Estimates of asymmetries in polarized charmonia production in Generalized Parton Model at NICA

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Generalized Parton Model (GPM)

The GPM kinematics (in a region of its application $\sqrt{\langle q_T^2\rangle} \ll \mu$):

$$q_1^\mu = x_1 p_1^\mu + y_1 p_2^\mu + q_{1T}^\mu, \qquad q_2^\mu = x_2 p_2^\mu + y_2 p_1^\mu + q_{2T}^\mu, \qquad q_{iT}^\mu = (0, \vec{q}_{iT}, 0)\,. \tag{1}$$

Parts of initial partons momenta are

$$\begin{aligned} q_1^\mu &= \left(\frac{x_1\sqrt{s}}{2} + \frac{\vec{q}_{1T}^{\,2}}{2\sqrt{s}x_1}, \vec{q}_{1T}, \frac{x_1\sqrt{s}}{2} - \frac{\vec{q}_{1T}^{\,2}}{2\sqrt{s}x_1} \right)^\mu, \\ q_2^\mu &= \left(\frac{x_2\sqrt{s}}{2} + \frac{\vec{q}_{2T}^{\,2}}{2\sqrt{s}x_2}, \vec{q}_{2T}, -\frac{x_2\sqrt{s}}{2} + \frac{\vec{q}_{2T}^{\,2}}{2\sqrt{s}x_2} \right)^\mu. \end{aligned} \tag{2}$$

According to the factorisation hypothesis in GPM the cross section are factorised:

$$d\sigma(pp \to \mathcal{C}X) = \int dx_1 \int d^2q_{1T} F(x_1, \mu_{\mathsf{F}}^2, q_{1T}^2) \int dx_2 \int d^2q_{2T} F(x_2, \mu_{\mathsf{F}}^2, q_{2T}^2) d\hat{\sigma}. \tag{3}$$

We use the following ansatz for PDF in GPM [U. D'Alesio, L. Maxia, et al., Phys. Rev. D, 102, 094011 (2020)]:

$$F(x, \mu_{\mathsf{F}}^2, q_T^2) = f(x, \mu_{\mathsf{F}}^2) \frac{e^{-q_T^2/\langle q_T^2\rangle}}{\pi\langle q_T^2\rangle}, \qquad \langle q_T^2\rangle = 1 \text{ GeV}^2. \tag{4}$$

Nonrelativistic Quantum Chromodynamics (NRQCD)

The NRQCD allows us to expand the charmonium state wave function to the series with small parameter v, relative non-relativistic speed of constituent heavy quarks [G. T. Bodwin, E. Braaten, and G. P. Lepage, Phys. Rev. D, 51, 1125 (1995)]:

$$|J/\psi\rangle = \mathcal{O}(v^0)\,|c\bar{c}[^3S_1^{(1)}]\rangle + \mathcal{O}(v^1)\,|c\bar{c}[^3P_J^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^3S_1^{(1,8)}]gg\rangle + \\ + \mathcal{O}(v^2)\,|c\bar{c}[^1S_0^{(8)}]g\rangle + \mathcal{O}(v^2)\,|c\bar{c}[^1D_J^{(1,8)}]gg\rangle + \dots \quad (5)$$

The NRQCD factorisation theorem gives expression for cross section as

$$d\sigma(a+b\to\mathcal{C}+X) = \sum_n d\hat{\sigma}(a+b\to c\bar{c}[n]+X)\langle\mathcal{O}^{\mathcal{C}}[n]\rangle \quad (6)$$

where $\langle\mathcal{O}^{\mathcal{C}}[n]\rangle$ are non-perturbative matrix elements.

Momentum shift in case of charmonium state decay $(\mathcal{C}_1\to\mathcal{C}_2+X)$ [B. Gong, L.-P. Wan, et al., Phys. Rev. Lett., 2014, Vol. 112. — P. 032001] is

$$\langle p_T(\mathcal{C}_2)\rangle \approx \frac{M_{\mathcal{C}_2}}{M_{\mathcal{C}_1}} p_T(\mathcal{C}_1). \quad (7)$$

