Moscow Institute of Physics and Technology

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Simulation of the setup for studying the kinematics of Compton scattering of annihilation photons in entangled and decoherent states

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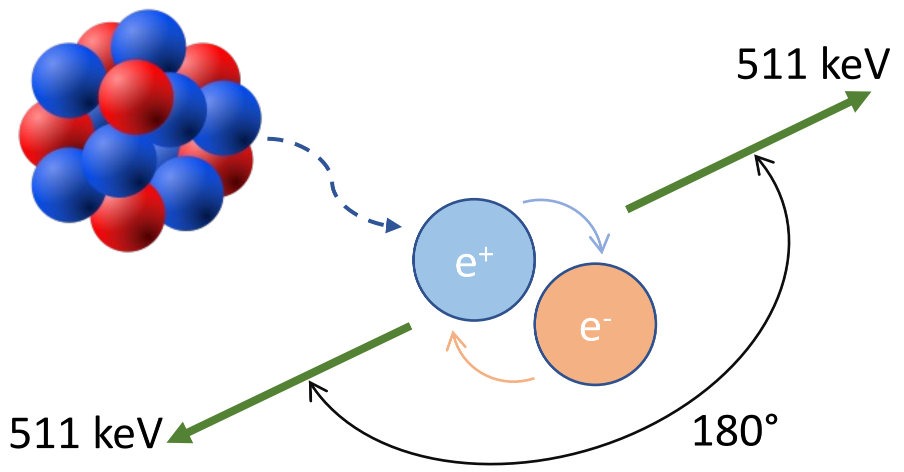
Importance

- Compton scattering of entangled and decoherent photons has not been studied thoroughly enough;
- Theoretical calculations in the scattering cross sections of entangled and decoherent photons lead to contradictory results;
- The differences in the Compton scattering of entangled and decoherent photons are planned to be used in a new generation PET tomographs.

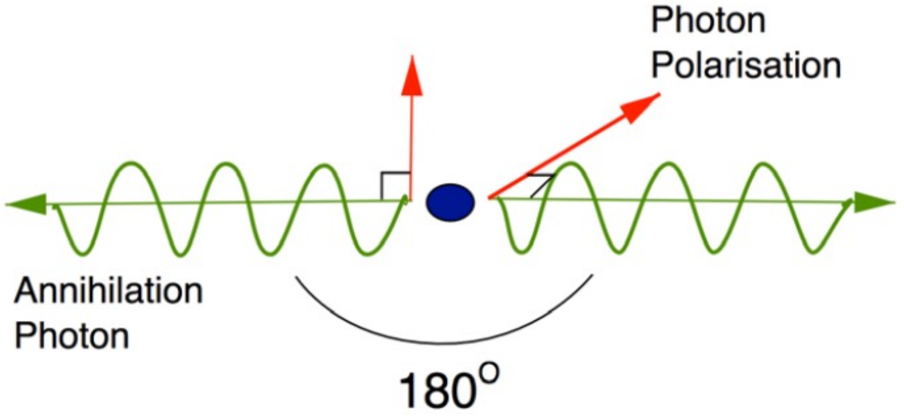
Annihilation photons



Electron-positron annihilation at rest

According to angular momentum conservation and parity symmetry the state vector of annihilation pair is:

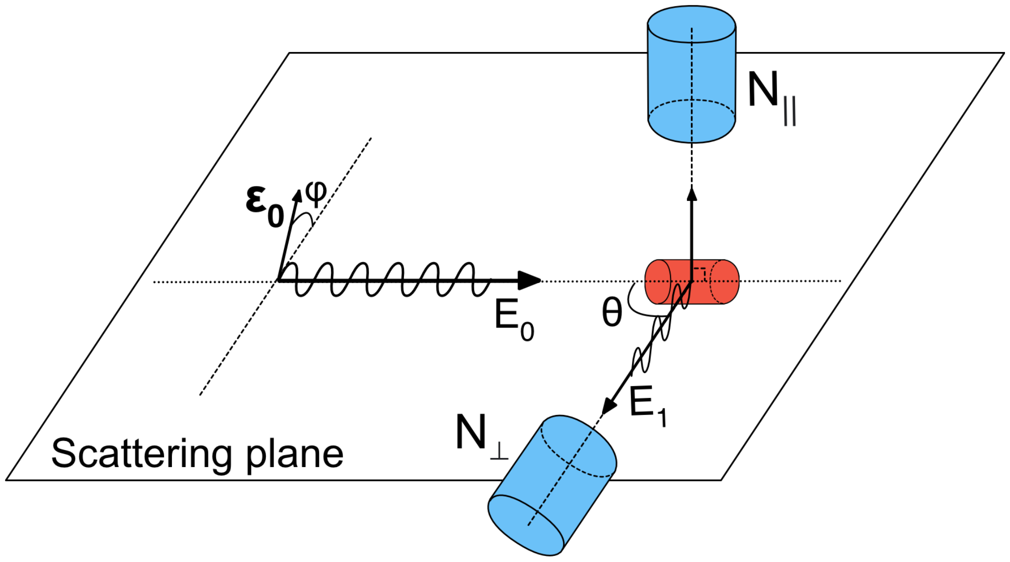
$$\Psi = |H\rangle_1 |V\rangle_2 + |V\rangle_1 |H\rangle_2$$



Each photon in pair has no definite polarization but polarizations are orthogonal for photons in pair.

According to the theory the annihilation photons are maximally entangled.

Compton polarimeter



Differential cross-section of Compton is given by Klein-Nishina formula:

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = \frac{1}{2}\cdot\frac{e^2}{m_e c^2}\cdot\frac{E_{\gamma_1}^2}{E_\gamma^2}\cdot\left(\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - 2\,\mathbf{sin^2}\,\boldsymbol{\theta}\,\mathbf{cos^2}\,\boldsymbol{\phi}\right)$$

Cross-section is maximum for $\boldsymbol{\phi} = \boldsymbol{\pi/2}$

Analyzing power is an asymmetry in scattering of gammas:

$$A = \frac{N_{\parallel} - N_{\perp}}{N_{\parallel} + N_{\perp}}$$

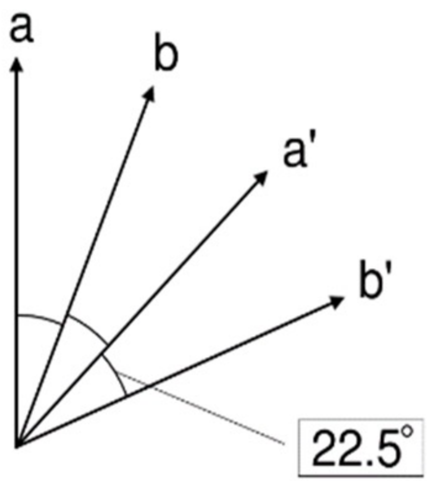
Analyzing power for Compton polarimeter:

$$A = \frac{\frac{d\sigma}{d\Omega}(\theta,\phi=90^0) - \frac{d\sigma}{d\Omega}(\theta,\phi=0^0)}{\frac{d\sigma}{d\Omega}(\theta,\phi=90^0) + \frac{d\sigma}{d\Omega}(\theta,\phi=0^0)} = \frac{\sin^2\theta}{\frac{E_{\gamma_1}}{E_\gamma}+\frac{E_\gamma}{E_{\gamma_1}} - \sin^2\theta}$$

Maximum A=0.7 for 511 keV gammas in ideal case (scattering angle=82^0).
A is significantly smaller than 1.
It provides the problems in polarization measurements for annihilation photons.

