

Optical simulation of PMT

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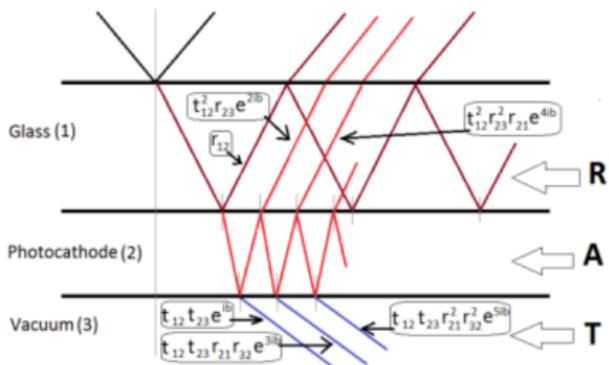
Joint Institute for Nuclear Research



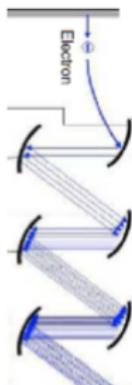
Formulation of a problem

- JUNO reactor antineutrino experiment aims to determine the neutrino mass hierarchy at $(3 - 4)\sigma$.
- To achieve this goal JUNO has to reconstruct $\bar{\nu}_e$ energy with accuracy better than $3\%/\sqrt{E_{\text{vis}}/\text{MeV}}$.
- One of the ingredients of this unprecedented energy resolution is high quantum efficiency PMT (Hamamatsu (Japan) and North Vision (China)).
- The optical model is a very important ingredient in the program of the energy reconstruction.

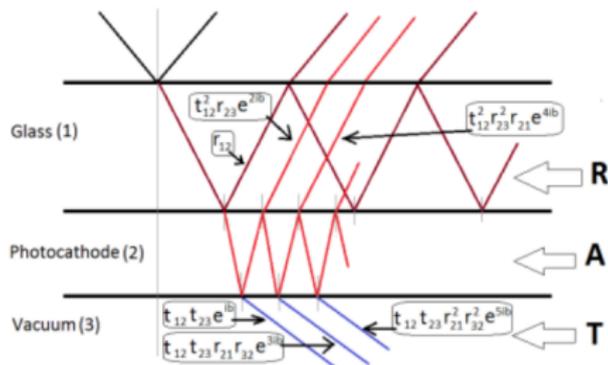
New optical model. PMT scheme.



not in scale



New optical model. PMT scheme.



$$r_{if}^s = \frac{n_i \cos(\theta_i) - n_f \cos(\theta_f)}{n_i \cos(\theta_i) + n_f \cos(\theta_f)}, \quad t_{if}^s = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_f \cos(\theta_f)},$$

$$r_{if}^p = \frac{\frac{1}{n_i} \cos(\theta_i) - \frac{1}{n_f} \cos(\theta_f)}{\frac{1}{n_i} \cos(\theta_i) + \frac{1}{n_f} \cos(\theta_f)}, \quad t_{if}^p = \frac{\frac{2}{n_i} \cos(\theta_i)}{\frac{1}{n_i} \cos(\theta_i) + \frac{1}{n_f} \cos(\theta_f)}.$$

$$R_s = |r_{if}^s|^2 = \left(\frac{n_i \cos(\theta_i) - n_f \cos(\theta_f)}{n_i \cos(\theta_i) + n_f \cos(\theta_f)} \right)^2, \quad T_s = \frac{n_f \cos(\theta_f)}{n_i \cos(\theta_i)} |t_{if}^s|^2,$$

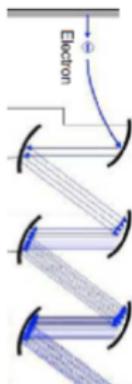
$$R_p = |r_{if}^p|^2 = \left(\frac{\frac{1}{n_i} \cos(\theta_i) - \frac{1}{n_f} \cos(\theta_f)}{\frac{1}{n_i} \cos(\theta_i) + \frac{1}{n_f} \cos(\theta_f)} \right)^2, \quad T_p = \frac{\frac{1}{n_f} \cos(\theta_f)}{\frac{1}{n_i} \cos(\theta_i)} |t_{if}^p|^2.$$

not in scale

$$r = \frac{r_{12} + r_{23}e^{2i\beta}}{1 + r_{12}r_{23}e^{2i\beta}}, \quad t = \frac{t_{12}t_{23}e^{i\beta}}{1 + r_{12}r_{23}e^{2i\beta}},$$

