Compton scattering of annihilation photons in entangled and decoherent polarization states

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TASKS:

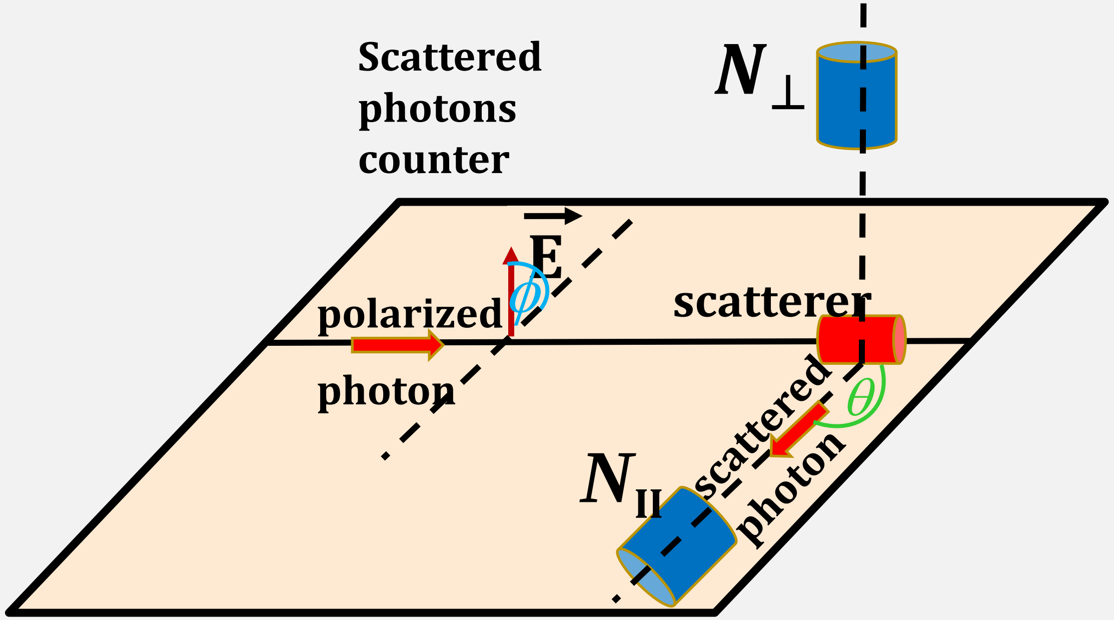
- Study of the Compton scattering kinematics for the pairs of decoherent annihilation gammas;
- Comparison of the scattering kinematics for the pairs of entangled and decoherent photons;

Importance:

- Compton scattering of entangled photons was not studied in details;
- Compton scattering of decoherent photons was not studied at all;
- Contradictions in the theoretical calculations of the cross-sections for entangled and decoherent photons;
- Possible applications for new generation of the positron emission tomography.

Compton polarimeter

Polarization of incident photon can be determined by registering scattered photon.



Differential cross-section of Compton is given by Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{2}\cdot\frac{e^2}{m_e c^2}\cdot\frac{E_{\gamma_1}^2}{E_\gamma^2}\cdot\left(\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - 2\sin^2\theta\cos^2\phi\right)$$

Angle between the scattering and polarization planes.

Cross-section is maximum for ϕ =π/2 angle!

Analyzing power is asymmetry in Compton scattering:

$$A = \frac{N_\parallel - N_\perp}{N_\parallel + N_\perp}$$

Analyzing power of Compton polarimeter:

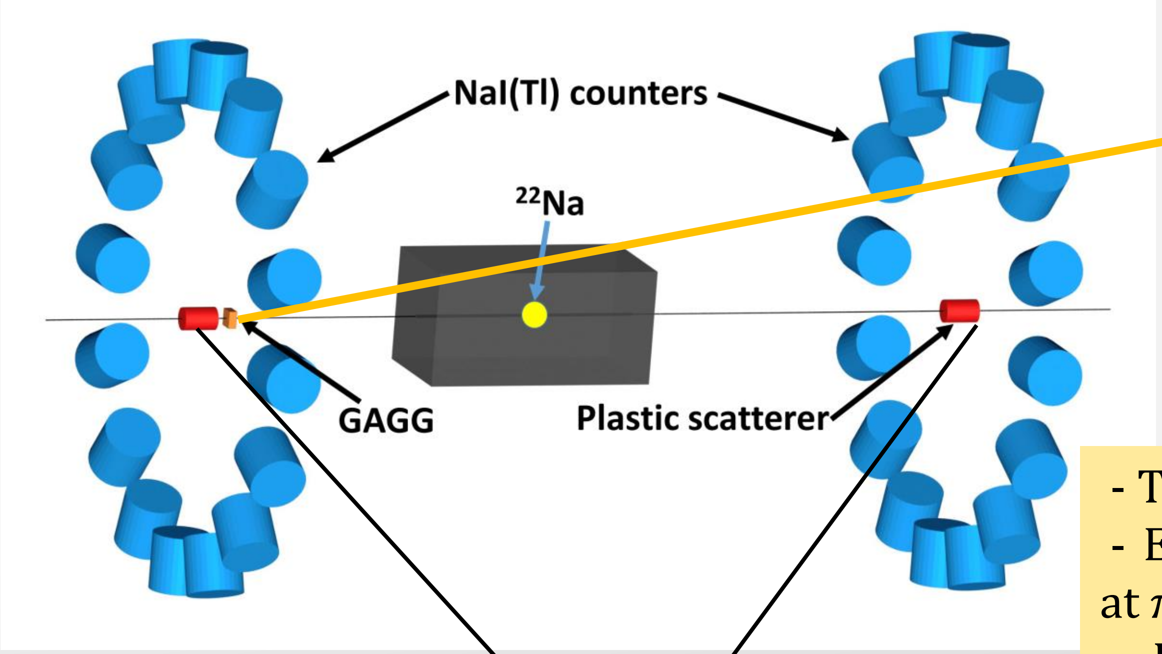
$$A = \frac{\frac{d\sigma}{d\Omega}(\theta,\phi=90^0) - \frac{d\sigma}{d\Omega}(\theta,\phi=0^0)}{\frac{d\sigma}{d\Omega}(\theta,\phi=90^0) + \frac{d\sigma}{d\Omega}(\theta,\phi=0^0)} = \frac{\sin^2\theta}{\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - \sin^2\theta}$$

A_{Max} = 0.7 for energy 511 keV (scattering angle 82^0).

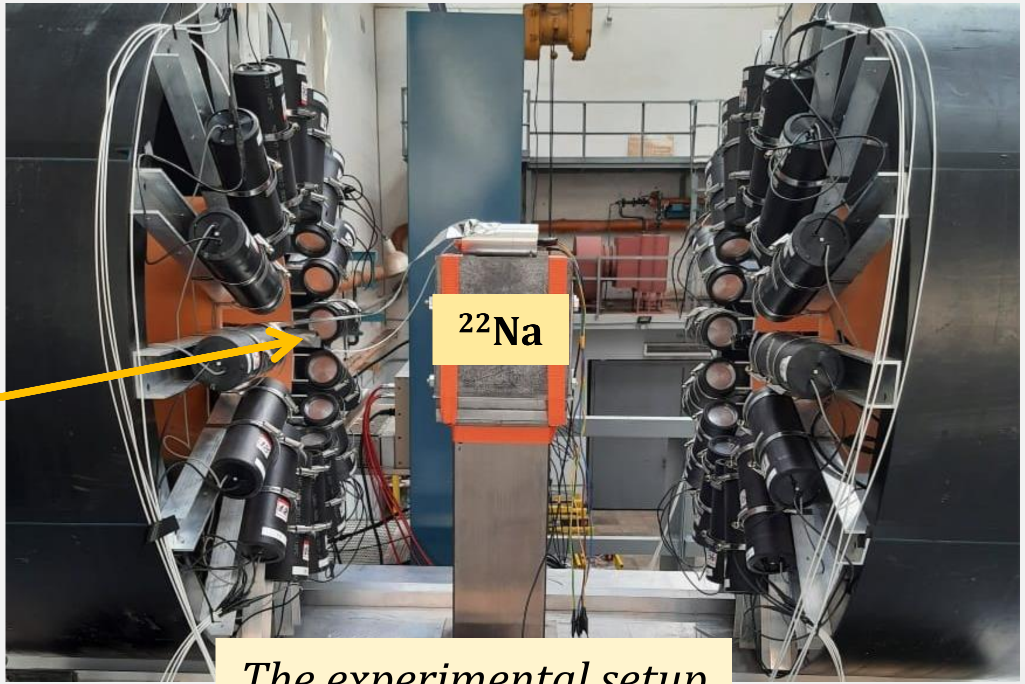
A much less than 1. (For optical polarimeters ~1)

Experimental Setup

The experimental setup constructed in INR RAS (Moscow) will help to research scattering of both entangled and decoherent photons.



