Light-like paths of anyon in magnetic field

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Model

We consider the model of a point classical spinning particle with the mass m and spin s travelling in $3d$ Minkowski space. Equations of motion in homogeneous magnetic field with the field strength vector $H = (H, 0, 0)$ read [DK, Retuntsev, 2022],

$$\frac{1}{e}\frac{d}{d\tau}\left(x - \rho\frac{[\xi, P]}{(\xi, P)}\right) = \left(1 + \frac{1}{2}\frac{(g-2)z\alpha(z)}{g + (g+1)z\alpha(z)}\right)P + \frac{g(g-2)(1+z\alpha(z))\alpha(z)}{g+(g+1)z\alpha(z)}\frac{sH}{2m}\,; \quad (1)$$

$$\frac{1}{e}\frac{dP}{d\tau} = \left(1 - \frac{1}{2}\frac{(g-2)z\alpha(z)}{g + (g+1)z\alpha(z)}\right)\Big[P, H\Big], \quad (2)$$

Here, ρ - constant with the length dimension, g - gyromagnetic ratio, $\xi = (1, \sin\varphi, \cos\varphi)$. The speed of light $c = 1$, and electric charge $e = 1$. The function $\alpha(z)$ is determined by the rule

$$\frac{1}{\alpha(z)} = cos\left(\frac{1}{3}arcsin\frac{3\sqrt{3}}{2}z\right) + \frac{1}{\sqrt{3}}sin\left(\frac{1}{3}arcsin\frac{3\sqrt{3}}{2}z\right), \qquad z = \frac{gs(P, H)}{2m^3}\,. \quad (3)$$

The particle position is x, the particle extended momentum is p, and φ is angular variable. The Minkowski metric is $\eta_{\mu\nu} = \mathrm{diag}\{-, +, +\}$. The mass shell condition reads

$$(p, p) + \mu^2 = 0\,, \qquad \mu = m\sqrt{1 + z\alpha(z)}\,. \quad (4)$$

Classical dynamics

The total angular momentum J and spin angular momentum S of the particle are determined by the formula

$$J = \left[x - \rho\frac{[\xi, p]}{(\xi, p)}, p\right] - \frac{s}{m}\alpha(z)p\,, \qquad S \equiv J - [x, p] = -\left[\rho\frac{[\xi, p]}{(\xi, p)}, p\right] - \frac{s}{m}\alpha(z)p\,. \tag{5}$$

The causal evolution is for the components of momentum p, and the special position in space-time (the mass center),

$$y = x - \rho\frac{[\xi, P]}{(\xi, P)}\,. \tag{6}$$

The variable φ can be arbitrary function of proper time. This is interpreted as the zitterbewegung. All the trajectories with various evolution of angular variable φ are connected by the gauge transformations. In the model (1), (2), the zitterbewegung phenomenon is preserved even at the **interacting** level.

The class of gauge equivalence of particle paths is given by the cylindrical surface of radius ρ centered at the trajectory $y(\tau)$. Depending on the evolution variable φ, the classical trajectories can be time-like, light-like, or space-like. [Nersessian,Ramos,1999; DK, Lyakhovich,2017]

