Spin nature of the energy gap in superconductors of the second kind

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Introduction

The paper discusses the estimation of the second critical field through a superconducting gap.
The second critical field is evaluated in two ways.

- The first one is thermodynamical. There is a Clogston' s estimation of the second critical field $H_{c2} = 1{,}84 \cdot T_c$
- The second one is coherence length. There is an expression $H_{c2} = \dfrac{\Phi_0}{2\pi\xi^2}$, where $\Phi_0 = \dfrac{\pi\hbar c}{e}$ — quantum of magnetic flow, ξ — coherence length.

In this work an estimate of the field is obtained with Zeeman' s effect of splitting energy levels of cooper pairs.

Methodology

In essence, 4 quantum states of paired electrons are possible, differing in spin projections, forming an orthonormal basis in the state space:

$$|++>$$
$$|+->$$
$$|-+>$$
$$|-->$$

To characterize the state of an isolated system of two electrons, we introduce the Hamilton operator. In the model we consider difference in energies due to spin only with magnetic field:

$$\widehat{H} = A\widehat{\sigma_1}\,\widehat{\sigma_2} - \mu_{B1}\sigma_1 B - \mu_{B2}\sigma_2 B \text{ , where}$$

$\widehat{\sigma_1}$, $\widehat{\sigma_2}$ — spin operators for both particles and

μ_{B1}, μ_{B2} — Bohr magnetons for both particles,

$A = \dfrac{\Delta}{4}$ the energy difference of the two states with $B = 0$.

Due to the orthogonality of the basis vectors to each other, we have the following matrix:

$$H = \begin{pmatrix} A - \mu_{B1}B - \mu_{B2}B & 0 & 0 & 0 \\ 0 & -A - A(\mu_{B1} - \mu_{B2})B & 2A & 0 \\ 0 & 2A & -A + A(\mu_{B1} - \mu_{B2})B & 0 \\ 0 & 0 & 0 & A + \mu_{B1}B + \mu_{B2}B \end{pmatrix}$$

We are looking for a solution to the Schrodinger' s equation in the form $\Psi(\overrightarrow{x}, t) = \psi(\overrightarrow{x}) \cdot e^{-i\omega t}$. Then our task is reduced to stationary and we obtain the following distribution of energy by states:

$$E_1 = A - (\mu_{B1} + \mu_{B2})B \text{ for } |1> = |++>$$

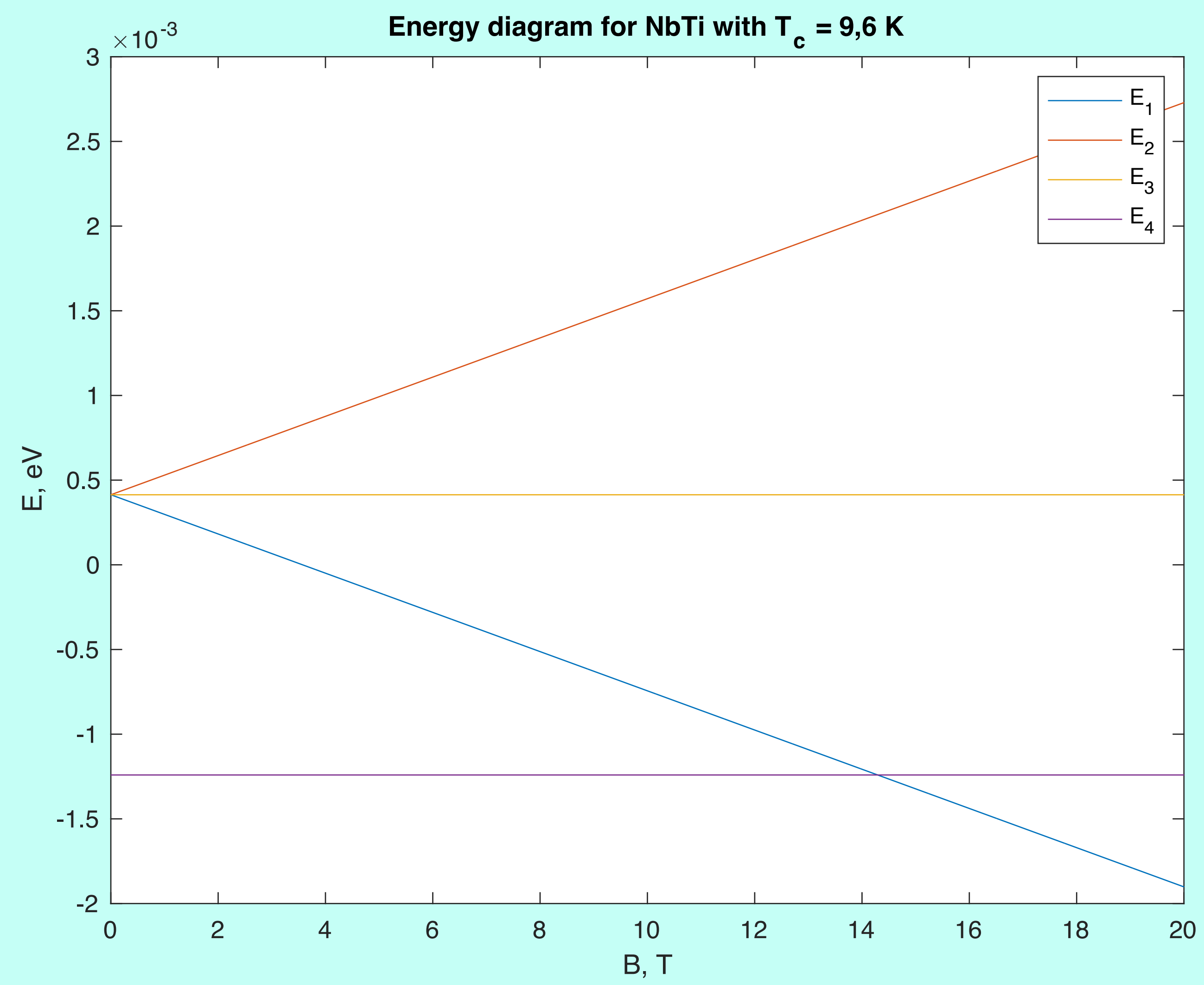
$$E_2 = A + (\mu_{B1} + \mu_{B2})B \text{ for } |2> = |-->$$

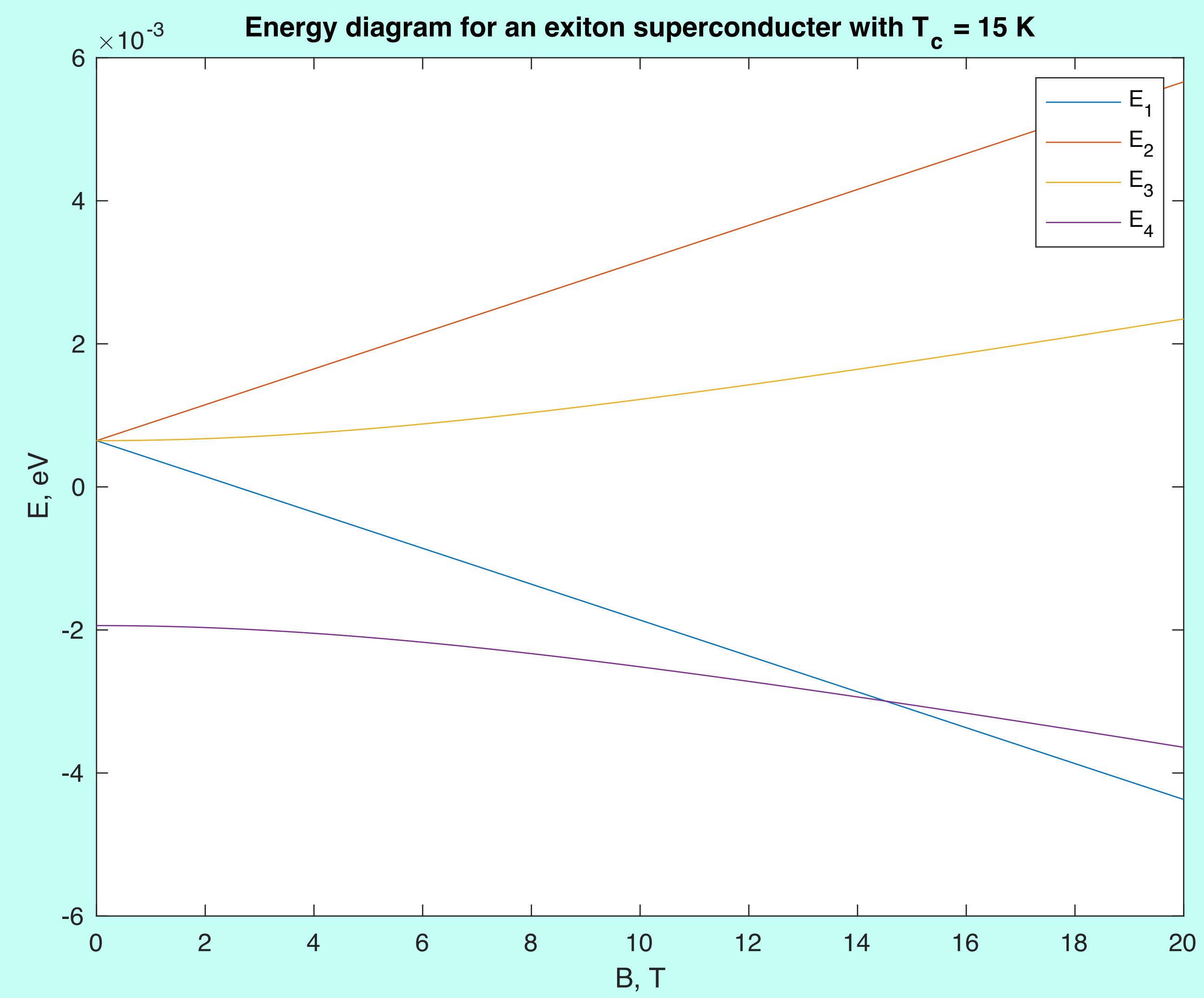
$$E_3 = A(-1 + 2C) \text{ for } |3> = \frac{1}{\sqrt{2}}(|+-> + |-+>)$$