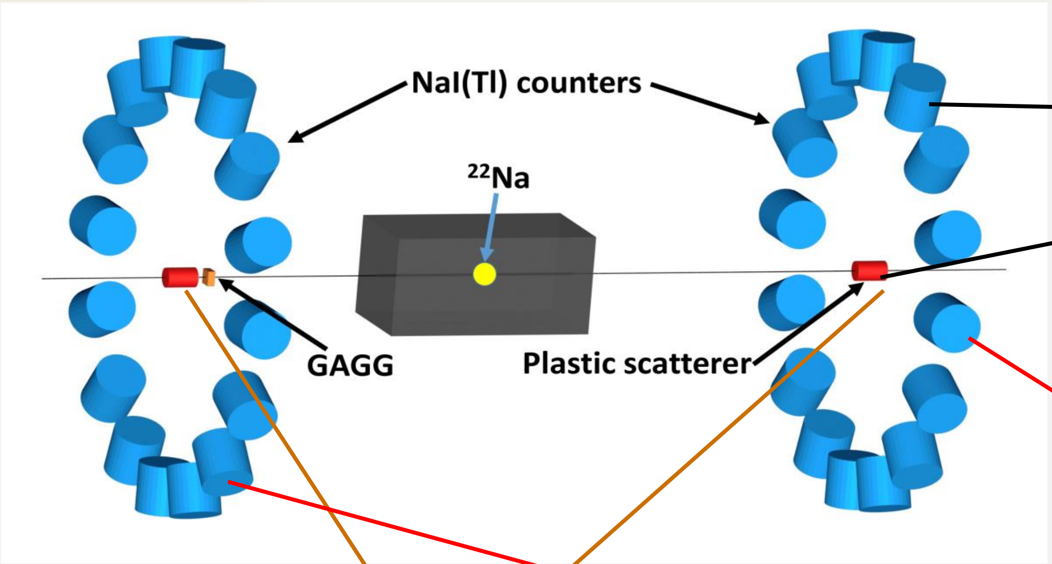
ADC ← Trigger — Coincidence

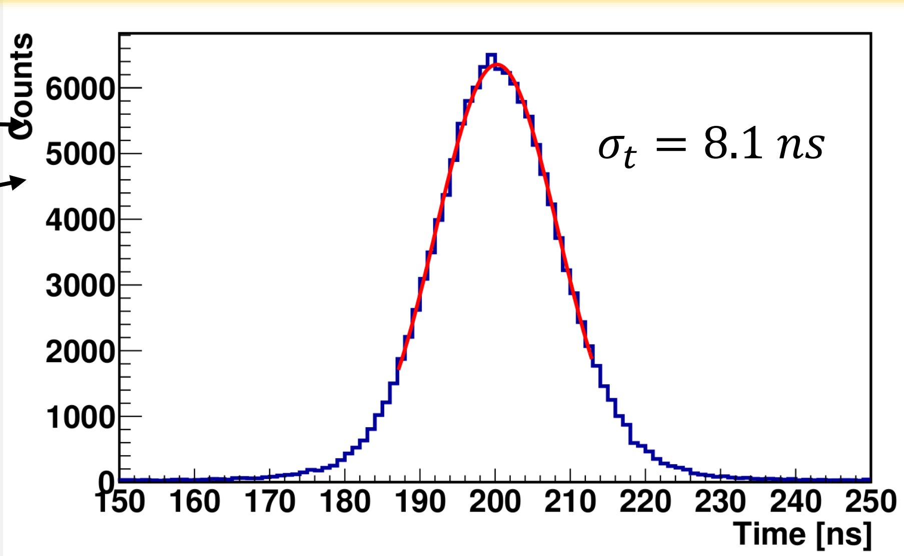


The experimental setup

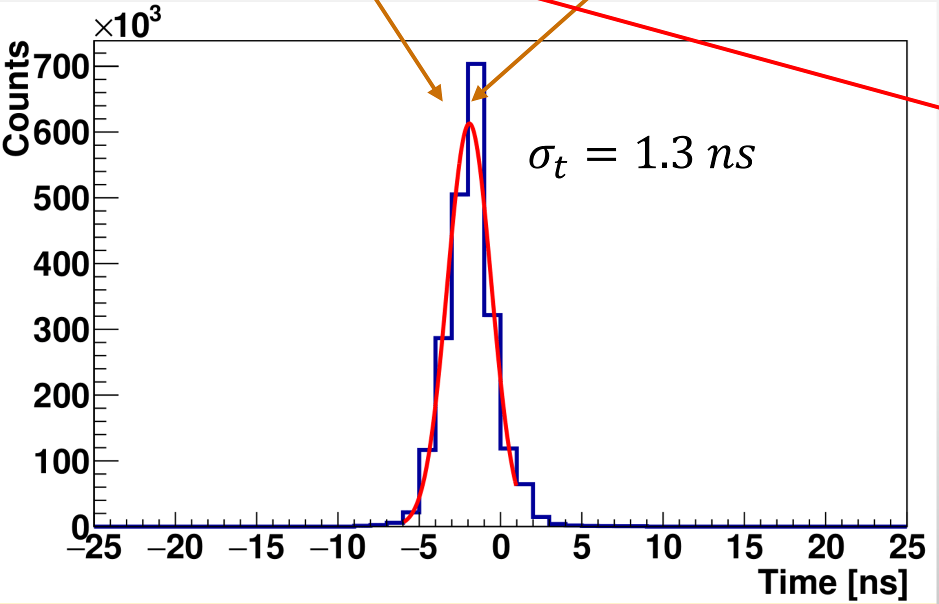
- The setup consists of 2 arms
- Each arms comprises 16 scintillation detector (NaI) positioned at $\pi/8$ angles to each other
- In each arm 16 independent Compton polarimeters can be distinguished (pair of perpendicularly positioned scintillation detectors (NaI))
- In front of one of the arms an additional intermediate GAGG scatterer is placed to produce pairs of decoherent photons.

Time spectra

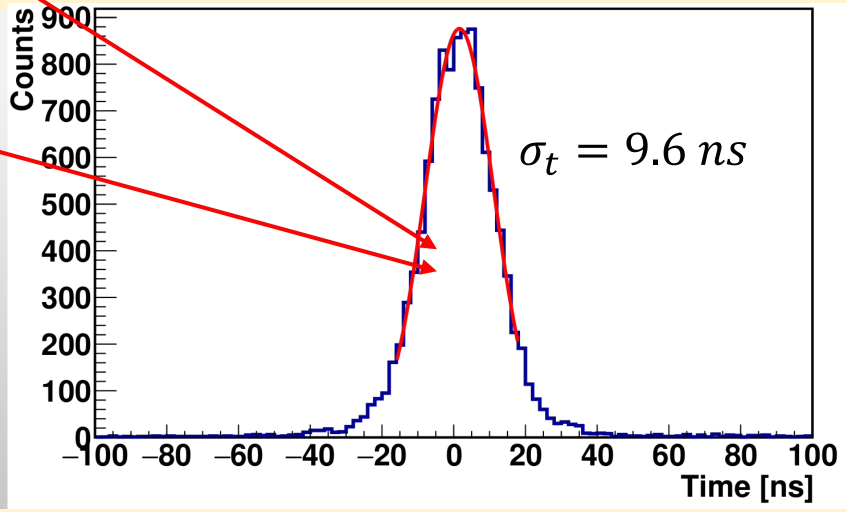




Time difference between one of the NaI counters and the main scatterer of the same arm

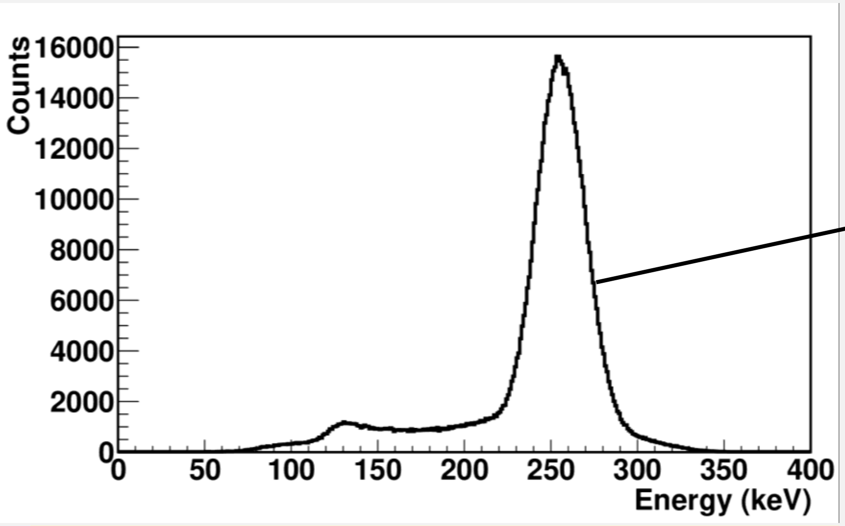


Time difference between main scatterers of the opposite arms

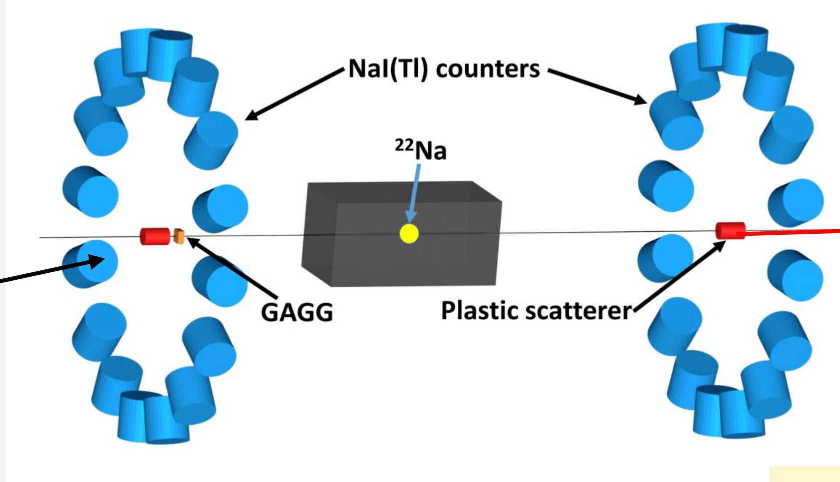


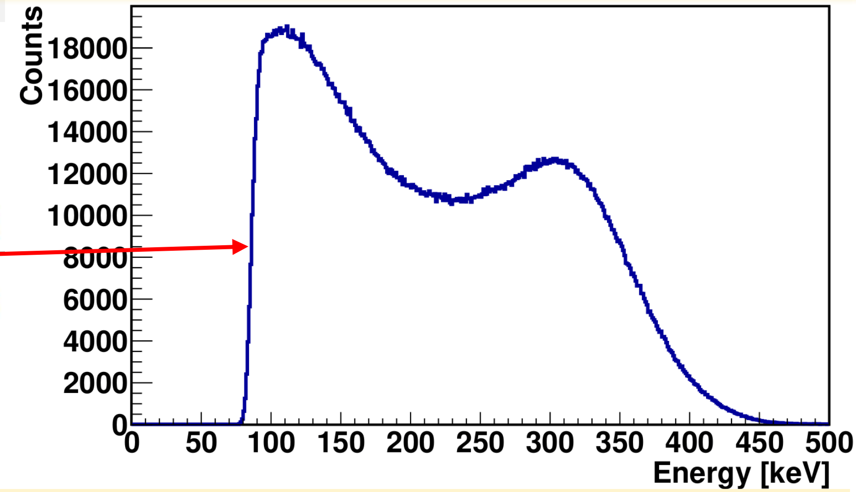
Time difference between two NaI counters of the opposite arms

Typical amplitude spectra

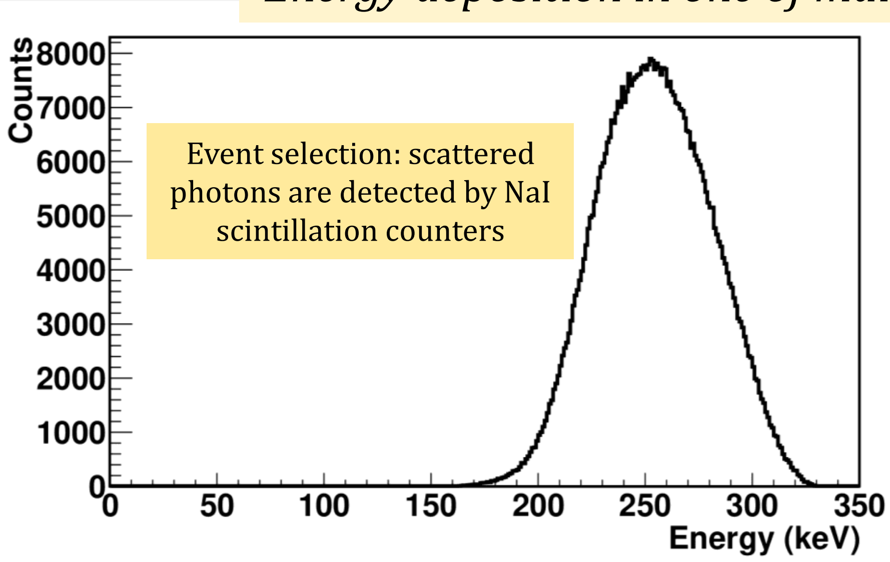


Energy deposition in NaI counter





Energy deposition in one of main scatterers



Energy deposition in one of the plastic scatters for photons detected by NaI scintillation detectors of the same arm