Evaluation of amplitudes: details

Amplitudes of $c\bar{c}$ pairs production are projected onto the charmonium production amplitude with [A. Leibovich, P. Cho, Phys. Rev. D, Vol. 53, P. $150 - 162$ (1996)]
spin-state projectors:

$$\Pi_0 = \frac{1}{\sqrt{8m_c^3}} \left(\frac{\hat{P}}{2} - \hat{q} - m_c \right) \gamma_5 \left(\frac{\hat{P}}{2} + \hat{q} + m_c \right), \qquad S = 0,$$
$$\Pi_1^\alpha = \frac{1}{\sqrt{8m_c^3}} \left(\frac{\hat{P}}{2} - \hat{q} - m_c \right) \gamma^\alpha \left(\frac{\hat{P}}{2} + \hat{q} + m_c \right), \qquad S = 1; \tag{8}$$

color-state projectors:

$$C_1 = \frac{\delta_{ij}}{\sqrt{N_c}}, \qquad C_8 = \sqrt{2} T^a_{ij}; \tag{9}$$

orbital momentum projecting expressions:

$$\mathcal{M}(a + b \to c\bar{c}[^3S_1^{(1)}]) = \mathrm{Tr}[C_1 \Pi_1^\alpha \mathcal{M}(a + b \to c\bar{c}) \varepsilon_\alpha(J_z, p)]\big|_{q=0},$$
$$\mathcal{M}(a + b \to c\bar{c}[^3P_1^{(1)}]) = \frac{d}{dq^\beta} \mathrm{Tr}[C_1 \Pi_1^\alpha \mathcal{M}(a + b \to c\bar{c}) \varepsilon_{\alpha\beta}(J_z, p)]\big|_{q=0}. \tag{10}$$

Matrix elements of parton subprocesses $2 \to 1$

In Color Singlet Model (CSM) we consider the $2 \to 1$ effective subprocesses with the following producting states of charmonium $\chi_{c0}[{}^3P_0^{(1)}]$, $\chi_{c2}[{}^3P_2^{(1)}]$ [P. Cho, A. Leibovich (1996)]:

$$\overline{|\mathcal{M}(g+g \to \mathcal{C}[{}^3P_0^{(1)}])|^2} = \frac{8}{3}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{C}}[{}^3P_0^{(1)}]\rangle}{M^3}, \qquad \langle \mathcal{O}^{\chi_{c0}}[{}^3P_0^{(1)}]\rangle = 8.9 \cdot 10^{-2}\ \text{GeV}^5, \tag{11}$$

$$\overline{|\mathcal{M}(g+g \to \mathcal{C}[{}^3P_2^{(1)}])|^2} = \frac{32}{45}\pi^2\alpha_s^2 \frac{\langle \mathcal{O}^{\mathcal{C}}[{}^3P_2^{(1)}]\rangle}{M^3}, \qquad \langle \mathcal{O}^{\chi_{c2}}[{}^3P_2^{(1)}]\rangle = 5 \cdot 8.9 \cdot 10^{-2}\ \text{GeV}^5, \tag{12}$$

$$\overline{|\mathcal{M}(g+g \to \mathcal{C}[{}^3P_1^{(1)}])|^2} = 0, \tag{13}$$

$$\langle \mathcal{O}^{\chi_{cJ}}[{}^3P_J^{(1)}]\rangle = (2J+1)\langle \mathcal{O}^{\chi_{c0}}[{}^3P_0^{(1)}]\rangle. \tag{14}$$

Matrix elements of parton subprocesses $2 \to 2$

In CSM we consider the $2 \to 2$ effective subprocesses with the following producting states of charmonium $J/\psi[^3S_1^{(1)}]$, $\psi'[^3S_1^{(1)}]$, $\chi_{c1}[^3P_1^{(1)}]$ [R. Gastmans et al., Phys. Lett. B 184 (1987)]:

$$\overline{|\mathcal{M}(g+g \to \mathcal{C}[^3S_1^{(1)}]+g)|^2} = \pi^3\alpha_s^3 \frac{\langle \mathcal{O}^{\mathcal{C}}[^3S_1^{(1)}]\rangle}{M^3} \frac{320M^4}{81(M^2-\hat{t})^2(M^2-\hat{u})^2(\hat{t}+\hat{u})^2} \times \tag{15}$$

$$\times\left[M^4\hat{t}^2 - 2M^2\hat{t}^3 + \hat{t}^4 + M^4\hat{t}\hat{u} - 3M^2\hat{t}^2\hat{u} + 2\hat{t}^3\hat{u} + M^4\hat{u}^2 - 3M^2\hat{t}\hat{u}^2 + 3\hat{t}^2\hat{u}^2 - 3M^2\hat{u}^3 + 2\hat{t}\hat{u}^3 + \hat{u}^4\right],$$

