Charge gap in SU(3) Yang-Mills with nonlinear spinor field

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Summary

- The equations of non-Abelian SU(3) Yang-Mills theory with a source in the form of a nonlinear spinor field have regular particlelike solutions with non-Abelian $SU(2) \subset SU(3)$ magnetic and Abelian $U(1) \subset SU(3)$ electric fields.
- Asymptotically, the color electric field exhibits the Coulomb behavior; this enables one to introduce the corresponding charge.
- The asymptotic behavior of the color magnetic field is the same as that of a magnetic dipole in Maxwell's electrodynamics - enables to determine the color magnetic moment.
- The profiles of the color charge and magnetic moment have global minima – charge gap.

Introduction

- The particlelike solutions can be configured in SU(3) Yang-Mills theory with color electric and magnetic fields created by a nonlinear spinor field.
- It can be shown that the electric field expresses the Coulomb asymptotic behavior, whereas one of color components of the magnetic field behaves asymptotically like the field of a magnetic dipole.
- The corresponding charge and magnetic moment can be determined
- The profiles of the color charge and magnetic moment have global minimum, which may be called charge and magnetic moment gaps
- The relationship between the total energy of the system and the color charge is provided

Methods

- The Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\hbar c\bar{\psi}\gamma^\mu D_\mu\psi - m_f c^2\bar{\psi}\psi + \frac{l_0^2}{2}\hbar c\left(\bar{\psi}\psi\right)^2 .$$

- The corresponding field equations:

$$\begin{aligned} D_\nu F^{a\mu\nu} &= j^{a\mu} = \frac{g\hbar c}{2}\bar{\psi}\gamma^\mu\lambda^a\psi, \\ i\hbar\gamma^\mu D_\mu\psi - m_f c\psi + l_0^2\hbar\psi\left(\bar{\psi}\psi\right) &= 0. \end{aligned}$$

- The Ansätze:

$$A^a_i = \frac{1}{g}\left[1 - f(r)\right]\begin{pmatrix} 0 & \sin\varphi & \sin\theta\cos\theta\cos\varphi \\ 0 & -\cos\varphi & \sin\theta\cos\theta\sin\varphi \\ 0 & 0 & -\sin^2\theta \end{pmatrix},$$

$$A^8_t = \frac{\chi(r)}{gr},$$

$$i = r, \theta, \varphi \text{ (in spherical coordinates)}, \quad a = 1, 2, 3,$$

$$\psi^T = \frac{e^{-i\frac{Et}{\hbar}}}{gr\sqrt{2}}\left\{\begin{pmatrix} 0 \\ -u \\ 0 \end{pmatrix}, \begin{pmatrix} u \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} iv\sin\theta e^{-i\varphi} \\ -iv\cos\theta \\ 0 \end{pmatrix}, \begin{pmatrix} -iv\cos\theta \\ -iv\sin\theta e^{i\varphi} \\ 0 \end{pmatrix}\right\}.$$

Acknowledgements

The work was supported by the program No. AP14869140 (The study of QCD effects in non-QCD theories) of the Ministry of Education and Science of the Republic of Kazakhstan. We are also grateful to the Research Group Linkage Programme of the Alexander von Humboldt Foundation for the support of this research.

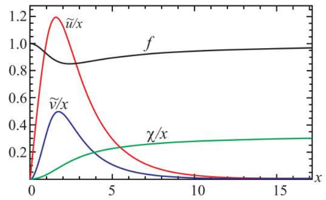
Numerical solutions

- For the Ansätze, we have the following nonvanishing components of electric and magnetic field

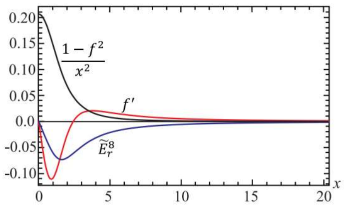
$$\begin{aligned} E^8_r &= \frac{\chi - r\chi'}{gr^2}, \\ H^1_i &= \frac{1}{g}\left\{\frac{\sin\theta\cos\varphi}{r^2}\left(1 - f^2\right), -\cos\theta\cos\varphi f', \sin\theta\sin\varphi f'\right\}, \\ H^2_i &= \frac{1}{g}\left\{\frac{\sin\theta\sin\varphi}{r^2}\left(1 - f^2\right), -\cos\theta\sin\varphi f', -\sin\theta\cos\varphi f'\right\}, \\ H^3_i &= \frac{1}{g}\left\{\frac{\cos\theta}{r^2}\left(1 - f^2\right), \sin\theta f', 0\right\}, \end{aligned}$$

- Substituting the Ansätze into the field equations we can obtain equations for the unknown functions

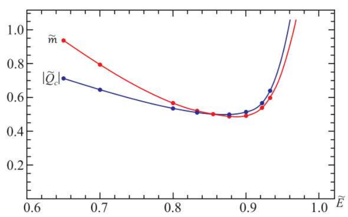
$$\begin{aligned} -f'' + \frac{f\left(f^2 - 1\right)}{x^2} &= -\tilde{g}^2\frac{\tilde{u}\tilde{v}}{x}, \\ -\frac{\chi''}{x} &= -\tilde{g}^2\frac{\tilde{u}^2 + \tilde{v}^2}{2\sqrt{3}x^2}, \\ \tilde{v}' + \frac{f\tilde{v}}{x} &= \tilde{u}\left(-1 + \tilde{E} + \frac{\chi}{2\sqrt{3}x} + \frac{\tilde{u}^2 - \tilde{v}^2}{x^2}\right), \\ \tilde{u}' - \frac{f\tilde{u}}{x} &= \tilde{v}\left(-1 - \tilde{E} - \frac{\chi}{2\sqrt{3}x} + \frac{\tilde{u}^2 - \tilde{v}^2}{x^2}\right), \end{aligned}$$



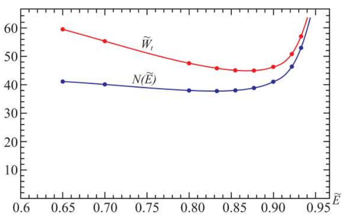
The gauge potentials f,χ and the spinor functions u, v



The radial component of the color electric field $\tilde{E}^8_r$, the radial part $(1 - f^2)/x^2$ of the radial components of the color magnetic field $H^{1,2,3}_r$, and the radial part f' of the tangential components of the color magnetic field $H^{1,2,3}_{\theta,\varphi}$



The dependence of the modulus of the color charge $|\tilde{Q}_c|$ and of the color magnetic dipole moment $\tilde{m}_c$ on the parameter $\tilde{E}$.



The energy spectrum of the solutions $\tilde{W}_t$ and the normalization integral $N(\tilde{E}) = 4\pi\tilde{E}\int_0^\infty(\tilde{u}^2 + \tilde{v}^2)dx$

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AYSS-2022

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