Photon energy after Compton scattering at θ angle: $E_{\gamma`}(\theta) = \frac{E_\gamma}{1+\frac{E_\gamma}{m_e c^2}(1-cos\theta)}$

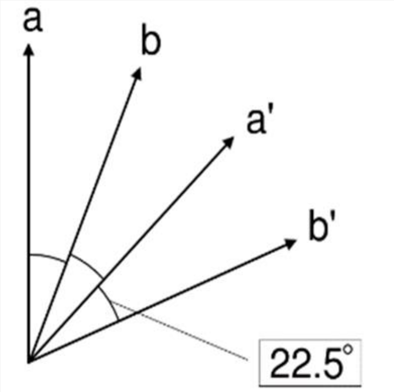
$E_{\gamma(\gamma`)}$ - energy of incident (scattered) photon.

In the experiment $E_\gamma = m_e c^2$.

$\Rightarrow E_{\gamma`}(\theta = 90^0) = \frac{m_e c^2}{2} \cong 255{,}5$ keV

How to prove the entanglement of annihilation photons?

Traditional method: CHSH inequality (not applicable)



N (***a,b***) – coincidences in counters ***a*** and ***b***

$$E\left(\vec{a},\vec{b}\right)=\frac{N\left(\overrightarrow{a_{||}},\overrightarrow{b_{||}}\right)+N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{\perp}}\right)-N\left(\overrightarrow{a_{||}},\overrightarrow{b_{\perp}}\right)-N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{||}}\right)}{N\left(\overrightarrow{a_{||}},\overrightarrow{b_{||}}\right)+N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{\perp}}\right)+N\left(\overrightarrow{a_{||}},\overrightarrow{b_{\perp}}\right)+N\left(\overrightarrow{a_{\perp}},\overrightarrow{b_{||}}\right)}$$

Correlation function for ideal polarimeter (A=1):

$$S=E\left(\vec{a},\vec{b}\right)-E\left(\vec{a},\vec{b}`\right)+E\left(\vec{a},\vec{b}`\right)+E\left(\vec{a}`,\vec{b}`\right)$$

According to Bell's (CHSH) inequality:

- S<2 for non-entangled system
- Maximum $S=2\sqrt{2}$ for entangled system.

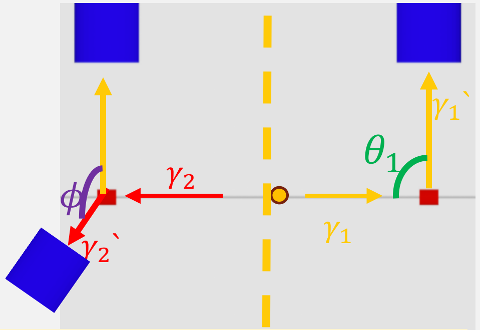
For non-ideal polarimeter:

$$S \quad \Rightarrow \quad S` = S * A^2 \quad \Rightarrow$$

$S`$<2 for Compton polarimeters (A^2<0.5, ***always***)!

Azimuthal correlations of scattered gammas

Snyder H *et al* - 1948 Angular correlation of scattered annihilation radiation Phys. Rev. 73 440–8.



$$P_{12}(E_1,E_2,\phi)=\left(\frac{d\sigma}{d\Omega_1}\right)_{NP}\left(\frac{d\sigma}{d\Omega_2}\right)_{NP}[1-\alpha(\theta_1)\alpha(\theta_2)cos(2\phi)]$$

Ratio of the numbers of scattered annihilation photons 90°, 0° :

$$R(\theta)=\frac{N\left(\phi=\frac{\pi}{2}\right)}{N(\phi=0)}=1+\frac{2\sin^4\theta}{\gamma^2-2\gamma\sin^2\theta};$$

$$\gamma=2-cos\theta+(2-cos\theta)^{-1}$$

$$R=2.83\ for\ \theta=82^0;\ R=2.6\ for\ \theta=90^0$$

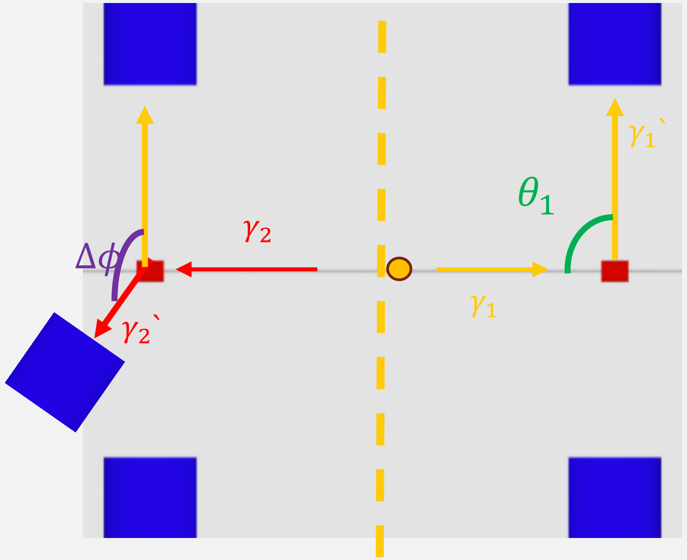
D. Bohm and Y. Aharonov (Phys. Rev. (1957) 108, 1070: if **R>2 ⇒gamma pair is entangled**.

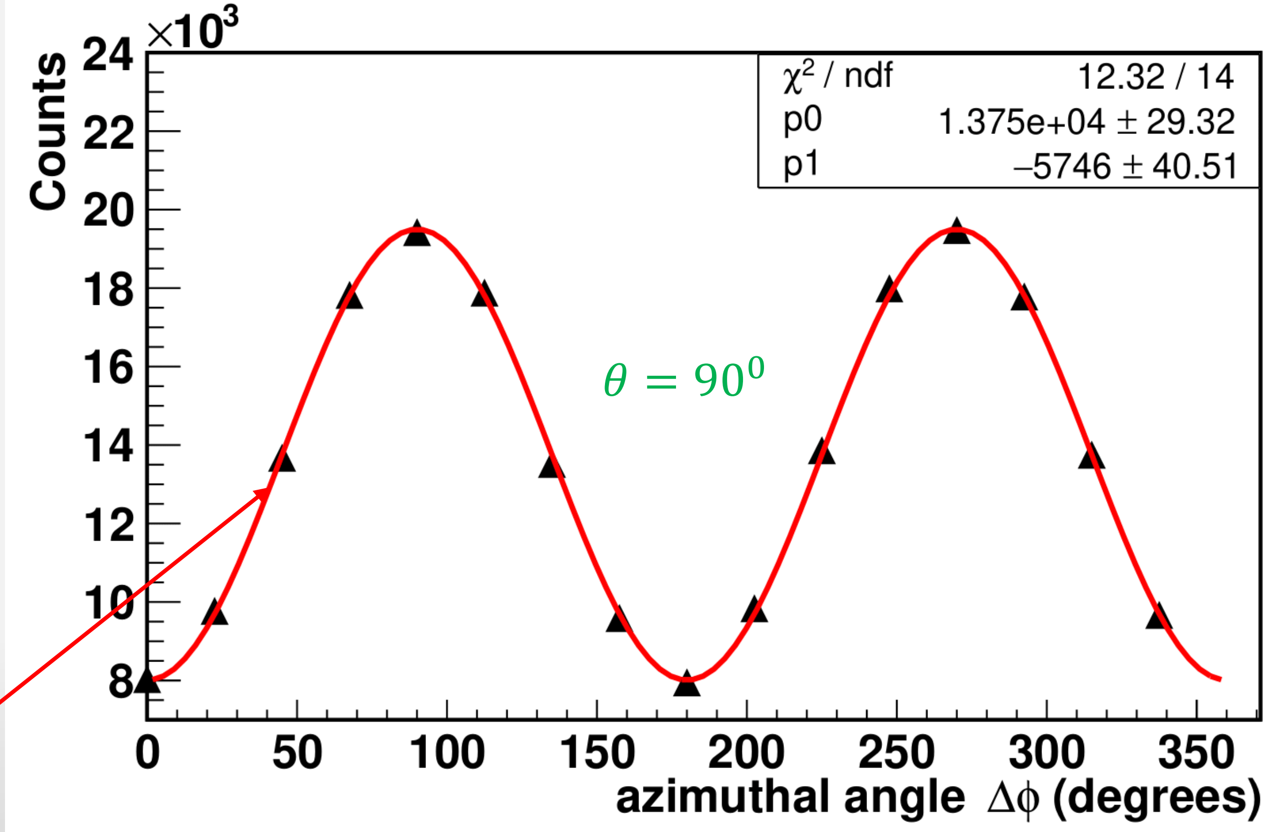
For decoherent R=1; for non-entangled photons R<2

H. Langhof, Zeitschrift fur Physik 160, 186-193 (1960) $R(82^0)=2.47\pm0.07$

Asymmetry in angular distribution of entangled scattered photons

Every shoulder contains 16 detectors ⇒ for each angle there are 16 different pairs of scintillation detectors, which add to the total number of coincidences for chosen angle(N)





$$\boldsymbol{P_{12}(E_1, E_2, \Delta\phi)} = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP}\left(\frac{d\sigma}{d\Omega_2}\right)_{NP}[1 - \alpha(\theta_1)\alpha(\theta_2)cos(2\Delta\phi)]$$

$$\Rightarrow N(\phi) = A + B \cdot \cos((2\Delta\phi))$$

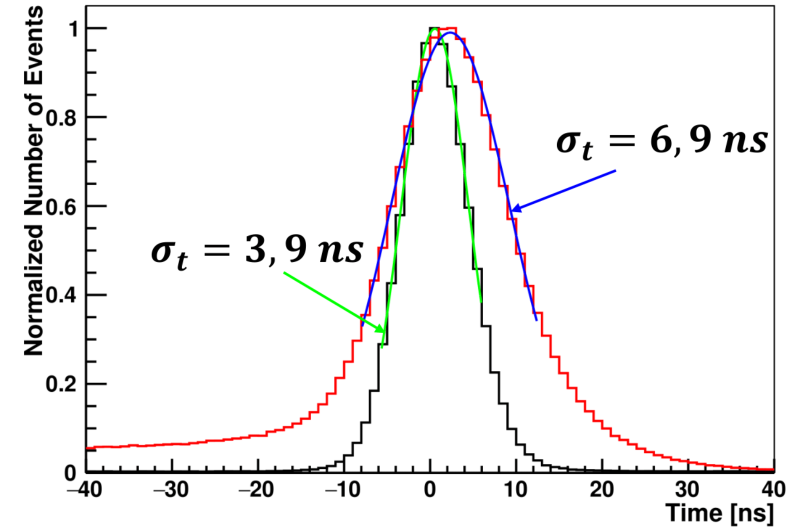
$\boldsymbol{\alpha(\theta_i)}$ – analyzing power of a Compton polarimeter