$$R = |r|^2 = \frac{|r_{12}|^2 + |r_{23}|^2 e^{-4v_2\eta} + 2\Re(r_{12}r_{23}^* e^{i2v_2\eta} e^{-2v_2\eta})}{1 + |r_{12}|^2 |r_{23}|^2 e^{-4v_2\eta} + 2\Re(r_{12}r_{23} e^{i2v_2\eta} e^{-2v_2\eta})},$$

$$T = C_1 |t|^2 = C_1 \frac{|t_{12}|^2 |t_{23}|^2 e^{-2v_2\eta}}{1 + |r_{12}|^2 |r_{23}|^2 e^{-4v_2\eta} + 2\Re(r_{12}r_{23} e^{i2v_2\eta} e^{-2v_2\eta})}.$$

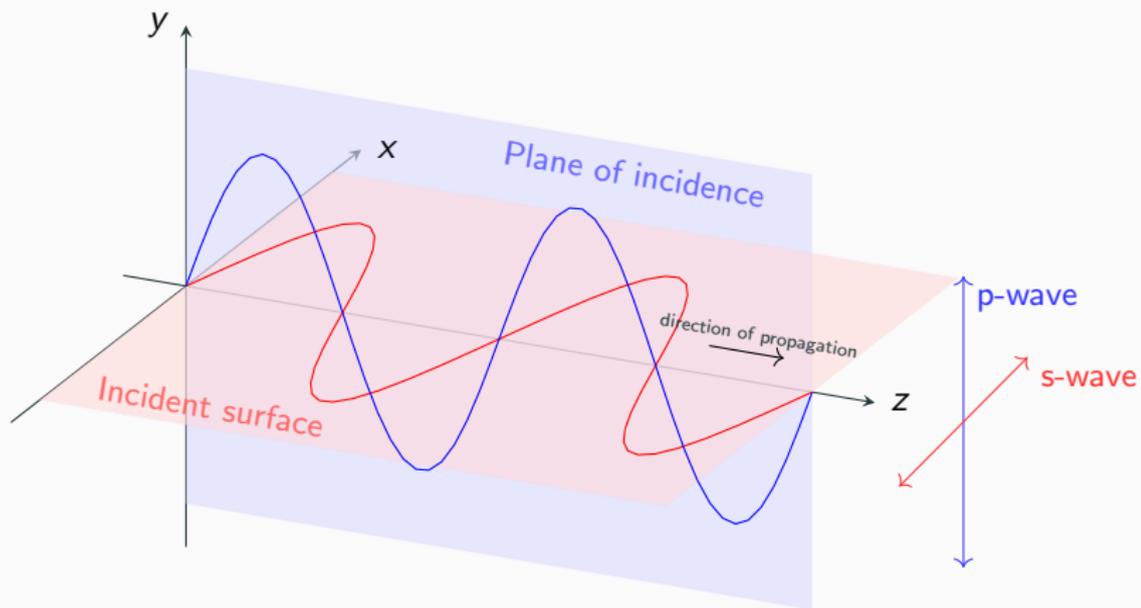


New optical model. PDE.

$$\langle \text{PDE} \rangle = \langle \underbrace{\text{CE}}_1 \int_0^d dz \underbrace{\left[1 - \int_0^z dx \frac{dA}{dx} \right]}_2 \underbrace{\sigma_{pe}}_3 \underbrace{\frac{df}{dz}(d-z)}_4 \rangle$$

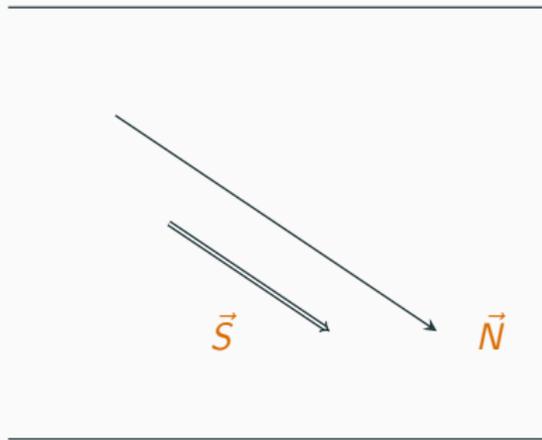
1. Collection efficiency
2. Non-absorption (optics)
3. photo-effect cross-section
4. Escape function

Polarisation.