$$\overline{|\mathcal{M}(g+g \to \mathcal{C}[^3P_1^{(1)}]+g)|^2} = \pi^3\alpha_s^3 \frac{\langle \mathcal{O}^{\mathcal{C}}[^3P_1^{(1)}]\rangle}{M^3} \frac{128A^2}{9(B-M^2A)^4} \times \tag{16}$$

$$\times\left[2B\left(-M^8+5M^4A+A^2\right) + M^2A^2(M^4-4B) - 15M^2B^2\right], \quad A = \hat{s}\hat{u}+\hat{s}\hat{t}+\hat{t}\hat{u}, \quad B = \hat{s}\hat{t}\hat{u},$$

$$\overline{|\mathcal{M}(g+g \to \mathcal{C}[^3S_1^{(1)}]_{\mathsf{L}}+g)|^2} = \pi^3\alpha_s^3 \langle \mathcal{O}^{\mathcal{C}}[^3S_1^{(1)}]\rangle \times \tag{17}$$

$$\times \frac{320M^3\hat{s}\hat{t}\hat{u}\left[(x_1-x_2)^2\hat{s}^2 + \hat{t}^2x_1^2 + \hat{u}^2x_2^2\right]}{81(\hat{t}+\hat{u})^2(M^2-\hat{t})^2(M^2-\hat{u})^2\left[M^4(x_1-x_2)^2 + (\hat{u}x_1+\hat{t}x_2)^2 - 2(x_1-x_2)(\hat{u}x_1-\hat{t}x_2)M^2\right]},$$

$$\langle \mathcal{O}^{J/\psi}[^3S_1^{(1)}]\rangle = 1.3 \text{ GeV}^3, \qquad \langle \mathcal{O}^{\psi'}[^3S_1^{(1)}]\rangle = 0.65 \text{ GeV}^3, \qquad \langle \mathcal{O}^{\chi_{c1}}[^3P_1^{(1)}]\rangle = 3\cdot 8.9\cdot 10^2 \text{ GeV}^5. \tag{18}$$

Polarization parameter and asymmetry

Angular distribution of charmonium leptonic decay is

$$W(\theta,\varphi) \sim 1 + \lambda_\theta \cos^2\theta + \lambda_\varphi \sin^2\theta \, \cos 2\varphi + \lambda_{\theta\varphi} \cos\varphi, \qquad \lambda_\theta = \frac{\sigma_{\mathsf{T}} - 2\,\sigma_{\mathsf{L}}}{\sigma_{\mathsf{T}} + 2\,\sigma_{\mathsf{L}}} = \frac{\sigma - 3\,\sigma_{\mathsf{L}}}{\sigma + \sigma_{\mathsf{L}}}. \tag{19}$$

The Sivers function is described unpolarized partons distribution in a polarized proton:

$$F_g^{\uparrow}(x,\mu_{\mathsf{F}}^2,\vec{q}_T) = F_g(x,\mu_{\mathsf{F}}^2,q_T) + \frac{1}{2}\Delta^N F_g^{\uparrow}(x,\mu_{\mathsf{F}}^2,q_T)\,\frac{\vec{s}\cdot[\,\vec{p}\times\vec{q}_T]}{|\,\vec{p}|\cdot|\vec{q}_T|}. \tag{20}$$

Transverse single-spin asymmetries (TSSA) are

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \equiv \frac{d\Delta\sigma}{2\,d\sigma}, \tag{21}$$

$$d\Delta\sigma \sim \int dx_1 \int d^2q_{1T} \int dx_2 \int d^2q_{1T}\left[F_g^{\uparrow}(x,\mu_{\mathsf{F}}^2,\vec{q}_T) - F_g^{\downarrow}(x,\mu_{\mathsf{F}}^2,\vec{q}_T)\right] F_g(x,\mu_{\mathsf{F}}^2,q_T)\,d\hat{\sigma}. \tag{22}$$

Unpolarized charmonia production in pp processes

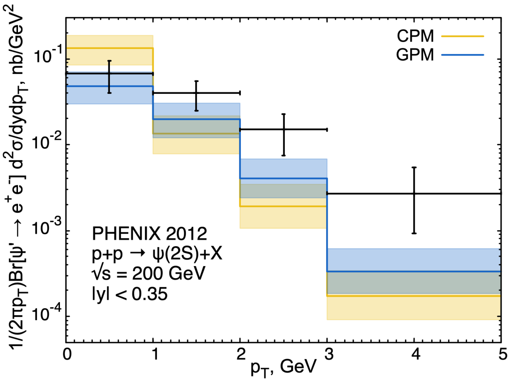

Figure 1: $\psi(2S)$ transverse momentum distribution in CPM and GPM within CSM. GPM parameter $\langle q_T^2 \rangle = 1$ GeV^2. Experimental data from PHENIX RHIC collaboration [A. Adare, et al., Phys. Rev. D 85 (2012): 092004-1].

