Analysis of composite operators of the dynamic model A

Davletbaeva D.A.[1*], Hnatic M.[2], Mizisin L.[2], Nalimov M.Yu.[1]

[1] Saint-Petersburg State University, Russia

[2] Joint Institute for Nuclear Research, Russia

*Email: st064737@student.spbu.ru

Our work is devoted to the description of the dynamic critical properties of a superfluid, that is, the influence of equilibrium fluctuations on critical behavior. We interested in the critical dimension of viscosity of λ-point during the transition of the system to the superfluid state.

The analysis carried out in [2] led to the fact that the dynamics of critical behavior in the vicinity of the λ-point is described by a model equivalent to the A-model of stochastic dynamics with action $S_A = -\alpha\phi'^2 + \phi'(\partial_t\phi - \alpha\Delta\phi + \alpha\tau\phi + \frac{\alpha g}{6}\phi^3)$ with $\tau = 0$ and with the rescaling of the fields and parameters.

It was found out that the critical dimension of viscosity is determined by the most significant critical dimension of composite operators of dimension 8. We consider operators at zero pulses and frequencies. During renormalization, operators of dimensions 2, 4, 6 are mixed with them, the missing dimension is compensated by degrees of τ.

The critical dimension of the operator F – Δ_F is the sum of the canonical dimension and anomalous dimension:

$$\Delta_F = d_F + \gamma_F$$

The critical dimension of the corresponding source is $d + z - \Delta_F$.

Due to spatial and temporal invariance of action [1], consider independent spatial d^p and temporal d^ω canonical dimensions: $d^p_x = d^\omega_t = -1$, $d^p_t = d^\omega_x = 0$.

The action of the model determines the dispersion relation: $i\omega \sim k^2$.Therefore, the full canonical dimension is $d_F = 2d^\omega_F + d^p_F$.

In the massless renormalization scheme [1], operators of the same canonical dimension are mixed during renormalization. The normalization formula has the form

$$F_i(Z_\Phi \Phi) = Z_{ik} F^R_k(\Phi) \tag{1}$$

Here Φ is the whole set of fields.

1 Composite operators of dimension 2

The only operator of dimension 2 is ϕ^2. From the correspondence of the critical behavior of dynamic models with a known static limit and the corresponding static models, the critical dimension can be calculated in statics [1]. Recall the corresponding section from [1]. Consider the generating functional of renormalized Green functions in static ϕ^4 theory:

$$G(A) = \int D\varphi e^{-S_{st}}$$

$$S_{st} = \frac{1}{2} Z^2_\varphi \partial\varphi\partial\varphi + \frac{\tau}{2} Z^2_\varphi Z_\tau \varphi^2 + \frac{1}{4!} Z^4_\varphi g\mu^\epsilon Z_g \varphi^4 - \varphi A \tag{2}$$

This functionality produces only UV final diagrams. So

$$\int D\varphi \frac{1}{2} Z^2_\varphi Z_\tau \varphi^2 e^{-S_{st}} = finite$$

Substituting here from (1) $Z^2_\varphi \varphi^2 = Z_{\varphi^2}[\varphi^2]^R$, in MS schema we obtain $Z_\tau Z_{\varphi^2} = 1$. It follows that $\Delta_{\varphi^2} = d - \Delta_\tau = d - 1/\nu$. Concerning the critical dimension of viscosity, we are interested in the dimension of the composite operator $\tau^3\varphi^2$, $\Delta\tau = 1/\nu$, the critical dimension of the source is $\Delta_1 = d + z - 3\Delta_\tau - \Delta_{\varphi^2} = z - 2/\nu$.

2 Composite operators of dimension 4

Composite operators of dimension 4 include $\varphi\Delta\varphi$, φ^4 and $\varphi'\varphi$. According to the structure of dynamic diagrams, static $\varphi\Delta\varphi$, φ^4 do not mix with dynamic $\varphi'\varphi$. The dimension of $\varphi'\varphi$ can be calculated similarly to the dimension of φ^2: $\Delta_{\varphi\varphi'} = d + z - \Delta_\tau - \Delta_\alpha = d - 2 + 2z - 1/\nu$. Critical dimension of the source of $\tau^2\varphi\varphi'$ is

$$\Delta_4 = d + z - 2\Delta_\tau - \Delta_{\varphi\varphi'} = 2 - z - 1/\nu$$

The critical dimensions of $\varphi\Delta\varphi$ and φ^4 can be established from consideration of purely static theory. Consider the Schwinger equation for (2) at $\tau = 0$ (according to the rules of dimensional regularization [1] $\delta(0) = 0$) and derivative (∂_g) of the generating functional of renormalized functions with inserts of compound operators.