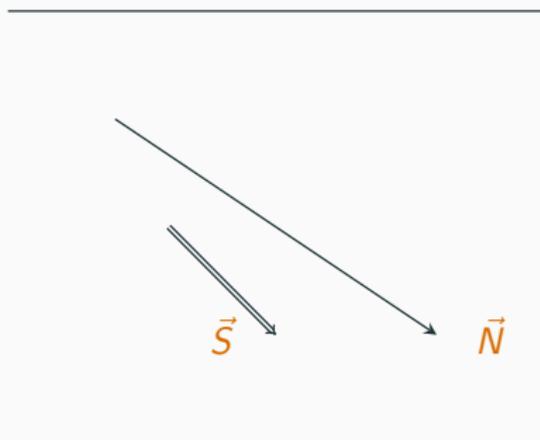


s- and p-waves

s-wave

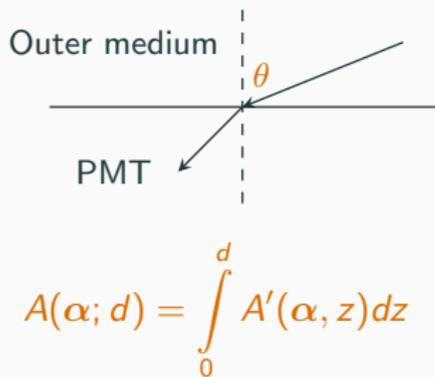
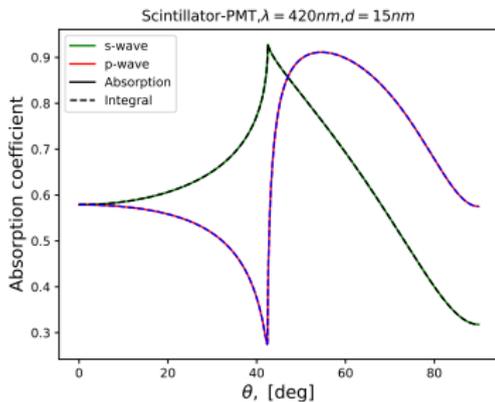
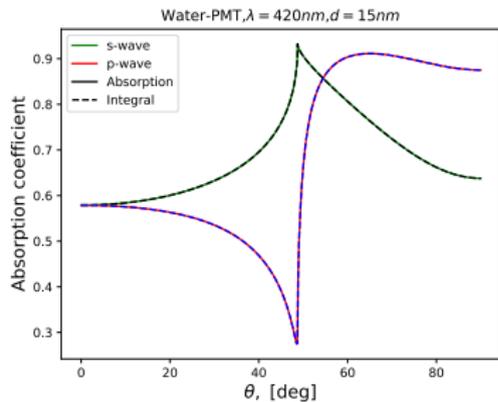
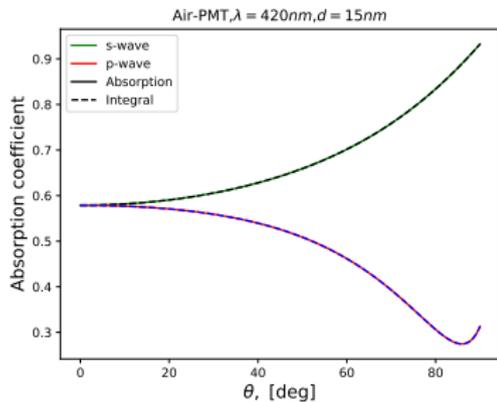