For comparison: optical polarimeters have A~1.

Probing entanglement (by measuring S-function of Bell's inequations)



Optimal angles between two-channels polarimeters

Traditional method: construction of correlation coefficients and functions based on the (CHSH-) Bell inequality :

Correlation coefficients $$E(\vec{a},\vec{b}) = \frac{N(\overrightarrow{a_{||}},\overrightarrow{b_{||}}) + N(\overrightarrow{a_{\perp}},\overrightarrow{b_{\perp}}) - N(\overrightarrow{a_{||}},\overrightarrow{b_{\perp}}) - N(\overrightarrow{a_{\perp}},\overrightarrow{b_{||}})}{N(\overrightarrow{a_{||}},\overrightarrow{b_{||}}) + N(\overrightarrow{a_{\perp}},\overrightarrow{b_{\perp}}) + N(\overrightarrow{a_{||}},\overrightarrow{b_{\perp}}) + N(\overrightarrow{a_{\perp}},\overrightarrow{b_{||}})}$$

N - number of coincidences between the corresponding counters of two two-channel polarimeters.

Correlation function for ideal polarimeter (A=1):

$$S = E(\vec{a},\vec{b}) - E(\vec{a},\vec{b}`) + E(\vec{a},\vec{b}`) + E(\vec{a}`,\vec{b}`)$$

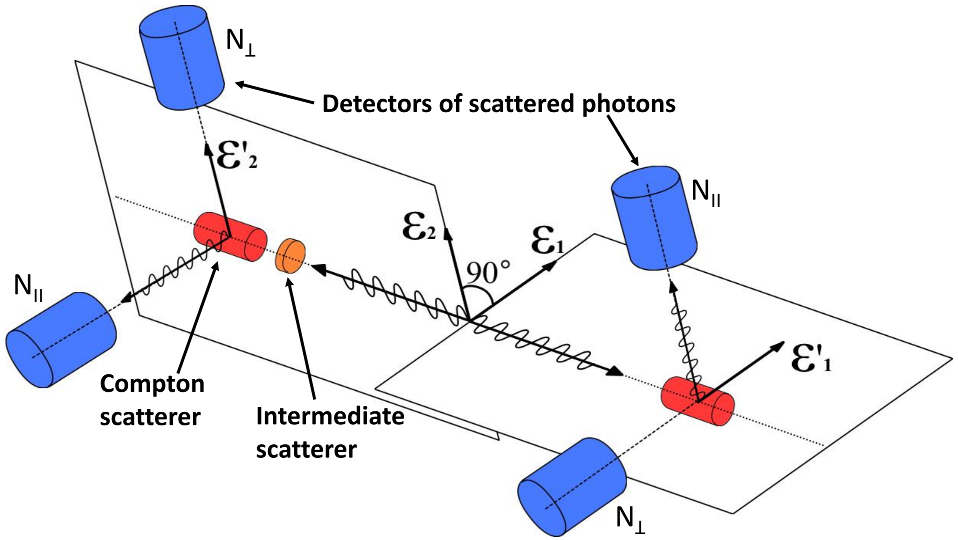
According to Bell's (CHSH) inequality:
- S<2 for **non-entangled** system
- Maximum $S = 2\sqrt{2}$ for **entangled** system if the angles between polarimeters are multiples of 22.5 degrees.

For non-ideal polarimeter: $S \quad \Rightarrow \quad S` = S * A^2 \quad \Rightarrow$
$S` < 2$ for Compton polarimeters ($A^2 < 0.5$) and annihilation photons!

The entanglement can not be proven directly from Bell's (CHSH) inequality because of low analyzing power (A~0.7)

Azimuthal asymmetry in angular distributions of scattered gammas (instead of Bell's inequality) was used to prove the entanglement of annihilation photons

Probing entanglement (by measuring angular correlations)



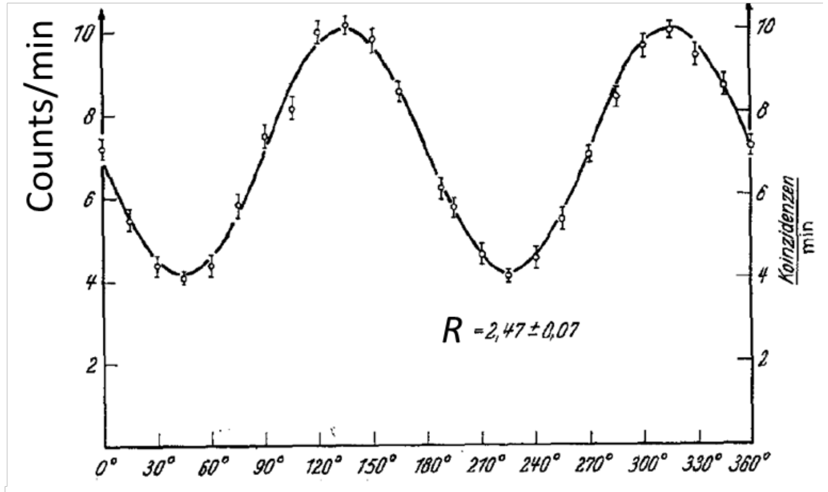
$$P_{12}(E_1, E_2, \phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP} \left(\frac{d\sigma}{d\Omega_2}\right)_{NP} [1 - \alpha(\theta_1)\alpha(\theta_2)\cos(2\phi)]$$

Ratio of the numbers of scattered annihilation photons:

$$R_{theory}(\theta) = \frac{N\left(\phi = \frac{\pi}{2}\right)}{N(\phi = 0)} = 1 + \frac{2\sin^4\theta}{\gamma^2 - 2\gamma\sin^2\theta}; \qquad \gamma = 2 - \cos\theta + (2 - \cos\theta)^{-1}$$

$$R = 2.6 \;\; for\; \theta = 90^0$$

According to D. Bohm and Y. Aharonov (Phys. Rev. (1957) 108, 1070) the measurements of angular correlations would provide the experimental test of the **entanglement** if R>2. For decoherent photons ***R=1*** for non-entangled photons ***R<2***



The best experimental values:

H. Langhof, Zeitschrift fur Physik 160, 186-193 (1960) $\quad R = 2.47 \pm 0.07$

L.Kasday, J.Ullman and C.Wu C1971–1996, Nuovo Cimento B 25 633–61 (1975) $\quad R = 2.33 \pm 0.10$

For a long time, it was assumed that these results confirmed entanglement of annihilation photons.
The ***decoherent*** annihilation photons ***were not measured*** at all!

