The XXVI International Scientific Conference of Young Scientists and Specialists

Parton distribution functions of positron in electron in QED

Uliana Voznaya
in collaboration with A.B. Arbuzov

BLTP JINR, Dubna State University

2022

Outline

- Radiative corrections and future colliders
- Parton distribution functions approach
- Convolutons and regularization
- Evolution equation
- Running coupling α
- Splitting functions
- Positron in electron PDFs
- Conclusion

Radiative corrections and future colliders

Calculation of the radiative corrections $\rightarrow$ accurate predictions of the high energy processes: $e^+e^- \rightarrow e^+e^-, \quad e^+e^- \rightarrow \mu^+\mu^-, \quad e^+e^- \rightarrow 2\gamma$

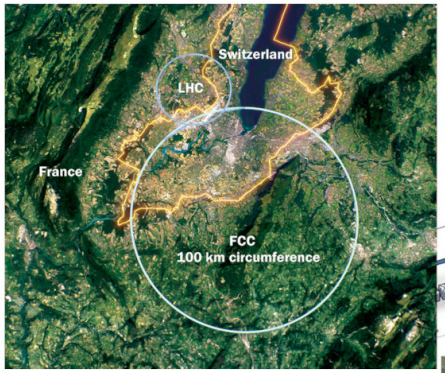
Future e^+e^- colliders

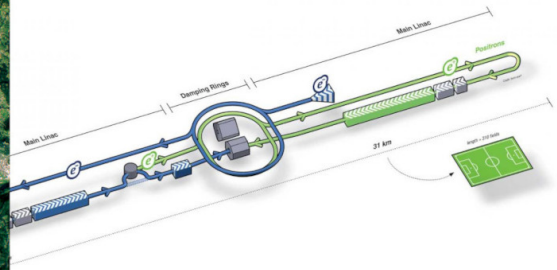
ILC

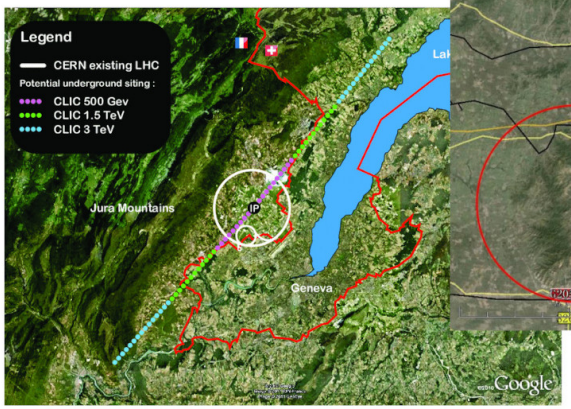
500-1000 GeV

FCC-ee

45-175 GeV







CEPC

240 GeV

CLIC

3 TeV

CLIC

Parton distribution functions approach

DGLAP (Dokschitzer-Gribov-Lipatov-Altarelli-Parisi) equation in QCD

$$\frac{\partial}{\partial \ln Q^2} f_{ih}(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dz}{z} P_{ij}\left(\frac{x}{z}\right) f_{jh}(z, Q^2)$$

G. Altarelli, G. Parisi (1977), V.N. Gribov, L.N. Lipatov (1972), Y.L. Dokshitzer (1977)

Evolution equation of parton distribution functons (PDFs) in QED

E.A. Kuraev, V.S. Fadin (1985)

$$D_{ba}(x, \mu^2, \mu_0^2) = \delta(1-x)\delta_{ba} + \sum_{i=e,\bar{e},\gamma} \int_{\mu_0^2}^{\mu^2} \frac{dt\alpha(s')}{2\pi s'} \int_x^1 \frac{dy}{y} D_{ia}(y, s', \mu_0) P_{bi}\left(\frac{x}{y}\right)$$

$D_{ba}(x, s)$ - parton distribution functions, $P_{bi}(x)$ - splitting functions

Parton distribution functions approach

Parton distribution functions

Function $D_{ba}(x, s)$ describes the probability density to find the massless parton b in the initial (massive) particle a with the of energy fraction x of the initial particle energy.

