E-models of inflation towards describing formation of primordial black holes

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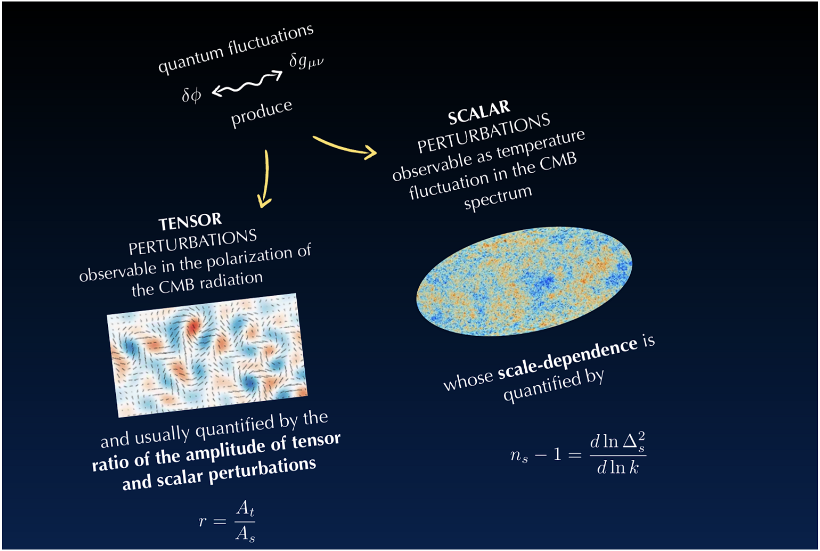
Motivation

Formation of primordial black holes (PBH):

- PBH are non-particle candidates of dark matter, as well as candidates for gravitational wave events. Also they constraint other dark matter models.
- Our aim is to get a single-field model that describes the formation of primordial black holes and keeps successes of inflation and the standard cosmology.

What inflation model should we consider for it?

- The Starobinsky model (1980) perfectly fits current observations of the CMB radiation, but does not lead to PBH production, so we should consider more general inflation model.
- We need model with double inflation for large scalar perturbations collapsing to PBH later.


quantum fluctuations
$\delta\phi$
$\delta g_{\mu\nu}$
produce
SCALAR
PERTURBATIONS
observable as temperature
fluctuation in the CMB
spectrum
TENSOR
PERTURBATIONS
observable in the polarization of
the CMB radiation
and usually quantified by the
ratio of the amplitude of tensor
and scalar perturbations
$r = \frac{A_t}{A_s}$
whose **scale-dependence** is
quantified by
$n_s - 1 = \frac{d \ln \Delta_s^2}{d \ln k}$

Starobinsky model

Current precision measurements of the CMB spectral tilt n_s of scalar perturbations and tensor-to-scalar ratio r:

$$\begin{aligned} n_S &= 0.9649 \pm 0.0042 \qquad && (68\%\,\text{C.L.})\,; \\ r &< 0.0036 && (95\%\,\text{C.L.})\,. \end{aligned} \tag{1}$$

Up to an uncertainty in the duration of inflation measured by the number of e-folds:

$$r_s \approx \frac{12}{N_e^2}\,, \quad \text{where} \quad N_e = \int_{t_{\text{in}}}^{t_{\text{end}}} H(t)dt\,, \tag{2}$$

with $H(t)$ being the Hubble function, and N_e is expected at 55 ± 10.

This estimate comes from the predicted value of n_s in the Starobinsky model via the Mukhanov-Chibisov formula

$$n_s \approx 1 - \frac{2}{N_e}\,. \tag{3}$$

The scalar potential of the canonical inflaton field ϕ in the Starobinsky model reads

$$V_S(\phi) = \frac{3}{4} M_{\rm PL}^2 M^2 (1 - y_S)^2 \,, \tag{4}$$

where we introduce the dimensionless field

$$y_S = \exp\left(- \sqrt{\frac{2}{3}} \frac{\phi}{M_{\rm PL}} \right) . \tag{5}$$

Here, $M_{\rm Pl} \sim 10^{18}$ GeV is the Planck mass, and $M \sim 10^{-5} M_{\rm Pl}$.

The E-model is simple generalisation of Starobinsky model with a new variable

$$y = \exp\left(- \sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{\rm PL}} \right) . \tag{6}$$

It leads to significantly change of the tilt r ,

$$r \approx \frac{12\alpha}{N_e^2} \,. \tag{7}$$

The inflationary observables for CMB will be essentially the same after a generalization of the scalar potential to

$$V(\phi) = \frac{3}{4} M_{\mathrm{PL}}^2 M^2 \left[1 - y + y^2 \zeta(y)\right]^2 , \tag{8}$$

where $\zeta(y)$ is a function regular at $y = 0$.

