Second order viscous hydrodynamics within an effective kinetic theory and thermal particles from QGP

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- Results are presented in

  Second order hydrodynamics based on effective kinetic theory and electromagnetic signals from QGP,
  Lakshmi J. Naik and V. Sreekanth
  arXiv : 2207.05310 [nucl-th]
  under review in Journal of Physics G.

Introduction

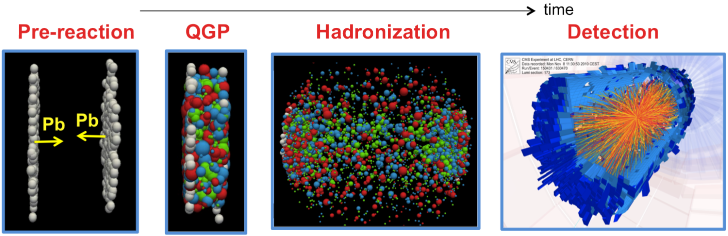

Figure: Schematic sketch of relativistic heavy ion collisions.[http://wl33.web.rice.edu/research.html]

- Quark-Gluon Plasma (QGP) has extreme low value of η/s $(= 1/4\pi)$. [KSS, Phys. Rev. Lett. 94, 111601 (2005)]
- The expansion of QGP has to be modelled using relativistic viscous hydrodynamics.

Introduction

- **First-order Navier Stokes theory** $\longrightarrow$ acausal behaviour [L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Vol. 6 (1987)]
- **Second order theories** $\longrightarrow$ no unique prescription to derive existing evolution equations for the dissipative quantities and there exist several successful formalisms.
- Recently, a new second order viscous hydrodynamics was developed within the effective fugacity quasiparticle model (EQPM) for the hot QCD medium [S. Bhadury et al., JPhysG 48, 105104 (2021)]

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- Recently, a new second order viscous hydrodynamics was developed within the effective fugacity quasiparticle model (EQPM) for the hot QCD medium [S. Bhadury et al., JPhysG 48, 105104 (2021)]

  $\rightarrow$ consistent analysis of evolution and particle spectra within this formalism

Effective fugacity Quasi-Particle Model

- Model initiates with an ansatz that the lattice QCD EoS can be interpreted in terms of non-interacting quasi-partons with effective fugacity parameters, $z_{q,g}$ encoding the interaction effects.

[V. Chandra and V. Ravishankar, EPJC 64, 63-72 (2009); V. Chandra and V. Ravishankar, PRD 84, 074013 (2011)]

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- Equilibrium momentum distribution functions of the quasi-particles are given by

$$f_k^0 = \frac{z_k \exp[-\beta(u_\mu p_k^\mu)]}{1 \pm z_k \exp[-\beta(u_\mu p_k^\mu)]},$$

$k \equiv (q, g)$ represent the quarks and gluons.

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Effective fugacity Quasi-Particle Model

- Quasi-particle 4−momenta is given by the dispersion relation

$$\tilde{p}^{\mu}_{g,q} = p^{\mu}_{g,q} + \delta\omega_{g,q} u^{\mu}; \qquad \delta\omega_{g,q} = T^2 \partial_T \ln(z_{g,q})$$

$\delta\omega_{g,q}$ is the modified part of dispersion relation.

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  $\delta\omega_{g,q}$ is the modified part of dispersion relation.
- Determination of equilibrium distribution function is achieved by fixing the temperature dependencies of z_k from lattice QCD EoS [M. Cheng et al., Phys. Rev. D 77, 014511 (2008); S. Borsanyi et al., Phys. Lett. B 730, 99–104 (2014)]

Form of viscous correction

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$$f_k = f_k^0 + \delta f_k$$
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- Effective Boltzmann equation within the framework of EQPM [S. Mitra and V. Chandra, PRD 97, 034032 (2018)]

$$\tilde{p}_k^{\mu}\partial_{\mu} f_k^0(x, \tilde{p}_k) + F_k^{\mu}\partial_{\mu}^{(p)} f_k^0 = -\frac{\delta f_k}{\tau_R}\omega_k,$$

where τ_R is the relaxation time and $F_k^{\mu} = -\partial_{\nu}(\delta\omega_k u^{\nu} u^{\mu})$.