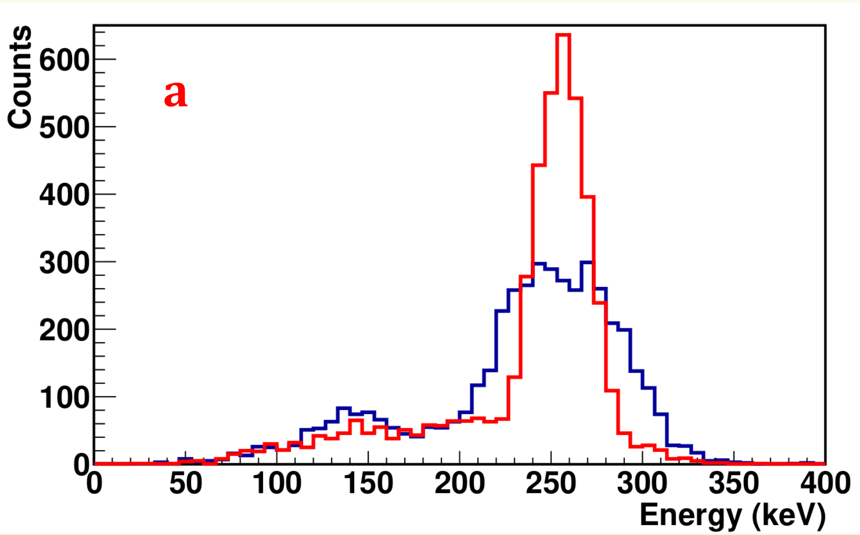
$$\mathrm{R}_{theory}(\theta = 90^0) = 2{,}6$$

$$\boldsymbol{R_{exp}(\theta = 90^0 \pm 7^0) = 2{,}435 \pm 0{,}018}$$

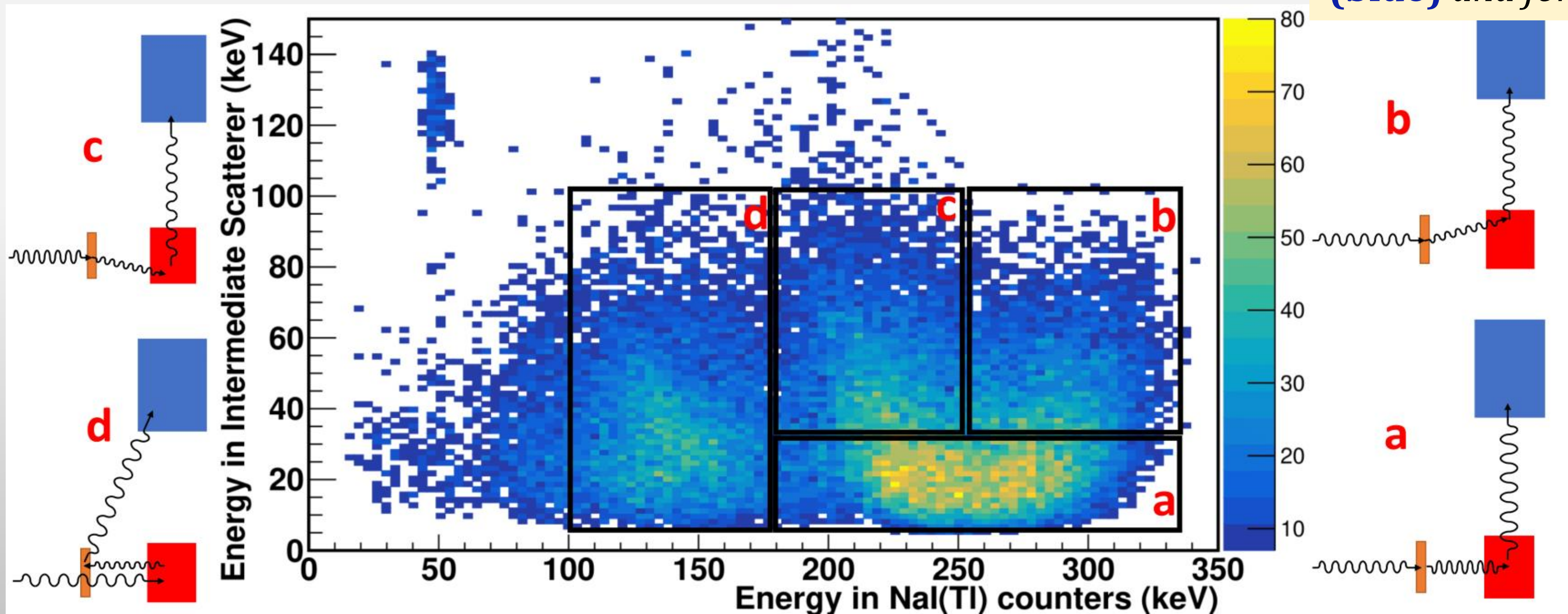
Selection of decoherent pairs

Spectra of time difference between intermediate and main scatterers of the same arm ***(for the full energy spectrum in (red histogram)*** *and for* ***E > 10 keV (black histogram))***



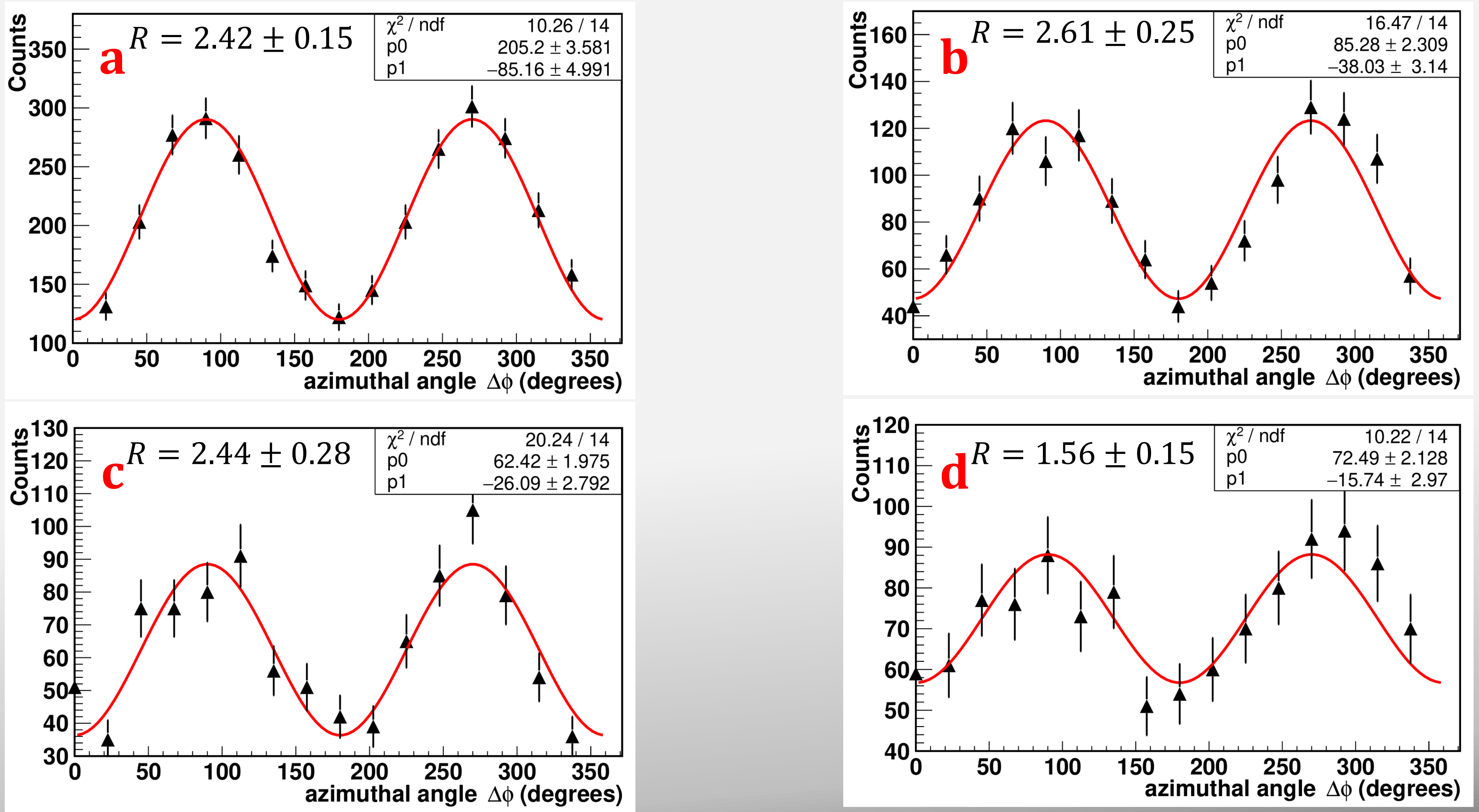


Energy deposition in NaI(Tl) for events of group **a (blue)** *and for entangled photons* **(red)**



Events of group **a** *have the closest scattering geometry to that of the entangled photons*

Asymmetry in angular distribution of decoherent photons for different event classes



