

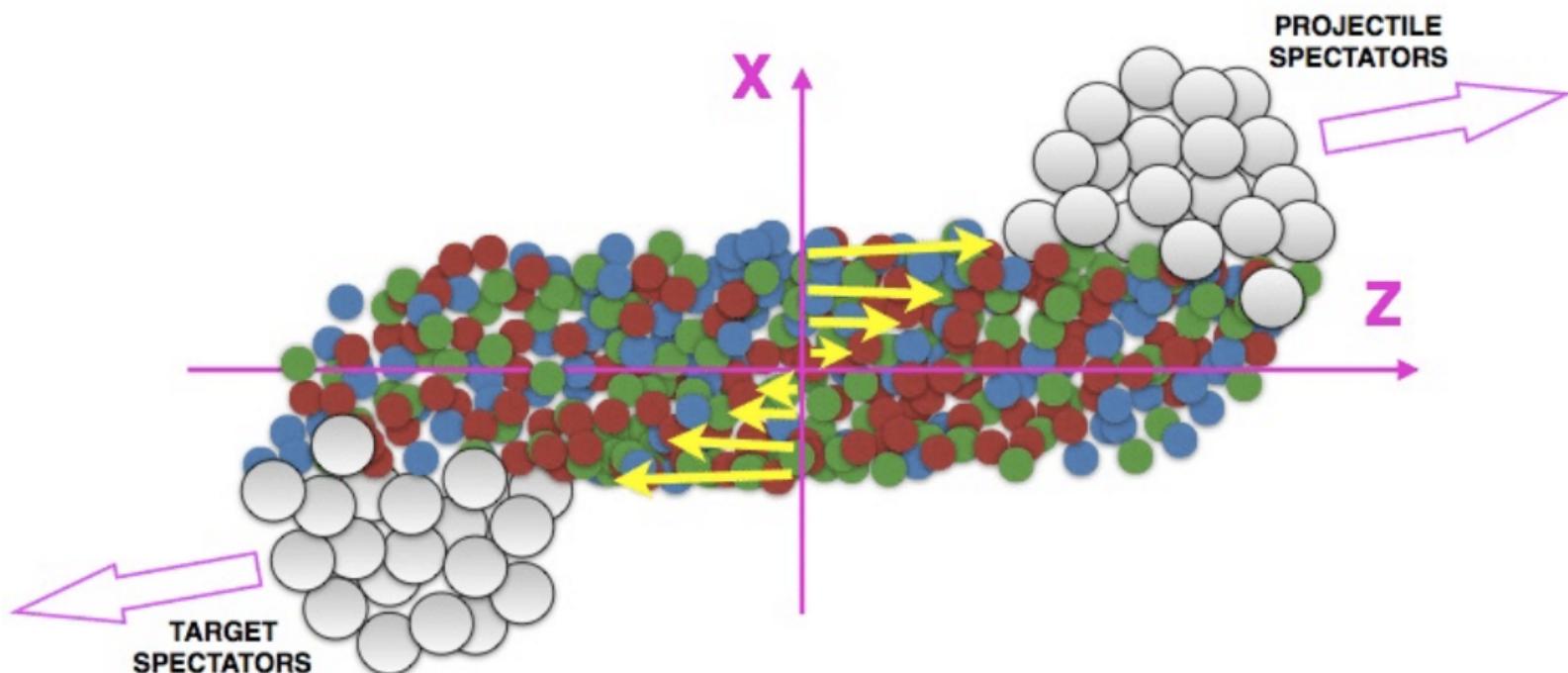
Influence of relativistic rotation on the equation of state of gluodynamics

D.A.Sychev
in collaboration with
V.V.Braguta, A.Yu.Kotov, A.A.Roenko

BLTP JINR, MIPT

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Motivation



Metric in rotating coordinates

$$x \equiv (x_0, x_1, x_2, x_3) = (t, r \sin \varphi, r \cos \varphi, z) \quad (1)$$

$$t = t_{\text{lab}}, \quad r = r_{\text{lab}}, \quad z = z_{\text{lab}}, \quad \varphi \sim (\varphi_{\text{lab}} - \Omega t) \quad (2)$$

$$g_{\mu\nu}^{(\text{lab})} = \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \quad (3)$$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = (1 - r^2 \Omega^2) dt^2 - 2r^2 \Omega dt d\varphi - dr^2 - r^2 d\varphi^2 - dz^2 \quad (4)$$