Form of viscous correction

δf is obtained from an iterative Chapman-Enskog like solution of the Boltzmann equation in RTA [S. Bhadury et al., JPhysG 48, 105104 (2021)]

$$\delta f_q = \tau_R \left[\tilde{p}_q^{\mu} \partial_\mu \beta + \frac{\beta\, \tilde{p}_q^{\mu}\, \tilde{p}_q^{\nu}}{u \cdot \tilde{p}_q} \partial_\mu u_\nu - \beta \Theta(\delta\omega_q) - \beta \dot{\beta} \left(\frac{\partial(\delta\omega_q)}{\partial\beta} \right) \right] f_q^0 \bar{f}_q^0$$

with $\bar{f}_q^0 = 1 - a\bar{f}_q^0$ and $a = \pm 1$ for quarks/gluons.

Hydrodynamic evolution equations

- The evolution equations for shear stress tensor and bulk viscous pressure are obtained as

$$
\begin{aligned}
\dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_R} &= 2\beta_\pi \sigma^{\mu\nu} + 2\pi_\phi^{\langle\mu}\omega^{\nu\rangle\phi} - \delta_{\pi\pi}\pi^{\mu\nu}\theta \\
&\quad - \tau_{\pi\pi}\pi_\phi^{\langle\mu}\sigma^{\nu\rangle\phi} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu}, \\
\dot{\Pi} + \frac{\Pi}{\tau_R} &= -\beta_\Pi\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}.
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Here, $\omega^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu)$ denotes the vorticity tensor.

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- The second order transport coefficients are obtained in terms of different thermodynamic integrals
  [S. Bhadury et al., JPhysG 48, 105104 (2021)]

1D Boost Invariant Flow

Geometry of QGP expansion: Bjorken's prescription to describe the evolution of QGP: [J. D. Bjorken, PRD 27, 140-151 (1983)]

- coordinates are parameterized using proper time $\tau = \sqrt{t^2 - z^2}$ and space-time rapidity $\eta_s = \frac{1}{2} \ln \left[\frac{t+z}{t-z} \right]$
- in the local rest frame of the fluid, $u^\mu = (\cosh \eta_s, 0, 0, \sinh \eta_s)$
- Under these assumptions, hydrodynamic evolution equations become

$$
\begin{aligned}
\frac{d\epsilon}{d\tau} &= -\frac{1}{\tau} \left(\epsilon + P + \Pi - \pi \right), \\
\frac{d\pi}{d\tau} + \frac{\pi}{\tau_\pi} &= \frac{4}{3} \frac{\beta_\pi}{\tau} - \left(\frac{1}{3} \tau_{\pi\pi} + \delta_{\pi\pi} \right) \frac{\pi}{\tau} + \frac{2}{3} \lambda_{\pi\Pi} \frac{\Pi}{\tau}, \\
\frac{d\Pi}{d\tau} + \frac{\Pi}{\tau_\Pi} &= -\frac{\beta_\Pi}{\tau} - \delta_{\Pi\Pi} \frac{\Pi}{\tau} + \lambda_{\Pi\pi} \frac{\pi}{\tau},
\end{aligned}
$$

$\pi = \pi^{00} - \pi^{zz}$.

Hydrodynamic evolution for Bjorken flow

- Hydrodynamic evolution equations can be solved by specifying the relaxations times
- As a result of RTA, we obtain a single relaxation time-scale for both shear and bulk, $\tau_\pi = \tau_\Pi = \tau_R$
- We choose different temperature dependent forms of τ_R

$$\tau_R = 2(\eta/s)/T, \quad 1.5(\eta/s)/T, \quad (\eta/s)/T$$

- We use the lower bound of shear viscosity to entropy ratio: $\eta/s = 1/4\pi$
- Initial conditions relevant for RHIC energies: $\tau_0 = 0.5$ fm/c and $T_0 = 0.31$ GeV