p-wave



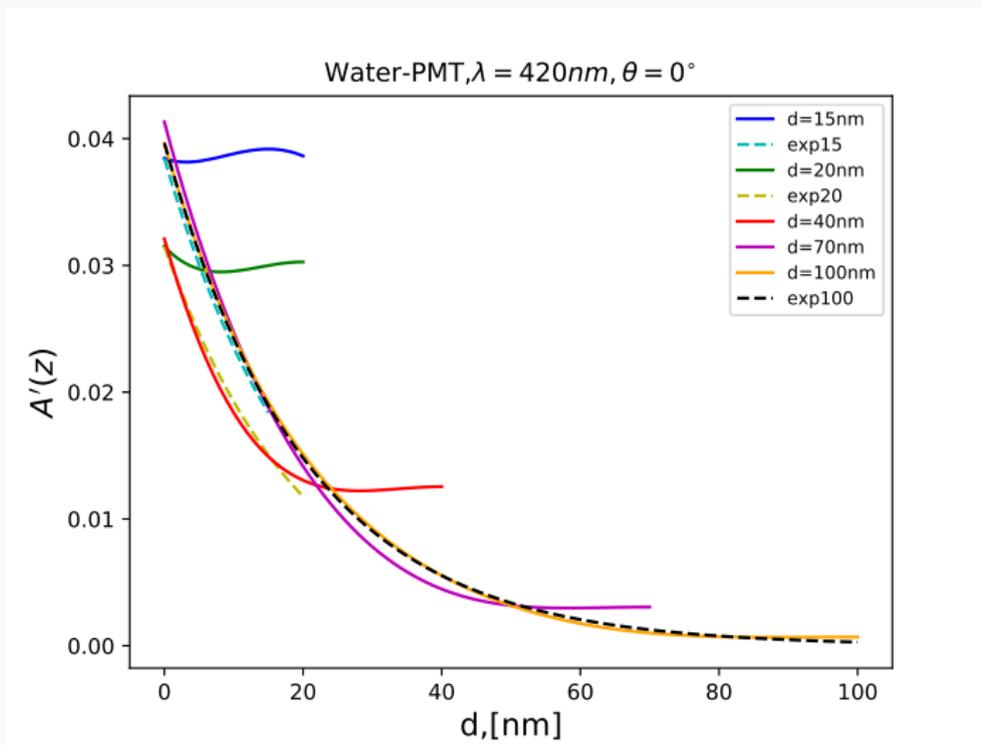
- \vec{N} is an optic ray
- \vec{S} is a Poynting vector (describes the energy flow)

Differential absorption function.

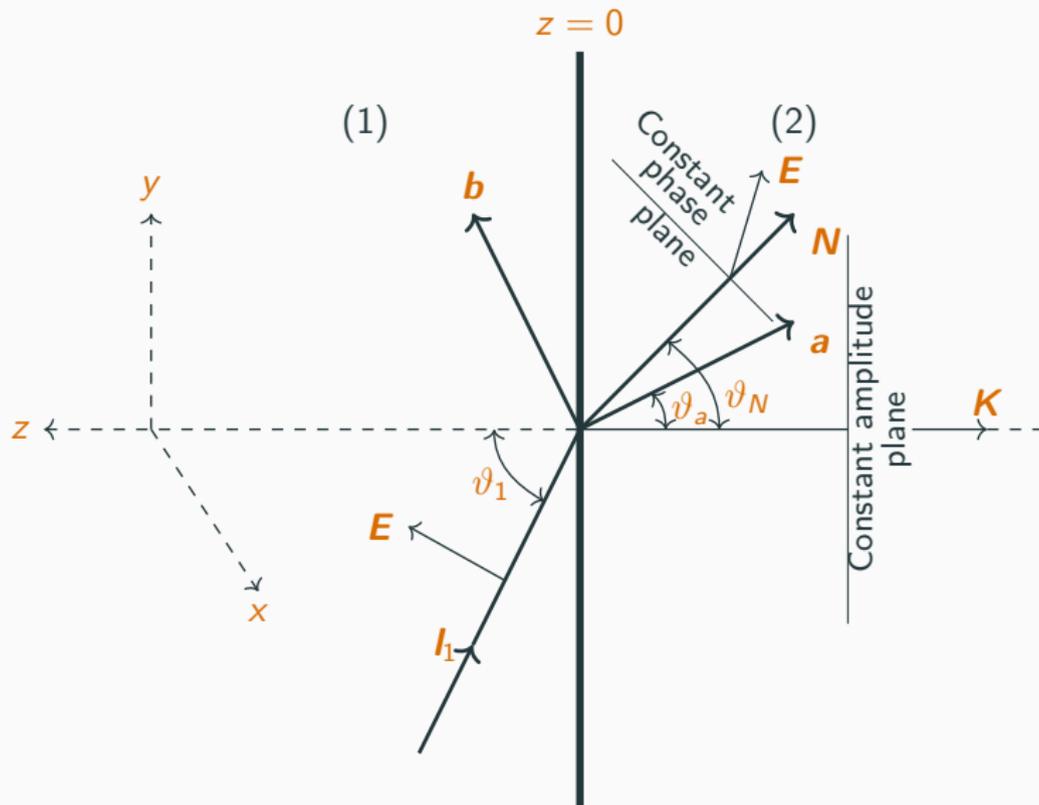


Interference effect.

