DUALIZATIONS FOR THE SPIN TWO FIELDS

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The general procedure for including Stueckelberg fields with reducible gauge symmetry allows us to construct higher derivative dual formulations for the same spin. By the choice of gauge fixing conditions in the Stueckelberg action, we can switch between them.

Reducible Stueckelberg symmetry

Involutive closure:

$$\partial_i S(\phi)=0\,,\quad \tau_\alpha(\phi)=0\,. \tag{1}$$

Gauge identities:

$$\Gamma_\alpha{}^i(\phi)\partial_i S(\phi)+\tau_\alpha(\phi)\equiv 0\,. \tag{2}$$

Identities between $\tau_\alpha(\phi)$:

$$Z_A{}^\alpha(\phi)\tau_\alpha(\phi)\equiv 0\,;\quad Z_{1A_1}{}^A(\phi)Z_A{}^\alpha(\phi)\equiv 0\,. \tag{3}$$

Stueckelberg action:

$$S_{St}(\phi,\xi)=\sum_k S_k\,,\quad S_0(\phi)=S(\phi)\,,\\ S_k(\phi,\xi)=W_{\alpha_1\dots\alpha_k}(\phi)\xi^{\alpha_1}\dots\xi^{\alpha_k}\,,\quad k>0\,, \tag{4}$$

where the first order on Stueckelberg fields is defined by

$$W_\alpha(\phi)=\frac{\delta S_{St}}{\delta\xi^\alpha}\Big|_{\xi=0}=\tau_\alpha(\phi)\,. \tag{5}$$

Stueckelberg action is gauge-invariant,

$$\delta_\epsilon S_{St}(\phi,\xi)\equiv 0\,,\quad \forall\,\epsilon^\alpha\,,\,\epsilon^A\,. \tag{6}$$

Reducible gauge symmetry transformations:

$$\delta_\epsilon\phi^i=\Gamma^i{}_\alpha\epsilon^\alpha+\dots\,,\quad \delta_\epsilon\xi^\alpha=\epsilon^\alpha+Z^\alpha{}_A\epsilon^A+\dots\,; \tag{7}$$

$$\delta_\omega\epsilon^\alpha=Z^\alpha{}_A\omega^A+\dots\,,\quad \delta_\omega\epsilon^A=-\omega^A+Z_1{}^A{}_{A_1}\omega^{A_1}\,; \tag{8}$$

$$\delta_\eta\omega^A=Z_1{}^A{}_{A_1}\eta^{A_1}+\dots\,;\quad \delta_\eta\omega^{A_1}=\eta^{A_1}+\dots\,. \tag{9}$$

Massive spin 1

Proca model in $d=4$ Minkowski space:

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{m^2}{2}A_\mu A^\mu\,,\quad F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\,. \tag{10}$$

Involutive closure:

$$\Box A_\mu-\partial_\mu\partial^\nu A_\nu+m^2A_\mu=0\,;\\ \tau\equiv m^2\partial_\mu A^\mu=0\,;\quad \tau_{\mu\nu}\equiv\frac{1}{2}(\Box+m^2)F_{\mu\nu}=0\,. \tag{11}$$

Stueckelberg action:

$$S_{St}=-\frac{1}{2}\int d^4x\Big[\partial_\mu A_\nu F^{\mu\nu}+\partial_\mu\partial^\lambda\xi_{\nu\lambda}(\partial^\mu\partial_\rho\xi^{\nu\rho}+2\partial^\mu A^\nu)\\ -m^2\big((A_\mu A^\mu+\partial_\mu\xi\partial^\mu\xi+\partial^\nu\xi_{\mu\nu}\partial_\lambda\xi^{\mu\lambda})+2A_\mu(\partial^\mu\xi+\partial_\nu\xi^{\mu\nu})\big)\Big]\,.$$

Reducible gauge symmetry transformations:

$$\delta_\epsilon A^\mu=-\partial^\mu\epsilon-\partial_\nu\epsilon^{\mu\nu}\,,\ \delta_\epsilon\xi=\epsilon\,,\ \delta_\epsilon\xi^{\mu\nu}=\epsilon^{\mu\nu}+\varepsilon^{\mu\nu\lambda\rho}\partial_\lambda\epsilon_\rho\,;\\ \delta_\omega\epsilon=0\,,\ \delta_\omega\epsilon^\mu=-\omega^\mu-\partial^\mu\omega\,;\ \delta_\eta\omega=\eta\,,\ \delta_\eta\omega^\mu=-\partial^\mu\eta\,. \tag{12}$$