$$E_4 = -A(1 + 2C) \text{ for } |4> = \frac{1}{\sqrt{2}}(|+-> - |-+>),$$

where $C = \sqrt{1 + (\mu_{B1} - \mu_{B2})^2 \cdot \dfrac{B^2}{4A^2}}$.

Results





Thus, we offer to estimate the second critical field B_{c2} through the intersection of the curves E_1 and E_4 of the energy diagram like curves of two different energy conditions with $B = 0$. If you equate energy curves and try to find the point of their intersection, you can find out a new estimate for the second critical field:

$$B_{c2} = A\frac{(\mu_{B1} + \mu_{B2})}{\mu_{B1}\mu_{B2}}$$

Superconducting materials. Estimate with $\Delta = 2kT_c$

Material	T_c , K	B_{c2}^{theor} , T	B_{c2}^{exper} , T
NbTi	9,6	14,29	14
Nb_3Ge	23,2	34,54	38
Nb_3Sn	18,3	27,23	24
$La_{1,85}Sr_{0,15}CuO_4$	38	56,55	62
$YBa_2Cu_3O_7$	93	138,39	120
$HgBa_2Ca_2Cu_3O_{10}$	135	200,88	190
Rb_3C_{60}	29,5	43,89	44
K_3C_{60}	19,5	29,02	30
MgB_2 ($\xi = 51$nm)	39	58,03	39
$PbMo_6S$	15	22,32	60
ZrV_2	8,5	12,65	16,5
NbN	16	23,82	22

with Bohr magneton for both particles (two electrons or electron and positron).

So for similar particles in Cooper pair, for example, electrons we have:

$$B_{c2} = \frac{2A}{\mu_B}$$

Let's remember, that $A = \dfrac{\Delta}{4} = \dfrac{kT_c}{2}$. So it turns out to $B_{c2} = \dfrac{kT_c}{\mu_B}$, which is similar with Klogston' s estimation.

Conclusion

Thus, the received estimate for superconductors' second critical field sufficiently consistent with experimental data for compounds from the table: superconducting alloys, metallic compounds, fullerides, nitride and Laves phase. The estimation is not good in accuracy of calculations for Chevrel phase and MgB_2.

According to the research we have an analogy between condensed, uncondensed Cooper pair and model of Zeeman' s splitting of energy levels of the pair.