predominantly in the same direction.

Correlation width



Correlation strength

- Neglecting final state (Coulomb, strong) interactions, the correlation function $C_2(0, K) = 2$
- Experimentally, limitations on two track resolution prevents correlation measurements at $q = 0$
- The correlation function is measured at $q \neq 0$ and then **extrapolated** to $q = 0$
- The extrapolated value can in general be different from the exact value at $q = 0$
- To quantify this value, define

$$\lambda(K) = \lim_{q \to 0} C_2(q, K) - 1$$

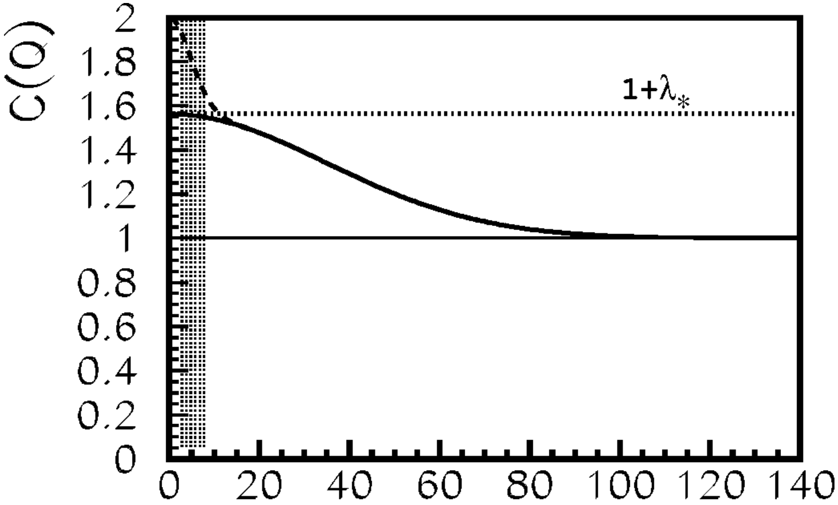
$\lambda(K)$ is also known as the **chaoticity parameter**

Resolution

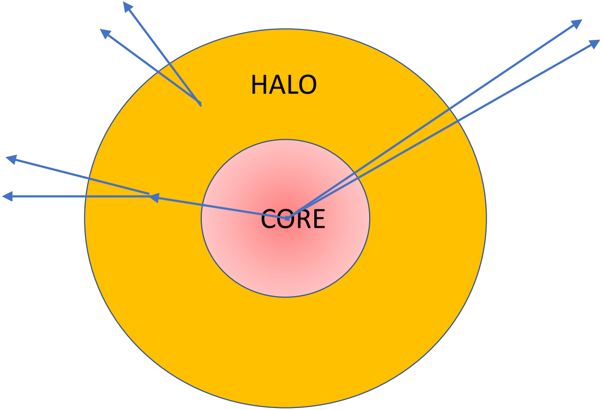
- For a totally chaotic source, $\lambda = 1$
- Partially coherent sources (possible contributions of a Bose-Einstein condensate) produce $\lambda < 1$

Even if the source is completely chaotic, since $R \sim 1/q$ maximum radius that can be resolved $R_{\mathrm{max}} \sim 1/q_{\mathrm{min}} \sim$ 25-30 fm

Resolution



Core halo picture



Correlation strength

- Physical assumption: the phase space emitting source is made of two components

$$S = S_{\text{core}} + S_{\text{halo}}$$

- If the pions are correlated and their correlation function is resolved, **both pions need to come from the core**
- Each component has a Fourier Transform

$$\begin{aligned} \widetilde{S}_{\text{core}}(q, K) &\equiv \int d^4x\, e^{iqx} S_{\text{core}}(x, K) \\ \widetilde{S}_{\text{halo}}(q, K) &\equiv \int d^4x\, e^{iqx} S_{\text{halo}}(x, K) \end{aligned}$$

Correlation strength

We notice that

$$N_{\text{core}}(K) \equiv \int d^4x\, S_{\text{core}}(x,K) = \widetilde{S}_{\text{core}}(0,K)$$
$$N_{\text{halo}}(K) \equiv \int d^4x\, S_{\text{halo}}(x,K) = \widetilde{S}_{\text{halo}}(0,K)$$

Therefore

$$\widetilde{S}(0,K) = N_{\text{core}}(K) + N_{\text{halo}}(K)$$

Thus, for the experimentally resolvable values of q

$$\widetilde{S}(q,K) \simeq \widetilde{S}_{\text{core}}(q,K)$$

Correlation strength

Thus, the correlation function can be expressed as

$$C_2(q,K) = 1 + \left(\frac{N_{\text{core}}(K)}{N_{\text{core}}(K) + N_{\text{halo}}(K)}\right)^2 \frac{\left|\widetilde{S}_{\text{core}}(q,K)\right|^2}{\left|\widetilde{S}_{\text{core}}(0,K)\right|^2}$$