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2 \Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (5)$$

Free energy density

$$E = E^{(\text{lab})} - J\Omega \quad (6)$$

$$F = E - TS \quad (7)$$

$$J = - \left(\frac{\partial F}{\partial \Omega} \right)_T \quad (8)$$

$$Z = \int \mathcal{D}[U] e^{-S_E[U]} \quad (9)$$

$$F = -T \log Z \quad (10)$$

$$\langle f \rangle = \frac{F}{V} \quad (11)$$

EoS expansion coefficients

$$\frac{\langle f \rangle(T, v_I, \dots)}{T^4} = c_0(T, \dots) + c_2(T, \dots)v_I^2 + c_4(T, \dots)v_I^4 + \dots \quad (12)$$

$$F = VT^4 \frac{\langle f \rangle}{T^4} = VT^4 [c_0 - c_2 v^2 + c_4 v^4 + \dots] \quad (13)$$

$$J = -\partial_\Omega F = VT^4 c_2 \frac{N_r^2 a^2}{2} \Omega - O(\Omega^3) \quad (14)$$

$$I = \partial_\Omega J|_{\Omega=0} = VT^4 c_2 \frac{N_r^2 a^2}{2} = \boxed{\frac{1}{2} c_2 T^4 V L_r^2 = I} \quad (15)$$

Gluodynamics in rotating coordinates

$$S_G = \int d^4x \sqrt{\det g_{\alpha\beta}} \frac{1}{2g_{\text{YM}}^2} g^{\mu\nu} g^{\rho\sigma} \text{tr} F_{\mu\rho} F_{\nu\sigma} \quad (16)$$

$$\begin{aligned} S_G = & \frac{1}{g_{\text{YM}}^2} \int d^4x \text{tr} [(1-r^2\Omega^2)F_{xy}F_{xy} + (1-y^2\Omega^2)F_{xz}F_{xz} + (1-x^2\Omega^2)F_{yz}F_{yz} + F_{x\tau}F_{x\tau} + F_{y\tau}F_{y\tau} + \\ & + F_{z\tau}F_{z\tau} - 2iy\Omega(F_{xy}F_{y\tau} + F_{xz}F_{z\tau}) + 2ix\Omega(F_{yx}F_{x\tau} + F_{yz}F_{z\tau}) - 2xy\Omega^2 F_{xz}F_{zy}] \end{aligned} \quad (17)$$

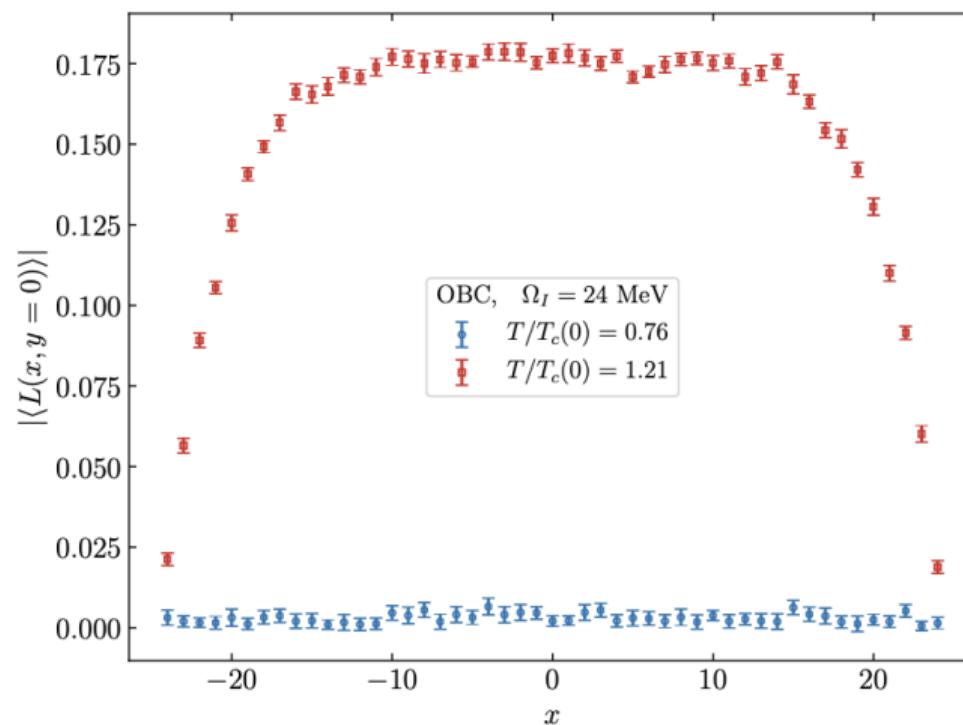
Simulations with a sign problem:

- Analytical continuation $\Omega_I = -i\Omega$ (this presentation)
- Expansion coefficients at $\Omega = 0$

Boundary conditions

- PBC are used for T and Z directions.
- PBC or OBC are used for X,Y directions.
- Typical lattice sizes: 40^4 and $\{4\dots8\} \times 40^3$.
- N_t^4 numerical factor practically forbids high N_t .
- Screening of BC due to high mass of glueballs.

Screening of boundary conditions



Lattice action density

$$S_G = -\beta \sum_{p \in \Lambda} s(p, N_t, N_r, N_z, v(p), Y) \quad (18)$$

$$\langle s \rangle = \frac{1}{|\Lambda|} \sum_{p \in \Lambda} s(p, \dots) \quad (19)$$

$$v = \frac{N_r}{2} a \Omega \quad (20)$$

$$v_I = \frac{N_r}{2} a \Omega_I \quad (21)$$

$$\langle \Delta s(\beta, \dots) \rangle = \left[\langle s \rangle(\beta, N_t, \dots) - \langle s \rangle(\beta, N_t^{(\infty)}, \dots) \right] \quad (22)$$

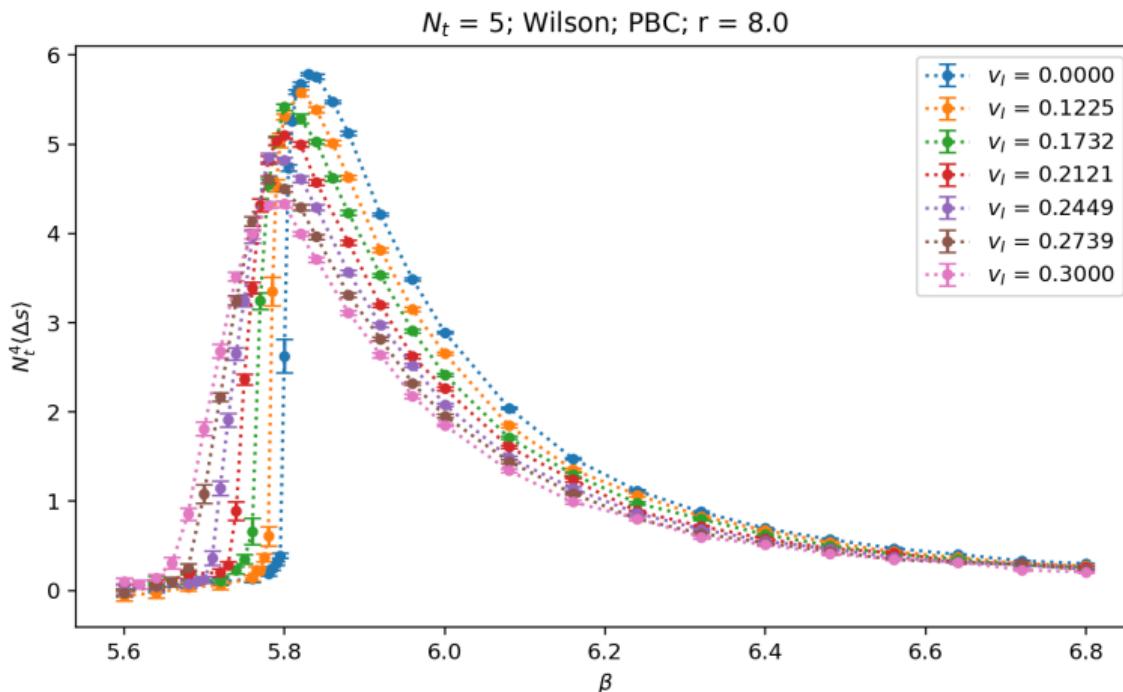
Free energy density from lattice data

$$\langle s \rangle = \frac{1}{Z} \int \mathcal{D}[U] \frac{1}{|\Lambda|} \partial_\beta e^{-S_G[U]} = \frac{1}{|\Lambda|} \frac{\partial \log Z(\beta, N_t, N_r, N_z, v_I, Y)}{\partial \beta} \quad (23)$$