If there is no interference in a thin layer than absorption to a given z should look like ordinary exponent



p-wave polarisation



p-wave polarisation

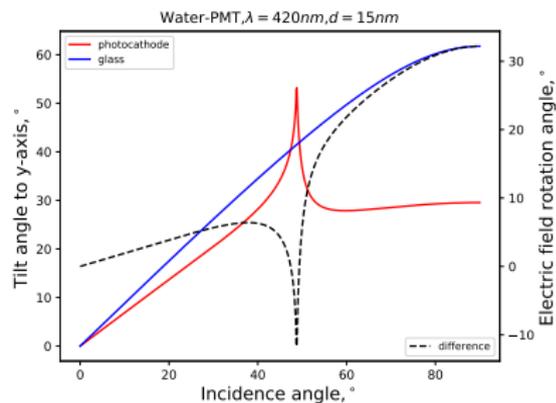
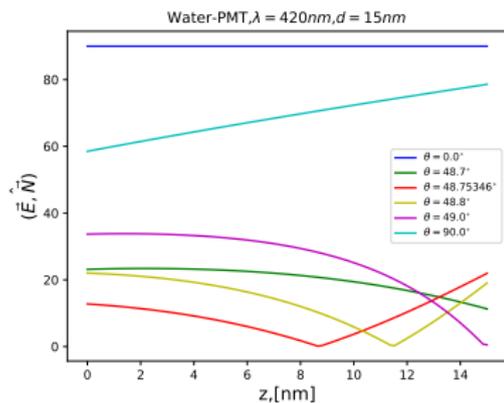
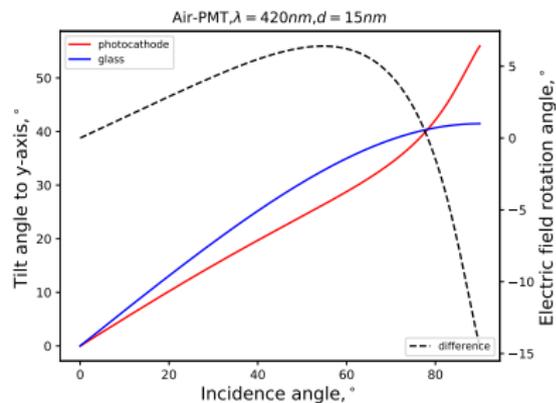
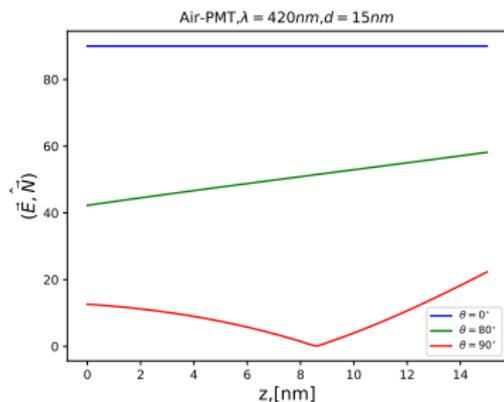
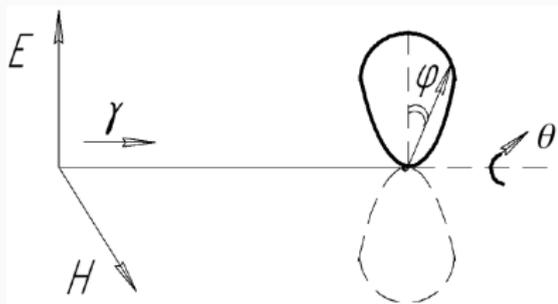
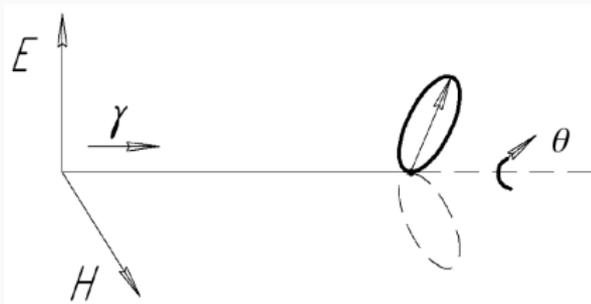


Photo electron production and the direction of its propagation.