Geometry of particle path

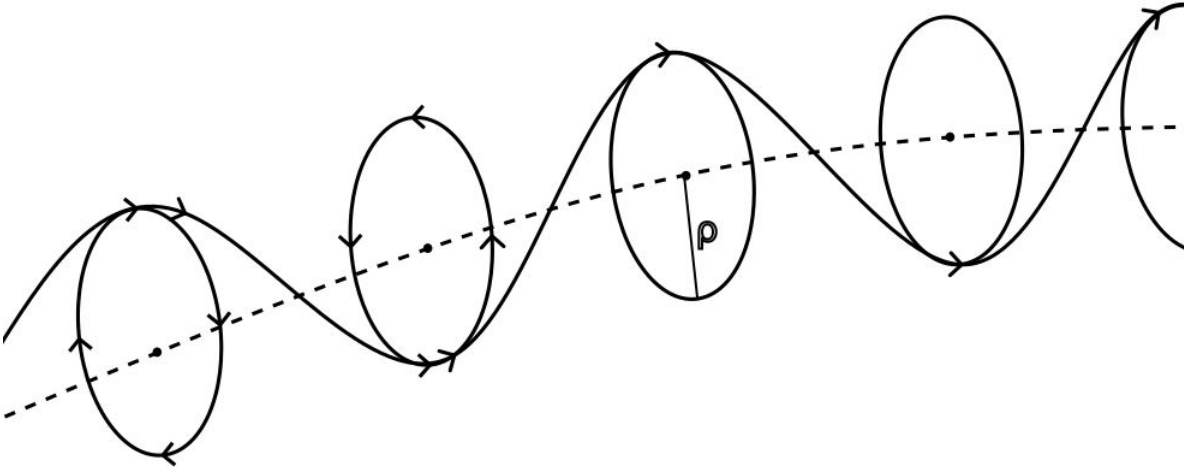

Figure: The trajectory of the particle

Motion of the mass center

It is convenient to solve the equations of motion (1), (20 using $1+2$ decomposition for the coordinates and momenta, $x=(t,\boldsymbol{x})$, $y=(t',\boldsymbol{y})$, $p=m\gamma(1,\boldsymbol{\beta})$:

$$\boldsymbol{\beta}=\frac{\boldsymbol{p}}{\sqrt{\mu^2+\boldsymbol{p}^2}}\,,\qquad \gamma=\frac{1}{\sqrt{1-\beta^2}}\,. \tag{7}$$

The equations of motion of the mass center position read

$$\frac{d\boldsymbol{\beta}}{dt'}=\Omega_c\Big[\boldsymbol{\beta},n\Big],\qquad \frac{d}{dt'}\left(\frac{H\mathbf{y}}{\gamma\mu}\right)=\Omega_c\boldsymbol{\beta}\,. \tag{8}$$

Here, $\mathbf{n}=(1,0,0)$, and Ω_c is the cyclotron frequency,

$$\Omega_c=\frac{gH\gamma}{\mu}\frac{3\alpha(z)z+1}{(2+(3\gamma^2-1)g)z\alpha(z)+g\gamma^2}=\frac{H}{m\gamma}\left(1+\frac{5}{2}z-\frac{2+g(3\gamma^2-1)}{g\gamma^2}z+o(z)\right). \tag{9}$$

The general solution to the space trajectory (8) gives a circular path:

$$\boldsymbol{y}=\boldsymbol{y}_0+\frac{\mu\gamma}{H}\Big[-\boldsymbol{\beta}_0\sin(\Omega_c y^0)+[\boldsymbol{\beta}_0,n]\cos(\Omega_c y^0)\Big]\,. \tag{10}$$

Light-like paths of anyon

The light like trajectories of anyon are determined by the condition

$$\left(\frac{dx}{d\tau}, \frac{dx}{d\tau}\right) = 0\,. \tag{11}$$

In the general case, this condition determines a quadratic equation with respect to the derivative of the angular variable $d\varphi/d\tau$.

In the weakly relativistic limit, relation (12) takes the form

$$\frac{d\varphi}{dt'} = \Omega_c(\boldsymbol{\xi}, \boldsymbol{\beta}) \pm \frac{1}{\rho}\Big[1 + (\boldsymbol{\xi}, \boldsymbol{\beta})(\Omega_c\rho - 1)\Big] + o(\beta)\,. \tag{12}$$

Here, Ω_c is the cyclotron frequency. The plus-minus sign determines two possible values of helicity of path. The corresponding trajectories are right handed and left-handed "helical" lines.

In the free limit, the trajectories of particle are left-handed and right-handed helical lines with a light-like tangent vector.

Non-relativistic light-like trajectories

The non-relativistic model is characterised by the condition $\beta \ll 1$. In this setting we have identification $t = t'$, and

$$\boldsymbol{x} = \boldsymbol{y} + \rho(\boldsymbol{i}\cos\varphi - \boldsymbol{j}\sin\varphi)\,, \qquad \boldsymbol{i} = (1,0)\,, \qquad \boldsymbol{j} = (0,1)\,. \tag{13}$$

The variable ϕ is linear function of time,

$$\frac{d\varphi}{dt} = \pm\frac{1}{\rho} \qquad \Longrightarrow \qquad \varphi(t) = \pm\frac{t}{\rho} + \varphi_0\,. \tag{14}$$

Thus, we have the following solution for the trajectory $\boldsymbol{x}(t) = (x^1(t), x^2(t))$ in space,

$$x^1 = \frac{\mu}{H}(\beta_0^2\cos(\Omega_c t) - \beta_0^1\sin(\Omega_c t)) + y_0^1 + \rho\cos(\frac{t}{\rho} + \varphi_0)\,; \tag{15}$$

$$x^2 = \frac{\mu}{H}(\beta_0^1\cos(\Omega_c t) - \beta_0^2\sin(\Omega_c t)) + y_0^2 \mp \rho\sin(\frac{t}{\rho} + \varphi_0)\,. \tag{16}$$

The dynamics of each coordinate is a bi-harmonic oscillation with frequencies Ω_c and $1/\rho$. The trajectory is closed if $\rho\Omega_c \in \mathbb{Q}$.