Gauge-fixing

- $\xi=0\,,\quad \xi^{\mu\nu}=0$:

$$\Box A_\mu-\partial_\mu\partial^\nu A_\nu+m^2A_\mu=0\,; \tag{13}$$

- $\xi=0\,,\quad A_\mu=0\,,\quad \varepsilon_{\mu\nu\lambda\rho}\partial^\nu\xi^{\lambda\rho}=0\,;$

$$(\Box+m^2)\partial^\nu\xi_{\mu\nu}=0\,. \tag{14}$$

Massive spin 2

Massive linearized gravity in $d=4$ Minkowski space:

$$\mathcal{L}=\frac{1}{2}\big(h\Box h-2h^{\mu\nu}\partial_\mu\partial_\nu h-h^{\mu\nu}\Box h_{\mu\nu}+2h^{\mu\nu}\partial_\nu\partial^\lambda h_{\mu\lambda}\\ -m^2(h^{\mu\nu}h_{\mu\nu}-h^2)\big)\,,\quad h=\eta_{\mu\nu}h^{\mu\nu}\,. \tag{15}$$

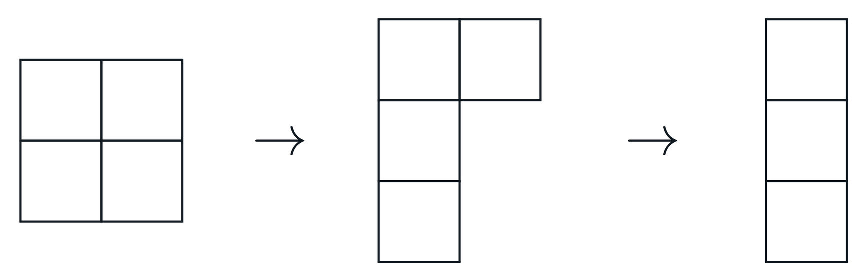
Involutive closure:

$$(\Box+m^2)h_{\mu\nu}=0\,;\quad \partial^\nu h_{\mu\nu}=0\,;\quad h=0\,. \tag{16}$$

Dual formulation:

$$(\Box+m^2)\partial^\lambda\partial^\rho\xi_{\mu\nu\lambda\rho}=0\,. \tag{17}$$

The equations are gauge-invariant under reducible gauge symmetry transformations with parameters



Massless spin 2

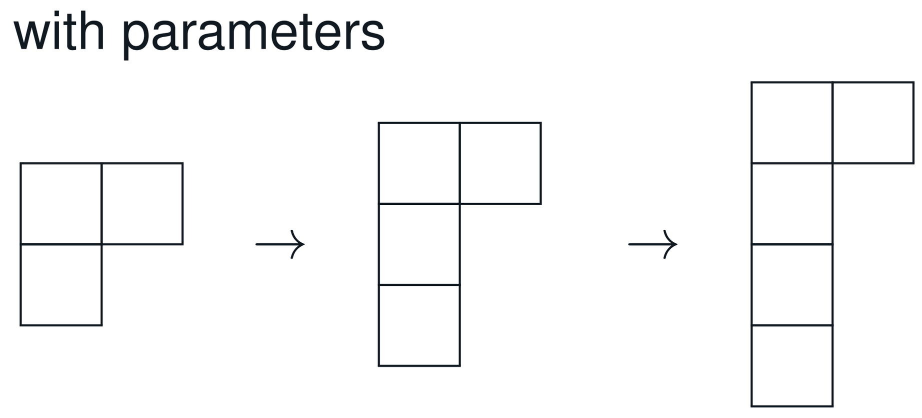
Einstein linearized gravity in $d=4$ Minkowski space:

$$\mathcal{L}=\frac{1}{4}\big(\partial_\mu h_{\nu\lambda}\partial^\mu h^{\nu\lambda}+2\partial^\mu h\partial^\nu h_{\mu\nu}-2\partial^\mu h^{\nu\lambda}\partial_\lambda h_{\mu\nu}-\partial_\mu h\partial^\mu h\big)\,. \tag{18}$$

Dual formulation:

$$\partial^\lambda\big[\partial_{(\mu}\partial^\rho H_{\nu)\lambda\rho}-\Box H_{\mu\nu\lambda}-\eta^{\alpha\beta}\big(\partial_\mu\partial_\nu-\Box\big)H_{\alpha\beta\lambda}\big]=0\,. \tag{19}$$

The equations are gauge-invariant under reducible gauge symmetry transformations with parameters



Degrees of freedom (DoF) count

$$\mathcal{N}_{DoF}=\frac{1}{2}\sum_n n\big(t_n-\sum_m(-1)^m(l_n^m+r_n^m)\big)\,, \tag{20}$$

where t_n is the number of equations of order n, and l_n^m and r_n^m are the numbers of gauge identities and gauge symmetries of total order n and order of reducibility m, respectively.

- for massive spin 2 $t_4=9$, $l_5^0=4$, $r_2^0=9$, $r_3^1=4$:

$$\mathcal{N}_{DoF}=\frac{1}{2}\big(4\cdot 9-5\cdot 4-2\cdot 9+3\cdot 4\big)=5$$

- for massless spin 2 $t_3=9$, $l_4^0=4$, $r_1^0=15$, $r_2^1=4$:

$$\mathcal{N}_{DoF}=\frac{1}{2}\big(3\cdot 9-4\cdot 4-1\cdot 15+2\cdot 4\big)=2$$

References

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3. V. A. Abakumova, D. Frolovsky, *et al.* EPJ C, 2022