Asymmetry in angular distribution of decoherent photons

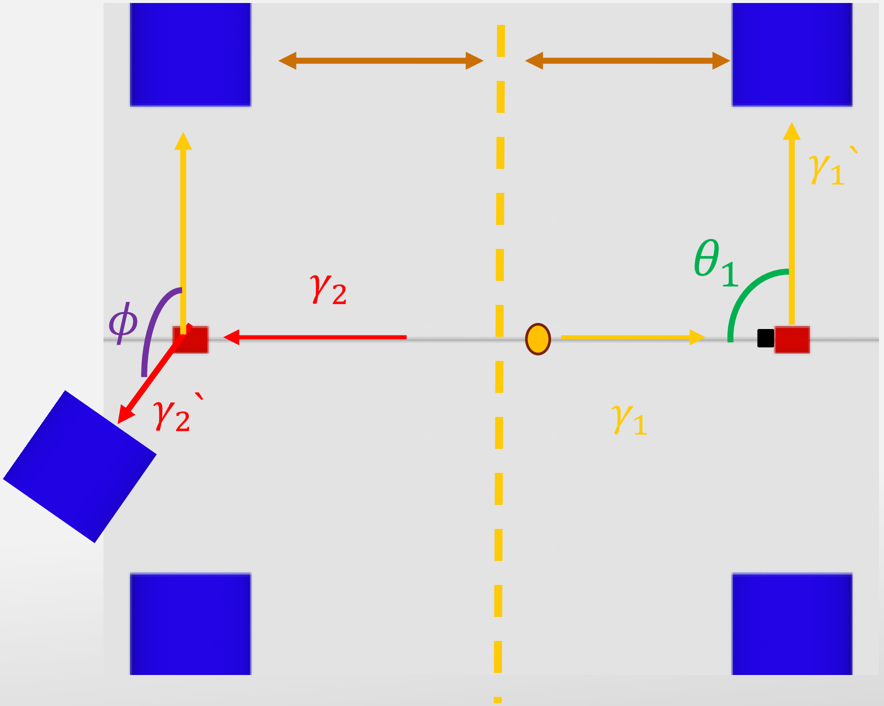
Na-22 source is closer to the arm with intermediate scatterer

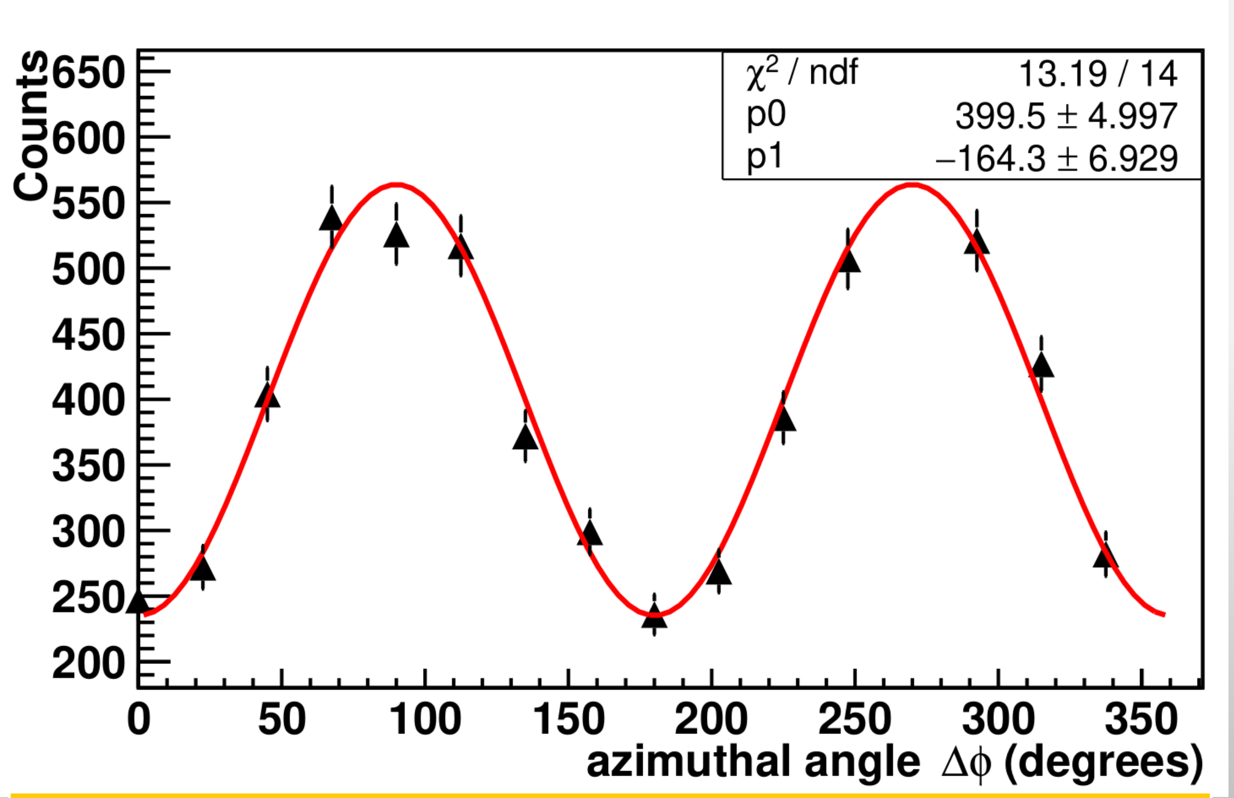
- If there is an interaction in intermediate scatterer ⇒ the pair is decoherent

Event selection: events belong to groups **a**, **b**, **c**

$$180\ keV < E_{NaI} < 340\ keV$$

$$2keV < E_{GAGG} < 100\ keV$$





$$R(\theta = 90^0) = 2,40 \pm 0,10$$

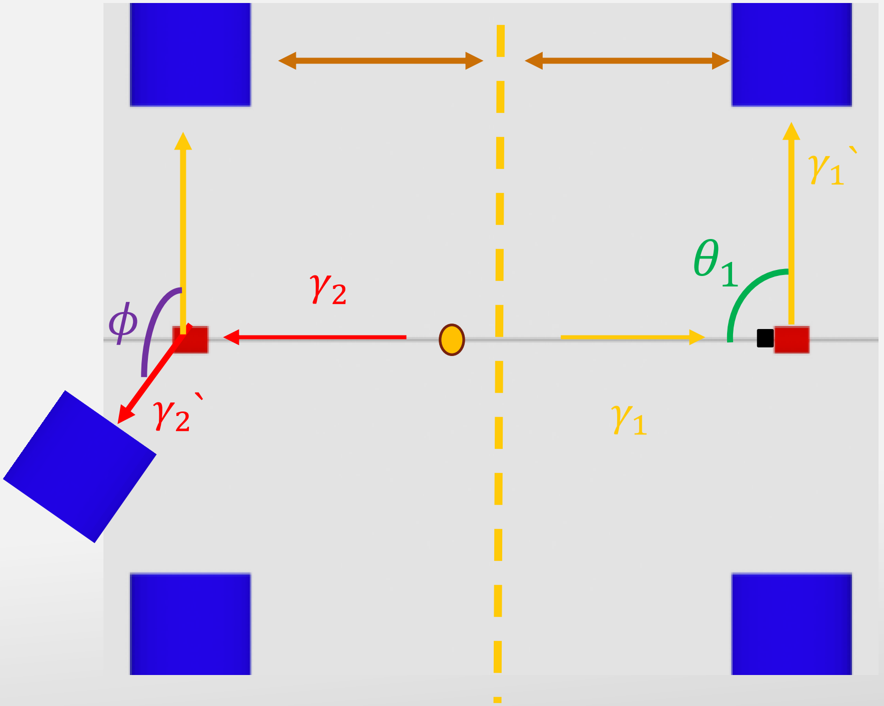
Asymmetry in angular distribution of entangled scattered photons

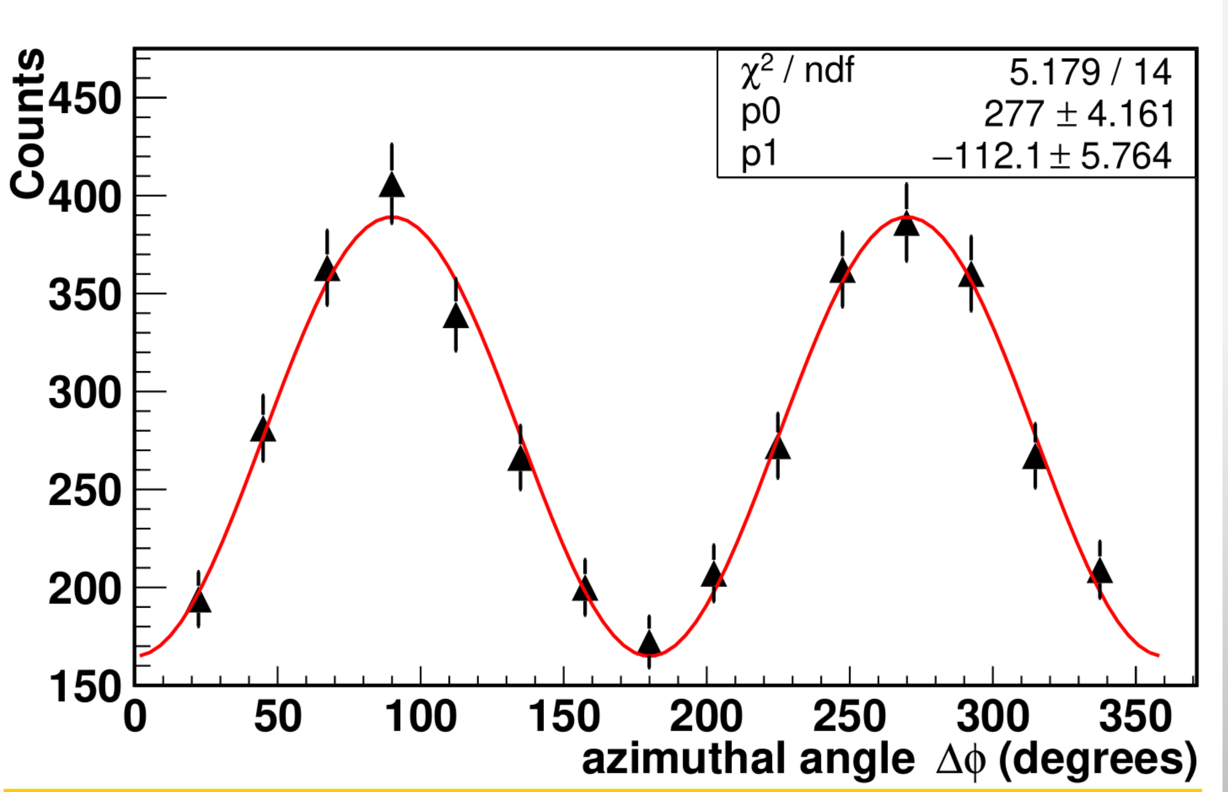
Na-22 source is closer to the arm without intermediate scatterer

- If there is an interaction in intermediate scatterer ⇒ the pair is scattered entangled