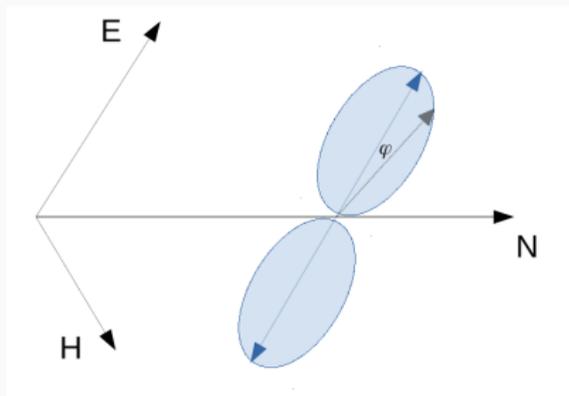
$$E_\gamma \ll m_e c^2$$



$$E_\gamma \gg m_e c^2$$



Photoelectric cross-section is $\sigma_{pe} \propto \sin^2 \theta \cos^2 \phi$



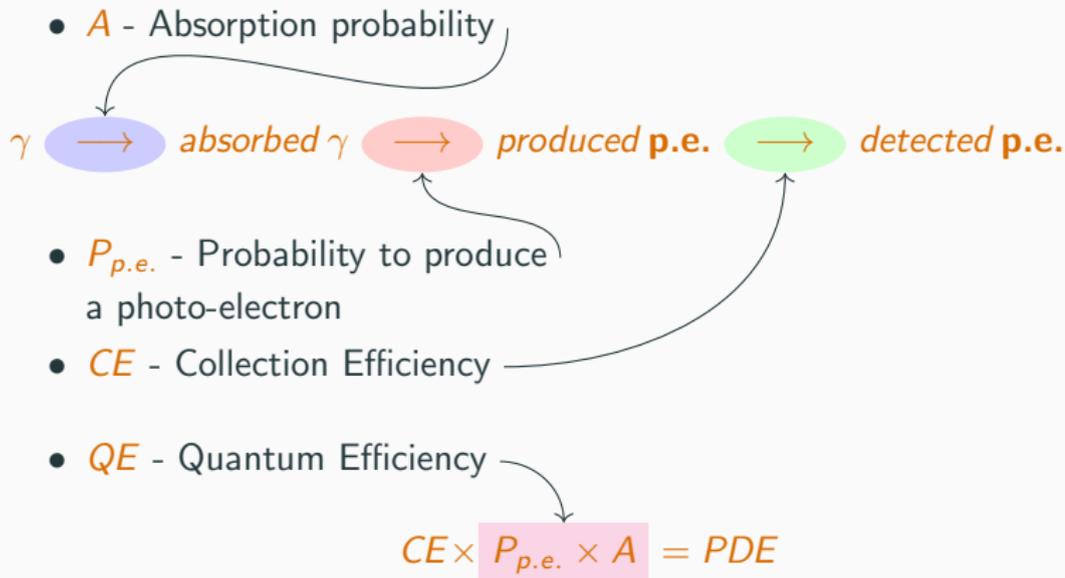
Our case is a low energy γ but \vec{E}
isn't normal to optic ray \vec{N}

- to evaluate the escape function with Geant4
- to submit a paper

- improved optical model:
 - interference in thin photocathode
 - first principles calculation
 - s and p waves
- increase in photodetection efficiency compared to naive mode

Backup

Attempts to develop the PMT optical model.



Both PDE and QE **do** depend on illumination.

Attempts to develop the PMT optical model.

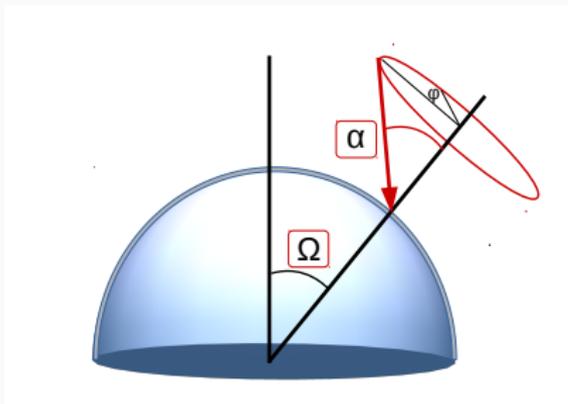


Figure 1: Definition of solid angles.