An opportunity of changing the inflaton potential by arbitrary function $\zeta(y)$ can be exploited in order to generate PBH.

Technically, the PBH production can be engineered by demanding a near-inflection point in the potential within the double inflation scenario.

The PBHs formation in the very early Universe should lead to a stochastic background of gravitational waves (GW) at present.

The frequency of those GW can be estimated as

$$f_{GW} \approx \left(\frac{M_{\mathrm{PBH}}}{10^{16} g}\right)^{-1/2} \mathrm{Hz} \, . \tag{9}$$

The model

Let us consider the following potential of the canonical inflaton ϕ:

$$V(\phi) = \frac{3}{4} M_{\mathrm{PL}}^2 M^2 \left[1 - y + y^2(\beta - \gamma y)\right]^2 , \qquad (10)$$

where

$$y = \exp\left(-\sqrt{\frac{2}{3\alpha}}\frac{\phi + \phi_0}{M_{\mathrm{PL}}}\right) ; \qquad (11)$$

$$\beta = \frac{1}{1 - \xi^2} \exp\left(\sqrt{\frac{2}{3\alpha}}\frac{\phi_i + \phi_0}{M_{\mathrm{PL}}}\right), \ \gamma = \frac{1}{3(1 - \xi^2)} \exp\left(2\sqrt{\frac{2}{3\alpha}}\frac{\phi_i + \phi_0}{M_{\mathrm{PL}}}\right) .$$

If $\xi = 0$, V has an inflection point at ϕ_i, and if $0 < \xi \ll 1$, V has a local minimum and maximum,

$$y_{ext}^{\pm} = y_i(1 \pm \xi) . \qquad (12)$$

The shape of potential $V(\phi)$ for selected values of inflaton field ϕ:

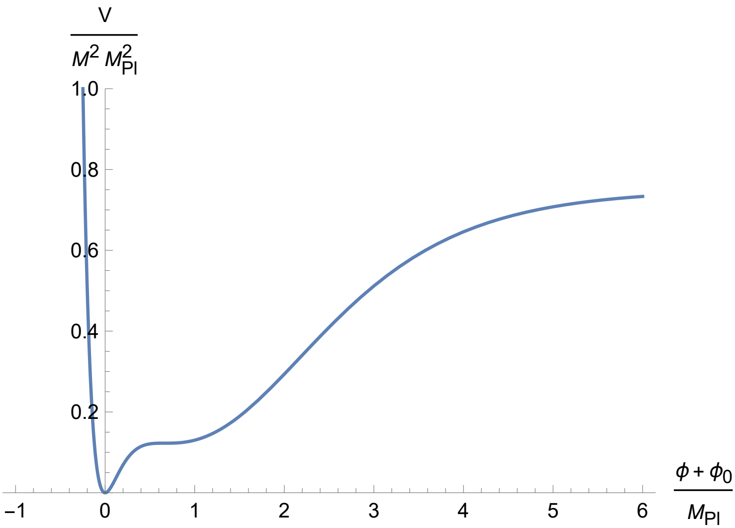


Figure: 1

The (running) number of e-folds in the slow-roll approximation:

$$N_e = \int_t^{t_{\text{end}}} H(t)dt \approx \frac{1}{M_{\text{PL}}^2} \int_{\phi_{end}}^{\phi} \frac{V(\phi)}{V'(\phi)} d\phi \,. \tag{13}$$

The standard slow-roll parameters:

$$\epsilon = \frac{M_{\text{PL}}^2}{2} \left(\frac{V'(\phi)}{V(\phi)} \right)^2 , \quad \eta = M_{\text{PL}}^2 \frac{V''(\phi)}{V(\phi)} \,. \tag{14}$$

It yields

$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{2}{N_e} + a \frac{3 \ln N_e}{2N_e^2} + \frac{b}{N_e^2} \,, \quad \text{and} \quad r = \frac{12\alpha}{N_e^2} \,, \tag{15}$$

where

$$\begin{aligned} a &= \alpha \left[1 - 2 \exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_i}{M_{\text{PL}}} \right) \right] , \\ b &= \frac{\alpha}{2} \left\{ \left[1 - 2 \exp \left(\sqrt{\frac{2}{3\alpha}} \frac{\phi_i}{M_{\text{PL}}} \right) \right] \ln \frac{4}{3\alpha} - 3 \right\} . \end{aligned} \tag{16}$$