Current situation

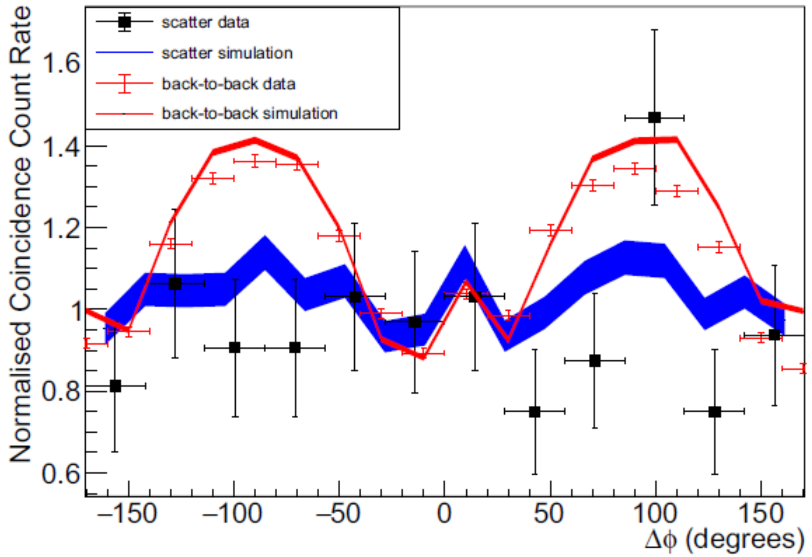
Hiesmayr B.C. and Moskal P. Witnessing entanglement in Compton scattering processes via mutually unbiased bases Sci. Rep.9 8166 (2019)

The Compton scattering of annihilation photons is the same for both entangled and decoherent states. There is NO the experimental proof of the entanglement.

Peter Caradonna *et al.* Probing entanglement in Compton interactions *J. Phys. Commun.* **3** 105005 (2019)

The Compton scattering of annihilation photons is principally different for entangled and decoherent states. There is no need to prove the entanglement. But…
The measurements of decoherent photons are needed!

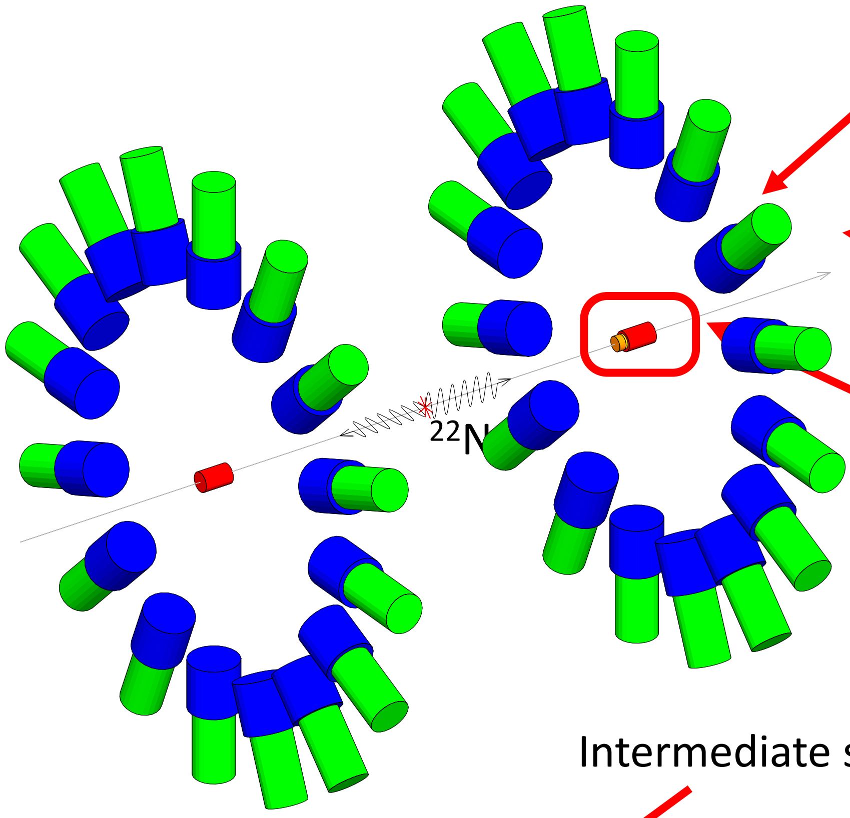
Watts, D.P., Bordes, J., Brown, J.R. *et al.* Photon quantum entanglement in the MeV regime and its application in PET imaging. *Nat Commun* **12,** 2646 (2021)



First measurement of decoherent annihilation photons was done last year with decoherent photons. The sensitivity of experimental setup and the poor statistics do not allow the comparison of Compton scattering of photons in entangled and decoherent states.