Event selection: events belong to groups **a**, **b**, **c**

$$180\ keV < E_{NaI} < 340\ keV$$
$$2keV < E_{GAGG} < 100\ keV$$





$$R(\theta = 90^0) = 2,36 \pm 0,12$$

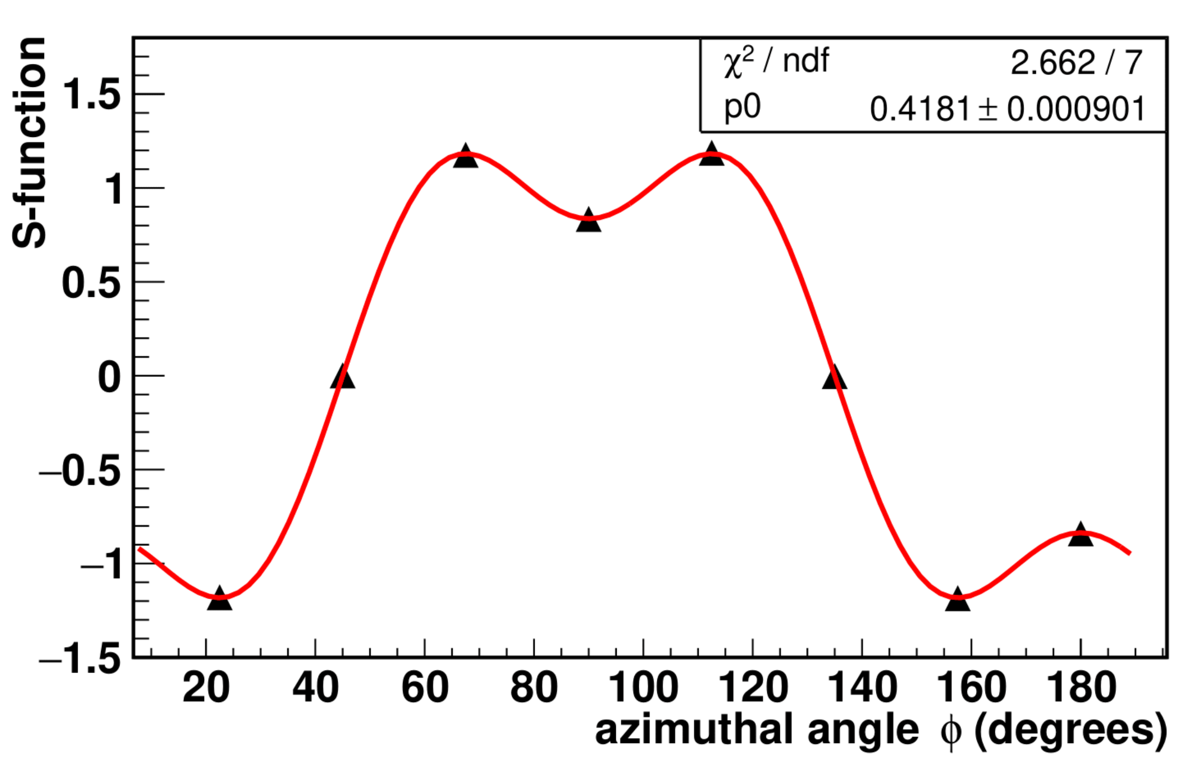
Correlation S-function of annihilation photons

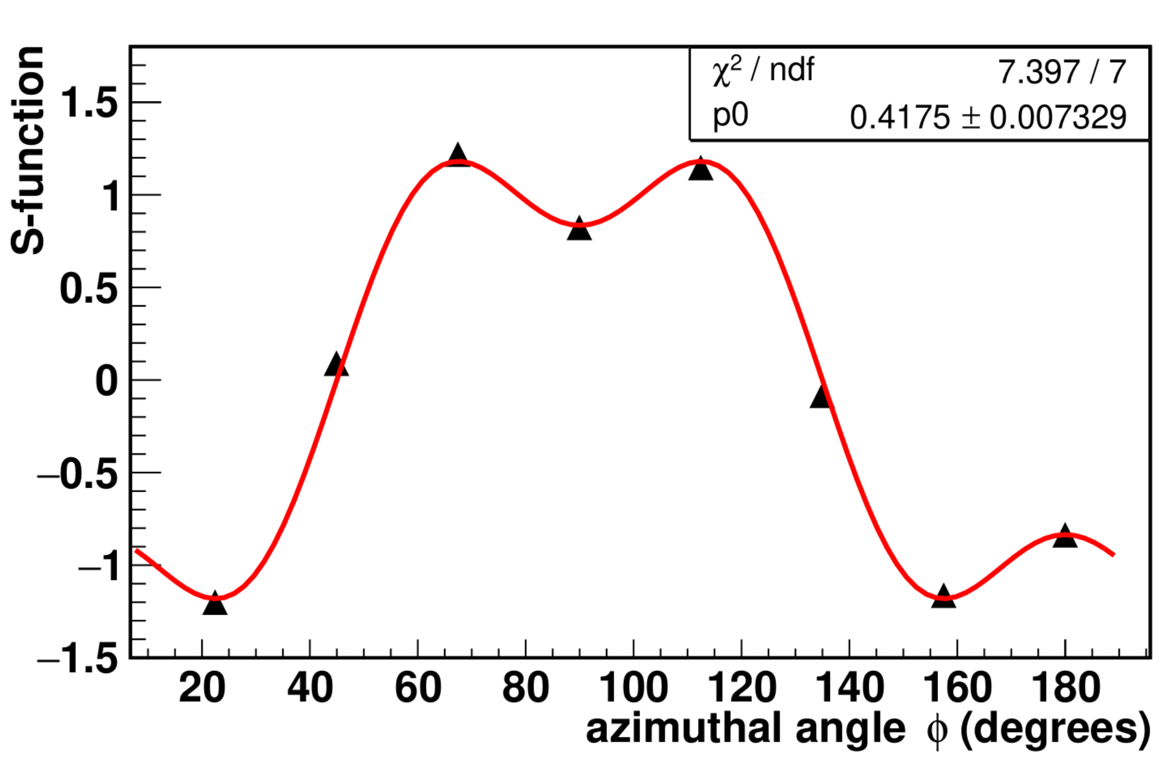
Entangled photons

Decoherent photons

$180\ keV < E_{NaI} < 340keV$
$2keV < E_{GAGG} < 30keV$

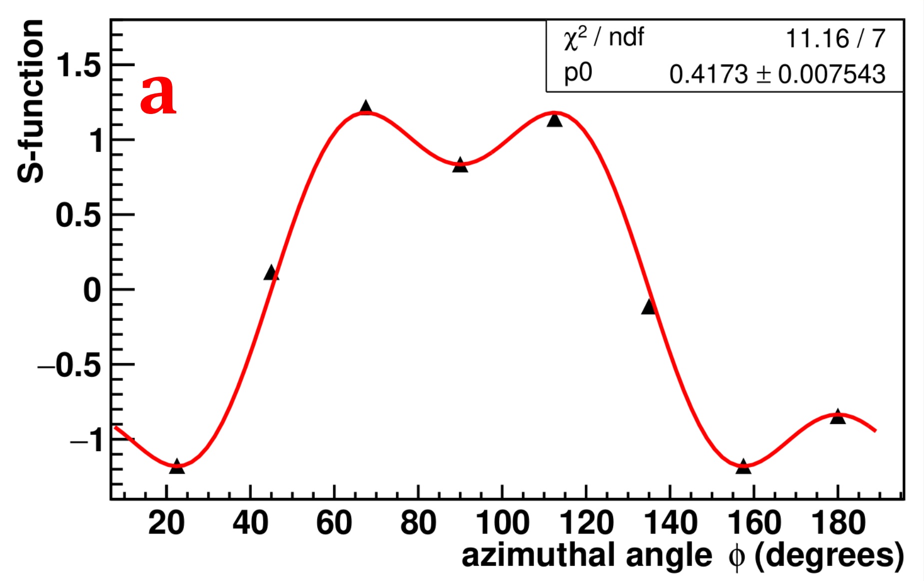
$$\boldsymbol{S} = E\left(\vec{a},\vec{b}\right) - E\left(\vec{a},\vec{b}'\right) + E\left(\vec{a},\vec{b}'\right) + E\left(\vec{a}',\vec{b}'\right) = \boldsymbol{p_0}\cdot(\mathbf{cos}(\mathbf{6}\boldsymbol{\phi}) - \mathbf{3}\,\mathbf{cos}(\mathbf{2}\boldsymbol{\phi}))$$