Splitting functions

$P_{bi}(x)$ describe a perturbative transformation of parton b into parton i which takes the energy fraction x

Parton Distribution Approach

Corrections to the differential cross section of the process:

$$\frac{d\sigma_{e^+e^-}}{ds'} = \frac{1}{s}\sigma^{(0)}\left\{1 + \sum_{k=1}^{\infty} a_0^k \sum_{l=0}^{k} c_{k,l} L^l\right\}$$

$$a_0 = \frac{\alpha_0}{2\pi}, \quad L = \ln\frac{\mu^2}{\mu_0^2}$$

μ and μ_0 are the factorization and renormalization scales
J.Blumlein et al., Nucl. Phys. B 855 (2012), 508-569

$$\frac{d\sigma_{e^+e^-}}{ds'} = D_{\bar{e}\bar{e}} \otimes D_{ee} \otimes \tilde{\sigma}_{\bar{e}e} +$$
$$+ D_{\gamma e} \otimes D_{\bar{e}\bar{e}} \otimes \tilde{\sigma}_{\bar{e}\gamma} + D_{ee} \otimes D_{\gamma\bar{e}} \otimes \tilde{\sigma}_{\gamma e} + D_{\gamma\bar{e}} \otimes D_{\gamma e} \otimes \tilde{\sigma}_{\gamma\gamma}$$

F.A. Berends , W.L. van Neerven, G.J.H. Burgers, Nucl. Phys. B. 297 (1988

Convolutions and regularization

Convolution:

$$F_{ab} \otimes F_{ij} = \int_0^1 \frac{dz}{z} F_{ab}(z) F_{ij}(\frac{x}{z}).$$

Regularization of functions with a pole $z = 1$

$$P_\Delta = -\int_0^{1-\Delta} dz\ P(z),$$

$$P_\Theta = P(z)\Big|_{z<1}.$$

Plus-prescription:

$$[P(z)]_+ = \lim_{\Delta \to 0} \left[P_\Theta(z)\Theta(1-z-\Delta) + \delta(1-z)P_\Delta\right],$$

where $\Theta(x)$ is the Heaviside function.
Delta-part of a convolution:

$$(F_1 \otimes F_2)^\Delta = F_1^\Delta F_2^\Delta - \int_{1-\Delta}^{1} \int_{1-\Delta}^{\frac{1-\Delta}{y}} F_1(y) F_2(x) dx$$

Evolution equation

Iterative method:

$$D_{e\bar{e}}^{(k)} = D_{e\bar{e}}^{(0)} + \frac{\alpha}{2\pi}\left(P_{ee} \otimes D_{e\bar{e}}^{(k-1)} + P_{e\bar{e}} \otimes D_{\bar{e}\bar{e}}^{(k-1)} + P_{e\gamma} \otimes D_{\gamma\bar{e}}^{(k-1)}\right)$$

$$P_{ji}(x) = P_{ji}^{(0)}(x) + \frac{\alpha}{2\pi} P_{ji}^{(1)}(x) + \mathcal{O}(\alpha^2)$$

Initial conditions:

$$D_{ee}^{(0)} = \delta(1-x) + d_{ee}^{(1)}$$

$$D_{\gamma e}^{(0)} = d_{\gamma e}^{(1)}$$

$$d_{ee}^{(1)}(x) = \frac{1+x^2}{1-x}(-1 - 2\ln(1-x))$$

$$d_{\gamma e}^{(1)} = -\frac{1+(1-x)^2}{2x}(2\ln x + 1)$$

$$D_{e\bar{e}}^{(0)}(x, \mu^2) = 0$$

Running coupling α

$$\alpha(q^2) = \frac{\alpha_0}{1 + \overline{\Pi}(\frac{-q^2}{\mu^2}, \frac{\overline{m}}{\mu}, \alpha_0)}$$

$$\overline{\Pi} = 2\alpha_0 \left(\left(\frac{5}{9} - \frac{L}{3} \right) + 4\alpha_0^2 \left(\frac{55}{48} - \zeta_3 - \frac{L}{4} \right) + 8\alpha_0^3 \left(\frac{-L^2}{24} \right) \right) + \dots$$