Double inflation

The equations of motion are given by

$$\begin{aligned} &\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\,; \\ &H^2 = \frac{1}{3M_{\rm Pl}^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right); \\ &\dot{H} = -\frac{1}{2M_{\rm Pl}^2}\dot{\phi}^2\,, \end{aligned} \tag{17}$$

where $\phi_{\rm in} + \phi_0 = \phi(0) = 5.938 \cdot M_{\rm Pl}$, $\phi'(0) = 0$.

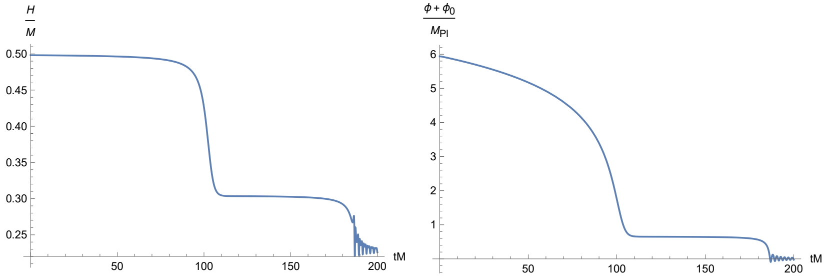


Figure: 2

Power spectrum of perturbations and PBH masses

The standard formula for the power spectrum of scalar perturbations in the slow-roll approximation:

$$P_R(t) = \frac{H^2(t)}{8M_{\rm Pl}^2\pi^2\epsilon(t)}\,, \quad \epsilon(t) = -\frac{\dot{H}}{H^2}\,, \quad k = aH = \dot{a}\,. \tag{18}$$

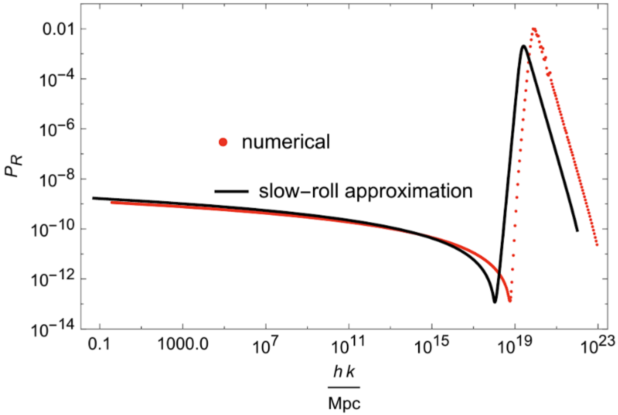


Figure: 3

$$M_{\rm PBH} \simeq \frac{M_{\rm Pl}^2}{H(t_{\rm peak})} \exp\left[2(N_{\rm total} - N_{\rm peak}) + \int_{t_{\rm peak}}^{t_{\rm end}} \epsilon(t) H(t) dt\right] \tag{19}$$

n_s	r	α	ξ	$\phi_i + \phi_0$	ΔN	$M_{\rm PBH}$
0,95452	0,00307	0,5	0,0102	0,606	20,62	$1,06 \cdot 10^{19}$ g
0,95491	0,00360	0,6	0,0106	0,633	20,93	$1,04 \cdot 10^{19}$ g
0,95658	0,00409	0,74	0,0122	0,664	18,76	$1,89 \cdot 10^{17}$ g
0,95672	0,00439	0,8	0,0115	0,677	19,23	$7,75 \cdot 10^{17}$ g
0,95650	0,00496	0,9	0,0111	0,696	18,99	$8,84 \cdot 10^{17}$ g

The values of $n_s > 0.9565$ are in good agreement with CMB observations at the 95% C.L.

The values of the tensor-to scalar ratio r are well inside the current observational bound.

Conclusion

- We modified the scalar potential of single-field E-models for double inflation and PBH production.
- The PBH can have masses $10^{17} - 10^{19}$ g, so that they can also survive in the present universe and may form part of cold dark matter (CDM).
- Our results agree with the current measurements of cosmic microwave background radiation but require fine-tuning of the parameters.
- GW from PBH formation may be detectable by the future space-based gravitational interferometers such as LISA, TAIJI, TianQin and DECIGO.

THANK YOU FOR YOUR ATTENTION!