Therefore, **in the core-halo picture**

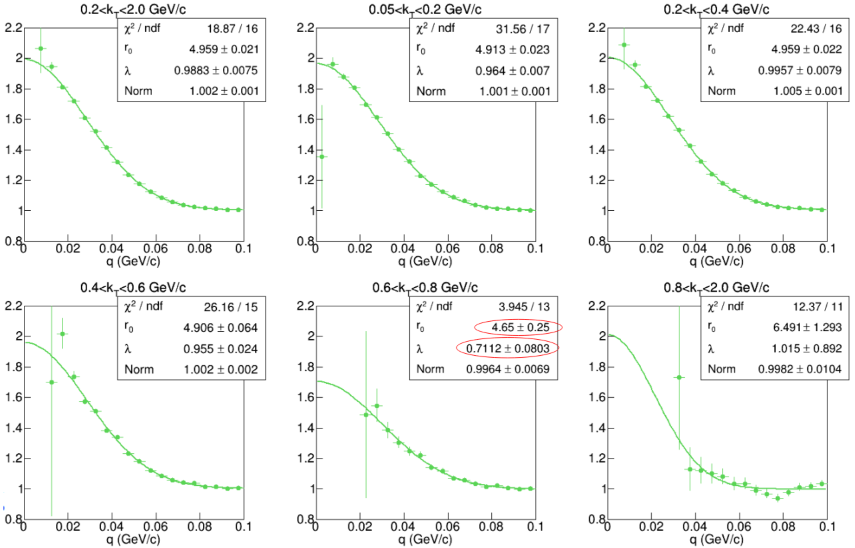
$$\lambda = \left(\frac{N_{\text{core}}(K)}{N_{\text{core}}(K) + N_{\text{halo}}(K)}\right)^2$$

λ carries indirect information on the decay of long-lived resonances (η, η', ω, K_s^0). In particular of η' that is considered a messenger if $U_A(1)$ symmetry restoration.

Possible effects on λ measurements

- Single-track momentum resolution produces smearing of the correlation function
- Track miss-identification decreases the maximum of the correlation.
- Track merging produces a lack of data at low q and has a strong effect for $k_T > 0.6$ GeV/c.
- Two-track effects, such as track splitting, can be corrected by increasing the number of hits for track selection.
- To measure λ for $k_T > 0.6$ GeV/c we need to increase the statistics to have a similar number of pairs at low q

C_2 determination



Efficiency

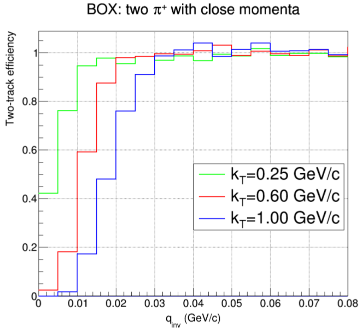
k_T (GeV/c)	λ	$\delta\lambda$ (%)
0.2 - 2.0	0.9883 ± 0.0075	0.8
0.05 - 0.2	0.964 ± 0.007	0.7
0.2 - 0.4	0.9957 ± 0.0079	0.8
0.4 - 0.6	0.955 ± 0.024	2.5
0.6 - 0.8	$0.7112 \pm 0.0.0803$	11.3
0.8 - 2.0	1.015 ± 0.892	87.9

Table: λ parameter for different k_T intervals

Efficiency

To estimate the number of events needed to improve the determination of λ, take the two-track reconstruction efficiency for primary tracks in Au+Au (Bi+Bi) and consider that for each k_l bin, the efficiency $\varepsilon_i(k_T)$ is given by

$$\varepsilon_i(k_T) = \frac{N_{rec_i}(k_T)}{N_{gen_i}(k_T)}$$



Estimate of the number of events

- N_{rec_i} is the number of pairs reconstructed and N_{gen_i} is the number of pairs generated in the $i-th$ q bin for each k_T.
- A rough estimate of the number of events needed to improve the statistics at low q, can be made assuming that the number of pairs generated in each bin is proportional to the total number of simulated events, $N_{gen_i} \propto N_{events}$.
- To have the same statistics that provides the same uncertainty in λ, we require that the number of reconstructed particles in some q bin at different k_T be the same

$$N_{rec_i}(k_{T_1}) = N_{rec_i}(k_{T_2})$$

- From this relation we get

$$\varepsilon_i(k_{T_1}) N_{gen_i}(k_{T_1}) = \varepsilon_i(k_{T_2}) N_{gen_i}(k_{T_2})$$

Estimate of the number of events

- Assuming the proportionality to the number of generated events

$$
\begin{aligned}
N_{gen_i}(k_{T_1}) &= \frac{\varepsilon_i(k_{T_2})}{\varepsilon_i(k_{T_1})} N_{gen_i}(k_{T_2}) \\
N_{events}(k_{T_1}) &= \frac{\varepsilon_i(k_{T_2})}{\varepsilon_i(k_{T_1})} N_{events}(k_{T_2})
\end{aligned}
$$

- We obtain that the number of generated events needed to improbe the statistics is ≈ 47.5 **million events** so that we have a similar number of pairs in the 2nd bin in the graph with $k_T \in (0.8, 2.0)$GeV/c as we have with 10 million events in the graph with $k_T \in (0.2, 0.4)$ GeV/c.
- As a first approach, **50 million events will produce a better fit for larger k_T**
- Also we need to recall that track reconstruction needs to be improved to avoid track merging and that the effect increases with k_T.

Conclusions

- Femtoscopy analyses lend themselves for first physics studies
- The study of the excitation function of the chaoticity parameter (correlation strength) λ is a handle to study chiral symmetry restoration.
- The core-halo picture may be a useful intuitive guide to interpret the results
- **50 million MC events** can provide a good statistics to improve the determination of λ

¡Muchas Gracias!

Backup

Variable dependence

- In general C_2 depends on the two four-momenta p_1 and p_2.
- However $q \cdot K = q_0 K_0 - \vec{q} \cdot \vec{K} = 0$
- This implies $q_0 = \frac{\vec{q} \cdot \vec{K}}{K_0}$
- We may then transform the q-dependence into a dependence on $\vec{q}$
- Moreover, if the pair is of similar energy then K is approximately on-shell and the correlation function becomes a function of $\vec{q}$ and $\vec{K}$

$$C_2(q, K) \to C_2(\vec{q}, \vec{K})$$