P. A. Baikov, K. G. Chetyrkin, J. H. Kuhn and C. Sturm, Nucl. Phys. B ***867*** *(2013), 182-202*

Leading logarithmic approximation: $\alpha^k L^k$

Next-to-leading logarithmic approximation: $\alpha^k L^{k-1}$

Splitting functions

3 types of partons e^+, e^- и γ Splitting functons:

$$P_{ee}^{(0)}(x) = \frac{1+x^2}{1-x}$$
$$P_{e\gamma}^{(0)}(x) = x^2 + (1-x)^2$$
$$P_{\gamma e}^{(0)}(x) = \frac{1+(1-x)^2}{x}$$
$$P_{\gamma\gamma}^{(0)}(x) = 0$$
$$P_{e\bar{e}}^{(0)}(x) = 0$$

Because of the charge conjugation:

$$P_{ee}^{(0)}(x) = P_{\bar{e}\bar{e}}^{(0)}(x)$$
$$P_{\bar{e}\gamma}^{(0)}(x) = P_{e\gamma}^{(0)}(x)$$
$$P_{\gamma\bar{e}}^{(0)}(x) = P_{\gamma e}^{(0)}(x)$$
$$D_{ee} = D_{\bar{e}\bar{e}}$$

Splitting functions

$$P_{eg}^{(1)}(x) = C_f T_f \left(4 - 9x - (1-4x)\ln x - (1-2x)\ln^2 x + 4\ln(1-x)\right.$$
$$\left. + \left(2(\ln(1-x) - \ln x)^2 - 4(\ln(1-x) - \ln x) - 4\zeta(2) + 10\right) p_{qg}\right)$$

$$P_{ge}^{(1)}(x) = C_f^2 \left(-\frac{5}{2} - \frac{7}{2}x + \left(2 + \frac{7}{2}x\right)\ln x - \left(1 - \frac{1}{2}x\right)\ln x^2 - 2x\ln(1-x)\right.$$
$$\left. -(3\ln(1-x) + \ln^2(1-x))p_{gq}\right) + C_f \left(-\frac{4}{3}x - \left(\frac{20}{9} + \frac{4}{3}\ln(1-x)\right) p_{gq}\right)$$

$$P_{ee}^{(V1)} = C_f^2 \left(\left(-2\ln x \ln(1-x) - \frac{3}{2}\ln x\right)\frac{1+x^2}{1-x} - \left(\frac{3}{2} + \frac{7}{2}x\right)\ln x -\right.$$
$$\left. -\frac{1}{2}(1+x)\ln^2 x - 5(1-x) + + C_f T_f \left(-\frac{2}{3}\ln x - \frac{10}{9}\frac{1+x^2}{1-x} - \frac{4}{3}(1-x)\right)\right)$$

$$P_{ee}^{(S1)} = C_f T_f \left(\frac{20}{9x} - 2 + 6x - \frac{56}{9}x^2 + (1 + 5x + \frac{8}{3}x^2)\ln x - (1+x)\ln^2 x\right)$$

$$P_{ee}^{(1)} = P_{ee}^{(V1)} + P_{ee}^{(S1)}$$

Splitting functions

$$P^{(1)}_{e\bar{e}}(x) = C_f^2 \left(2\frac{1+x^2}{1+x} S_2(x) + 2(1+x)\ln x + 4(1-x) \right), \quad (1)$$

$$S_2(x) = \int\limits_{\frac{x}{1+x}}^{\frac{1}{1+x}} \frac{dz}{z} \ln \frac{1-z}{z}$$

$$C_f = 1, \; T_f = 1,$$

$$p_{qg} = x^2 + (1-x)^2, \; p_{gq} = \frac{1+(1-x)^2}{x}$$

Positron in electron PDFs

- $P^{(1)}_{e\bar{e}}$ appear only from $D^{(\mathrm{II})}$
- Convolutions with $P^{(1)}_{e\bar{e}}$ appear from $D^{(\mathrm{III})}$
- In cross-section $D_{e\bar{e}}$ give contribution from the order $\alpha^4 L^4$