Turned out, that this experiment was already running at INR RAS

Experimental setup at INR RAS, Moscow

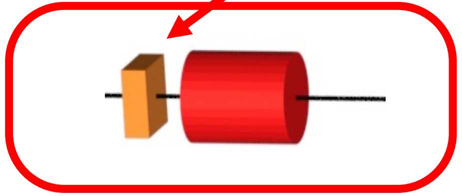


NaI(Tl) scintillation detectors

Photomultipliers

Main Compton scatterer

Intermediate scatterer GAGG



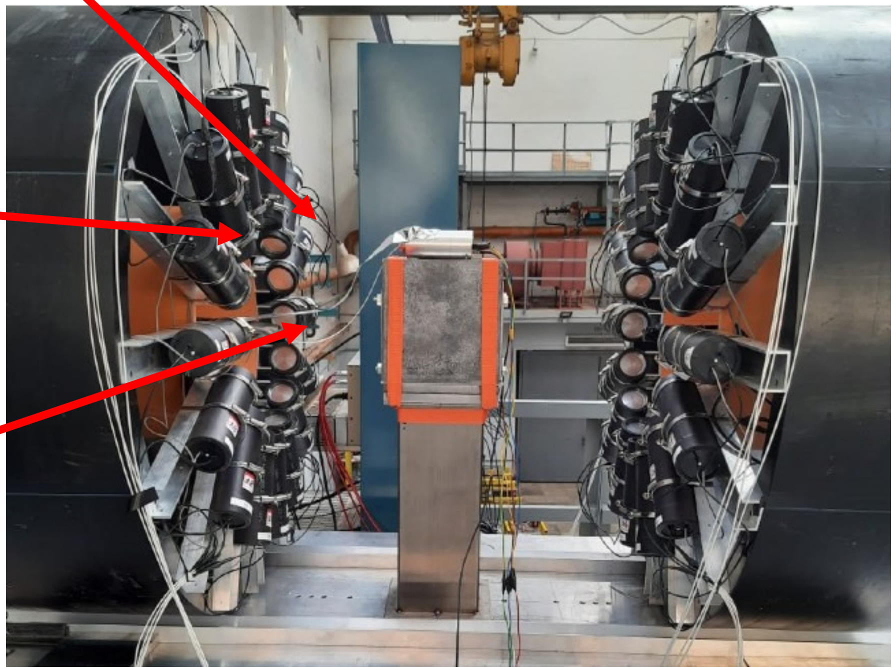


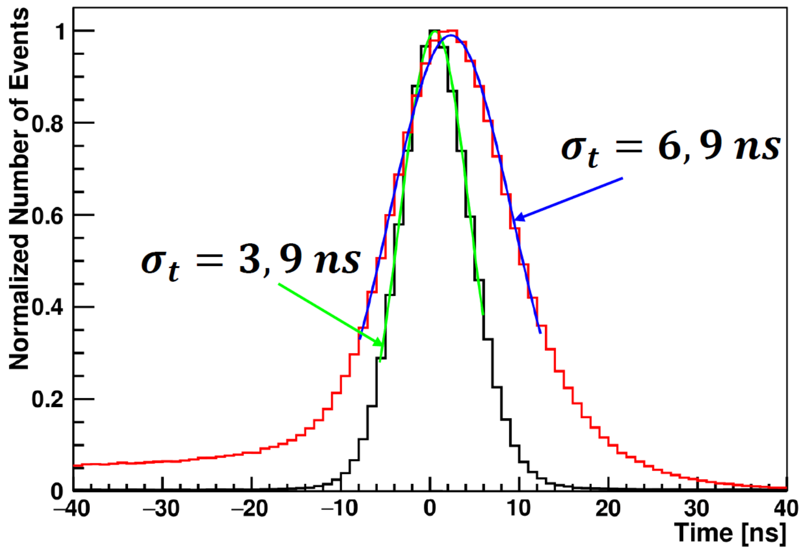
Photo of experimental setup at INR RAS

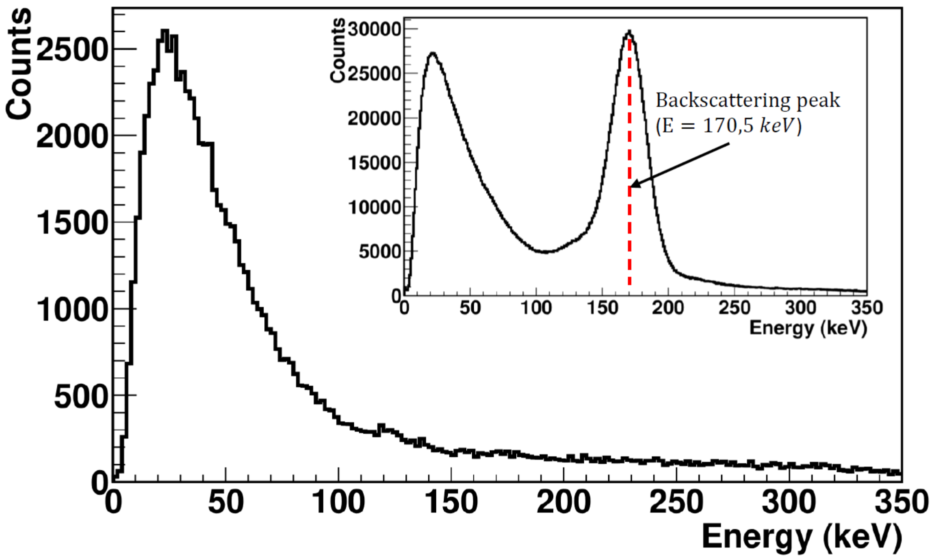
16 NaI(Tl) detectors form 16 independent Compton polarimeters in each arm

Decoherent photons

Intermediate scatterer :
GAGG scintillator

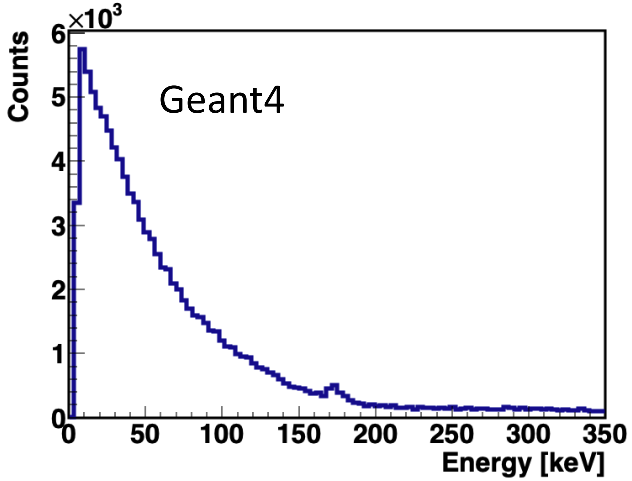
Interaction in the intermediate scatterer leads to the collapse of the entangled state and the development of a decoherent pair (mixed state).