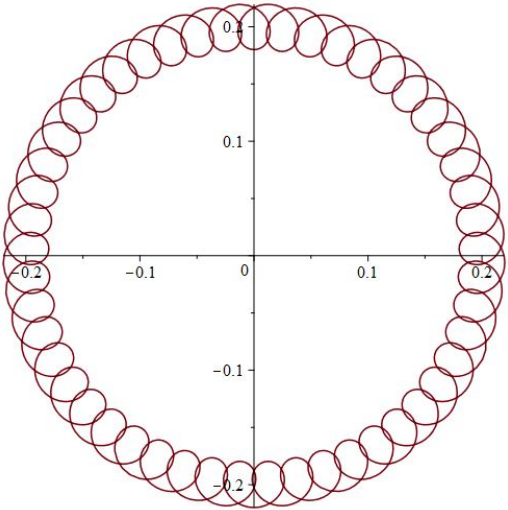

Figure: $\beta_0 = 0.2$, $\Omega_c = 1$, $\rho = 0.02$, left-handed

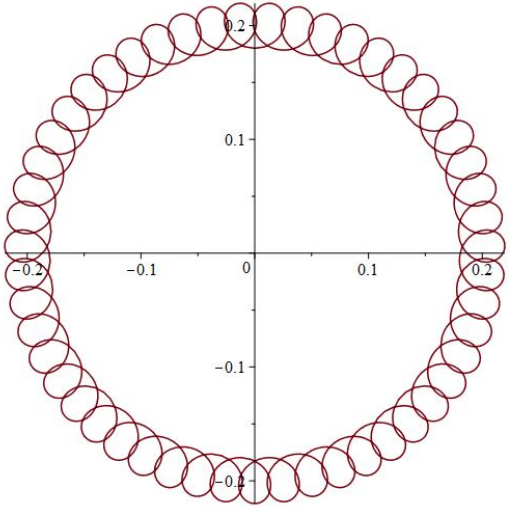

Figure: $\beta_0 = 0.2$, $\Omega_c = 1$, $\rho = 0.02$, right-handed

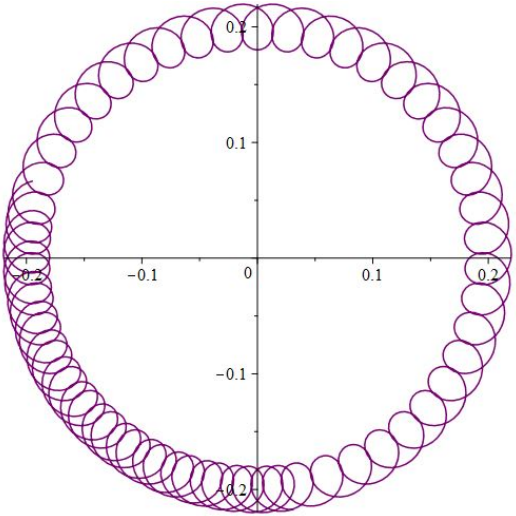

Figure: $\beta_0 = 0.21$, $\Omega_c = 1.05$, $\rho = 0.02$, 1.3 turns

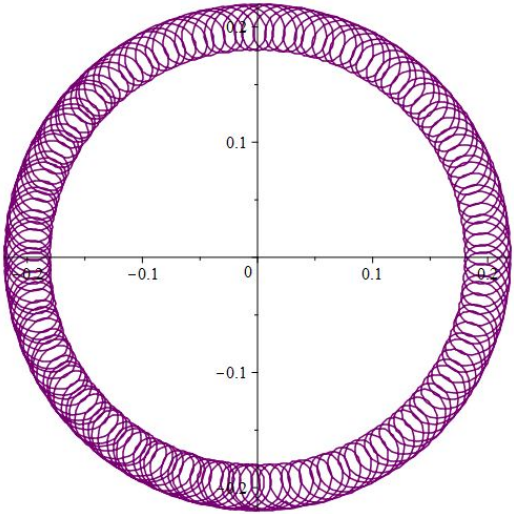

Figure: $\beta_0 = 0.21$, $\Omega_c = 1.05$, $\rho = 0.02$, 2.3 turns