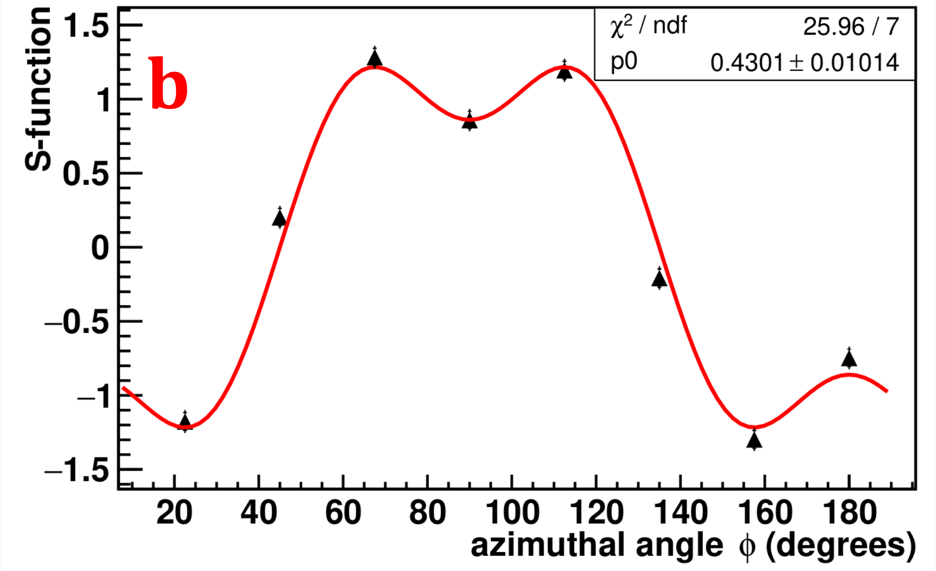


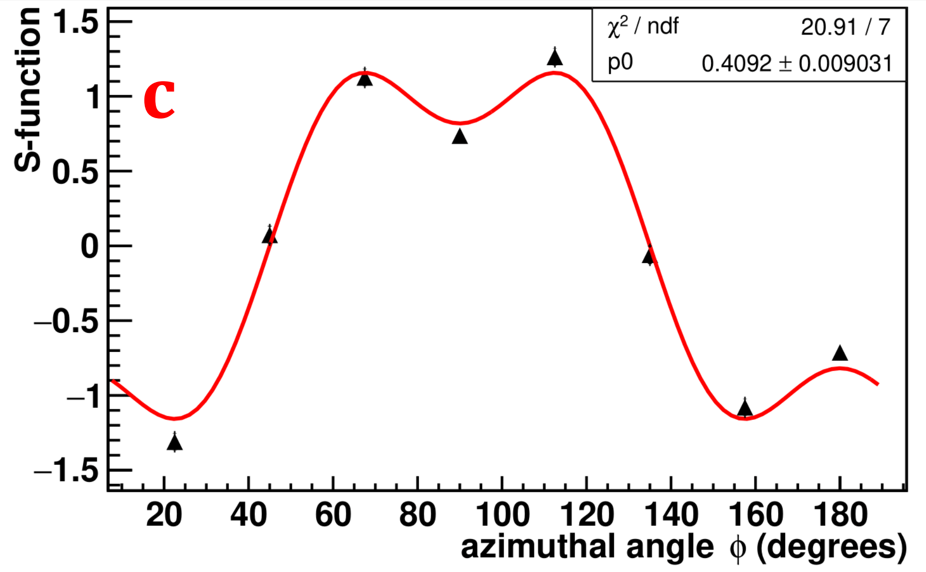


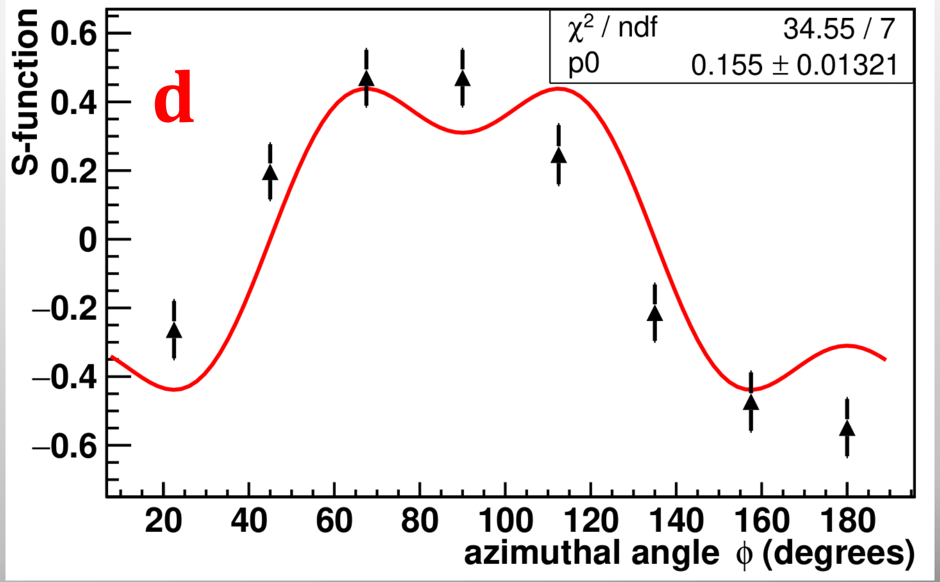
$S_{Max} = \mathrm{p}_0 \cdot 2\sqrt{2} < 2 \Rightarrow$ impossible to tell if entangled
No difference between correlation functions of entangled and decoherent photons was found

Correlation S-function of decoherent annihilation photons for different event classes









Conclusion

- Experimental setup to study the Compton scattering of entangled and decoherent annihilation photons was constructed;

- The dependence of the number of detected gammas on the angle between the scattered photons is obtained for entangled and decoherent gammas;

- The angular dependence corresponds to the theoretical predictions for the entangled photons;

- No difference in scattering kinematics of entangled and decoherent photons was found;

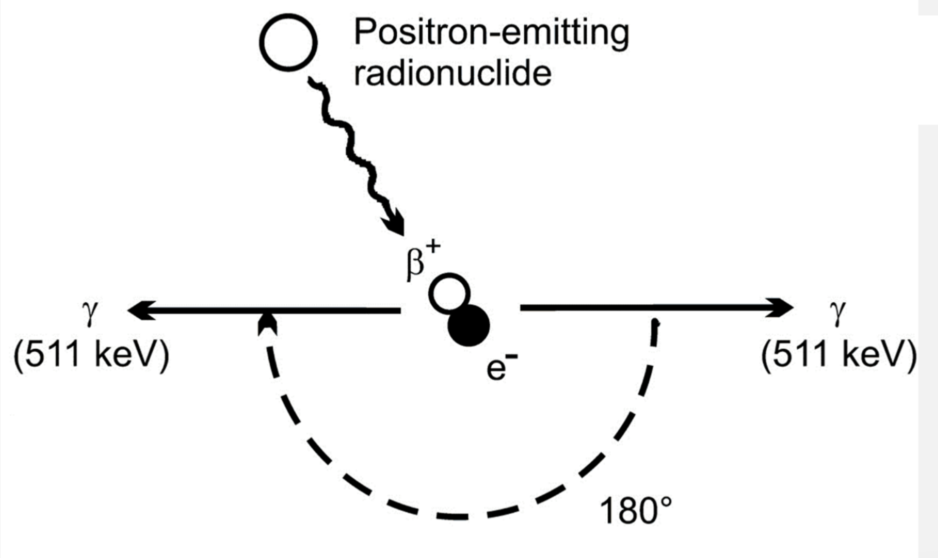
- As follows from the above results, the entanglement of annihilation photons cannot be proven from angular distributions;

- New methods should be developed to prove the entanglement of the annihilation photons.

Thank you for your attention

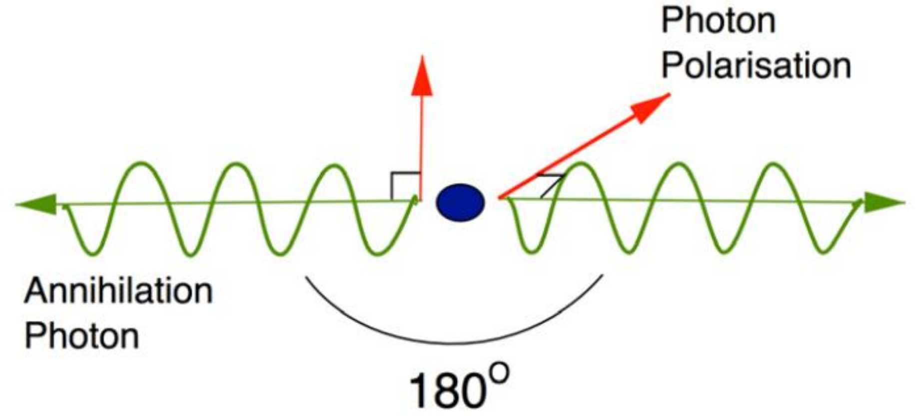
Entangled annihilation photons

(two-photon electron-positron annihilation at rest)



According to angular momentum conservation and parity symmetry the state vector of annihilation pair is:

$$\Psi = |H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2$$



Each photon in pair has no definite polarization but polarizations are orthogonal for photons in pair.
According to the <u>theory</u> the annihilation photons are maximally entangled.
But it was never <u>experimentally</u> proven!
The reason: difficulties in polarization measurements for high energy gammas.

CURRENT SITUATION WITH ANNIHILATION PHOTONS

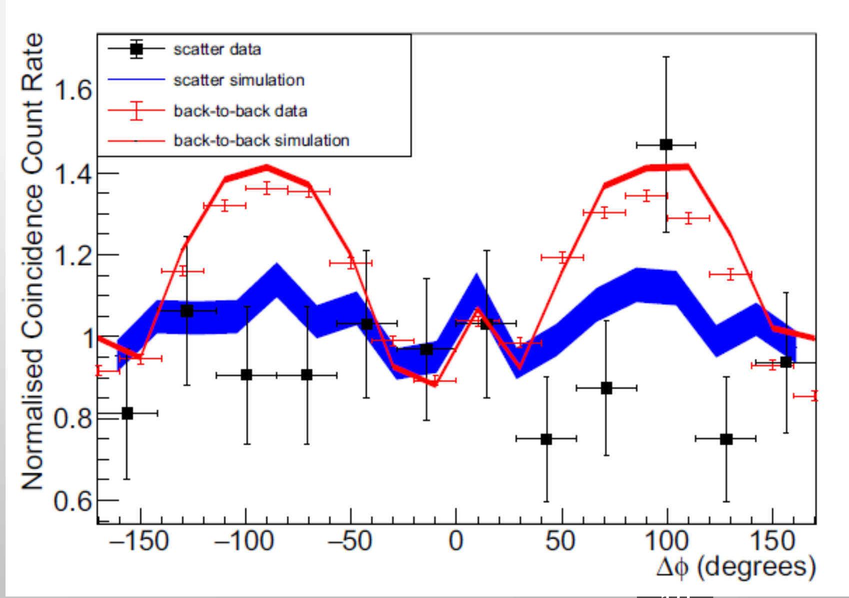
Hiesmayr B.C. and Moskal P. Witnessing entanglement in Compton scattering processes via mutually unbiased bases Sci. Rep.9 8166 (2019)

The Compton scattering of annihilation photons is the same for both entangled and decoherent states. There is NO the experimental proof of the entanglement.

Peter Caradonna *et al.* Probing entanglement in Compton interactions *J. Phys. Commun.* **3** 105005 (2019)

The Compton scattering of annihilation photons is principally different for entangled and decoherent states. There is no need to prove the entanglement. But…
The measurements of decoherent photons are needed!

Watts, D.P., Bordes, J., Brown, J.R. *et al.* Photon quantum entanglement in the MeV regime and its application in PET imaging. *Nat Commun* **12,** 2646 (2021)

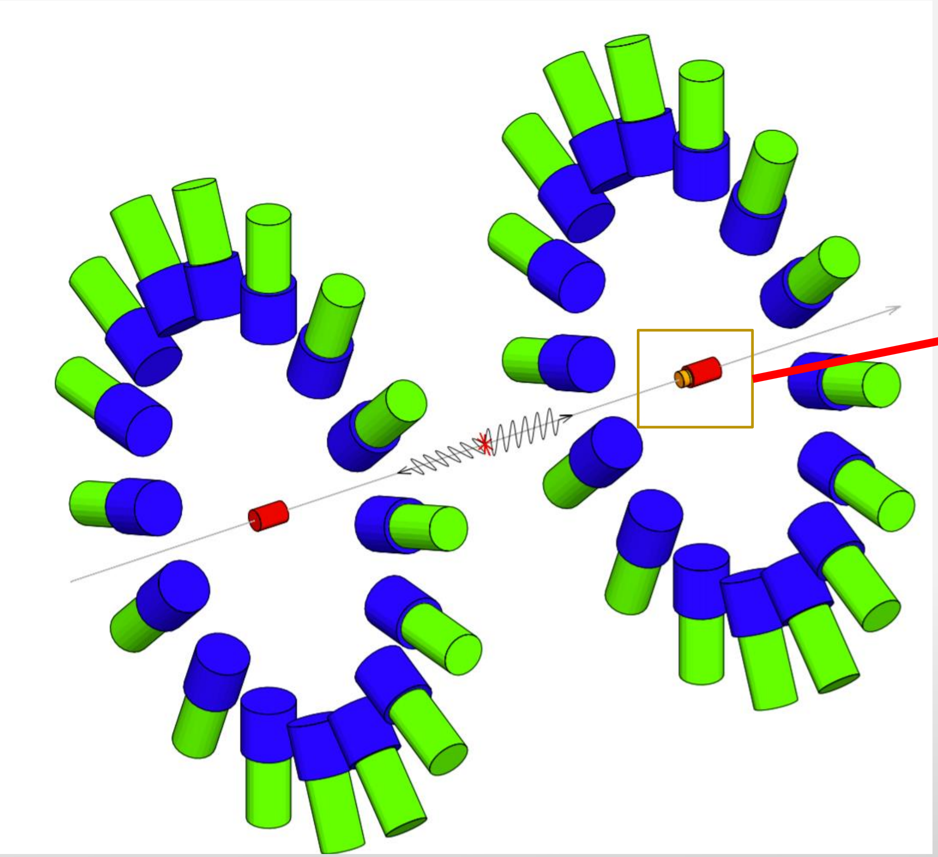


First measurement of decoherent annihilation photons was done this year with decoherent photons. The sensitivity of experimental setup and the poor statistics do not allow the comparison of Compton scattering of photons in entangled and decoherent states.