Polarization parameter in GPM: PHENIX collaboration

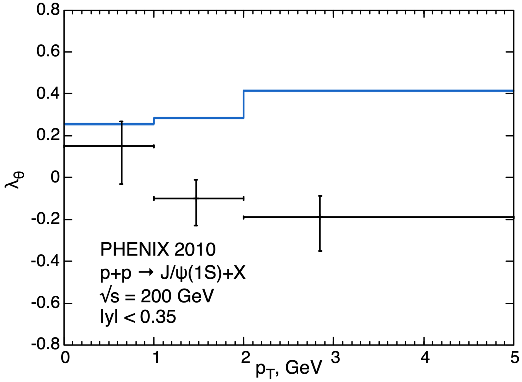

Figure 2: Direct $J/\psi(1S)$ transverse momentum distribution. GPM parameter $\langle q_T^2 \rangle = 1$ GeV^2. Experimental data from PHENIX RHIC collaboration [A. Adare, et al., Phys. Rev. D 82 (2010): 012001].

Predictions of polarization parameter: NICA collider

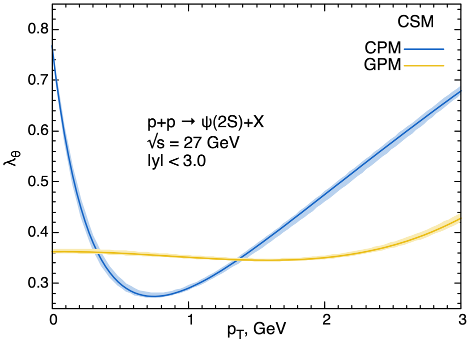

Figure 3: Polarization parameter λ_θ as a function of charmonium transverse momentum p_T in ψ' production.

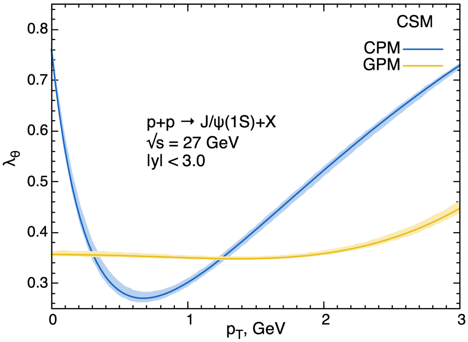

Figure 4: Polarization parameter λ_θ as a function of charmonium transverse momentum p_T in direct J/ψ production.

Asymmetry: PHENIX collaboration

Experimental data taken from [C. Aidala, Y. Akiba, M. Alfred, V. Andrieux, Phys. Rev. D, Vol. 98. — P. 012006 (2018)].

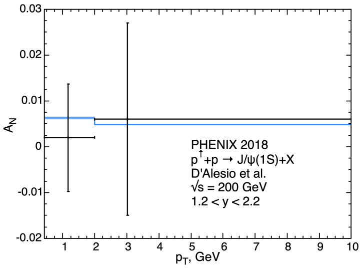

Figure 5: Asymmetry in unpolarized J/ψ direct production in D'Alesio et al. parametrization.

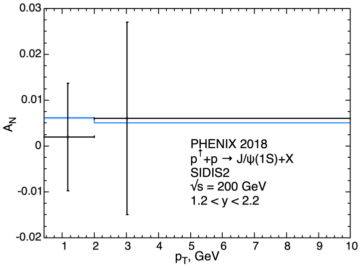

Figure 6: Asymmetry in unpolarized J/ψ direct production in SIDIS2 parametrization.

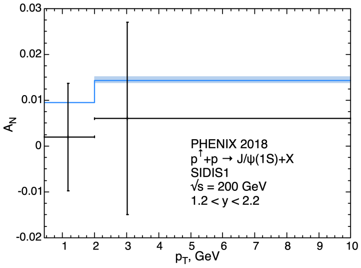

Figure 7: Asymmetry in unpolarized J/ψ direct production in SIDIS1 parametrization.