- γ field in each point is $\frac{d^2\Phi}{d\Omega d\alpha}$

$$\overline{PDE} = \frac{\iint PDE(\Omega, \alpha) \frac{d^2\Phi}{d\Omega d\alpha} d\Omega d\alpha}{\iint \frac{d^2\Phi}{d\Omega d\alpha} d\Omega d\alpha}$$

- we can parameterise the light field by the following way

$$\iint \frac{d^2\Phi}{d\Omega d\alpha} d\Omega d\alpha = 1$$

$$\overline{PDE} = \iint_{\Omega, \alpha} A(\Omega, \alpha) P_{p.e.}(\Omega, \alpha) CE(\Omega, \alpha) \frac{d^2\Phi}{d\Omega d\alpha} d\Omega d\alpha$$

"New" optical model.

Let's consider area dS

$$\frac{dN_\gamma}{d\alpha dt} = \frac{d\Phi}{d\alpha} dS$$

adding the photocathode width d

$$\frac{dN_e^{prod}(z)}{dt} = N_\gamma(z) \sigma_{pe} = \left[1 - \int_0^z \frac{dA(\bar{z})}{d\bar{z}} d\bar{z} \right] N_\gamma(0) \sigma_{pe}$$

then

$$\begin{aligned} \frac{dN_{p.e.}^{obs}}{dt} &= \int_{\alpha} d\alpha \int_0^d dz \left[1 - \int_0^z d\bar{z} \frac{dA(\bar{z})}{d\bar{z}} \right] \frac{d\Phi_\gamma(0)}{d\alpha} \sigma_{pe} CE(\alpha) \frac{df}{dz}(d-z) \\ \text{PDE} = \frac{\frac{dN_{p.e.}^{obs}}{dt}}{\frac{dN_\gamma(0)}{dt}} &= \frac{\int_{\alpha} d\alpha CE(\alpha) \int_0^d dz \left[1 - \int_0^z d\bar{z} \frac{dA(\bar{z})}{d\bar{z}} \right] \sigma_{pe} \frac{df}{dz}(d-z) \frac{d\Phi_\gamma(0)}{d\alpha}}{dS \int_{\alpha} \frac{d\Phi_\gamma(0)}{d\alpha} d\alpha} \end{aligned}$$

"New" optical model. Comparison of two models.

Let's name

$$d\bar{\sigma}_{pe}(\alpha) = \int_0^d dz \left[1 - \int_0^z d\bar{z} A'(\bar{z}) \right] \sigma_{pe} \frac{df}{dz}(d-z)$$

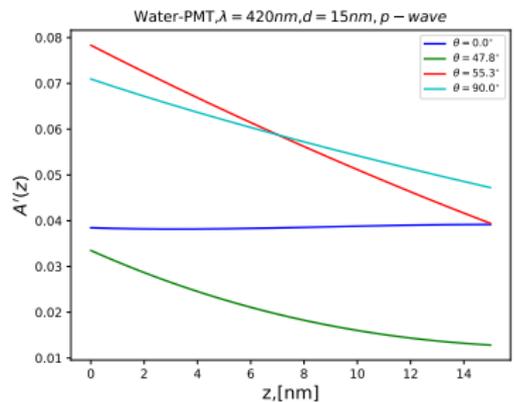
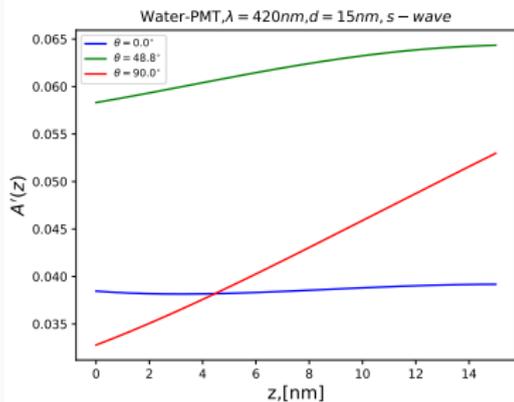
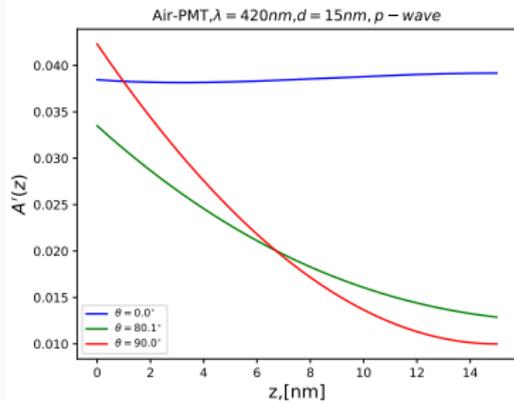
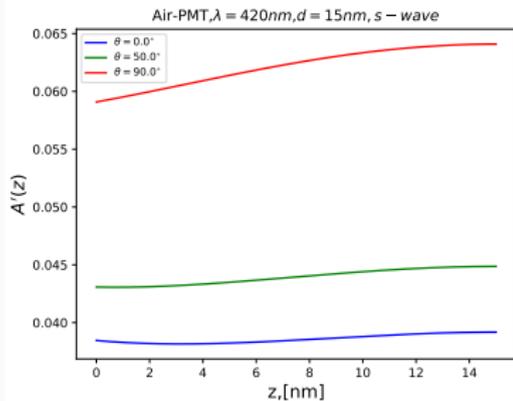
If we name $\frac{d\bar{\sigma}_{pe}}{dS} = \mathcal{P}_{p.e.}$, then