$$D^{(\mathrm{I})}_{e\bar{e}} = D^{(0)}_{e\bar{e}} + \frac{\alpha}{2\pi}(P_{ee} \otimes D^{(0)}_{e\bar{e}} + P_{e\bar{e}} \otimes D^{(0)}_{ee} + P_{e\gamma} \otimes D^{(0)}_{e\gamma}) = 0$$

$$D^{(\mathrm{II})}_{e\bar{e}}(x,\mu) = \left(\frac{\alpha}{2\pi}\right)^2 L \left(P^{(1)}_{e\bar{e}} + d^{(1)}_{\gamma e} \otimes P^{(0)}_{e\gamma}\right) + \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left(\frac{1}{2} P^{(0)}_{e\gamma} \otimes P^{(0)}_{\gamma\bar{e}}\right)$$

Because of the charge conjugation:

$$P_{ee} = P_{\bar{e}\bar{e}}$$
$$P_{e\bar{e}} = P_{\bar{e}e}$$
$$P_{e\gamma} = P_{\bar{e}\gamma}$$
$$P_{\gamma e} = P_{\gamma\bar{e}}$$

Positron in electron PDFs

$$D_{e\bar{e}}^{(\mathrm{III})}(x,\mu) = \left(\frac{\alpha}{2\pi}\right)^2 L \left(P_{e\bar{e}}^{(1)} + d_{\gamma e}^{(1)} \otimes P_{e\gamma}^{(0)}\right)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left(\frac{1}{2} P_{e\gamma}^{(0)} \otimes P_{\gamma\bar{e}}^{(0)}\right) + \left(\frac{\alpha}{2\pi}\right)^3 L^2 \left(\frac{2}{3} P_{e\bar{e}}^{(1)} + \frac{1}{2} P_{\gamma\bar{e}}^{(0)} \otimes P_{e\gamma}^{(1)}\right.$$
$$-\frac{10}{9} P_{\gamma\bar{e}}^{(0)} \otimes P_{e\gamma}^{(0)} + \frac{1}{2} P_{\gamma\bar{e}}^{(1)} \otimes P_{e\gamma}^{(0)} + \frac{1}{3} d_{\gamma e}^{(1)} \otimes P_{e\gamma}^{(0)} + \frac{1}{2} d_{\gamma e}^{(1)} \otimes P_{\gamma\gamma}^{(0)} \otimes P_{e\gamma}^{(0)}$$
$$+\frac{1}{2} d_{ee}^{(1)} \otimes P_{\gamma\bar{e}}^{(0)} \otimes P_{e\gamma}^{(0)} + \frac{1}{2} P_{\bar{e}\bar{e}}^{(0)} \otimes P_{e\bar{e}}^{(1)} + \frac{1}{2} P_{ee}^{(0)} \otimes P_{e\bar{e}}^{(1)}$$
$$\left.+\frac{1}{2} P_{ee}^{(0)} \otimes d_{\gamma e}^{(1)} \otimes P_{e\gamma}^{(0)}\right) + \left(\frac{\alpha}{2\pi}\right)^3 L^3 \left(\frac{1}{3} P_{e\gamma}^{(0)} \otimes P_{\gamma\bar{e}}^{(0)}\right.$$
$$\left.+\frac{1}{6} P_{e\gamma}^{(0)} \otimes P_{\gamma\bar{e}}^{(0)} \otimes P_{\gamma\gamma}^{(0)} + \frac{1}{6} P_{\bar{e}\bar{e}}^{(0)} \otimes P_{\gamma\bar{e}}^{(0)} \otimes P_{e\gamma}^{(0)} + \frac{1}{6} P_{ee}^{(0)} \otimes P_{\gamma\bar{e}}^{(0)} \otimes P_{e\gamma}^{(0)}\right)$$