Evolution: Shear

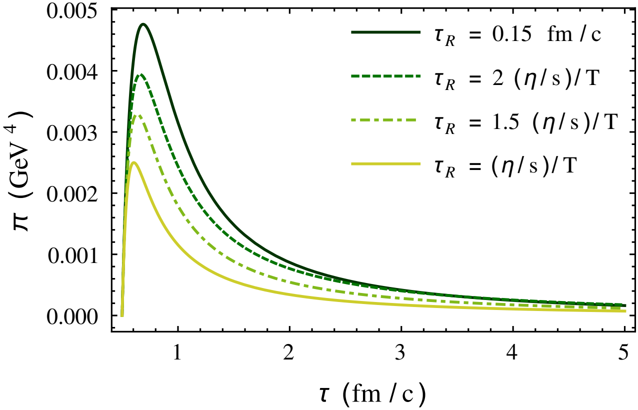

Figure: Proper time evolution of shear stress tensor for different temperature dependent forms of τ_R.

Evolution: Bulk

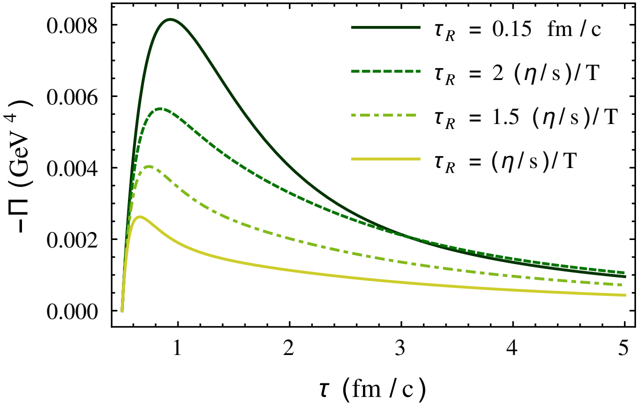

Figure: Proper time evolution of bulk viscous pressure for different temperature dependent forms of τ_R.

Evolution: Pressure anisotropy

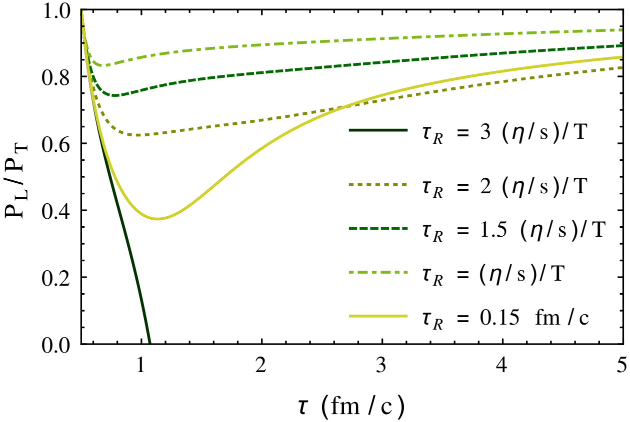

Figure: Proper time evolution of pressure anisotropy P_L/P_T with various temperature dependent relaxation times.

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- Thermal photons/dileptons can be used as a tool to measure the shear viscosity [J. Bhatt and V. Sreekanth IJMPE 19, 299–306 (2010), K Dusling NPA 839, 70–77 (2010)], bulk viscosity [J. Bhatt et. al JHEP 11 106 (2010), J. Bhatt et. al. NPA 875 181-196 (2012)] of the strongly interacting matter produced in the collisions

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- Thermal particle production with various non-equilibrium scenarios have been explored in EQPM [V Chandra and V Sreekanth, PRD (2015) & EPJC (2017); Lakshmi J Naik et. al J Phys G (2022)]

Thermal dilepton production rate

- Viscosity affects the thermal particle production via T and viscous corrections entering through rate calculations

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$$\frac{dN}{d^4x d^4p} = \iint \frac{d^3\vec{p}_1}{(2\pi)^3} \frac{d^3\vec{p}_2}{(2\pi)^3} \frac{M_{\rm eff}^2\, g^2\, \sigma(M_{\rm eff}^2)}{2\omega_1\omega_2} f(\vec{p}_1) f(\vec{p}_2) \delta^4(\tilde{p} - \tilde{p}_1 - \tilde{p}_2).$$