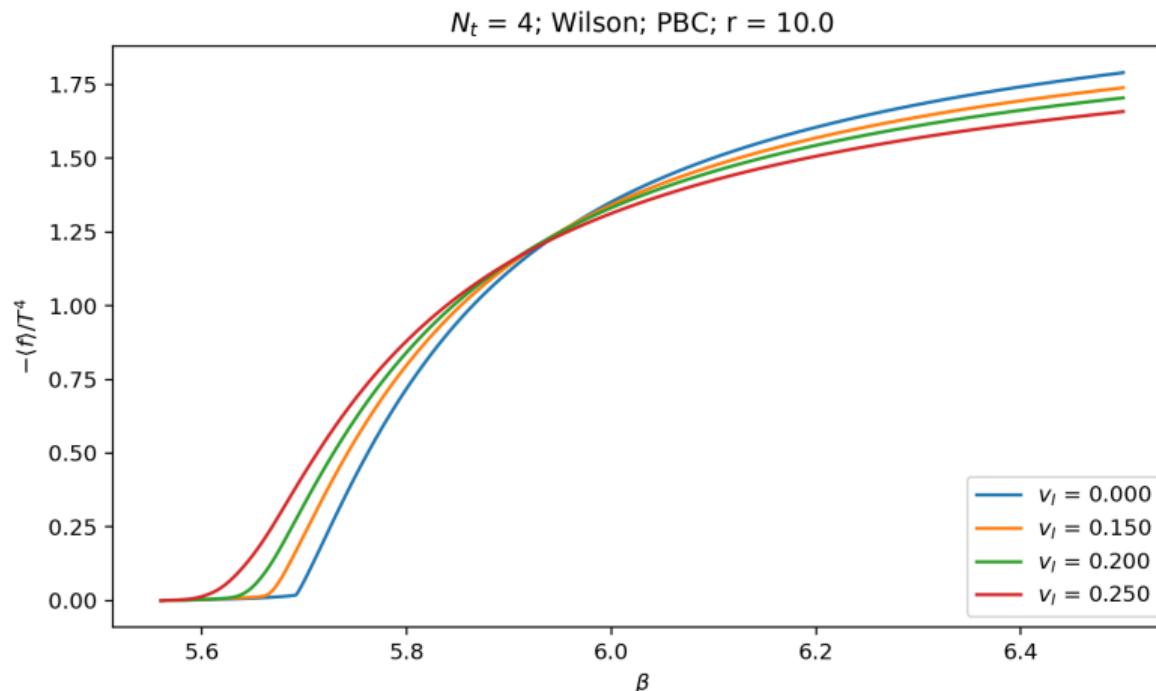
$$\begin{aligned} \frac{\langle f \rangle(\beta, N_t, N_r, N_z, v_I, Y)}{T(N_t, \beta)} &= -\frac{1}{V(N_r, N_z, \beta)} \log Z(\beta, N_t, N_r, N_z, v_I, Y) = \\ &= -\frac{1}{N_r^2 N_z a^3(\beta)} \int_0^\beta d\tilde{\beta} (\partial_\beta \log Z)(\tilde{\beta}, N_t, N_r, N_z, v_I, Y) = \\ &= -N_t^4 T^3(N_t, \beta) \int_0^\beta d\tilde{\beta} \langle s \rangle(\tilde{\beta}, v_I, N_t, \dots) \end{aligned}$$

$$\boxed{\frac{\langle f \rangle(\beta, v_I, N_t, \dots)}{T^4(N_t, \beta)} = -N_t^4 \int_0^\beta d\tilde{\beta} [\langle s \rangle(\tilde{\beta}, v_I, N_t, \dots) - \langle s \rangle(\tilde{\beta}, v_I, N_t \rightarrow \infty, \dots)]} \quad (24)$$