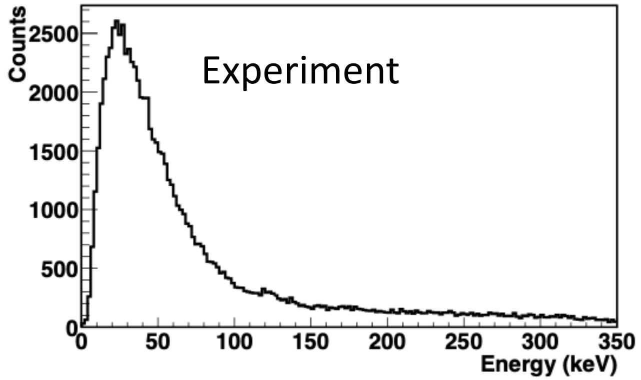


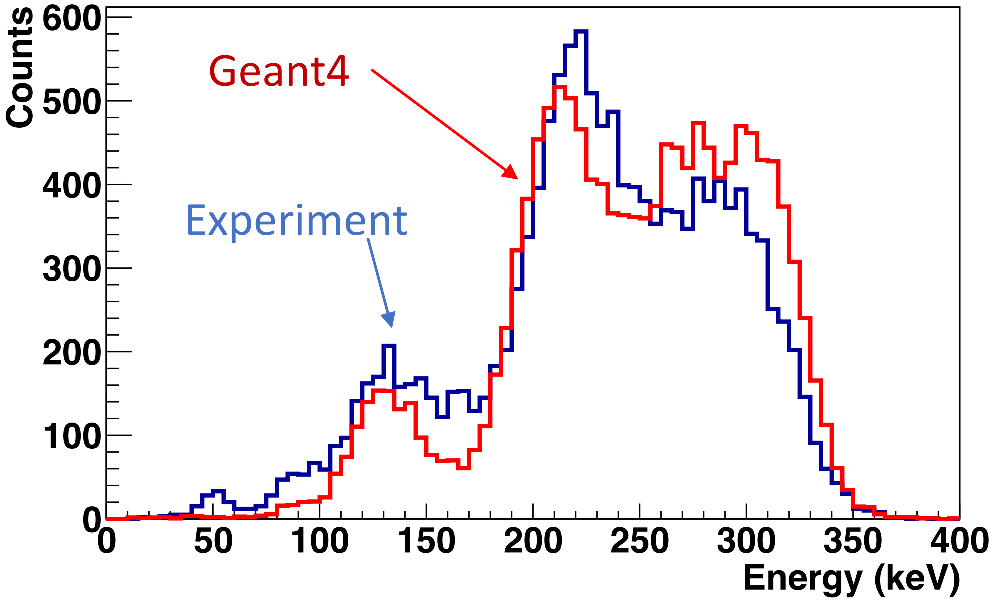
An event is considered decoherent if there is an energy deposition in the GAGG, and the signal is in the required time window.

Event selection with interaction in GAGG

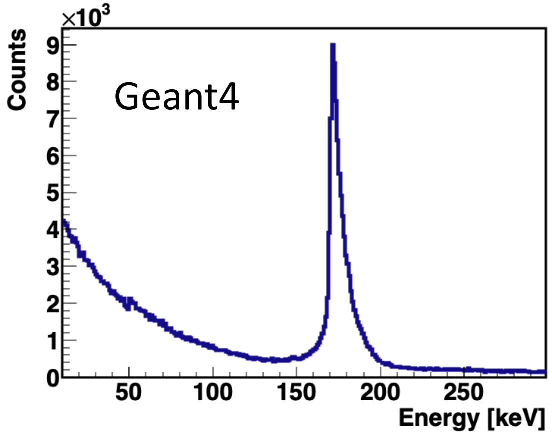


Energy spectra in GAGG
(with hit to NaI)

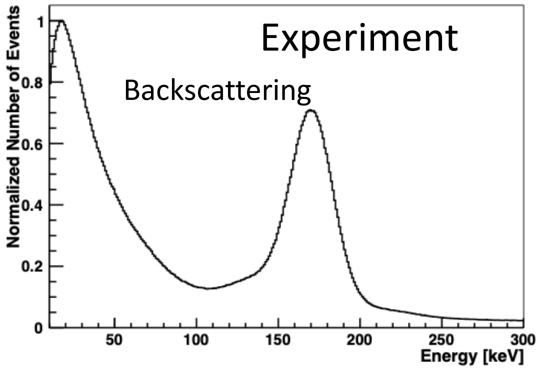




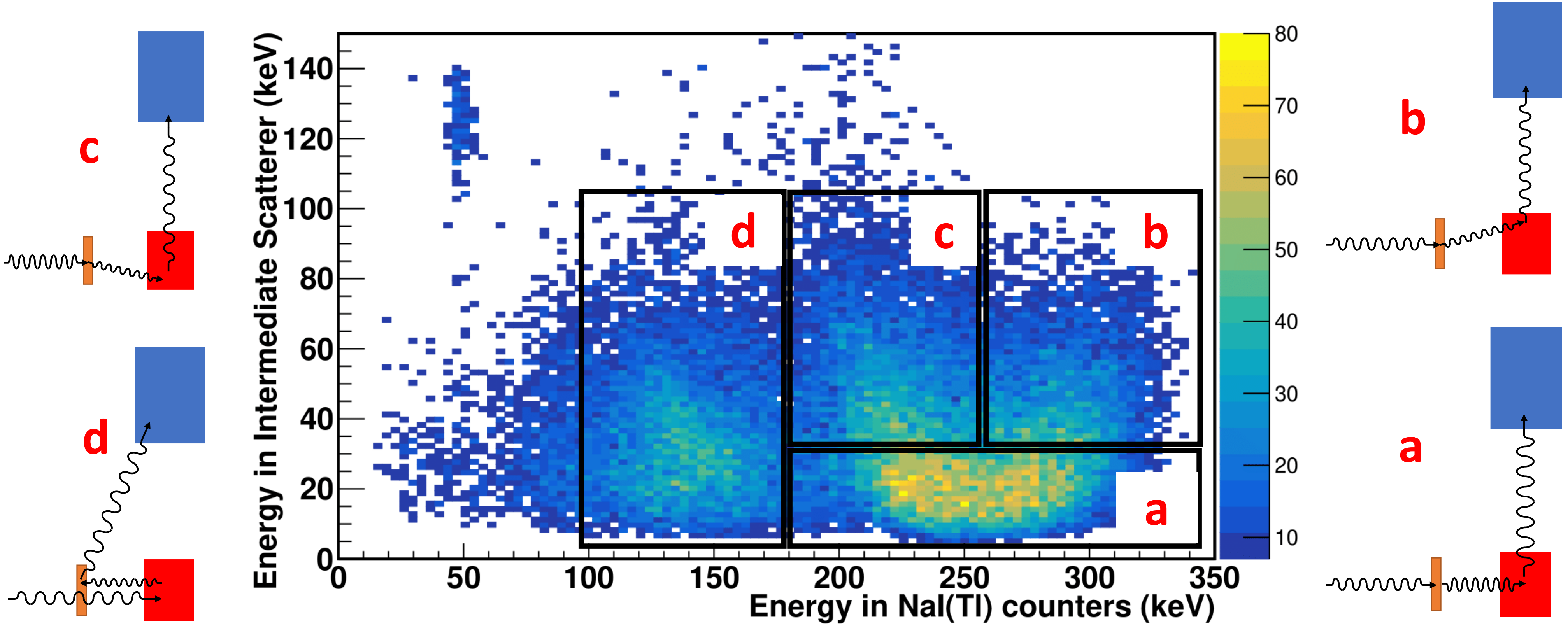
Energy deposition in NaI(Tl) in events with energy deposition in GAGG



Energy spectra in GAGG



Energy deposition correlation between NaI(Tl) and GAGG



Four types of Compton scattering in GAGG:
a) scattering at very small angles $\theta<100^0$;
b) scattering at large angles $100^0 < \theta < 200^0$ in the direction of NaI(Tl);
c) scattering at large angles $100^0 < \theta < 200^0$ in the opposite direction from NaI(Tl);
d) backscattering at 180^0.