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- $M_{\rm eff}^2 = (\omega_1+\omega_2)^2 - (\vec{p}_1+\vec{p}_2)^2$ represents the modified effective mass of the virtual photon in the interacting QCD medium.
- Keeping the terms up to linear order in $\delta\omega_q$, we get [Lakshmi J. Naik et al., J Phys G. (2022)]

$$M_{\rm eff}^2 \approx M^2\left(1+\frac{4\,\delta\omega_q\,(E_1+E_2)}{M^2}\right).$$

- In the limit of ideal EoS, $M_{\rm eff} \to M$

Distribution function

- Viscous modified momentum distribution functions : $f(\vec{p}) \equiv f_q^0 + f_q^0 \bar{f}_q^0 \delta f_q$, where

$$\begin{aligned} \delta f &= \delta f_\pi + \delta f_\Pi \\ &= \frac{\beta}{2\beta_\pi (u\cdot\tilde{p})} \tilde{p}^\mu \tilde{p}^\nu \pi_{\mu\nu} + \frac{\beta\Pi}{\beta_\Pi}\Big[\xi_1 - \xi_2 (u\cdot\tilde{p})\Big], \end{aligned}$$

where

$$\begin{aligned} \xi_1 &= \beta c_s^2 \frac{\partial \delta\omega_q}{\partial\beta} + \delta\omega_q, \\ \xi_2 &= \left(c_s^2 - \frac{1}{3}\right) + \frac{\delta\omega_q}{3(u\cdot\tilde{p})^2}\left[2(u\cdot\tilde{p}) - \delta\omega_q\right]. \end{aligned}$$

Thermal dilepton production rate

Contribution due to shear and bulk viscosities:

$$\frac{dN^{(\pi)}}{d^4xd^4p} = \frac{dN^{(0)}}{d^4xd^4p}\left\{\frac{\beta}{\beta_\pi}\frac{1}{2|\vec{p}|^5}\left[\frac{(u\cdot\tilde{p})|\vec{p}|}{2}\left(2|\vec{p}|^2-3M_{\rm eff}^2\right)\right.\right.$$
$$\left.\left.+\frac{3}{4}M_{\rm eff}^4\ln\left(\frac{(u\cdot\tilde{p})+|\vec{p}|}{(u\cdot\tilde{p})-|\vec{p}|}\right)\right]\tilde{p}^\mu\tilde{p}^\nu\pi_{\mu\nu}\right\},$$

$$\frac{dN^{(\Pi)}}{d^4xd^4p} = \frac{dN^{(0)}}{d^4xd^4p}\frac{2\beta\Pi}{\beta_\Pi}\left\{\beta c_s^2\frac{\partial\delta\omega_q}{\partial\beta}-\frac{2}{3}\delta\omega_q-\left(c_s^2-\frac{1}{3}\right)\frac{(u\cdot\tilde{p})}{2}\right.$$
$$+\frac{\delta\omega_q^2}{3}\frac{1}{2|\vec{p}|^5}\left[\frac{(u\cdot\tilde{p})|\vec{p}|}{2}\left(2|\vec{p}|^2-3M_{\rm eff}^2\right)\right.$$
$$\left.\left.+\frac{3}{4}M_{\rm eff}^4\ln\left(\frac{(u\cdot\tilde{p})+|\vec{p}|}{(u\cdot\tilde{p})-|\vec{p}|}\right)\right]\right\}.$$

Thermal dilepton yield

- Dilepton yield within Bjorken expansion is calculated as

$$\frac{dN}{dM^2 d^2p_T dy} = A_\perp \int_{\tau_0}^{\tau_f} d\tau\, \tau \int_{-\infty}^{\infty} d\eta_s\, \chi(T,\eta_s) \left(\frac{1}{2} \frac{dN}{d^4x d^4p} \right),$$

where $\chi(T,\eta_s) = \left[1 + \frac{2}{m_T} \cosh(y-\eta_s) \delta\omega_q \right]$.