Positron in electron PDFs

$$D_{e\bar{e}}^{(\mathrm{I})}(z) = 0$$

$$D_{e\bar{e}}^{(\mathrm{II})}(z) = \left(\frac{\alpha}{2\pi}\right)^2 L \left[1 - \frac{2}{z} + 3z + 2\zeta_2 \frac{1+z^2}{1+z} - 2z^2 + \ln^2(1+z)\frac{1+z^2}{1+z}\right.$$

$$\left. + \ln z \left(-3 - \frac{8}{3z} + 3z + \frac{8}{3}z^2\right) - 2\ln z \ln(1+z)\frac{1+z^2}{1+z} + \ln^2 z(-3-3z)\right]$$

$$+ \left(\frac{\alpha}{2\pi}\right)^2 L^2 \left[\frac{1}{2} + \frac{2}{3z} - \frac{1}{2}z - \frac{2}{3}z^2 + (1+z)\ln z\right]$$

Positron in electron PDFs

$$
\begin{aligned}
D_{e\bar{e}}^{(\mathrm{III})}(z) \quad &= \left(\frac{\alpha}{2\pi^2}\right)^2 L\Bigg[1-\frac{2}{z}+3z-2z^2+2\zeta_2\frac{1+z^2}{1+z}+\ln^2(1+z)\frac{1+z^2}{1+z} \\
&+\ln z\left(-3-\frac{8}{3z}+3z+\frac{8}{3}z^2\right)-2\ln z\ln(1+z)\frac{1+z^2}{1+z}+\ln^2 z(-3-3z)\Bigg] \\
&+\left(\frac{\alpha}{2\pi^2}\right)^2 L^2\left[\frac{1}{2}+\frac{2}{3z}-\frac{1}{2}z-\frac{2}{3}z^2+(1+z)\ln z\right]+\left(\frac{\alpha}{2\pi^2}\right)^3 L^2\Bigg[\frac{169}{36}+\frac{59}{27z}-\frac{97}{36}z \\
&-\frac{113}{27}z^2-4z\ln(2)+(2+2z)\ln^2(2)-\frac{2}{3}\frac{1+z^2}{1+z}\ln^3(2)+\zeta_3\left(-\frac{35}{4}+\frac{22}{1+z}+\frac{35}{4}z\right) \\
&+\zeta_2\left(\frac{22}{3}-\frac{8}{3z}+\frac{26}{3(1+z)}+\frac{19}{3}z+\frac{8}{3}z^2-4\ln(2)\frac{1+z^2}{1+z}\right)-\left(\frac{8}{1+z}+4z\right)\mathrm{S}_{1,2}(1-z) \\
&+\mathrm{Li}_2(1-z)\Bigg\{\frac{2}{3}+\frac{2}{3}z+\frac{4}{3}z^2+\ln(1+z)\left(6-\frac{16}{1+z}-6z\right)+\ln(1-z)(-2-2z) \\
&+(-6-6z)\ln z\Bigg\}+4\Bigg\{\left(\mathrm{Li}_2\left(\frac{1+z}{2}\right)+\mathrm{Li}_2\left(\frac{2z}{1+z}\right)\right)(-\ln(1+z)+\ln z) \\
&+\mathrm{Li}_2\left(\frac{2z}{1+z}\right)\ln(2)\Bigg\}\frac{1+z^2}{1+z}+4\big(\mathrm{Li}_2(-z)+\mathrm{Li}_2(z)\big)\left(1+\ln(1+z)\left(1-\frac{4}{1+z}-z\right)+\ln z(1-z)\right) \\
&+(2+2z)\mathrm{Li}_3(1-z)+\mathrm{Li}_3(1+z)\left(2-\frac{8}{1+z}-2z\right)+\left(\mathrm{Li}_3(z)+\mathrm{Li}_3(-z)\right)(-2+2z) \\
&-4\left(\mathrm{Li}_3\left(\frac{z}{1+z}\right)+\mathrm{Li}_3\left(\frac{2z}{1+z}\right)\right)\frac{1+z^2}{1+z}+\ln(1+z)\Bigg(2+2z+4\frac{1+z^2}{1+z}\ln^2(2) \\
&+\left(-10+\frac{28}{1+z}+10z\right)\zeta_2\Bigg)+\ln^2(1+z)\left(-\frac{25}{6}+\frac{13}{3(1+z)}+\frac{1}{6}z+4\ln(2)\frac{1+z^2}{1+z}\right) \\
&+\frac{4}{3}\ln(1+z)^3\frac{1+z^2}{1+z}+\ln(1-z)\Bigg(\frac{1}{2}-\frac{2}{3z}+\frac{3}{2}z-\frac{4}{3}z^2+2\ln^2(2)\frac{1+z^2}{1+z}+4\frac{1+z^2}{1+z}\zeta_2\Bigg) \\
&+\ln^2(1-z)\left(-\frac{1}{2}-\frac{2}{3z}+\frac{1}{2}z+\frac{2}{3}z^2\right)+\ln z\Bigg(\frac{425}{36}-\frac{1}{36}z+\frac{28}{9}z^2+2\ln^2(2)\frac{1+z^2}{1+z} \\
&+\left(4-\frac{12}{1+z}-12z\right)\zeta_2\Bigg)+\ln z\ln(1+z)\left(\frac{25}{3}-\frac{26}{3(1+z)}-\frac{1}{3}z-4\ln(2)\frac{1+z^2}{1+z}\right) \\
&+\ln z\ln^2(1+z)\left(5-\frac{12}{1+z}-5z\right)+\ln z\ln(1-z)\left(-\frac{4}{3}-\frac{8}{3z}+\frac{8}{3}z+4z^2\right) \\
&+\ln z\ln(1-z)\ln(1+z)\left(6-\frac{16}{1+z}-6z\right)+(-1-z)\ln z\ln^2(1-z)+\ln^2 z\left(-\frac{1}{6}-\frac{14}{3}z-4z^2\right) \\
&+\ln^2 z\left(2\ln(1+z)\frac{1+z^2}{1+z}+\ln(1-z)(-6+\frac{4}{1+z}-4z)\right)+\frac{7}{6}\ln^3 z(1+z)\Bigg] \\
&+\left(\frac{\alpha}{2\pi^2}\right)^3 L^3\Bigg[-\frac{19}{18}+\frac{8}{27}+\frac{19}{18}z-\frac{8}{27}z^2+\left(\frac{4}{3}z+\frac{4}{3}\right)\mathrm{Li}_2(1-z) \\
&+\ln(1-z)\left(\frac{8}{9z}+\frac{2}{3}-\frac{2}{3}z-\frac{8}{9}z^2\right)+\ln z\left(\frac{4}{9}+\frac{10}{9}z+\frac{8}{9}z^2\right) \\
&+\ln z\ln(1-z)\left(\frac{4}{3}+\frac{4}{3}z\right)+\left(-\frac{1}{3}-\frac{1}{3}z\right)\ln^2 z\Bigg]
\end{aligned}
$$