Angular correlations for entangled photons

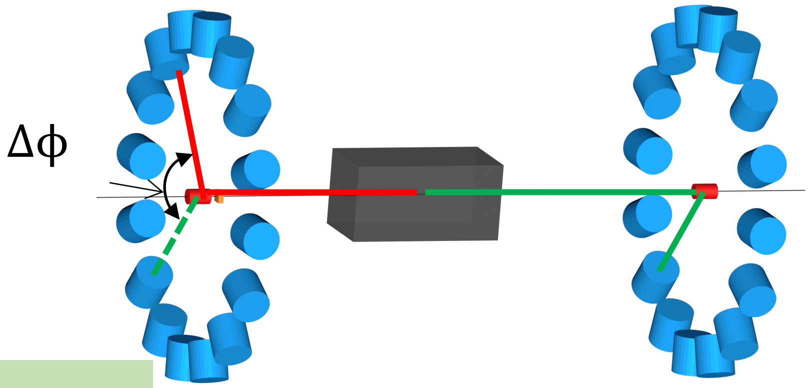
Theory:

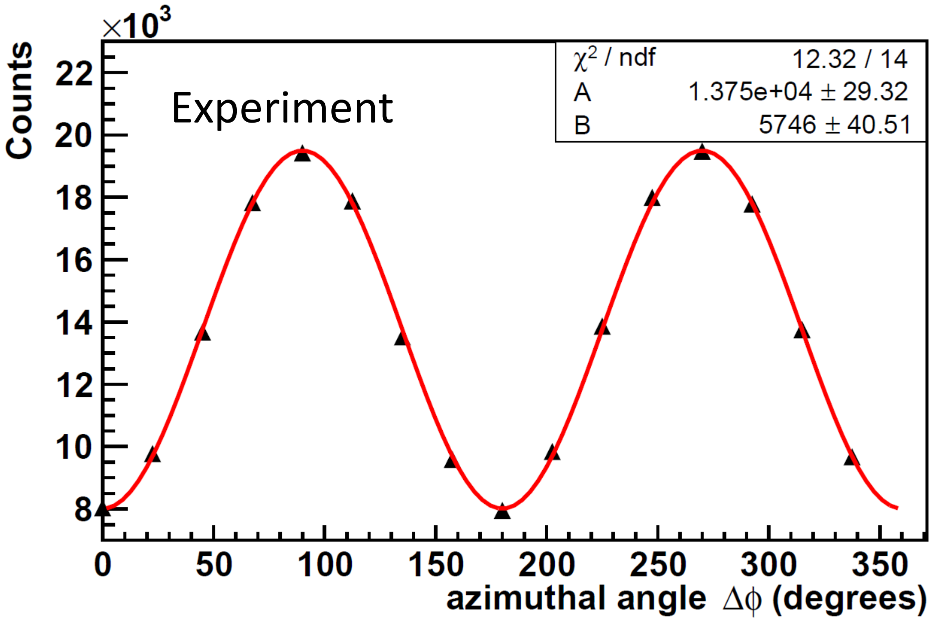
$$P_{12}(E_1, E_2, \phi) = \left(\frac{d\sigma}{d\Omega_1}\right)_{NP} \left(\frac{d\sigma}{d\Omega_2}\right)_{NP} [1 - \alpha(\theta_1)\alpha(\theta_2)cos(2\phi)]$$

$$\Rightarrow \quad N(\phi) = A - B \cdot \cos(2\phi)$$

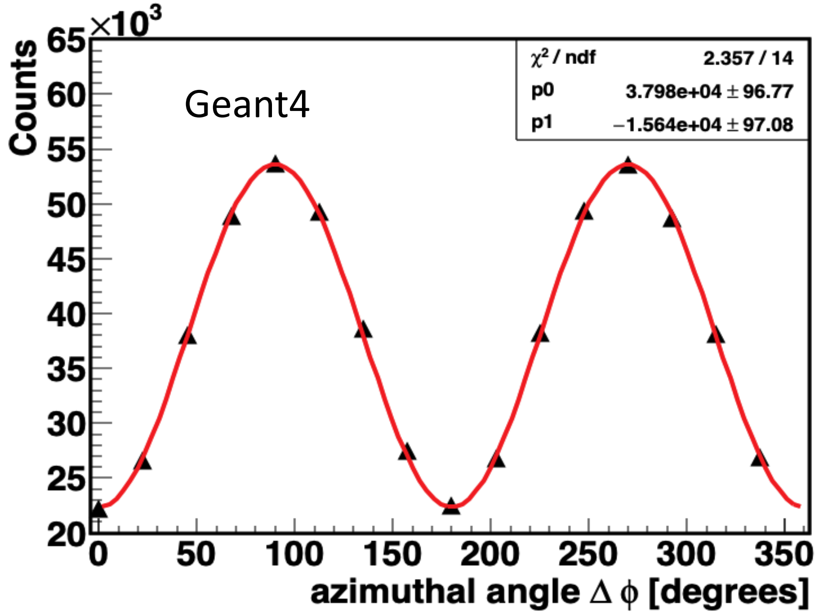
$$R_{theory}(\theta) = \frac{N\left(\phi = \frac{\pi}{2}\right)}{N(\phi = 0)}$$