$$\int D\varphi\varphi \frac{\delta}{\delta\varphi} e^{-S_{st}(\tau=0)} = \int D\varphi (Z^2_\varphi \partial\varphi\partial\varphi + \frac{1}{3!} Z^4_\varphi g\mu^\epsilon Z_g \varphi^4 - \varphi A) e^{-S_{st}} = fin. \tag{3}$$

$$\int D\varphi \partial_g e^{-S_{st}(\tau=0)} = \int D\varphi (\frac{1}{2}\partial_g Z^2_\varphi \partial\varphi\partial\varphi + \frac{1}{4!}\partial_g (Z^4_\varphi g\mu^\epsilon Z_g)\varphi^4) e^{-S_{st}} = fin. \tag{4}$$

Substituting in (3,4) the relations (1), $Z_{i,k} = \delta_{i,k} + [Z_{i,k}]/\epsilon$ and choosing the first order by $\frac{1}{\epsilon}$ we obtain a system of equations. System solution is: $[Z_{\varphi^4,\partial\varphi\partial\varphi}] = -4!\partial_g[Z_\varphi]$, $[Z_{\partial\varphi\partial\varphi,\partial\varphi\partial\varphi}] = 4g\partial_g[Z_\varphi]$, $[Z_{\varphi^4,\varphi^4}] = -[Z_g] - g\partial_g[Z_g] - 4g\partial_g[Z_\varphi]$, $[Z_{\partial\varphi\partial\varphi,\varphi^4}] = g^2\partial_g[Z_g]/6 + 2g^2\partial_g[Z_\varphi]/3$.

Matrix of critical dimensions was calculated in six-loop approximation in [3]. In the first nontrivial order of perturbation theory it is:

$$\begin{pmatrix} d - 16b_2 g^2_* & -g_*\epsilon/3 \\ 48b_2 g_* & 2d - 4 + 2\epsilon \end{pmatrix}$$

Here $d = 4-\epsilon$, $Z_g = 1 + a_2 g/\epsilon$, $g_* = \epsilon/a_2$, $Z_\varphi = 1 + b_2 g^2/\epsilon$, $a_2 = \lambda(1-ib)$, $b_2 = \lambda u$, λ is the kinetic coefficient, b – inter-mode coupling constant, u – the nonperturbative charge of the model F.

The eigenvalues of this matrix: $4-\epsilon$, $4 - 16b_2 g^2_*$. Corresponding operators of dimension 8 are $\mu^2\partial\varphi\partial\varphi$ and $\mu^2\varphi^4$. Critical dimensions of sources are

$$\Delta_2 = z - \frac{2}{\nu}, \quad \Delta_3 = z + \epsilon - \frac{2}{\nu} + 16b_2 g^2_*$$

3 Composite operators of dimension 6

Composite operators of dimension 6 include ϕ'^2,$\phi'\partial_t\phi$, $\phi'\Delta\phi$, $\phi'\phi^3$,$\phi\partial_t\phi^3$, $\phi\partial^2_t\phi$, $\phi\partial_t\Delta\phi$, ϕ^6, $\phi\Delta\phi^3$, $\phi\Delta^2\phi$. Operators $\phi\partial_t\phi^3$, $\phi\partial_t\Delta\phi$ turn to zero after integration by x and t, therefore, they may be excluded from consideration.

For φ'^2, $\varphi'\partial_t\phi$, $\varphi'\Delta\phi$, $\varphi'\phi^3$, $Z = 1 + [Z]/\epsilon$, we can write a system of Schwinger equations and derivatives of renormalized parameters from the generating functional of renormalized functions similarly to (3,4) for massless theory ($\tau = 0$). In the first nontrivial order of perturbation theory, in the first order by $\frac{1}{\epsilon}$ the matrix of critical dimensions of these composite operators is:

$$\begin{pmatrix} d+2 & 0 & 0 & 0 \\ 0 & d+z & 0 & 0 \\ 0 & & d+2 & 0 \\ 0 & 0 & 0 & 2d-2+\epsilon \end{pmatrix}$$

The corresponding critical dimensions of the sources are

$$\Delta_5 = z - 2 - 1/\nu, \quad \Delta_6 = -1/\nu, \quad \Delta_7 = z - 2 - 1/\nu$$

In future we intend to calculate calculate the critical dimensions of composite operators of dimension 8.

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