- Total dilepton yield,

$$\frac{dN}{dM^2 d^2p_T dy} = \frac{dN^{(0)}}{dM^2 d^2p_T dy} + \frac{dN^{(\pi)}}{dM^2 d^2p_T dy} + \frac{dN^{(\Pi)}}{dM^2 d^2p_T dy}.$$

Dilepton Spectra

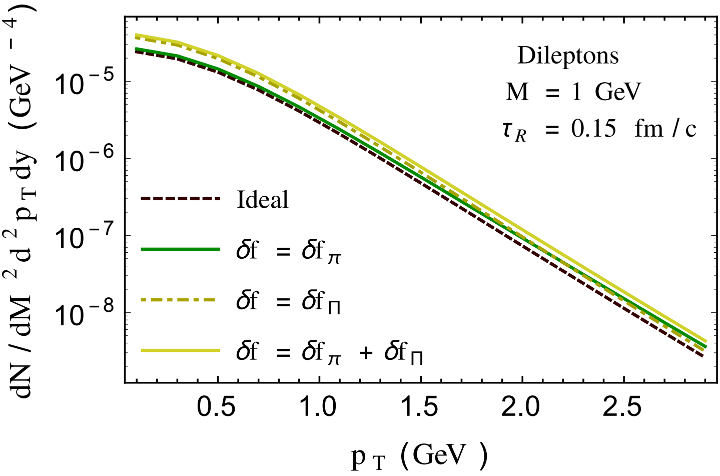

Figure: Thermal dilepton yields in the presence of viscous corrections corresponding to $\tau_R = 0.15$ fm/c and for $M = 1$ GeV.

Dilepton Spectra

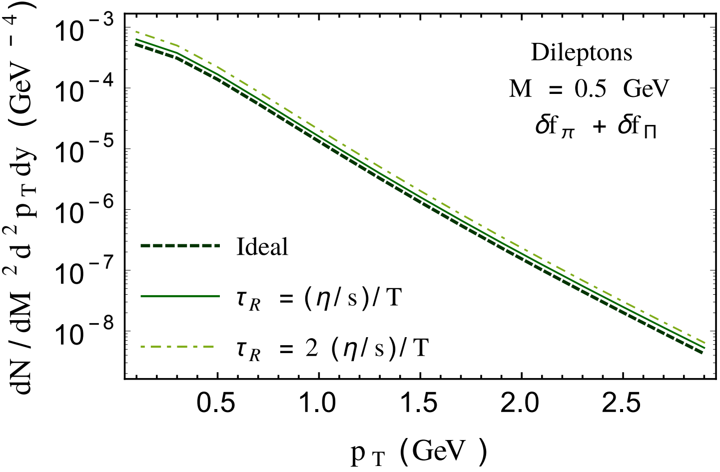

Figure: Dilepton spectra in the presence of viscous corrections by varying τ_R for $M = 0.5$ GeV

Dilepton Spectra

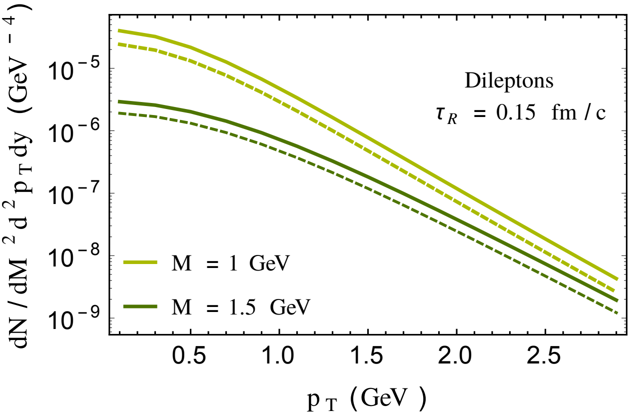

Figure: Comparison of dilepton spectra for different M values with $\tau_R = 0.15$ fm/c. The solid lines represent the total yields and dashed lines correspond to $\delta f = 0$ case.

Conclusions

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- The dissipative corrections to the phase-space distribution functions are obtained from the Chapman-Enskog like iterative solution of effective Boltzmann equation in the relaxation time approximation.
- The sensitivity of shear and bulk viscous pressures to the temperature dependence of relaxation time is analyzed within one dimensional boost invariant expansion of QGP

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- Particle emission yields are quantified for the longitudinal expansion of QGP with different temperature dependent relaxation times.
- Analysis indicates that the particle spectra is well behaved and sensitive to relaxation time.

Thank You