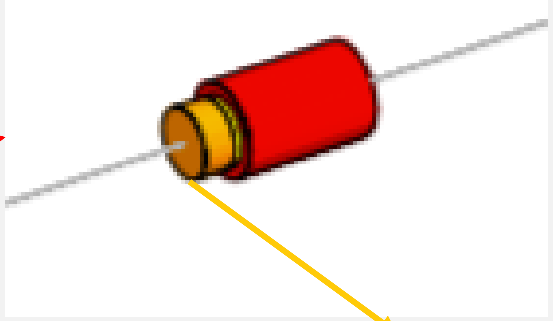
New experiment is needed to test the theoretical puzzle!

Principle of production of decoherent pairs

Decoherence is the transition from entangled to mixed quantum state as a result of interaction with environment (intermediate scatterer).



Intermediate scatterer: scintillator GAGG



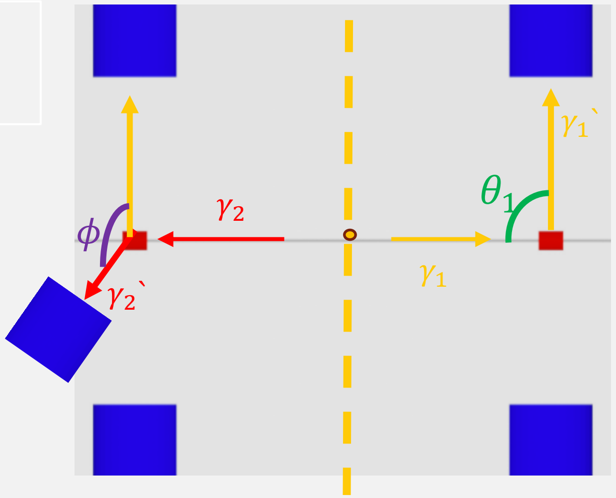
To produce decoherent photons ***intermediate GAGG scatterer*** is placed before one of the ***main scatterers***.

If first interaction in intermediate scatterer occurs then pair of gammas becomes decoherent. The decoherent pairs are easily distinguished from the entangled ones by analyzing time and energy spectra in GAGG scintillator.

AZIMUTHAL CORRELATIONS OF SCATTERED PHOTONS

Snyder H S, Pasternack S and Hornbostel J, - 1948 Angular correlation of scattered annihilation radiation Phys. Rev. 73 440–8

$$P_{12}(E_1,E_2,\phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP}\left(\frac{d\sigma}{d\Omega_2}\right)_{NP}[1 - \alpha(\theta_1)\alpha(\theta_2)cos(2\phi)]$$



Ratio of the numbers of scattered annihilation photons:

$$R_{theory}(\theta) = \frac{N\left(\phi = \frac{\pi}{2}\right)}{N(\phi = 0)} = 1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}; \qquad \gamma = 2 - cos\theta + (2 - cos\theta)^{-1}$$

$R = 2.83$ $for\ \theta = 82^0$ or $R = 2.6$ $for\ \theta = 90^0$

According to D. Bohm and Y. Aharonov (Phys. Rev. (1957) 108, 1070:
if R > 2 ⇒ gamma pair is entangled.

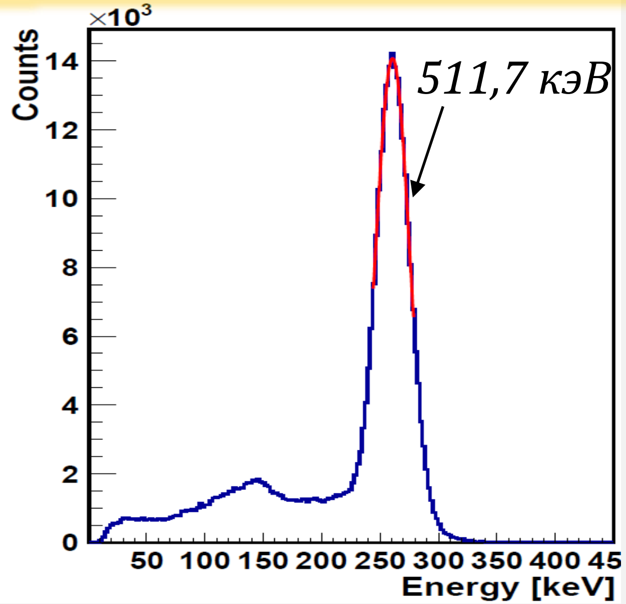
For decoherent photons ***R=1*** for non-entangled photons ***R<2***

The best experimental values:

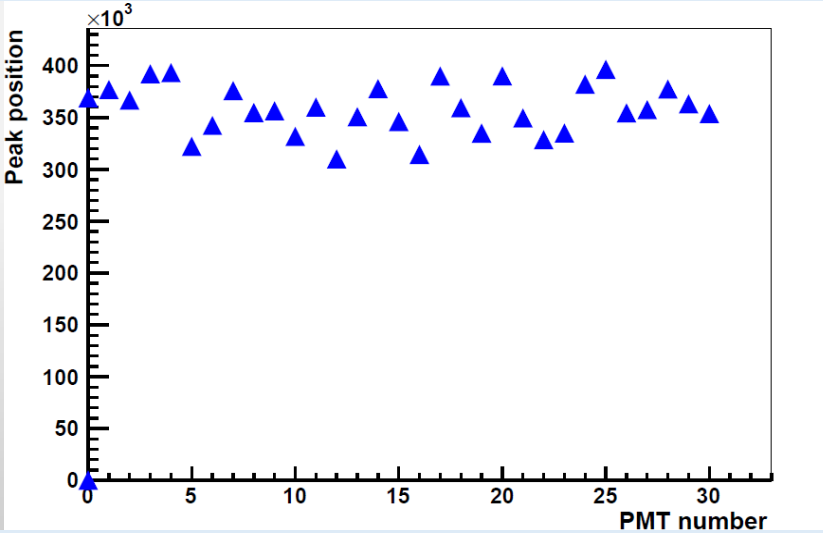
H. Langhof, Zeitschrift fur Physik 160, 186-193 (1960) $R = 2.47 \pm 0.07$

The above data confirmed (*to authors belief*) that ***the annihilation photons are entangled***!
The ***decoherent*** annihilation photons ***were not measured*** at all!

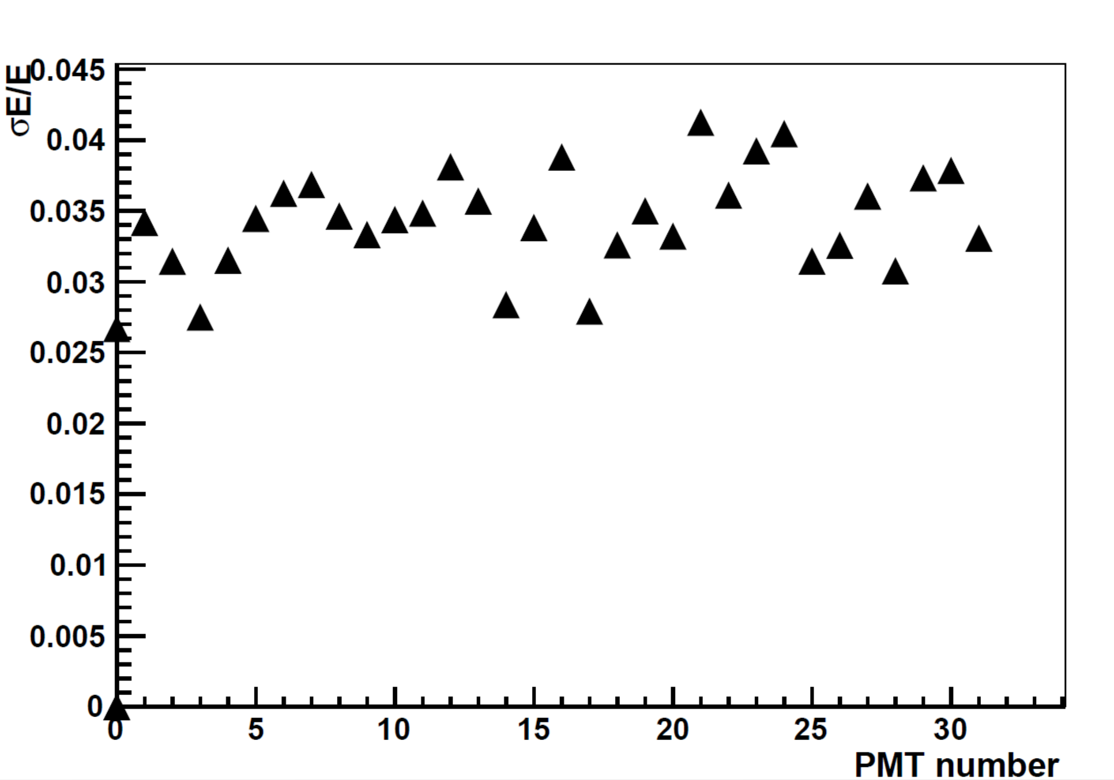
Calibration of energy resolution of scintillation detectors (NaI)



Energy spectrum of ^{22}Na in scintillation detector



Peak position for all scintillation detectors of the setup (NaI)



Energy resolutions of scintillation detectors (NaI)

Energy resolution FWHM/E of implemented PMTs claimed by Hamamatsu is nearly equal to 8% for the peak.

$$\frac{FWHM}{E} = 2{,}355 \cdot \frac{\sigma_E}{E} \cong 2{,}355 \cdot 0{,}034 = 0{,}08$$

As we can see, the resolution of our PNTs is equal to that number