$R_{theory} = 2,40 \pm 0,02$



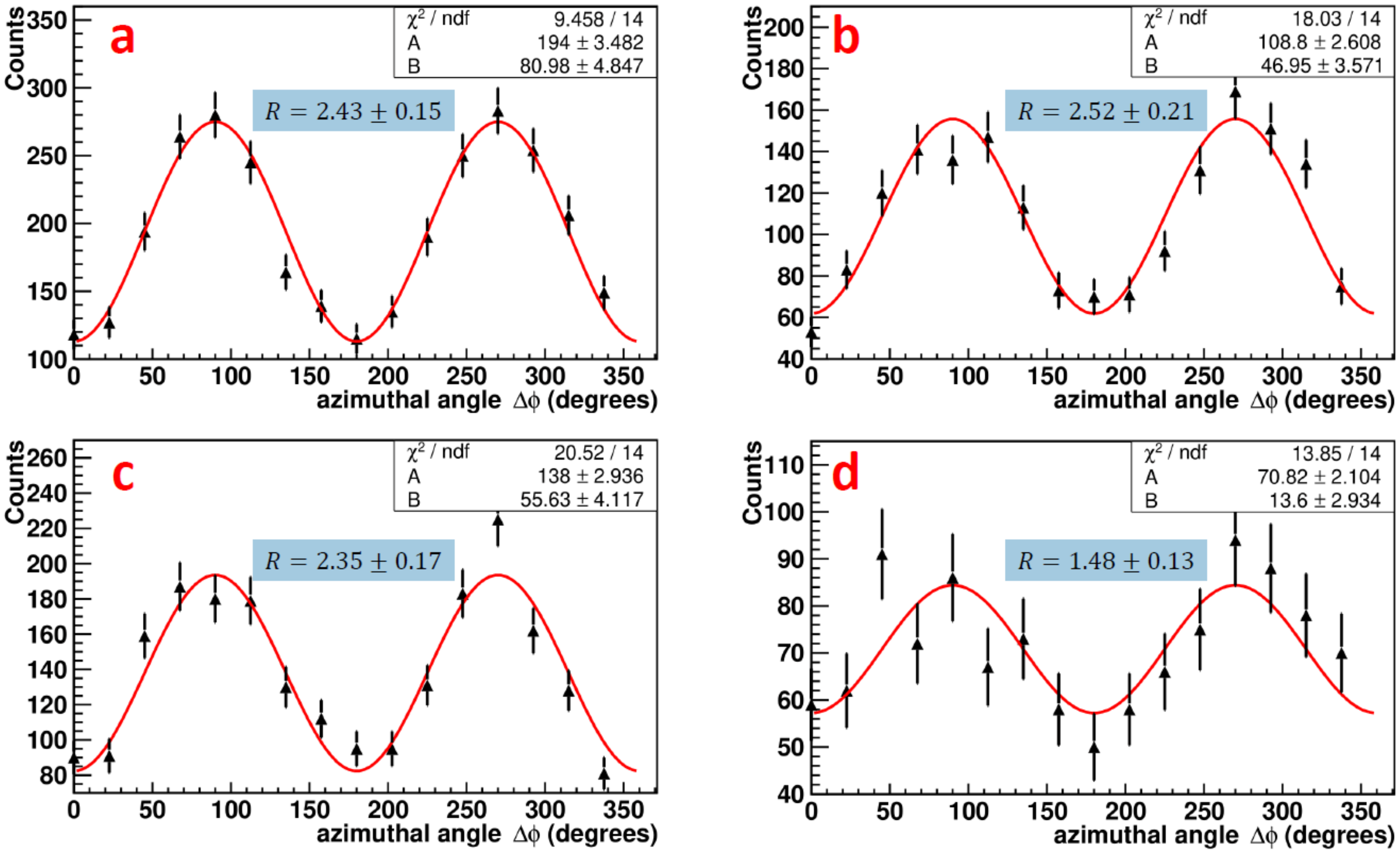


$R_{exp} = 2.44 \pm 0.02$

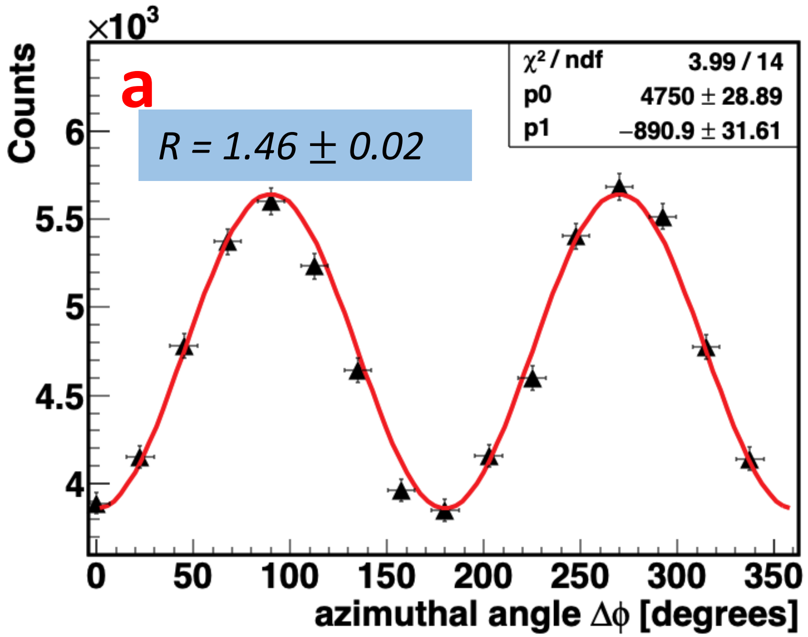


$R_{MC} = 2.399 \pm 0.015$

Angular correlations for decoherent photons



Experiment



Geant4

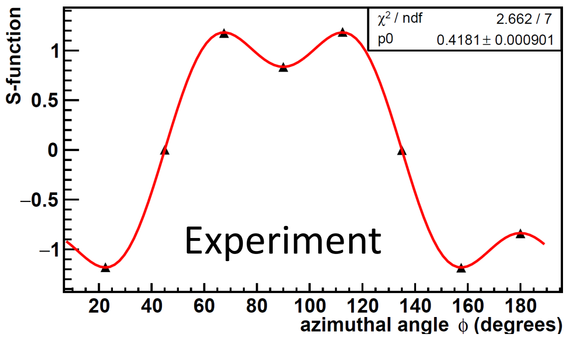
The angular correlations of entangled and decoherent photons coincide for all cases of Compton scattering in experiment, with the exception of backscattering, where photon depolarization occurs.

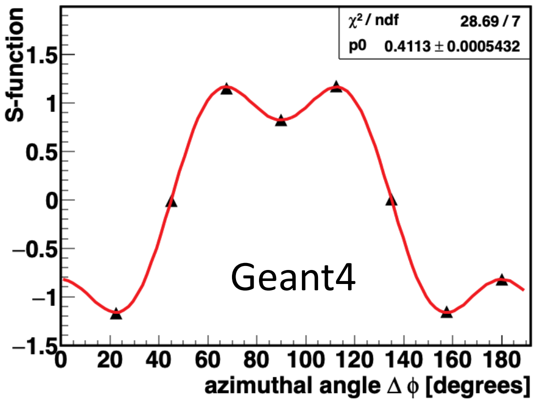
S-function in CHSH inequality

$$S = E(\vec{a},\vec{b}) - E(\vec{a},\vec{b'}) + E(\vec{a},\vec{b'}) + E(\vec{a'},\vec{b'}) \Rightarrow S = -p_0(3\cos(2\phi) - \cos(6\phi))$$

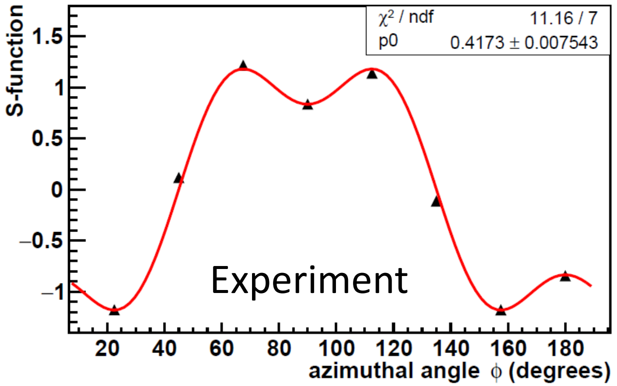
P_0 is the product of the efficiencies (analyzing abilities) of two polarimeters.