Summary

- PDFs in QED appear in parton distribution function approach used for calculation of radiative corrections
- Precise calculation of radiative corrections are important for accurate prediction of high energy processes
- PDFs and splitting functions are independent of the process
- We solve evolution equation of PDFs using the iterative method
- PDFs of positron in electron type appear from the order $\alpha^4 L^4$ in the cross-section
- Planning to apply the results to particular processes: $e^+e^- \to e^+e^-, \quad e^+e^- \to \mu^+\mu^-, muondecay$

References

- J. Ablinger, J. Blümlein, A. De Freitas and K. Schönwald, Nucl. Phys. B **955** (2020), 115045
- J. Blumlein, A. De Freitas and W. van Neerven, Nucl. Phys. B **855** (2012), 508-569
- P. A. Baikov, K. G. Chetyrkin, J. H. Kuhn and C. Sturm, Nucl. Phys. B **867** (2013), 182-202
- A. Arbuzov and K. Melnikov, Phys. Rev. D **66** (2002)
- W. Furmanski and R. Petronzio, Phys. Lett. B **97** (1980), 437-442
- R. K. Ellis and W. Vogelsang, [arXiv:hep-ph/9602356 [hep-ph]].
- Арбузов, А.Б. Ведущее и следующее за ведущим логарифмические приближения в КЭД : дис. ...д-ра физ.-мат. наук: 01.04.02 /Арбузов Андрей Борисович. Дубна, 2010. - 215 с.
- Berends:1987ab F. A. Berends, W. L. van Neerven and G. J. H. Burgers, Nucl. Phys. B **297** (1988), 429 [erratum: Nucl. Phys. B **304** (1988), 921]

Thank you for your attention!