Prediction of asymmetry for polarized charmonium: NICA collider

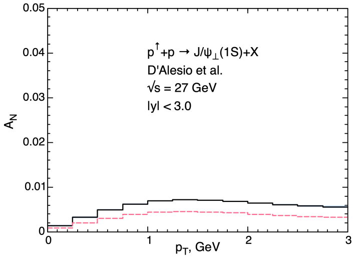

Figure 8: Asymmetry in transverse polarized J/ψ direct production in a D'Alesio et al. parametrization.

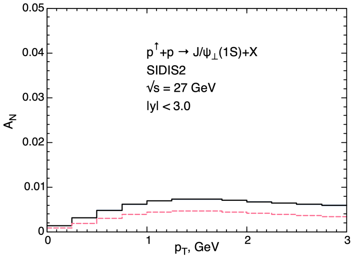

Figure 9: Asymmetry in transverse polarized J/ψ direct production in a SIDIS2 parametrization.

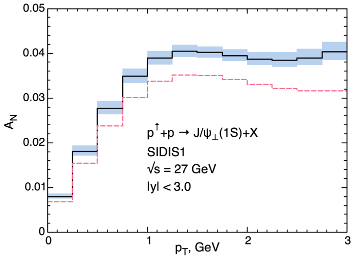

Figure 10: Asymmetry in transverse polarized J/ψ direct production in a SIDIS1 parametrization.

Dash pink line refers to the asymmetry in unpolarized charmonium direct production.

Prediction of asymmetry for polarized charmonium: NICA collider

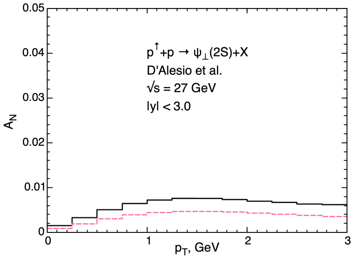

Figure 11: Asymmetry in transverse polarized ψ' production in a D'Alesio et al. parametrization.

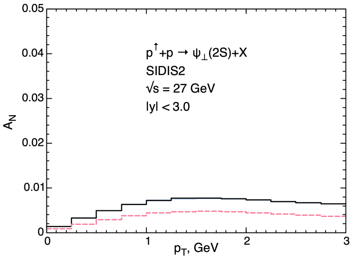

Figure 12: Asymmetry in transverse polarized ψ' production in a SIDIS2 parametrization.

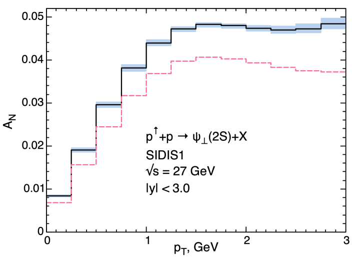

Figure 13: Asymmetry in transverse polarized ψ' production in a SIDIS1 parametrization.

Dash pink line refers to the asymmetry in unpolarized charmonium production.

Conclusion

- Within LO of the GPM and the CSM we obtained an estimation for the polarization parameter of the *direct* J/ψ production at $\sqrt{s} = 200$ GeV at RHIC. From comparison with the data we see the need to take into account a feed-down contribution at least. This is a task for following research.

- We calculated the polarization parameter for ψ' and direct contribution to the one for J/ψ mesons. For ψ' meson we see the significant discrepancy between predictions of the CPM and the GPM, and experimental data are very important for that case. As for λ_θ of J/ψ, we need to include the feed-down contribution before making any final conclusions.

- Some preliminary conclusion we can make for the asymmetry of the polarized charmonia production: the asymmetry for the polarized charmonia is bigger than for the unpolarized ones. But again for J/ψ we need to take into account the feed-down contribution.

Backup slides. Sivers function paramatrizations

$$\Delta^N F^{\uparrow}(x, \mu_{\mathsf{F}}^2, q_T) = 2q_T \frac{\sqrt{2e}}{\pi} \mathcal{N}_g(x) f_g(x, \mu_{\mathsf{F}}^2) \sqrt{\frac{1-\rho}{\rho}} \cdot \frac{e^{-q_T^2/\rho\langle q_T^2\rangle}}{\langle q_T^2\rangle^{3/2}}, \tag{23}$$

$$\mathcal{N}_g(x) = N_g\, x^{\alpha}(1-x)^{\beta} \frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^{\alpha}\beta^{\beta}}. \tag{24}$$

Parametrization	N_g	α	β	ρ	$\langle q_T^2\rangle$
D'Alesio et al.	0.25	0.6	0.6	0.1	1.0
SIDIS1	0.65	2.8	2.8	0.687	0.25
SIDIS2	0.05	0.8	1.4	0.576	0.25