$$\text{PDE}(\Omega) = \frac{\text{CE} \int d\alpha \frac{d\Phi_\gamma(0)}{d\alpha} \mathcal{P}_{p.e.}}{\int_\alpha \frac{d\Phi_\gamma(0)}{d\alpha} d\alpha}$$

Then the final formula for average photo-detection efficiency will be

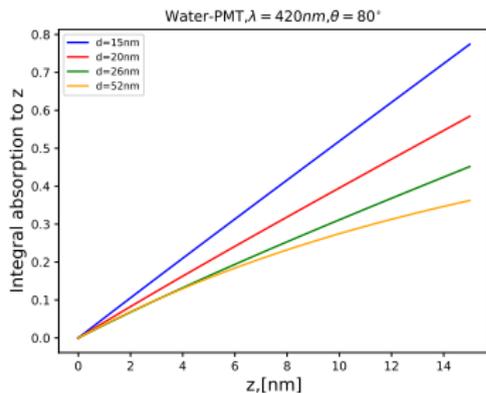
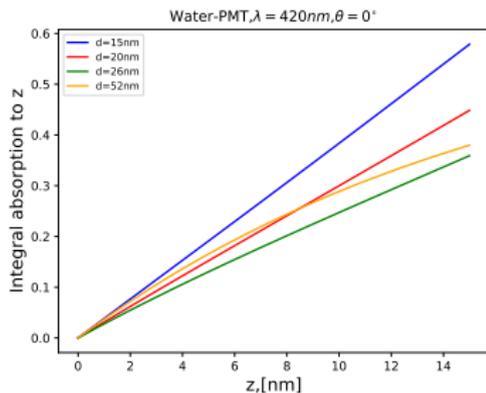
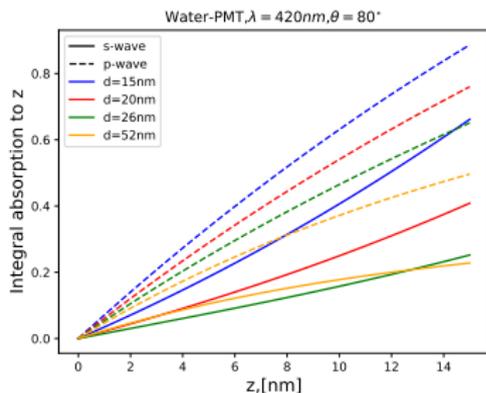
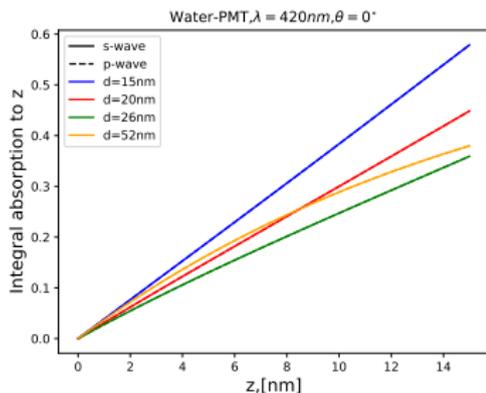
$$\overline{\text{PDE}} = \frac{\text{CE} \iint_{\Omega, \alpha} d\Omega d\alpha \frac{d^2\Phi_\gamma(0)}{d\Omega d\alpha} \mathcal{P}_{p.e.}}{\iint_{\Omega, \alpha} \frac{d^2\Phi_\gamma(0)}{d\Omega d\alpha} d\Omega d\alpha}$$

Differential absorption function.



Interference effect.

Integral absorption of the photocathode layer.



Interference effect.

Integral absorption of the photocathode layer.