Lattice action density



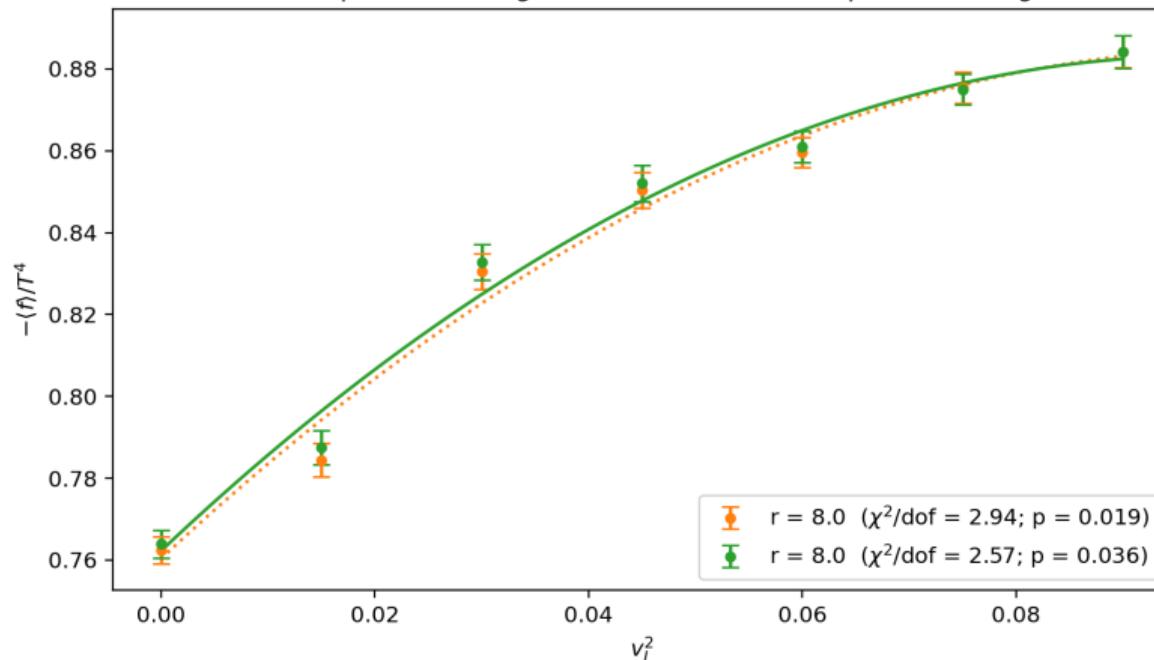
EoS from lattice data



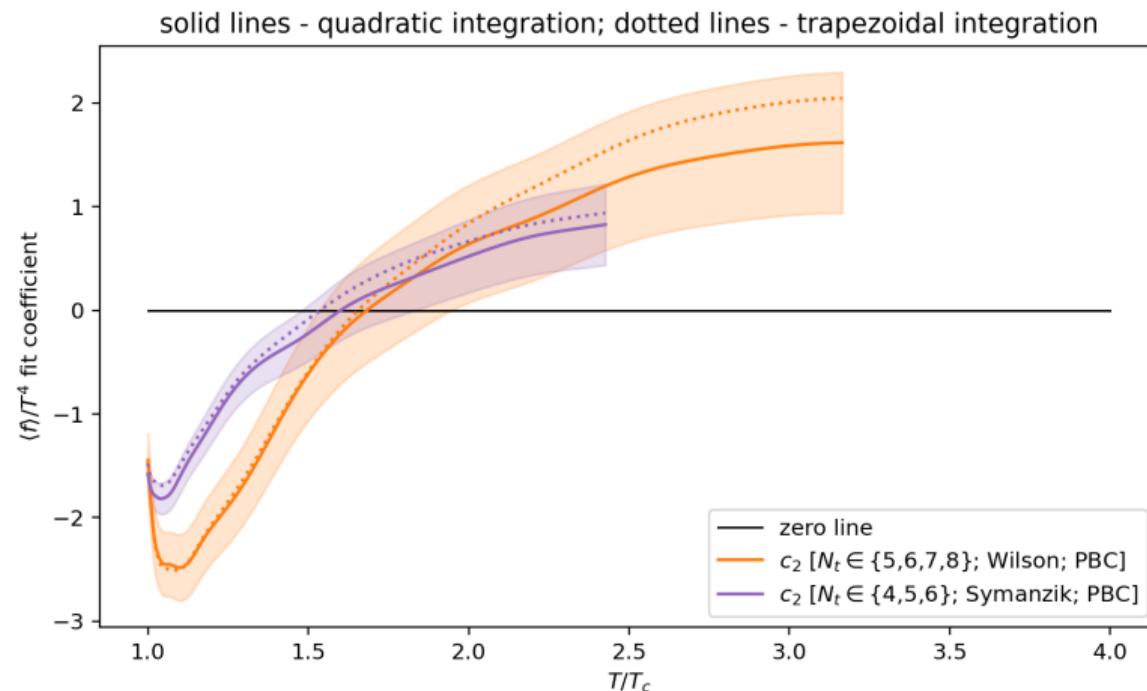
Fitting EoS

 $N_t = 5; N_s = 40$; Wilson; PBC; $\beta = 5.95$ ($T/T_c = 1.34$)

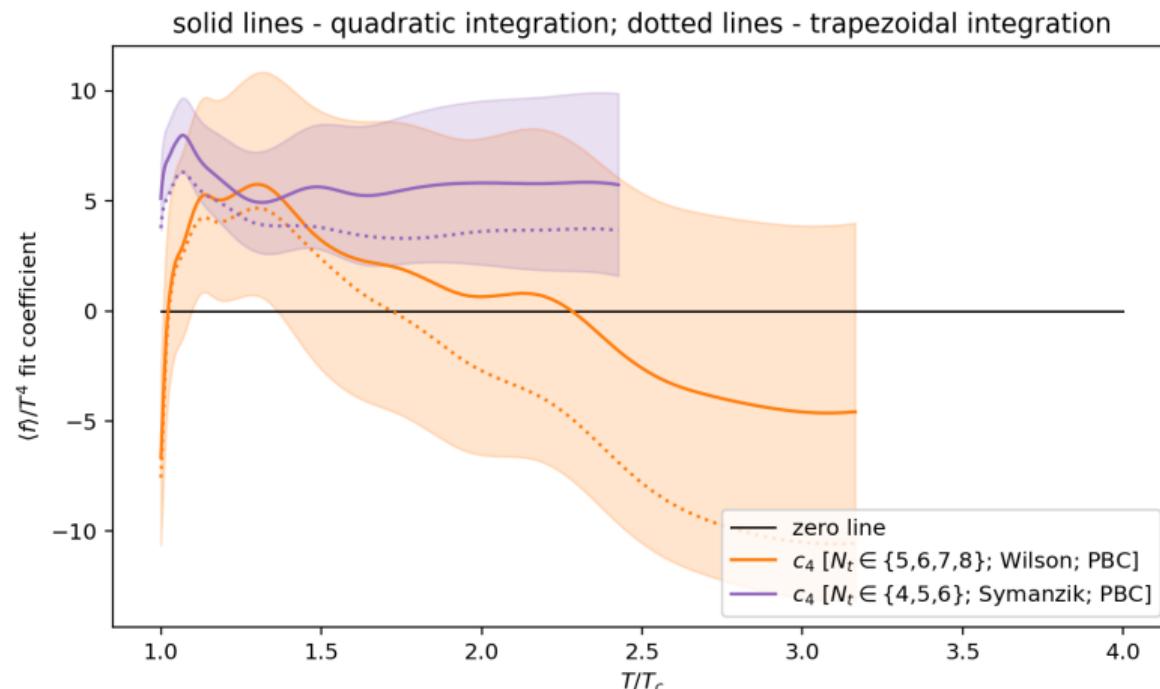
solid lines - quadratic integration; dotted lines - trapezoidal integration



EoS expansion coefficients. Results



EoS expansion coefficients. Results



Conclusions

- Method for extracting thermal contribution to the EoS dependence on rotation was introduced.
- Simulations show smaller moment of inertia near the critical temperature.
- Plans to compare with direct expansion coefficients computations.

Conclusions

Thanks for attention!