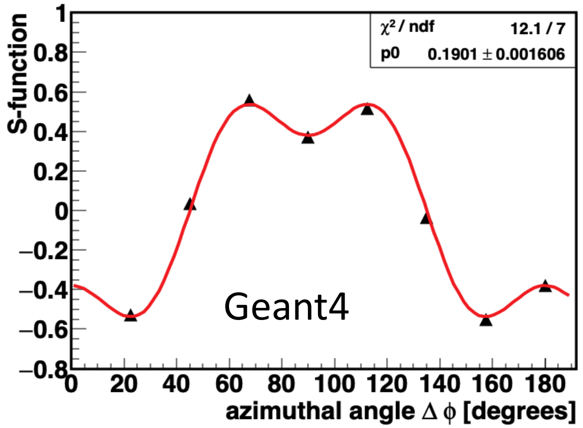
Entangled:





Decoherent:





In experiment:

$P_0 = \boldsymbol{\alpha(\theta_1)\alpha(\theta_2)}$ is the same in all dimensions, as in the azimuthal distributions and correlation functions of the bell inequalities. Bella.

The correlation functions are identical for the entangled and the mixed state!

Entangled and separable states show the same correlations!

Conclusion

- The high energy of annihilation photons (5 orders of magnitude higher than the energy of optical photons) makes it possible to use a controlled decoherence process and directly compare the polarization correlations of initial and decoherent photons.
- The correlation data turned out to be identical for both quantum states
- A contradiction between the calculations of the current Geant4 model, which describes the kinematics of decoherent photons, and the experimental data is found.
- The entanglement of annihilation photons has not been experimentally proven, since the correlations of initial and decoherent photons coincide.
- The developed positron emission tomographs using the entanglement of annihilation photons will not be able to suppress the background from parasitic scattering in the studied objects.

Publications:

Setup of Compton polarimeters for measuring entangled annihilation photons
Published in: *JINST* 17 (2022) 03, P03010 e-Print: 2204.04692 [physics.ins-det] DOI: 10.1088/1748-0221/17/03/P03010

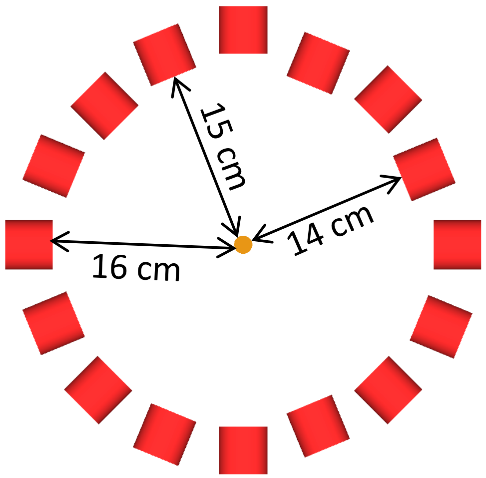
Study of the Compton Scattering of Entangled Annihilation Photons
Published in: *Phys.Part.Nucl.Lett.* 19 (2022) 5, 509-512 DOI: 10.1134/S1547477122050405

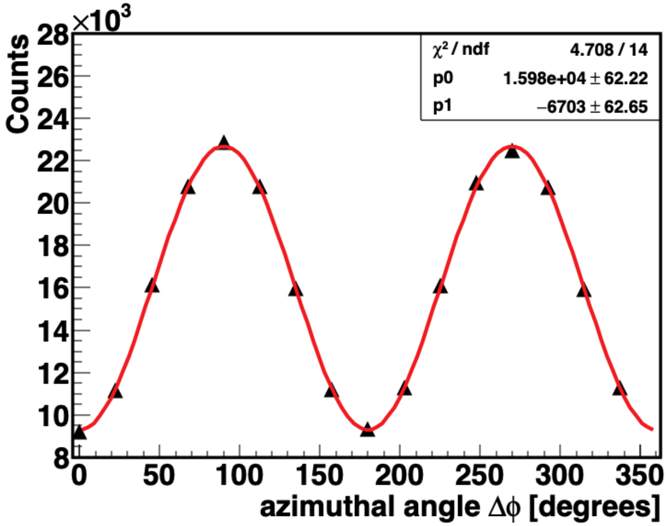
Entanglement of annihilation photons
arXiv:2210.07623v1 [quant-ph] 14 Oct 2022

Thank you for your attention

Systematic error study

Random distribution of distances from scatterer to NaI(Tl) detectors.





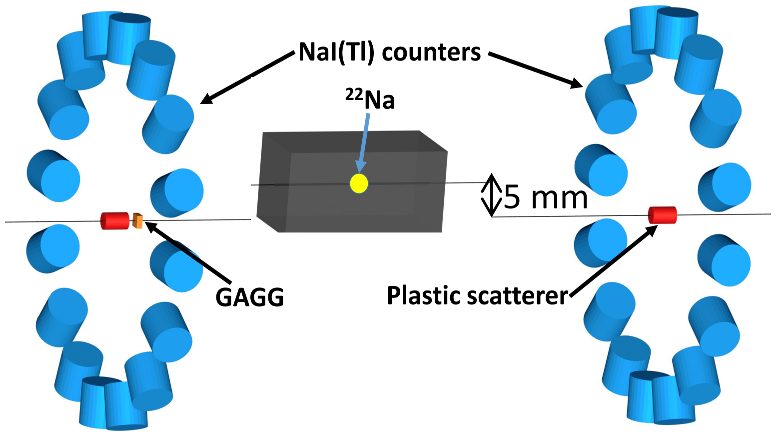
$R \quad = 2.445 \pm 0.025$

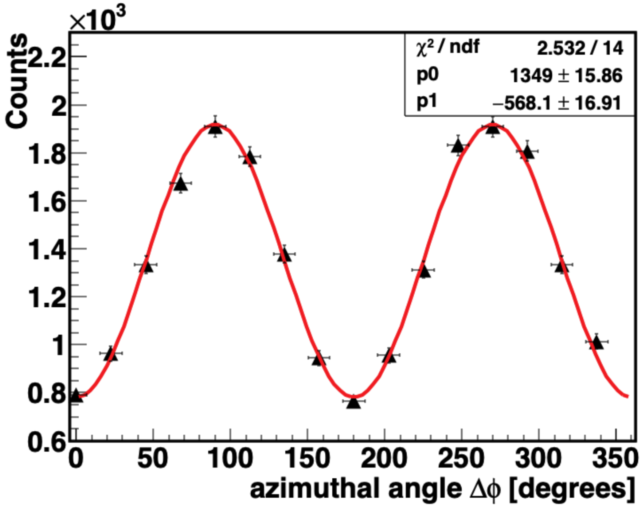
$R_{normal} = 2.399 \pm 0.015$

Given that precision is 10 times more accurate:

$\Delta R \sim 5 \cdot 10^{-3}$

Collimator shift.





$R \quad = 2.455 \pm 0.080$

$R_{normal} = 2.399 \pm 0.015$

Given that precision is 10 times more accurate:

$\Delta R \sim 6 \cdot 10^{-3}$

Systematic error is much smaller than statistical error